

Aligning Ambition and Incentives*

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Abstract

In many economic situations, several principals contract with the same agents sequentially. From observing agents' performance the first principal obtains information regarding their abilities that is not directly available to outsiders. This has profound implications for the design of incentive contracts. We show that the principal always strategically distorts information revelation to future principals about the performance of her agents. The second main result is that she can limit her search for optimal incentive schemes to the class of relative performance contracts that cannot be replicated by contracts based on individual performance only. This provides a new rationale for the optimality of such compensation schemes.

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1 Introduction

It has become a characteristic of modern economies that individuals work for many different employers during their life. Topel and Ward (1992) report that an average white male in the USA holds seven different full-time jobs during the first ten years of his career alone. Moreover, each month about 2.6 percent of all employed in the US switch to a new employer (Fallick and Fleischman 2004). In such labor markets characterized by high turnover, firms' hiring and remuneration decisions depend to a large extent on the employment history and track record of an individual. Good current performance enhances the labor market's perception of a worker's ability and increases his future earnings, providing a strong motivation for effort. As a consequence, the explicit incentive contract has two distinct functions. First, it is supposed to directly affect effort incentives through monetary transfers. Second, it is supposed to indirectly affect effort by controlling the flow of information to the market to create appropriate reputational incentives.

The current employer has at her disposal a variety of information about her workers that she can use as inputs in her reward scheme and that is not otherwise available to the labor market. For example, the firm can link compensation to the individual's own performance by implementing piece rates or make compensation contingent on measures of relative performance by creating a rank-order tournament or implementing a group bonus scheme. The labor market tries to back out such information by "inverting" the compensation formula. Therefore, the firm can shape its workers' reputational incentives through appropriate contractual design.

We model this in the context of a contracting problem between a principal and two heterogeneous agents. The more talented of the two is labelled "high skilled" and the other one "low skilled". Both agents work for the principal during one period. In period 2, they contract with other principals who draw inferences about their abilities from the publicly observable contracts and hard evidence on transfers received in period 1. Observing an agent's first-period output is informative of the agent's ability. Therefore, market inference relies on inverting the transfer scheme to back out the agent's performance. For example, the market can easily do so from a simple piece rate. In contrast, if a transfer is offered in several output states, the contract creates ambiguity about the worker's underlying performance in these states. Thus, a principal can design a contract to be perfectly revealing of the underlying performance in some output states (e.g., "rewarding" a high-skilled agent when he produces high output by revealing the agent's type) and to create ambiguity about performance in other output states (e.g., "punishing" a high-skilled agent who produces low output by giving him a transfer also associated with output states reached by low-skilled agents). In other words, contracts create lotteries over future reputation that depend on the agents' underlying

performance. In the model, we focus on the more realistic case where the principal can only write deterministic contracts, since stochastic contracts impose the strong requirement that the principal can commit to lotteries.

As we show, the principal can profit from using the transfer scheme to pool performance-related signals for low- and high-skilled agents in such a way that reputation increases with output. On the one hand, this provides agents with reputational incentives which allow scaling back monetary incentives. On the other hand, this also increases the total cost of implementing effort. Therefore, the two functions of an incentive contract discussed above conflict with each other and a trade-off between “good monetary incentives” and “good reputational incentives” arises. However, the principal cares only about her monetary cost of providing incentives. We prove that the reputational incentives created through imperfectly revealing contracts outweigh the increase in total implementation cost. Thus, optimal contracts always distort information revelation to the labor market about the performance of an employer’s workers. Moreover, we show that the principal can benefit from directly tying the incentives of her workers together by using relative performance measures, since these provide her with more flexibility in creating lotteries over reputation than contracts based on individual performance measures. This result offers a new rationale for the optimality of relative performance contracts in a setup where the extant reasons for the optimality of such compensation schemes are absent.

Related Literature

Career concerns models were introduced by Fama (1980) and Holmström (1982/99) and are typically cast in terms of *symmetric learning*: symmetrically informed firms try to infer the ability of an agent from publicly observable measures of his past performance. Agents interfere with the updating process by exerting effort to influence these performance measures. Ex ante, the parties cannot internalize the impact of agents’ actions on reputation, either because no formal compensation contracts can be written, or because of limited pre-commitment powers. This prevents the dynamic incentive problem from simply collapsing to a static one.

Under symmetric learning, the impact of current actions on future reputation, and thus the strength of reputational incentives, can either increase or decrease with improved information.¹ However,

¹Dewatripont, Jewitt, and Tirole (1999) characterize the impact of different information systems on implicit incentives for situations where explicit incentives are not possible. Meyer and Vickers (1997) show that better information can either enhance or weaken incentives in environments where implicit incentives are complemented by explicit incentives. Incentives from the reputation enhancing effect of effort can be outweighed by disincentives arising from the ratchet effect.

many economic situations are characterized by sequential contracting where there is *asymmetric learning*. Waldman (1984) is an early example for the impact of a firm's actions on workers' reputations. In his analysis, a firm learns about its workers' types during the first period of their employment and then decides on job assignments for period 2. Outsiders can observe the job offer that a worker receives. Promotion to a job that requires higher ability sends a favorable signal to the labor market and enhances the agent's outside options. Retaining such an agent is therefore more costly and, as a result, the firm sets the ability threshold for promotion too high compared to the socially efficient level.²

If a principal can use superior information about agents' abilities then the explicit compensation scheme provides agents with direct incentives as well as with signals that affect their reputation. The model of Zábajník and Bernhardt (2001) illustrates this dual role of explicit incentives. A firm sets up a tournament in which ex ante identical workers compete in human capital investments and are subject to a permanent human capital shock. The promotion scheme ranks workers by their realized human capital. Reputational incentives arise because the expected human capital shock for a tournament winner is larger than that for the next highest in rank, etc.³

A question that remains is whether tournaments can indeed be optimal contracts in such a sequential contracting environment where parties can only commit to spot contracts. Our model addresses this issue by excluding all the non-reputation based reasons for the use of relative performance contracts that the literature has identified. First, correlation between stochastic components in the outputs of different agents can be used to insure risk-averse agents against common performance shocks (e.g., Lazear and Rosen (1981), Holmström (1982/99), Nalebuff and Stiglitz (1983), Green and Stokey (1983), and Mookherjee (1984)).⁴ Second, relative performance contracts can help internalize production externalities (e.g., Itoh (1991)). Third, the principal can use relative performance contracts to create proper incentives when agents can monitor each others' efforts (e.g., Ma (1988), Che and Yoo (2001), and Laffont and Rey (2001)). Fourth, if agents with other-regarding preferences interact, the principal may find it optimal to design relative performance schemes that exploit the dependence of an agent's utility on other agents'

²Other models with asymmetric learning are Greenwald (1986), Ricart I Costa (1988), Bernhardt (1995), and Waldman (1990). In Lazear (1986) both the incumbent employer and outsiders obtain signals about workers' abilities.

³The analysis abstracts from the strategic impact of promotion that arises in Waldman (1984) by assuming that the firm can commit to its promotion rule ex ante. See Waldman (2003) for a discussion of conflicts between ex ante incentives and ex post optimal promotion rules, and the role of commitment.

⁴Meyer and Vickers (1997) show that this static insurance effect can be outweighed by the negative impact on implicit incentives of the ratchet effect in a dynamic model with career concerns.

transfers to enhance incentives (e.g., Itoh (2004)).

The outline of the paper is as follows. Section 2 sets up the model. Section 3 contains the analysis of the contracting problem. A discussion of our results is given in Section 4.

2 The Model

A principal (“she”) offers two agents (“he”) contracts to work for her during one period. It is common knowledge that the agents are heterogeneous. We then denote as *high-skilled* ($\theta = H$) the more talented of the two and as *low-skilled* ($\theta = L$) the other one.⁵ Both agents’ working lives last for two periods and they have outside options providing life-time utility $u = 0$. An agent who contracts with the principal in period 1 then faces contracting opportunities with other principals in period 2. All parties are risk neutral but agents are subject to wealth and credit constraints that prevent the principal from imposing negative transfers. Discount rates are normalized to one.

We begin by describing the first-period production technology. An agent of type $\theta \in \{L, H\}$ who works for the principal in period 1 can achieve two possible type-dependent output levels, a low one ($q_{\theta l}$) and a high one ($q_{\theta h}$). A high output level can only be reached if the agent exerts effort ($e_{\theta} = 1$) at a private cost ψ . Formally,

$$\text{Prob}(\tilde{q} = q_{\theta h} | e_{\theta} = 1) = P_{\theta}, \quad (1)$$

$$\text{Prob}(\tilde{q} = q_{\theta h} | e_{\theta} = 0) = 0. \quad (2)$$

Thus, both agents’ outputs depend only on their own effort and type.⁶ We also assume that the high-skilled agent has a larger productivity of effort than the low-skilled one. That is,

$$P_H > P_L > 0. \quad (3)$$

In sum, stochastic output accruing to the principal from an arbitrary agent can take on four possible realizations $\tilde{q} \in Q \equiv \{q_{Ll}, q_{Lh}, q_{Hl}, q_{Hh}\}$, where $q_{Ll} < q_{Lh} \neq q_{Hl} < q_{Hh}$.

Agents are initially privately informed about their own type. At the beginning of period 1, the principal offers contracts that agents can accept or reject. Upon accepting, an agent non-cooperatively chooses his effort level, which is not observable by any other party.⁷ At the end of period 1, output realizes and agents receive transfers from the principal.

⁵Introducing uncertainty about the agents’ types would not alter our qualitative results. One could also introduce a stage where the principal selects agents from a heterogeneous population to contract with (see our companion paper Koch and Peyrache (2005)).

⁶This eliminates production externalities as a rationale for relative performance contracts.

⁷This rules out mutual monitoring as a rationale for relative performance contracts.

We consider an environment where agents' outputs are *contractible* but not *publicly* observable.⁸ Compensation schemes use monetary transfers $t \in \mathbb{R}_+$ and cheap-talk messages $m \in M$. Contracts map agents' outputs to *transfer/message* (t/m) pairs which are *hard evidence*. Messages in the form of reference letters, job titles, honorific rewards, and medals are often observed in employment relations. While our model incorporates such messages, we do not attempt to capture their entire richness. In our setup, messages serve the technical purpose of guaranteeing existence of equilibrium by allowing the principal to make a distinction between two agents who receive the same monetary transfers.

In period 2, agents leave the first principal⁹ and face new contracting opportunities with different principals. We simply assume that $k_\theta > 0$ reflects an *experienced* agent's productivity in a competitive labor market and that $\Delta k \equiv k_H - k_L > 0$. Thus, the utility that an agent derives from such a contractual relationship is increasing in his expected type.

All our qualitative results obtain under the above assumptions alone. To strengthen our findings, we additionally introduce the possibility that the principal and agents can renegotiate about t/m pairs at the end of period 1. As we show, monetary transfers and cheap talk messages influence an agent's reputation. Therefore, the principal could for example agree to swap the monetary transfer which the agent is entitled to in exchange for a lower transfer or a cheap-talk message providing a higher reputation in the labor market. We carry out our analysis in this framework to guarantee that our findings are not an artefact of ruling out such renegotiation opportunities which are likely to occur in most labor market settings.

To summarize, the timing of the model is as follows. At date 0, the first principal makes contract offers to the two agents. Agents then accept or refuse the offer. If an agent rejects, he receives the outside utility $u = 0$. If he accepts, he gets hired and the market observes the contract. Agents who accepted the contract then non-cooperatively choose their effort levels and output realizes. At the end period 1, agents receive a t/m pair according to their contracts or the outcome of their renegotiation with the principal, and the relation with the first principal ends. In period 2, agents who worked for the first principal enter the market for experienced labor, where future employers

⁸In our companion paper (Koch and Peyrache 2005) we model the complementary case where the principal can only contract on variables that are publicly observable and her decision to reveal performance variables directly impacts her contracting possibilities.

⁹Calzolari and Pavan (2005) make a similar assumption in their sequential contracting model. This assumption is actually stronger than what we need for our results to hold. We extensively discuss its implications in Section 4.

meet at most one of the agents. Agents can show employers their t/m pair or conceal one or both components of it and get paid their expected productivity given the hard evidence they provided.

The above model structure is common knowledge. We solve for a Perfect Bayesian Equilibrium and restrict attention to contracts that incite both agents to exert effort.¹⁰

3 Analysis

We start our analysis by solving backwards, first focusing on the labor market for experienced labor, where agents solicit offers in period 2.

3.1 Belief formation

Upon meeting an agent who shows a transfer/message (t/m) pair, the market forms beliefs about the probability β of facing a low-skilled individual. Given such beliefs β , the market's posterior about an agent with t/m pair (t, m) is:¹¹

$$E[k_\theta|t, m] = \beta(t, m) k_L + [1 - \beta(t, m)] k_H. \quad (4)$$

Each contract induces a distribution over t/m pairs. Let $X(\phi)$ be the set of t/m pairs that are observed on the equilibrium path under contract ϕ . For such t/m pairs $(t, m) \in X(\phi)$ beliefs β above are formed using Bayes' rule. Off the equilibrium path ($(t, m) \notin X(\phi)$), Bayes' rule no longer applies. One requirement that we impose is that market participants account for the possibility of agents deliberately hiding from the labor market the t/m pair that they received. Therefore, the market assigns to any agent who shows up empty handed ($(t, m) = (0, \emptyset)$) the worst belief associated with an equilibrium t/m pair $(t, m) \in X(\phi)$.

The fact that t/m pairs (which are hard evidence) serve as a signal to the labor market leads to the first insight of our model: the value of a t/m pair to an agent does not only depend on its monetary component but also on its impact on the assessment of the market regarding the agent's ability. The *perceived transfer* is the combination of the direct monetary value t and the reputation $E[k_\theta|t, m]$ associated with a t/m pair.

¹⁰A sufficient condition for this is that $q_{Lh} - q_{Ll} \geq \frac{2\psi}{P_L}$. Then, even if there existed a contract that offered sufficiently high reputational incentives to incite the high-skilled agent to exert effort at no monetary cost but not the low-skilled one, it would pay to switch to a contract with no reputational incentives that implements effort by both agents (see footnote 13).

¹¹To avoid cumbersome notation, we do not index this expectation by the first-period contract.

3.2 The Key Trade-Offs

In the contract design stage the principal anticipates the market beliefs that a contract will induce. We start our analysis by first ignoring the possibility of renegotiation and focusing on two specific contracts that base t/m pairs on each agent's individual performance to provide the main intuition underlying our results. Subsequently, we will admit the possibility of renegotiation and derive our results considering case by case the admissible schemes in the contract space, moving from contracts based on individual performance measures to contracts utilizing relative performance measures.

In the space of contracts, *perfectly revealing* t/m pairs are a polar case. They induce market beliefs of facing either a high- or a low-skilled agent with probability one:

Definition 1 (Perfectly Revealing Transfer-Message Pairs and Contracts)

A transfer-message pair $(t, m) \in \mathbb{R}_+ \times M$ is perfectly revealing if $\beta(t, m) \in \{0, 1\}$. A contract is perfectly revealing if all of its t/m pairs are perfectly revealing.

Contract 1: A Perfectly Revealing Contract. Consider a contract that conditions only on an individual agent's output and stipulates distinct t/m pairs for each output state: $[(t_{Hh}, m_{Hh}), (t_{Hl}, m_{Hl}), (t_{Lh}, m_{Lh}), (t_{Ll}, m_{Ll})]$. To induce agents to exert effort, t/m pairs must satisfy the following incentive constraints:

$$\begin{aligned} P_\theta [t_{\theta h} + k_\theta] + (1 - P_\theta) [t_{\theta l} + k_\theta] - \psi &\geq t_{\theta l} + k_\theta, \\ \Leftrightarrow t_{\theta h} - t_{\theta l} &\geq \frac{\psi}{P_\theta}, \quad \theta = L, H. \end{aligned} \tag{5}$$

Since the contract is perfectly revealing, the anticipated second-period wage k_θ has no impact on incentives. The limited liability constraint binds because $k_L > 0$ and thus agents always receive more than their outside option value $u = 0$.¹² Thus, the optimal t/m pairs for this perfectly revealing contract are given by $(t_{Hh} = \frac{\psi}{P_H}, m_{Hh})$, $(t_{Lh} = \frac{\psi}{P_L}, m_{Lh})$, $(t_{Hl} = 0, m_{Hl})$, and $(t_{Ll} = 0, m_{Ll})$, where all messages are distinct. As shown in Table 1, no reputational incentives arise and the principal has to rely exclusively on monetary incentives, incurring an expected implementation cost of 2ψ .¹³

¹²The possibility of binding individual rationality constraints is analyzed in a related framework in our companion paper (Koch and Peyrache 2005). We do not treat this case here since it does not add much economic insight but greatly complicates expressions later in the analysis.

¹³From this we can derive the sufficient condition stated in footnote 10. The gain in expected output from making the low-skilled agent exert effort is $P_L (q_{Lh} - q_{Ll})$ and the expected cost of providing effort for both agents is bounded above by 2ψ .

output	t/m pair	$E[k_\theta t, m]$	
q_{Hh}	(t_{Hh}, m_{Hh})	\bar{k}	} no reputational incentives
q_{Hl}	$(0, m_{Hl})$	\bar{k}	
q_{Lh}	(t_{Lh}, m_{Lh})	\underline{k}	} no reputational incentives
q_{Ll}	$(0, m_{Ll})$	\underline{k}	

Table 1: Perfectly revealing contract (Contract 1)

Contract 2: A Performance Standard Contract. Now consider slightly altering the contract structure in Contract 1 by setting both $t_{Hl} = t_{Lh} = \tilde{t}$ and $m_{Hl} = m_{Lh} = \tilde{m}$. That is, we simply pool the t/m pairs for a high-skilled agent in a low-output state and a low-skilled agent in a high-output state. The contract sets two performance thresholds, q_{Lh} and q_{Hh} . Once the output of an agent reaches a given performance standard, he receives a different t/m pair. In contrast to Contract 1, it groups agents of different types in the intermediate tier of performances and therefore is not perfectly revealing. In general, the design of performance standards affects the probabilities of meeting the thresholds for the different types of agents, and controls how much information about agents' types is transmitted in each performance tier. As shown in Table 2, Contract 2 reveals a high-skilled agent as such if he produces high-output, guaranteeing him earnings \bar{k} in period 2. If he produces low output, he is “punished” by being pooled with a low-skilled agent who produces high-output, resulting in second-period earnings $E[k_\theta | \tilde{t}, \tilde{m}] < \bar{k}$. Similarly, a wedge is driven between the second-period earnings of a low-skilled agent. He is “rewarded” following high output by being made undistinguishable from a high-skilled agent with low output, raising the second-period earnings relative to the situation where he produces low output and is revealed as low skilled: $E[k_\theta | \tilde{t}, \tilde{m}] > \underline{k}$. Thus, the incentive constraint now becomes

$$\underbrace{t(q_{\theta h}) - t(q_{\theta l})}_{\text{monetary incentives}} \geq \frac{\psi}{P_\theta} - \underbrace{\left\{ E[k_\theta | t(q_{\theta h}), m(q_{\theta h})] - E[k_\theta | t(q_{\theta l}), m(q_{\theta l})] \right\}}_{\text{reputational incentives}}. \quad (6)$$

On the one hand, the reputational incentives that arise allow the principal to lower monetary incentives, as can be seen from (6). On the other hand, pooling of t/m pairs forces the principal to pay the monetary transfer \tilde{t} also to a high-skilled agent reaching a low-output state. Thus, a side effect of creating reputational incentives is raising the *total* implementation cost relative to the perfectly revealing contract in Contract 1. However, what matters to the principal is the *monetary* (and not the *total*) cost of implementing effort. Specifically, the above contract generates

output	t/m pair	$E[k_\theta t, m]$	
q_{Hh}	(t'_{Hh}, m_{Hh})	\bar{k}	↑ reputational incentives
q_{Hl}	(\tilde{t}, \tilde{m})	$E[k_\theta \tilde{t}, \tilde{m}] < \bar{k}$	
q_{Lh}	(\tilde{t}, \tilde{m})	$E[k_\theta \tilde{t}, \tilde{m}] > \underline{k}$	↑ reputational incentives
q_{Ll}	$(0, m_{Ll})$	\underline{k}	

Table 2: Performance standard contract (Contract 2)

the following reputational incentives: for the high-skilled agent we have

$$E[k_\theta | t'_{Hh}, m_{Hh}] - E[k_\theta | \tilde{t}, \tilde{m}] = k_H - \frac{(1 - P_H) k_H + P_L k_L}{1 - P_H + P_L} = \frac{P_L}{1 - P_H + P_L} \Delta k, \quad (7)$$

and, for the low-skilled agent, we have

$$E[k_\theta | \tilde{t}, \tilde{m}] - E[k_\theta | 0, m_{Ll}] = \frac{(1 - P_H) k_H + P_L k_L}{1 - P_H + P_L} - k_L = \frac{1 - P_H}{1 - P_H + P_L} \Delta k. \quad (8)$$

Thus, under this contract structure the principal optimally sets

$$\begin{aligned} t'_{Hh} &= \max \left\{ \tilde{t} + \frac{\psi}{P_H} - \frac{P_L}{1 - P_H + P_L} \Delta k, 0 \right\} \quad \text{and} \\ \tilde{t} &= \max \left\{ \frac{\psi}{P_L} - \frac{1 - P_H}{1 - P_H + P_L} \Delta k, 0 \right\}. \end{aligned}$$

The contract has an expected monetary implementation cost of $P_H t'_{Hh} + (1 - P_H + P_L) \tilde{t}$ which is decreasing in the heterogeneity of experienced agents' human capital, Δk . Consider the extreme situation where Δk is sufficiently large that the principal can incite agents to exert effort at no cost (i.e., $t'_{Hh} = \tilde{t} = 0$). By setting $t_{Hh} = t_{Lh} = t_{Hl} = t_{Ll} = 0$ and using three distinct messages m_{Hh} , \tilde{m} , and m_{Ll} the principal benefits from reputational incentives despite a flat monetary scheme to achieve a monetary implementation cost lower than under Contract 1.¹⁴

The comparison of Contracts 1 and 2 illustrates well the dichotomy between *monetary* and *perceived* transfers that is key to the results in this paper. The principal has two currencies with which she can reward agents: cash and reputation. However, there is a trade-off between “good monetary incentives” and “good reputational incentives”. While a perfectly revealing contract minimizes the total implementation cost, incentives must be provided entirely through monetary transfers. An imperfectly revealing contract increases the total implementation cost but creates

¹⁴Technically, the messages guarantee the existence of an equilibrium by serving as a means of distinguishing two identical monetary transfers in terms of the reputation that they confer. Without messages one would need to introduce a grid for such transfers to achieve existence of an equilibrium in a situation where monetary transfers differ just to create distinct reputations.

reputational incentives, which can lower the expected monetary implementation cost well below 2ψ .

In our subsequent analysis we will see that it is always optimal for the principal to create ambiguity about agents' types rather than design perfectly revealing contracts. The principal cares only about her monetary cost of providing incentives and the reputational incentives created through imperfectly revealing contracts outweigh the increase in total implementation cost. In the preceding examples agents are rewarded solely based on their own performance. In such *Individual Performance Measure* (IPM) contracts (defined more precisely below) the principal can *indirectly* tie the incentives of the agents together by pooling t/m pairs across different output states. Thus, IPM contracts create lotteries over future earnings that depend on the agent's individual performance alone. As we show, the principal can improve upon IPM contracts by conditioning on multiple agents' performances. Such a *Relative Performance Measure* (RPM) contract (defined more precisely below) enables the principal to *directly* tie together agents' incentives and induce lotteries over future earnings that depend on the combination of individual performances rather than on a single agent's performance. Therefore, RPM contracts provide the principal with more flexibility in creating lotteries over perceived transfers.

3.3 Renegotiation Proofness

Without any restrictions on contracts, an agent might offer to give up a monetary transfer that he is entitled to in exchange for a t/m pair with a lower monetary transfer but a high reputational value. Clearly, the principal would not refuse such offer. For this reason, we want contracts to be such that the parties would not agree to renegotiate even when they have the opportunity to do so. Requiring contracts to be renegotiation proof strengthens our findings because all results go through also if we exclude renegotiation. Since the details of the renegotiation game are not important, we simply present the renegotiation proofness conditions in the form of a definition:

Definition 2 (Renegotiation Proofness)

Given market beliefs β , a contract ϕ is renegotiation proof if $\forall (t', m') \in X(\phi)$ and $\forall (t'', m'') \in \mathbb{R}_+ \times M$, for which $(t'', m'') \neq (t', m')$, none of the following conditions is violated:

- (i) $t' > t'' \Rightarrow t' + E[k_\theta | t', m'] > t'' + E[k_\theta | t'', m'']$,
- (ii) $t'' + E[k_\theta | t'', m''] > t' + E[k_\theta | t', m'] \Rightarrow t'' > t' \quad \text{or} \quad t' = 0$,
- (iii) $t' = t'' > 0 \Rightarrow E[k_\theta | t', m'] = E[k_\theta | t'', m'']$.

The definition states that there is no deviation that would be preferred by both the principal and the agent from the t/m pairs stipulated by the contract ϕ to any other t/m pair. Condition (i)

requires perceived transfers to be strictly increasing in their monetary component. Otherwise, an agent could successfully renegotiate with the principal, offering her to replace the contractually guaranteed t/m pair (t', m') by a pair (t'', m'') involving a lower monetary transfer but a higher perceived transfer. The principal never renegotiates to a t/m pair with a higher monetary transfer because agents cannot commit to repay her anything after having used the t/m pair as a signal in the market. Conditions (ii) and (iii) guarantee that it is never profitable to buy a message from the principal if the agent has cash. If $t' = 0$, renegotiation is impossible since the agent lacks the funds to bribe the principal. Definition 2 imposes on out-of-equilibrium beliefs that they support an equilibrium where contracts are not renegotiated after output has realized.¹⁵ Note that the above conditions also guarantee that an agent never has an incentive to break apart a t/m pair and conceal one or all of its components.

To illustrate the impact of renegotiation proofness on contract design, reconsider the perfectly revealing Contract 1. Suppose both that $\frac{\psi}{P_L} + k_L < k_H$ and that market beliefs are such that the contract is not renegotiated. Then, a low-skilled agent receiving t/m pair (t_{Lh}, m_{Lh}) could renegotiate to obtain $(0, m_{Hl})$, which yields a higher perceived transfer to the agent and decreases the monetary cost for the principal. Thus, the assumed beliefs are inconsistent and renegotiation-proofness is violated.

3.4 Individual Performance Measure Contracts

In this section, we restrict attention to Individual Performance (IPM) schemes only and derive optimal contracts within this class of contracts (best IPM contracts), which provides the benchmark for proving that the principal can gain from using relative performance measures.

Definition 3 (Individual Performance Measure Contract)

An Individual Performance Measure (IPM) contract $\phi \in \Phi_{IPM}$ is a function from the set of outputs of a single agent to the set of transfer-message pairs, $\phi : Q \rightarrow \mathbb{R}_+ \times M$.

Contracts 1 and 2 are two specimens of IPM contracts. We saw that Contract 2 is more profitable than Contract 1 if the heterogeneity in experienced agents' productivities, Δk , is sufficiently large. Contract 2 can be shown to be the unique best IPM contract for some parameter values. We relegate the complete analysis of the set of IPM contracts to Appendix A and focus on the two other contracts that are best IPM contracts for some parameter values. This will help build the

¹⁵For example, the simplest out-of-equilibrium beliefs that sustain a contract ϕ that is renegotiation-proof on the equilibrium path, are: $\beta(t, m) = 1$ if $(t, m) \notin X(\phi)$.

intuition for our result that perfectly revealing contracts are not optimal.

Contract 3. Consider slightly altering Contract 1 by setting $m_{Hl} = m_{Ll} = m'$. This has two countervailing effects on incentives. Upon receiving t/m pair $(0, m')$ the expected productivity for the agent is $E[k_\theta|0, m'] \in (k_L, k_H)$. On the one hand, this creates reputational incentives for the high-skilled agent because moving from the low-output state to the (perfectly revealing) high-output state increases his second-period earnings from $E[k_\theta|0, m']$ to k_H . On the other hand, this leads to reputational disincentives because a low-skilled agent loses in terms of reputation by moving from the low-output state to the high-output state. This decreases his second-period earnings from $E[k_\theta|0, m']$ to k_L . The larger marginal product of effort for the high-skilled agent ($P_H > P_L$) means that he is more likely to reach a high-output state than the low-skilled agent. Intuitively, reducing the monetary transfer in such states should be more important than keeping the high-output state monetary transfer for the low-skilled agent low. Indeed, it can easily be shown that Contract 3 is always more profitable than the perfectly revealing Contract 1. That is, on balance the principal benefits from setting $m_{Hl} = m_{Lh}$ and not revealing the agents' types in the low-output states.¹⁶

Contracts 2 and 3 each create favorable reputational incentives and both can be shown to be uniquely optimal in the class of IPM contracts for some parameter values. Both offer three distinct t/m pairs, i.e., set two performance thresholds. Crossing a performance threshold increases the monetary transfer and/or the reputation of an agent. However, for some parameter values, one of the perceived transfers associated with intermediate and high output states becomes more attractive than the t/m pair with the highest monetary transfer, and therefore the contracts become vulnerable to renegotiation. Setting a single performance threshold and thereby limiting the number of distinct t/m pairs solves this problem.

Contract 4. It offers a unique (t, m) pair whenever the agent achieves one of the high-output states and a unique pair $(0, m')$ in any of the two low-output states. It can be shown that Contract 4 is uniquely optimal in the class of IPM contracts for some parameter values.¹⁷

By highlighting Contracts 2, 3, and 4 we have both shown how contract design affects reputational incentives and characterized the contracts that are optimal in the class of IPM contracts.

¹⁶See the first part of the proof of Proposition 1 (IPM₁ versus IPM₅) in Appendix A

¹⁷See the last part of the proof of Proposition 1 (IPM₆ uniquely optimal) in Appendix A

Proposition 1

In the class of Individual Performance Measure (IPM) contracts where both agents exert effort, the profit maximizing contracts (best IPM contracts) are imperfectly revealing contracts with (multiple) performance standards. Specifically, the best IPM contract has the structure of Contract 2, 3, or 4.

The details of the proof are relegated to Appendix A.¹⁸ The procedure is the following. Consider all feasible IPM contracts. Start with a candidate contract structure and determine the associated equilibrium perceived transfers. Then show that, given these transfers and their supporting market beliefs, the principal has no incentive to deviate to a different contract, agents indeed exert effort, and no renegotiation occurs.

Proposition 1 confirms the intuition that the principal always benefits from offering some form of *imperfectly* revealing IPM contract that creates ambiguity about agents' types. By using t/m pairs to pool across output states reached by different types of agents, the principal distorts the information transmission to the market and generates profit enhancing reputational incentives. The next section shows that this insight extends to the entire set of deterministic contracts.

3.5 Individual versus Relative Performance Measure Contracts

In this section we consider the entire set of deterministic contracts Φ which can be partitioned into two subclasses, *Individual Performance Measure* (IPM) contracts (the focus of Section 3.4) and *Relative Performance Measure* (RPM) contracts:

Definition 4 (Relative Performance Measure Contract)

A Relative Performance Measure (RPM) contract $\phi \in \Phi_{RPM}$ is a mapping from the set of outputs of both agents to the set of tuples of transfer-message pairs, that cannot be replicated using IPM contracts.¹⁹

An RPM contract can be represented by a matrix as in Table 3. A symmetric RPM contract ignores agents' identities and thus contains fewer distinct t/m pairs. As for IPM contracts, a given RPM contract ϕ induces a probability distribution over the t/m pairs $(t, m) \in X(\phi)$, which determines the market's equilibrium beliefs. Thus, the incentive constraint for the high-skilled agent, say he

¹⁸In the appendix, Contracts 2, 3 and 4 are denoted IPM_4 , IPM_5 and IPM_6 , respectively.

¹⁹In the contract proposal game, the principal needs to specify what happens if only one agent accepts the contract. Since in equilibrium both agents accept the contract, to keep things simple, we do not include this contingency in the definition of the contract. For example, the contract could stipulate to then apply the perfectly revealing benchmark IPM contract from Section 3.4.

RPM Contract ($\phi \in \Phi_{RPM}$)

Output of agent 1	Output of agent 2			
	q _{Ll}	q _{Lh}	q _{Hl}	q _{Hh}
q _{Hl}	(t ₁ , m ₁), (t ₂ , m ₂)	(t ₃ , m ₃), (t ₄ , m ₄)		
q _{Hh}	(t ₅ , m ₅), (t ₆ , m ₆)	(t ₇ , m ₇), (t ₈ , m ₈)		
q _{Ll}			(t' ₁ , m' ₁), (t' ₂ , m' ₂)	(t' ₅ , m' ₅), (t' ₆ , m' ₆)
q _{Lh}			(t' ₃ , m' ₃), (t' ₄ , m' ₄)	(t' ₇ , m' ₇), (t' ₈ , m' ₈)

Symmetric RPM Contract

Output combinations	q _{Hl}	q _{Hh}
q _{Ll}	(t ₁ , m ₁), (t ₂ , m ₂)	(t ₃ , m ₃), (t ₄ , m ₄)
q _{Lh}	(t ₅ , m ₅), (t ₆ , m ₆)	(t ₇ , m ₇), (t ₈ , m ₈)

Table 3: RPM contracts

is agent 1, under an RPM contract $\phi \in \Phi_{RPM}$ is given by²⁰

$$\begin{aligned}
 & \underbrace{[P_L t(q_{Hh}, q_{Lh}) + (1 - P_L) t(q_{Hh}, q_{Ll})] - [P_L t(q_{Hl}, q_{Lh}) + (1 - P_L) t(q_{Hl}, q_{Ll})]}_{\text{monetary incentives}} \\
 & \geq \frac{\psi}{P_H} - \left\{ \underbrace{\left[P_L E[k_\theta | \phi_1(q_{Hh}, q_{Lh})] + (1 - P_L) E[k_\theta | \phi_1(q_{Hh}, q_{Ll})] \right]}_{\text{reputational incentives}} \right. \\
 & \quad \left. - \left[P_L E[k_\theta | \phi_1(q_{Hl}, q_{Lh})] + (1 - P_L) E[k_\theta | \phi_1(q_{Hl}, q_{Ll})] \right] \right\}. \tag{9}
 \end{aligned}$$

The low-skilled agent's incentive constraint can be decomposed in a similar way.

Perfectly Revealing RPM Contracts. These contracts generate no reputational incentives in equilibrium so that all that matters to an agent is the expected monetary reward received in each of the possible output states. Because agents are risk neutral and their outputs are independent random variables, conditioning contracts on the other agent's output cannot decrease implementation cost. This is immediately apparent from a comparison of incentive constraints (6) and (9): a perfectly revealing RPM contract cannot have a strictly lower monetary implementation cost than the perfectly revealing IPM Contract 1, which itself is not a best IPM contract. As a direct implication of this and Proposition 1 we obtain the following important result.

²⁰We denote here by $\phi_1(q_1, q_2)$ the t/m pair received by agent 1 under contract ϕ for output pair (q_1, q_2) .

Proposition 2

Perfectly revealing contracts are not optimal.

This generalizes our finding in Proposition 1 that the principal wants to design the transfer scheme so that it creates ambiguity about agents' types ex post. The result that the optimal information transmission is always imperfect is reminiscent of Calzolari and Pavan (2005)'s model. In their sequential contracting model with pure asymmetric information, the first principal never fully discloses information since this would eliminate all information rents in the second contractual relationship. In our moral hazard model, full disclosure would eliminate all first-period reputational incentives and force the principal to provide additional incentives for effort in monetary terms. Strategic information revelation permits the principal to shift part of the moral hazard cost to the future labor market. Thus, reputational incentives can be interpreted as an information rent accruing to the first principal.

Asymmetric RPM Contracts (see Table 3). These contracts can only differ from symmetric RPM contracts by adding a randomization over t/m pairs based on agents' identities (and not types). Since such contractual contingencies do not affect reputational incentives, they cannot increase profits and the principal can restrict attention to symmetric RPM contracts only.

We have thus considerably narrowed the set of candidate optimal contracts to the best IPM contracts from Proposition 1 and the class of imperfectly revealing symmetric RPM contracts.

Rank-Order Tournaments. Rank-order tournaments are the most prominent example of symmetric RPM contracts. A tournament selects the agent with the highest output as the winner, who then receives the t/m pair $(B^e, \text{"winner"})$, consisting of an explicit bonus $B^e \geq 0$ and a message announcing the agent as the winner. The loser receives t/m pair $(0, \text{"loser"})$. If $q_{Hl} > q_{Lh}$ the high-skilled agent wins the tournament with certainty. That is, rank-order tournaments become perfectly revealing contracts and cannot strictly dominate IPM contracts. In contrast, if $q_{Hl} < q_{Lh}$ they create the following reputational incentives:

$$R(B^e) \equiv E[k_\theta | (B^e, \text{"winner"})] - E[k_\theta | (0, \text{"loser"})] = [1 - 2P_L(1 - P_H)] \Delta k. \quad (10)$$

The combination of explicit bonus and reputational incentives generates a perceived bonus $B = B^e + [1 - 2P_L(1 - P_H)] \Delta k$. Satisfying both agents' incentive constraints²¹ requires that $B \geq$

²¹Recall that the individual rationality constraint is always satisfied. A detailed derivation of the following results is available from the authors.

$\max \left\{ \frac{\psi}{P_L P_H}, \frac{\psi}{P_L (1 - P_H)} \right\}$. Since wealth constraints prevent the principal from imposing negative transfers, the explicit bonus is

$$B^e = \max \left\{ \max \left\{ \frac{\psi}{P_L P_H}, \frac{\psi}{P_L (1 - P_H)} \right\} - R(B^e), 0 \right\}. \quad (11)$$

Clearly, rank-order tournaments are optimal contracts whenever Δk is sufficiently large so that reputational incentives are sufficient to implement effort. In such a setting $B^e = 0$ and the principal obtains the maximum possible expected profit. However, a rank-order tournament does not *strictly* dominate the best IPM contracts, as the next result states:

Proposition 3

Rank-order tournaments are optimal if they implement effort using reputational incentives only. Then, there exists a payoff equivalent IPM contract. Otherwise, there exists an IPM contract that is strictly more profitable than rank-order tournaments.

The proof consists in showing that at least one of the best IPM contracts identified in the proof of Proposition 1 achieves the same profit if $B^e = 0$ and strictly dominates rank-order tournaments whenever $B^e > 0$.

Other Symmetric RPM Contracts. Proposition 3 raises two questions. Does there always exist an RPM contract that generates at least the same profit as the best IPM contracts? Can RPM contracts strictly dominate IPM contracts? Intuition suggests that both can be answered in the affirmative: RPM contracts provide the principal with a larger number of t/m pairs and thus more flexibility in designing lotteries over perceived transfers. Starting with the first question, if the best IPM contract involves some perfectly revealing t/m pairs it is quite straightforward to construct an RPM contract that guarantees the principal at least the same profit, while leaving each agent with the same expected payoff as under the best IPM contract:

Lemma 1

If at least one t/m pair associated with an IPM contract is perfectly revealing, then there exists an RPM contract that is payoff equivalent.

The intuition is simple: a payoff equivalent RPM contract can be obtained by adding noise to the fully revealing transfers by making transfers in this output state contingent on the output of the remaining agent. By doing so, the reputation effects and thus the expected cost of monetary transfers remain unchanged (see the proof in Appendix B.2). Even though other RPM contracts

can be found that also do at least as well as these IPM contracts (see the example in Appendix B.3) the result provides a useful short cut.

Two caveats remain. First, whenever the best IPM contract does not contain any perfectly revealing transfers (see Contract 4 in Section 3.4) we cannot rely on Lemma 1 to find a payoff equivalent RPM contract. Indeed, any slight modification of the contract terms leads to discrete changes in the reputation attached to the associated t/m pairs. Total implementation cost and reputational incentives then change in complex ways. Second, the trick used in Lemma 1 does not enable us to construct an RPM contract that *strictly* dominates the best IPM contract for some parameter values.

Note that it is impossible to find an RPM contract which *always* strictly dominates IPM contracts because the latter can have zero implementation cost and then are optimal contracts. Nevertheless, in the proof of the following result we show that there exists a renegotiation-proof RPM contract that overcomes the two caveats mentioned above. It is a simple “group bonus scheme” which rewards both agents in the same way when both produce high output and provides an individualized bonus to the high achiever if only one agent produces high output. This contract always produces at least the same profit as Contract 4 from Section 3.4 and strictly dominates it for (non-degenerate) parameter values for which it is the unique best IPM contract. From Proposition 1 we know that any other candidate best IPM contract has a perfectly revealing t/m pair and a payoff equivalent RPM contract exists by Lemma 1.

Proposition 4

Among the class of deterministic contracts which implement effort by all agents, RPM contracts weakly dominate IPM contracts. For a non-degenerate range of parameter values, RPM contracts are strictly more profitable than IPM contracts.

Proposition 4 tells us that the complex contracting problem can be reduced to a search on the subclass of imperfectly revealing symmetric RPM contracts. This provides a new rationale for the use of relative performance contracts since the assumptions in our model were deliberately chosen so that the known reasons for the use of such contracts are absent.

4 Discussion and Conclusion

Any reward scheme both provides incentives and transmits information to the labor market regarding an agent’s productivity. As a consequence the explicit incentive contract both directly affects

effort incentives through monetary transfers and indirectly impacts effort by controlling the flow of information to the market.

In our setup, the production process of one agent is independent of that of the other. Nevertheless, since output contains information about the agent's ability it is optimal for the principal to tie the incentives of the two agents together by pooling t/m pairs across different output states (Proposition 2). This permits her to create ambiguity about the agent's type because then the market cannot back out perfectly the output that the agent realized by inverting the incentive scheme. Under such an imperfectly revealing contract, the agent faces a lottery over future reputation that depends on the output he produces. Specifically, the principal chooses a contract that partitions the joint distribution of agents' t/m pairs in such a way that, for at least one type of agent, the reputation derived from using t/m pairs as a signal in the labor market is increasing in the output that he produces. Even though a trade-off between "good monetary incentives" (low total implementation cost) and "good reputational incentives" arises, the principal can create sufficient reputational incentives to more than compensate for the increase in total implementation cost resulting from pooling transfers across output states.

Ideally, the principal would want to write a stochastic contract to fine tune these lotteries over perceived transfers. However, the principal might only be able to credibly commit to deterministic incentive schemes (as in our setup) since in contrast to stochastic contracts these are easy to verify by third parties such as courts. Then the contract space is partitioned into Individual Performance Measure (IPM) contracts and Relative Performance Measure (RPM) contracts. The principal can benefit from using relative performance measures since they provide her with more flexibility²² in creating lotteries over perceived transfers than individual performance measures. Even though agents' wealth constraints and renegotiation proofness constrain contract choices, imperfectly revealing symmetric RPM contracts dominate IPM contracts (Proposition 4). Thus, the complex contracting problem can be reduced to a search on the subclass of imperfectly revealing symmetric RPM contracts. This provides a new rationale for the use of relative performance contracts based on the informational externalities created by compensation schemes.

Undoubtedly, a key assumption of our paper is that agents switch employers at the end of the first contracting period. We restricted our setup along this dimension to bring out clearly the hitherto unstudied informational spill-overs caused by compensation schemes in sequential contracting with moral hazard. The effects of asymmetric learning for turnover have been extensively analyzed.

²²However, remember that, by definition, RPM contracts cannot be replicated using IPM contracts only.

Typically, in such models the informational asymmetry between current employer and outsiders causes a lemons problem that prevents movements of workers across firms in equilibrium unless there are exogenous sources of turnover (e.g., Greenwald (1986)) or worker-firm matches have a random component unrelated to ability (e.g., Lazear (1986) and Owan (2004)). While there is some empirical support for adverse selection²³, both Baker, Gibbs, and Holmström (1994b) and Lazear and Oyer (2004) document substantial turnover at all hierarchy levels of firms. This suggests that in these environments adverse selection is not a severe problem since otherwise the market for experienced labor could not be active (Greenwald 1986). Gibbs, Ierulli, and Milgrom (2002) even reports a positive effect on income ensuing a move to another firm based on Swedish personnel records.

Our assumption that all agents move to new employers is in fact stronger than necessary for our results to hold. What is key for reputational incentives to arise in equilibrium is that there is some heterogeneity within the average cohort of agents leaving the principal. This then allows to affect agents' incentives by pooling t/m pairs across output states in the same way as in our model because the market does not expect all agents to have the same type. For example, production technology might impose "slot constraints" for positions higher in the hierarchy that prevent the firm from retaining all its workers once they have become too experienced for the introductory level job. Notably, in internal labor markets, constraints on positions very often imply that an upward career movement means switching to other departments. This restricts the scope for adverse selection on internal movements in the firm. Moreover, centrally set compensation rules can serve as a commitment device to credibly avoid adverse selection in employee turnover. Baker et al. (1994a, p.913) present evidence that such rules place a "wedge between an employee's pay and what pay would be in an external spot market" and thus prevent a firm from giving agents with favorable performance signals sufficiently large raises to retain them.

Another way to extend our model would be to allow for an agent's productivity to be determined both by ability and the match between the agent's human capital and the job that a principal can offer (e.g., Antel (1985) and McLaughlin (1991)). The turnover patterns from our setup would then arise endogenously if the match between a principal and skills for experienced agents were always better in a different segment of the labor market, *regardless* of agents' ability levels. For example, productive abilities and resources under control might be complements (Rosen 1982). Then, it would be efficient for experienced agents to move to a bigger firm if they all sufficiently enhanced their human capital through learning by doing in period 1 (while still differing in the

²³Gibbons and Katz (1991) find an adverse selection effect for white collar workers, which however is not apparent for workers with less than two years of tenure (p.367).

attained productivity levels).

Given the assumptions underlying our setup the implications of the model should apply mainly to markets where there are high rates of turnover, as in the professional service industry. Here employee turnover can be as high as 20 to 25 percent of the workforce per year (Maister (2003), p.15). Young professionals gather substantial amounts of experience and are generally viewed as “free agents” who invest primarily in general human capital (Groysberg and Nanda 2002). Despite the importance of reputational incentives for individuals’ careers, compensation is not tightly linked to individual’s performance in the early stages of their career. In fact, partnerships in human capital intensive professional services tend to avoid very informative measures of individual performance.²⁴ Instead firms employ up-or-out systems that resemble the simple imperfectly revealing performance standard contracts in this paper.²⁵ Adverse selection is not a serious issue in this market since individuals who leave professional service firms are generally not viewed as lemons but rather enter very attractive and highly remunerated positions (Maister 2003). Being employed for some time in such a firm provides young professionals with experience, training, and the cachet of a renowned firm. These credentials help them enter prime positions that they could not have obtained as fast by another route (Maister 2003). Despite their attractiveness, professional firms do not make young workers pay up front with an entrance fee. Instead, as our model suggests, they (partially)²⁶ extract these gains through low pay to young workers, measured relative to their qualifications (e.g., Rebitzer and Taylor (1995) and Tadelis and Levin (2005)).

The setup in this paper presents a first step in exploring the impact of informational externalities generated by compensation schemes on contract design. Interesting directions for future research are incorporating the job transition decision into the model and extending the analysis to more general production technologies.

²⁴E.g., see evidence on law firms in Gilson and Mnookin (1985).

²⁵Typically, a promotion decision has to be made after a set number of years, e.g, 6-10 years in law firms (Gilson and Mnookin 1989). Thus firms commit to not opportunistically keep qualified individuals at the associate level indefinitely (e.g., Gilson and Mnookin (1989) and Rebitzer and Taylor (1995)).

²⁶Rebitzer and Taylor (1995) provide evidence for substantial employment rents in prestigious large law firms.

IPM Contract ($\phi \in \Phi_{\text{IPM}}$)

Output ($q_{\theta i}$)	q_{Hh}	q_{Lh}	q_{Hl}	q_{Ll}
Label	a	b	c	d
$\phi(q_{\theta i})$	(t_a, m_a)	(t_b, m_b)	(t_c, m_c)	(t_d, m_d)

Table 4: IPM contracts

A Appendix: Individual Performance Measure Contracts

To easily identify the output states in which a t/m pair is given under an IPM contract, we use subscripts for the components of t/m pairs. These correspond to the letters assigned to the particular output states listed in Table 4. For example, t_a denotes the monetary transfer associated with output state a , and t_{ab} is a short-hand for $t_a = t_b$.

The following two rather straightforward results already greatly reduce the set of candidate IPM contracts.

Lemma 2

Under any IPM contract, if a t/m pair associated to low output is perfectly revealing, then the principal always sets the corresponding monetary transfer equal to zero.

Proof.

Note first that it can never be optimal to offer a larger monetary transfer in the low state than in the high state. Now, consider a positive monetary transfer for an agent of type θ who is in the low-output state. Reducing this monetary transfer relaxes the agent's incentive constraint and, therefore, the principal can also reduce the expected monetary transfer to the agent in the high-output state by the same amount. Since the t/m pair in the low-output state is perfectly revealing, this modification in monetary transfers does not change the reputation effect of moving from low to high output. ■

Lemma 3

Under any IPM contract that gives agents the same t/m pairs in all low-output states, the principal always sets the corresponding monetary transfer equal to zero.

Proof.

Decreasing the monetary transfer in low-output states has no impact on the reputation, but relaxes the agents' incentive constraints. ■

Transfer/message pairs	Profit
<i>Perfectly revealing IPM contracts</i>	
1. $[(t_a, m_a), (t_b, m_b), (t_c, m_c), (t_d, m_d)]$	$\Pi_{IPM_1} = \begin{cases} \hat{q} - 2\psi & \text{if } \frac{\Delta k}{\psi} \leq \frac{1}{P_L}, \\ \text{not renegotiation proof} & \text{otherwise.} \end{cases}$
<i>IPM contracts with three distinct t/m pairs</i>	
2. $[(t_{ab}, m_{ab}), (t_c, m_c), (t_d, m_d)]$	$\Pi_{IPM_2} = \begin{cases} \hat{q} - \frac{P_H+P_L}{P_H} \psi - P_L \Delta k & \text{if } \frac{\Delta k}{\psi} > \frac{P_H-P_L}{P_H P_L}, \\ \hat{q} - \frac{P_H+P_L}{P_L} \psi + P_H \Delta k & \text{otherwise.} \end{cases}$
3. $[(t_{ad}, m_{ad}), (t_b, m_b), (t_c, m_c)]$	$\Pi_{IPM_3} = \hat{q} - \frac{1+2P_H}{P_H} \psi - \Delta k.$
4. $[(t_a, m_a), (t_{bc}, m_{bc}), (t_d, m_d)]$	$\Pi_{IPM_4} = \begin{cases} \hat{q} - \frac{1+2P_L}{P_L} \psi + \Delta k & \text{if } P_H \leq \frac{1}{2} \text{ and } \frac{\Delta k}{\psi} < C_1, \\ & \text{or } P_H > \frac{1}{2} \text{ and } \frac{\Delta k}{\psi} \leq C_2, \\ \hat{q} - \psi + \frac{P_H P_L}{1+P_L-P_H} \Delta k & \text{if } P_H \leq \frac{1}{2} \text{ and } C_1 \leq \frac{\Delta k}{\psi} < C_2, \\ \text{not renegotiation proof} & \text{if } P_H > \frac{1}{2} \text{ and } C_2 < \frac{\Delta k}{\psi} < C_1, \\ \hat{q} & \text{if } \frac{\Delta k}{\psi} \geq \max\{C_1, C_2\}, \end{cases}$
$C_1 \equiv \frac{1+P_L-P_H}{P_L(1-P_H)}, \text{ and } C_2 \equiv \frac{1+P_L-P_H}{P_L P_H}.$	
5. $[(t_a, m_a), (t_b, m_b), (t_{cd}, m_{cd})]$	$\Pi_{IPM_5} = \begin{cases} \hat{q} - 2\psi + \frac{P_H-P_L}{2-P_H-P_L} \Delta k & \text{if } \frac{\Delta k}{\psi} < C_3, \\ \hat{q} - \psi - \frac{P_L(1-P_H)}{2-P_H-P_L} \Delta k & \text{if } C_3 \leq \frac{\Delta k}{\psi} < C_4, \\ \text{not renegotiation proof} & \text{otherwise,} \end{cases}$
$C_3 \equiv \frac{2-P_H-P_L}{P_H(1-P_L)} \text{ and } C_4 \equiv \frac{2-P_H-P_L}{P_L(1-P_L)}.$	
<i>IPM contracts with two distinct t/m pairs</i>	
6. $[(t_{ab}, m_{ab}), (t_{cd}, m_{cd})]$	$\Pi_{IPM_6} = \begin{cases} \hat{q} - \frac{P_L+P_H}{P_L} \psi + \frac{P_H-P_L}{2-P_H-P_L} \Delta k & \text{if } \frac{\Delta k}{\psi} < C_5, \\ \hat{q} & \text{otherwise,} \end{cases}$
$C_5 \equiv \frac{(P_H+P_L)(2-P_H-P_L)}{P_L(P_H-P_L)}.$	

Table 5: IPM contracts for which both agents exert effort

A.1 Proof of Proposition 1

We only consider contracts under which both types of agents exert effort.²⁷ Thus, all contracts ensure an expected output of

$$\hat{q} \equiv q_{Hl} + P_H (q_{Hh} - q_{Hl}) + q_{Ll} + P_L (q_{Lh} - q_{Ll}). \quad (12)$$

Using Lemmas 2 and 3 we can characterize all types of IPM contracts, and their respective profits are given in Table 5 (the derivation of this table is available from the authors).

The proof below consists in showing that one of the candidate contracts always dominates the other IPM contracts.

1. IPM₁ is strictly dominated by IPM₅.

If IPM₅ is not renegotiation proof, then IPM₁ is not either. This stems from the fact that $\frac{1}{P_L} < \frac{2-P_H-P_L}{P_L(1-P_L)} \equiv C_4$ since $\frac{2-P_H-P_L}{1-P_L} - 1 = \frac{1-P_H}{1-P_L} > 0$. Moreover, if

- $\frac{\Delta k}{\psi} < \min \left\{ C_3, \frac{1}{P_L} \right\}$, then $\Pi_5 - \Pi_1 = \frac{P_H-P_L}{2-P_H-P_L} \Delta k > 0$.
- $C_3 \leq \frac{\Delta k}{\psi} < \frac{1}{P_L}$, then $\Pi_5 - \Pi_1 = \psi - \frac{P_L(1-P_H)}{2-P_H-P_L} \Delta k > 0$ since $C_4 \equiv \frac{2-P_H-P_L}{P_L(1-P_L)} > \frac{1}{P_L} > \frac{\Delta k}{\psi}$.

2. IPM₂ is strictly dominated by IPM₅ or IPM₆.

1. Suppose that IPM₅ is renegotiation proof, i.e., $\frac{\Delta k}{\psi} < C_4$. Given that $C_4 - \frac{P_H-P_L}{P_H P_L} = \frac{P_H(1-P_H)+P_L(1-P_L)}{P_H P_L(1-P_L)} > 0$, the number of cases to consider are reduced. Then, if

- $\frac{\Delta k}{\psi} < \min \left\{ C_3, \frac{P_H-P_L}{P_H P_L} \right\}$, then $\Pi_5 - \Pi_2 = \frac{P_H-P_L}{P_L} \psi - P_H \Delta k + \frac{P_H-P_L}{2-P_H-P_L} \Delta k > \frac{P_H-P_L}{P_L} \psi - P_H \Delta k > 0$, since $\frac{\Delta k}{\psi} < \frac{P_H-P_L}{P_H P_L}$.
- $C_3 \leq \frac{\Delta k}{\psi} < \frac{P_H-P_L}{P_H P_L} < C_4$, then $\Pi_5 - \Pi_2 = \frac{P_H}{P_L} \psi - \frac{2P_H-2P_H P_L+P_L-P_H^2}{2-P_H-P_L} \Delta k$. Since $\psi > \frac{P_H P_L}{P_H-P_L} \Delta k$ this is greater than $\left[\frac{P_H^2}{P_H-P_L} - \frac{2P_H-2P_H P_L+P_L-P_H^2}{2-P_H-P_L} \right] \Delta k = \frac{P_L[P_H(1-P_L)+P_L(1-P_H)]}{(P_H-P_L)(2-P_H-P_L)} \Delta k > 0$.
- $\frac{P_H-P_L}{P_H P_L} \leq \frac{\Delta k}{\psi} < C_3$, then $\Pi_5 - \Pi_2 = -\frac{P_H-P_L}{P_H} \psi + P_L \Delta k + \frac{P_H-P_L}{2-P_H-P_L} \Delta k > -\frac{P_H-P_L}{P_H} \psi + P_L \Delta k \geq 0$, since $\frac{P_H-P_L}{P_H P_L} \leq \frac{\Delta k}{\psi}$.
- $\max \left\{ C_3, \frac{P_H-P_L}{P_H P_L} \right\} \leq \frac{\Delta k}{\psi} < C_4$, then $\Pi_5 - \Pi_2 = \frac{P_L}{P_H} \psi + P_L \Delta k - \frac{P_L(1-P_H)}{2-P_H-P_L} \Delta k = \frac{P_L}{P_H} \psi + \frac{P_L(1-P_L)}{2-P_H-P_L} \Delta k > 0$.

2. Suppose now that IPM₅ is not renegotiation proof, i.e., $\frac{\Delta k}{\psi} \geq C_4$, then if

- $C_4 \leq \frac{\Delta k}{\psi} < C_5$, then $\Pi_6 - \Pi_2 = -\frac{(P_H-P_L)(P_L+P_H)}{P_L P_H} \psi + \frac{P_H-P_L}{2-P_H-P_L} \Delta k + P_L \Delta k$. Since $\psi \leq \frac{1}{C_4} \Delta k = \frac{P_L(1-P_L)}{2-P_H-P_L} \Delta k$ this expression is greater or equal to $\left[-\frac{(1-P_L)(P_H-P_L)(P_L+P_H)}{P_H(2-P_H-P_L)} + \frac{P_H-P_L}{2-P_H-P_L} + P_L \right] \Delta k = \frac{P_L(1-P_L)(P_L+P_H)}{P_H(2-P_H-P_L)} > 0$.

²⁷See footnote 10 for a sufficient condition on the parameters.

- $C_5 \leq \frac{\Delta k}{\psi}$, then $\Pi_6 - \Pi_2 = \frac{P_H + P_L}{P_H} \psi + P_L \Delta k > 0$.

3. IPM₃ is strictly dominated by IPM₅ or by IPM₆.

1. Suppose IPM₅ is renegotiation proof, i.e., $\frac{\Delta k}{\psi} < C_4$, then if

- $\frac{\Delta k}{\psi} < C_3$, then $\Pi_5 - \Pi_3 = \frac{\psi}{P_H} + \Delta k + \frac{P_H - P_L}{2 - P_H - P_L} \Delta k > 0$.
- $C_3 \leq \frac{\Delta k}{\psi} < C_4$, then $\Pi_5 - \Pi_3 = \frac{1 + P_H}{P_H} \psi + \frac{(2 - P_H)(1 - P_L)}{2 - P_H - P_L} \Delta k > 0$.

2. Suppose now that IPM₅ is not renegotiation proof, i.e., $\frac{\Delta k}{\psi} \geq C_4$:

- $C_4 \leq \frac{\Delta k}{\psi} < C_5$. Then $\Pi_6 - \Pi_3 = \frac{P_L + P_H P_L - P_H^2}{P_H P_L} \psi + \frac{2(1 - P_L)}{2 - P_H - P_L} \Delta k > -\frac{P_H}{P_L} \psi + \frac{2(1 - P_L)}{2 - P_H - P_L} \Delta k$.
Since $\psi \leq \frac{\Delta k}{C_4} = \frac{P_L(1 - P_L)}{2 - P_H - P_L} \Delta k$ this expression is greater or equal to $\frac{(2 - P_H)(1 - P_L)}{2 - P_H - P_L} \Delta k > 0$.
- $C_5 \leq \frac{\Delta k}{\psi}$. Then $\Pi_6 - \Pi_3 = \frac{1 + 2P_H}{P_H} \psi + \Delta k > 0$.

Hence, only IPM₅, IPM₆, or IPM₄ can be optimal in the class of IPM contracts. Each of these contracts can be shown not to be always dominated by one of the other two candidate contracts:

1. Looking at the profits of IPM contracts in Table 5, it is obvious that IPM₄ is optimal whenever $\frac{\Delta k}{\psi} \geq \max\{C_1, C_2\}$ and that IPM₆ is optimal whenever $\frac{\Delta k}{\psi} \geq C_5$. For example, fix $P_L = 0.4$, then $\max\{C_1, C_2\} < C_5$ for $P_H < 0.76$ and IPM₄ is uniquely optimal among IPM contracts for values of $\frac{\Delta k}{\psi}$ between these thresholds.
2. Similarly, IPM₆ can be uniquely optimal since the above inequality is reversed for larger values of P_H .
3. Finally, IPM₅ is the best IPM contract for $P_H > \frac{1}{2}$ and $C_2 < \frac{\Delta k}{\psi} < \min\{C_1, C_4\}$:²⁸
 - IPM₅ strictly dominates IPM₆ for $\frac{\Delta k}{\psi} < C_4$. First, recall that $C_4 > C_3$ and note that $C_5 - C_4 = \frac{(2 - P_H - P_L)^2}{(P_H - P_L)(1 - P_L)} > 0$. Hence, there are only two cases to consider:
 - $\frac{\Delta k}{\psi} < C_4$, then $\Pi_{IPM_5} - \Pi_{IPM_6} = \frac{P_H - P_L}{P_L} \psi > 0$.
 - $C_3 \leq \frac{\Delta k}{\psi} < C_4$, then $\Pi_{IPM_5} - \Pi_{IPM_6} = \frac{P_H}{P_L} \psi - \frac{P_H(1 - P_L)}{2 - P_H - P_L} \Delta k > 0$, since $\frac{\Delta k}{\psi} < C_4 \equiv \frac{2 - P_H - P_L}{P_L(1 - P_L)}$.
 - IPM₄ is not renegotiation proof for $P_H > \frac{1}{2}$ and $C_2 < \frac{\Delta k}{\psi} < C_1$.

²⁸This interval is non-degenerate. One can easily see that for $P_H > \frac{1}{2}$ we have $C_1 > C_2$ and $C_4 - C_2 = [(2P_H - 1)(1 - P_L) - (P_H - P_L)(P_H + P_L - 1)]/[P_H P_L (1 - P_L)] > (2P_H - 1)(1 - P_H)/[P_H P_L (1 - P_L)] > 0$.

B Appendix: Relative Performance Measure Contracts

B.1 Proof of Proposition 3

The expected profit under a rank-order tournament (T) is given by

$$\Pi_T = \begin{cases} \hat{q} - \frac{\psi}{P_L P_H} + [1 - 2P_L(1 - P_H)] \Delta k & \text{if } P_H \leq \frac{1}{2} \text{ and } \frac{\Delta k}{\psi} < TC_1, \\ \hat{q} - \frac{\psi}{P_L(1 - P_H)} + [1 - 2P_L(1 - P_H)] \Delta k & \text{if } P_H > \frac{1}{2} \text{ and } \frac{\Delta k}{\psi} < TC_2, \\ \hat{q} & \text{if } \frac{\Delta k}{\psi} \geq \max\{TC_1, TC_2\}, \end{cases} \quad (13)$$

where $TC_1 \equiv \frac{1}{P_L P_H [1 - 2P_L(1 - P_H)]}$, and $TC_2 \equiv \frac{1}{P_L(1 - P_H) [1 - 2P_L(1 - P_H)]}$.

1. IMP_4 dominates T whenever it is renegotiation proof

First, consider $P_H \leq \frac{1}{2}$. Note that then $TC_1 = \max\{TC_1, TC_2\}$ and $C_2 = \max\{C_1, C_2\}$. Since

$$TC_1 - C_2 = \frac{P_H - P_L + 2P_L(1 - P_H)(1 + P_L - P_H)}{P_L P_H [1 - 2P_L(1 - P_H)]} > 0 \quad (14)$$

we have that $\Pi_T = \hat{q} \Rightarrow \Pi_{IPM_4} = \hat{q}$. Thus, what remains to be considered are the cases:

- $C_1 \leq \frac{\Delta k}{\psi} < C_2$. Then $\Pi_{IPM_4} - \Pi_T = \frac{1 - P_H(1 + 2P_L)}{P_L P_H} \psi + 2P_L(1 - P_H) \Delta k > 0$ since $1 - P_H(1 + 2P_L) > 1 - P_H(1 + 2P_H) \geq 0$ and $P_H \leq \frac{1}{2}$.
- $C_1 \leq \frac{\Delta k}{\psi} < C_2 (< TC_1)$. Then $\Pi_{IPM_4} - \Pi_T > 0$ if

$$\frac{\Delta k}{\psi} < C_2 \underbrace{\frac{1 - P_L P_H}{(1 - P_H) \{1 + P_L [1 - 2(1 + P_L - P_H)]\}}}_{>1}. \quad (15)$$

This condition is satisfied since the right-hand side (RHS) is larger than C_2 .

Now consider $P_H > \frac{1}{2}$. Then, $TC_2 - C_1 > 0$ and we again have that $\Pi_T = \hat{q} \Rightarrow \Pi_{IPM_4} = \hat{q}$.

Thus, what remains to be considered are the cases:

- $\frac{\Delta k}{\psi} \leq C_2 (< C_1 < TC_2)$. Then $\Pi_{IPM_4} - \Pi_T > 0$ if

$$\frac{\Delta k}{\psi} < C_1 \frac{1 - P_L(1 - P_H)}{1 - P_L(1 - P_H) - P_L[1 + 2(1 - P_H)(P_H - P_L)]}. \quad (16)$$

The denominator on the RHS is positive since

$$\begin{aligned} P_L(1 - P_H) - P_L[1 + 2(1 - P_H)(P_H - P_L)] &= 1 - P_L(2 - P_H) + 2(1 - P_H)(P_H - P_L) \\ &> 1 - P_L(2 - P_H) > (1 - P_H)^2 > 0, \end{aligned}$$

and it clearly is smaller than the numerator. Hence, the expression on the RHS of (16) is larger than C_1 and the condition is satisfied.

- $C_2 \leq \frac{\Delta k}{\psi} < C_1$, IPM_4 is not renegotiation proof.

2. IMP_6 dominates T whenever IPM_4 is not renegotiation proof

Two case need to be considered:

- $C_2 \leq C_5 \leq \frac{\Delta k}{\psi} < C_1$ ($< TC_2$). Then we have $\Pi_{\text{IPM}_6} = \hat{q} > \Pi_T$.
- $C_2 \leq \frac{\Delta k}{\psi} < C_1 < C_5$. Then, $\Pi_{\text{IPM}_6} - \Pi_T > 0$ if

$$\frac{\Delta k}{\psi} < \frac{(2 - P_H - P_L)[1 - (1 - P_H)(P_L + P_H)]}{2(1 - P_H)[1 - P_L(2 - P_H - P_L)]} \frac{1}{P_L(1 - P_H)}. \quad (17)$$

The expression on the RHS being greater than C_1 , the condition is satisfied. Indeed, we have that $\frac{1}{P_L(1 - P_H)} > C_1$ and the remaining fraction is larger than one since

$$\begin{aligned} & (2 - P_H - P_L)[1 - (1 - P_H)(P_L + P_H)] - 2(1 - P_H)[1 - P_L(2 - P_H - P_L)] \\ & = (P_H - P_L)[2P_H - 1 + P_L(1 - P_H) + P_H(1 - P_H)] > 0 \quad (\text{since } P_H > \frac{1}{2}). \end{aligned}$$

B.2 Proof of Lemma 1

Assume, that the contract promises the high-skilled agent a perfectly revealing t/m pair (t_{Hs}, m_{Hs}) in output state $s \in \{l, h\}$.²⁹ Suppose first that $t_{Hs} > 0$. Now construct an RPM contract by setting $t(q_{Hs}, q_{Lh}) = t_{Hs} + \eta$ and $t(q_{Hs}, q_{Ll}) = t_{Hs} - \alpha\eta$, while leaving all other transfers and messages as under the initial IPM contract. Obviously, the new transfers under the RPM are perfectly revealing, just as t_{Hs} was. Therefore, the expected perceived transfer that the agent receives in output state s is equal under both contracts if

$$P_L \eta - (1 - P_L) \alpha \eta = 0 \quad \Rightarrow \quad \alpha = \frac{P_L}{1 - P_L}.$$

For such an η , the expected perceived transfer for the agent and the expected profit of the principal are equal under both the RPM contract and the initial IPM contract.

If $t_{Hs} = 0$, the principal can easily construct an RPM contract by impacting messages rather than monetary transfers. Indeed let the principal offer $t(q_{Hs}, q_{Lh}) = t(q_{Hs}, q_{Ll}) = 0$ in conjunction with $m(q_{Hs}, q_{Lh}) \neq m(q_{Hs}, q_{Ll})$, which are distinct from other messages under the contract. Obviously, this keeps both t/m pairs perfectly revealing and the expected monetary cost remains unchanged. Finally, such RPM contracts inherit renegotiation proofness from the IPM contract for all parameter values.

²⁹The proof is analogous for the low-skilled agent.

B.3 Example: An RPM Contract that dominates IPM₅

As an illustration, we show that strict dominance can be obtained by an RPM contract relative to IPM₅ (a candidate best IPM contract with a t/m pair that is perfectly revealing).

Consider the following RPM contract:

RPM ₁		$\theta = \mathbf{H}$	
		q_{Hh}	q_{Hl}
$\theta = \mathbf{L}$	q_{Lh}	$[(t_2, m_2), (t_2, m_2)]$	$[(t_1, m_1), (0, m_3)]$
	q_{Ll}	$[(0, m_3), (t_2, m_2)]$	$[(0, m_3), (0, m_3)]$

Given that both agents exert effort under RPM₁, beliefs about the agents' types are:

$$E[k_\theta | t_1, m_1] = k_L, \quad (18)$$

$$E[k_\theta | t_2, m_2] = \frac{k_H + P_L k_L}{1 + P_L}, \quad (19)$$

$$E[k_\theta | 0, m_3] = \frac{(1 - P_H) k_H + (1 - P_L) k_L}{2 - P_H - P_L}. \quad (20)$$

The high-skilled agent's incentive and wealth constraints imply that³⁰

$$t_2 = \begin{cases} \frac{\psi}{P_H} - \frac{1 - 2P_L + P_H P_L}{(1 + P_L)(2 - P_H - P_L)} \Delta k & \text{if } \frac{\Delta k}{\psi} < C_6 \\ 0 & \text{otherwise,} \end{cases} \quad (21)$$

where $C_6 \equiv \frac{(1 + P_L)(2 - P_H - P_L)}{P_H(1 - 2P_L + P_H P_L)}$. In conjunction with the wealth constraint the low-skilled agent's incentive constraint determines t_1 :

$$t_1 = \begin{cases} \frac{1 - P_L}{P_L(1 - P_H)} \psi + \frac{1 - P_H}{2 - P_H - P_L} \Delta k & \text{if } \frac{\Delta k}{\psi} < C_6 \\ \frac{\psi}{P_L(1 - P_H)} + \frac{(1 - 3P_H + P_L + P_H^2)}{(1 - P_H)(2 - P_H - P_L)(1 + P_L)} \Delta k & \text{if } C_6 \leq \frac{\Delta k}{\psi} \text{ and } C_8 \geq 0 \\ & \text{or } C_6 \leq \frac{\Delta k}{\psi} < C_7 \text{ and } C_8 < 0 \\ 0 & \text{if } \frac{\Delta k}{\psi} \geq \max\{C_6, C_7\} \text{ and } C_8 < 0, \end{cases} \quad (22)$$

where $C_7 \equiv \frac{(2 - P_H - P_L)(1 + P_L)}{P_L(-1 + 3P_H - P_L - P_H^2)}$ and $C_8 \equiv 1 - 3P_H + P_L + P_H^2$. Recall that IPM₅ is renegotiation proof if and only if $\frac{\Delta k}{\psi} < \frac{2 - P_H - P_L}{P_L(1 - P_L)} \equiv C_4$. Therefore, we only have to consider this range of values.

- No renegotiation between t/m pairs (t_1, m_1) and (t_2, m_2)

We always have that $t_1 > t_2$ if $t_2 > 0$. Therefore, the conditions in Definition 2 require that $t_1 + k_L > t_2 + E[k_\theta | t_2, m_2]$. For $\frac{\Delta k}{\psi} < C_6$, we have $t_1 - t_2 + k_L - E[k_\theta | t_2, m_2] = \frac{P_H - P_L}{P_H P_L(1 - P_H)} \psi > 0$. If $\frac{\Delta k}{\psi} \geq C_6$ (i.e. $t_2 = 0$), we have $t_1 + E[k_\theta | t_1, m_1] - t_2 - E[k_\theta | t_2, m_2] \geq \frac{1}{P_L(1 - P_H)} \psi - \frac{1 - 2P_L + P_H P_L}{(1 - P_H)(1 + P_L)(2 - P_H - P_L)} \Delta k$. This expression is positive if $\frac{\Delta k}{\psi} < \frac{(2 - P_H - P_L)(1 + P_L)}{P_L(1 - 2P_L + P_H P_L)} \equiv C_9$, which is implied by the condition $\frac{\Delta k}{\psi} < C_4$ and $C_9 - C_4 = \frac{(2 - P_H - P_L)^2}{P_L(1 - 2P_L + P_H P_L)(1 - P_L)} > 0$.

³⁰Note that $1 - 2P_L + P_H P_L > 0$ for all possible values of P_H and P_L .

- No renegotiation between t/m pairs (t_1, m_1) and $(0, m_3)$

Definition 2 requires that $t_1 + E[k_\theta | t_1, m_1] > E[k_\theta | 0, m_3]$. For $\frac{\Delta k}{\psi} < C_6$, we have $t_1 + E[k_\theta | t_1, m_1] - E[k_\theta | 0, m_3] = \frac{1-P_L}{P_L(1-P_H)} \psi > 0$. For $\frac{\Delta k}{\psi} \geq C_6$, we have $t_1 + E[k_\theta | t_1, m_1] - E[k_\theta | 0, m_3] \geq \frac{1}{P_L(1-P_H)} \psi - \frac{P_H(1-2P_L+P_H P_L)}{(1-P_H)(1+P_L)(2-P_H-P_L)} \Delta k$. This expression is positive if $\frac{\Delta k}{\psi} < \frac{(2-P_H-P_L)(1+P_L)}{P_H P_L(1-2P_L+P_H P_L)} = \frac{1}{P_H} C_9$, which is implied by $\frac{\Delta k}{\psi} < C_4 < C_9$.

Hence, the expected profit for the range $\frac{\Delta k}{\psi} < C_4$ is:³¹

$$\Pi_{RPM_1} = \begin{cases} \hat{q} - 2\psi + \frac{P_H-P_L}{2-P_H-P_L} \Delta k & \text{if } \frac{\Delta k}{\psi} < \min\{C_4, C_6\} \\ \hat{q} - \psi - \frac{P_L(1-3P_H+P_L+P_H^2)}{(2-P_H-P_L)(1+P_L)} \Delta k & \text{if } C_6 \leq \frac{\Delta k}{\psi} < C_4. \end{cases} \quad (23)$$

Next, we compare the profits of RPM_2 and IPM_5 . Note that $C_4 > C_3$. Moreover, since $C_6 - C_3 = \frac{P_L(2-P_H-P_L)^2}{P_H(1-P_L)(1-2P_L+P_L P_H)} > 0$ only the following cases have to be considered:

- $\frac{\Delta k}{\psi} < C_3$, then $\Pi_{RPM_1} - \Pi_{IPM_5} = 0$.
- $C_3 \leq \frac{\Delta k}{\psi} < \min\{C_4, C_6\}$, then $\Pi_{RPM_1} - \Pi_{IPM_5} = -\psi + \frac{P_H(1-P_L)}{2-P_H-P_L} \Delta k \geq 0$, since $\frac{\Delta k}{\psi} \geq C_3$.
- $C_6 \leq \frac{\Delta k}{\psi} < C_4$, then $\Pi_{RPM_1} - \Pi_{IPM_5} = \frac{P_H P_L}{1+P_L} \Delta k > 0$.

B.4 Proof of Proposition 4

Proposition 1 lists IPM_4 , IPM_5 , and IPM_6 as best IPM contracts. By Lemma 1, there always exists a payoff-equivalent RPM contract to the two candidate contracts IPM_4 and IPM_5 . Thus, to prove the result it is sufficient to construct an RPM that always performs at least as well as IPM_6 and strictly dominates it for parameter values for which it is the best IPM contract.

RPM₂ dominates IPM₆.

RPM ₂		$\theta = \mathbf{H}$	
		q_{Hh}	q_{Hl}
$\theta = \mathbf{L}$	q_{Lh}	$[(t_1, m_1), (t_1, m_1)]$	$[(t_1, m_1), (0, m_2)]$
	q_{Ll}	$[(0, m_2), (t_1, m_1)]$	$[(0, m_3), (0, m_3)]$

Given that both agents exert effort under RPM_2 , beliefs about the agents' types are:

$$E[k_\theta | t_1, m_1] = \frac{P_H k_H + P_L k_L}{P_H + P_L}, \quad (24)$$

$$E[k_\theta | 0, m_2] = \frac{P_L(1-P_H)k_H + P_H(1-P_L)k_L}{P_H + P_L - 2P_H P_L}, \quad (25)$$

$$E[k_\theta | 0, m_3] = \frac{k_H + k_L}{2}. \quad (26)$$

³¹It can easily be shown that C_6 can be either larger or smaller than C_4 . Note that $t_2 > 0$ since $C_8 < 0 \Rightarrow C_7 > C_4 > \frac{\Delta k}{\psi}$.

The high-skilled agent's incentive constraint requires that

$$t_1 \geq \bar{t} \equiv \frac{\psi}{P_H} - \frac{(P_H - P_L)(P_H + P_L + P_L^2 - P_H P_L)}{2(P_H + P_L)(P_H + P_L - 2P_H P_L)} \Delta k. \quad (27)$$

Similarly, the low-skilled agent's incentive constraint requires that

$$t_1 \geq \underline{t} \equiv \frac{\psi}{P_L} - \frac{(P_H - P_L)(P_H + P_L + P_H^2 - P_H P_L)}{2(P_H + P_L)(P_H + P_L - 2P_H P_L)} \Delta k. \quad (28)$$

Note that $\bar{t} > \underline{t}$ if and only if $\frac{\Delta k}{\psi} > \frac{2(P_H + P_L - 2P_H P_L)}{P_H P_L (P_H - P_L)} \equiv C_{10}$.

Together, the incentive and wealth constraints imply that $t_1 \geq \max\{\underline{t}, \bar{t}, 0\}$. We have that $\underline{t} > 0$ if and only if $\frac{\Delta k}{\psi} < \frac{2(P_H + P_L)(P_H + P_L - 2P_H P_L)}{P_L(P_H - P_L)(P_H + P_L + P_H^2 - P_H P_L)} \equiv C_{11}$ and $\bar{t} > 0$ if and only if $\frac{\Delta k}{\psi} < \frac{2(P_H + P_L)(P_H + P_L - 2P_H P_L)}{P_H(P_H - P_L)(P_H + P_L + P_L^2 - P_H P_L)} \equiv C_{12}$. Conveniently, $C_{10} > C_{11} > C_{12}$, since $C_{10} - C_{11} = \frac{2(P_H + P_L)(P_H + P_L - 2P_H P_L)^2}{P_L P_H (P_H - P_L)(P_H + P_L + P_H^2 - P_H P_L)} > 0$ and $C_{11} - C_{12} = \frac{2(P_H + P_L)(P_H + P_L - 2P_H P_L)^2}{P_H P_L (P_H + P_L + P_L^2 - P_H P_L)(P_H + P_L + P_H^2 - P_H P_L)} > 0$. Hence,

$$t_1 = \begin{cases} \frac{\psi}{P_L} - \frac{(P_H - P_L)(P_H + P_L + P_H^2 - P_H P_L)}{2(P_H + P_L)(P_H + P_L - 2P_H P_L)} \Delta k & \text{if } \frac{\Delta k}{\psi} < C_{11}, \\ 0 & \text{otherwise.} \end{cases} \quad (29)$$

The contract is renegotiation proof if and only if for $\frac{\Delta k}{\psi} < C_{11}$ we have that $t_1 + E[k_\theta | t_1, m_1] > \max\{E[k_\theta | 0, m_2], E[k_\theta | 0, m_3]\}$. Note that $E[k_\theta | 0, m_3] - E[k_\theta | 0, m_2] = \frac{P_H - P_L}{2(P_H - P_L - 2P_L P_H)} \Delta k > 0$. Moreover, $t_1 + E[k_\theta | t_1, m_1] - E[k_\theta | 0, m_3] = \frac{\psi}{P_L} - \frac{P_H(P_H - P_L)}{2(P_H + P_L - 2P_H P_L)} \Delta k$. This expression is positive if $\frac{\Delta k}{\psi} < \frac{2(P_H + P_L - 2P_H P_L)}{P_H P_L (P_H - P_L)} \equiv C_{13}$. Hence, renegotiation proofness follows from $C_{13} - C_{11} = \frac{2(P_H + P_L - 2P_L P_H)^2}{P_H P_L (P_H - P_L)(P_H + P_L + P_H^2 - P_L P_H)} > 0$.

Using the above results, the expected profit is given by:

$$\Pi_{RPM_2} = \begin{cases} \hat{q} - \frac{P_L + P_H}{P_L} \psi + \frac{(P_H - P_L)(P_H + P_L + P_H^2 - P_H P_L)}{2(P_H + P_L - 2P_H P_L)} \Delta k & \text{if } \frac{\Delta k}{\psi} < C_{11}, \\ \hat{q} & \text{otherwise.} \end{cases} \quad (30)$$

Comparing this profit with that of IPM_6 a useful result is that

$$C_5 - C_{11} = \frac{(P_H + P_L)^2 (1 - P_H)}{P_L (P_H + P_L + P_H^2 - P_H P_L)} \geq 0. \quad (31)$$

This leaves the following cases to be considered:

- $\frac{\Delta k}{\psi} < C_{11}$, then $\Pi_{RPM_2} - \Pi_{IPM_6} = \frac{(P_H - P_L)^2 (P_H + P_L)(1 - P_H)}{2(P_H + P_L - 2P_H P_L)(2 - P_H - P_L)} \Delta k > 0$.
- $C_{11} \leq \frac{\Delta k}{\psi} < C_5$, then $\Pi_{RPM_2} - \Pi_{IPM_6} = \frac{P_L + P_H}{P_L} \psi - \frac{P_H - P_L}{2 - P_H - P_L} \Delta k > 0$, since $\frac{\Delta k}{\psi} < C_5$.
- $C_5 \leq \frac{\Delta k}{\psi}$, then $\Pi_{RPM_2} - \Pi_{IPM_6} = 0$.

It now remains to show that RPM_2 strictly dominates IPM_6 whenever the latter is the best IPM contract. First, notice that IPM_6 has zero implementation cost and therefore is an optimal contract

(i.e., cannot be beaten strictly by *any* other contract) if $\frac{\Delta k}{\psi} \geq C_5$. Similarly, RPM₂ is an optimal contract if $\frac{\Delta k}{\psi} \geq C_{11}$. Hence, we can potentially beat all IPM contracts in the range $[C_{11}, C_5)$. Now, recall that

1. IPM₅ is not renegotiation proof if $\frac{\Delta k}{\psi} \geq C_4$.
2. IPM₄ is not renegotiation proof if $C_2 < \frac{\Delta k}{\psi} < C_1$ (implying that $P_H > 1/2$).

Hence, IPM₆ is the best renegotiation-proof IPM but not (necessarily) an optimal contract if

$$\max\{C_2, C_4\} < \frac{\Delta k}{\psi} < \min\{C_1, C_5\}. \quad (32)$$

We can loosen this condition taking into account that $C_1 > C_2$ implies that $P_H > \frac{1}{2}$ and then $C_4 \geq C_2$ (see footnote 28). Therefore, RPM₂ is an optimal contract while IPM₆, which is the best IPM contract, does not attain the same profit if

$$\max\{C_4, C_{11}\} < \frac{\Delta k}{\psi} < \min\{C_1, C_5\}. \quad (33)$$

The interval in (33) is non-empty for a non-degenerate range of parameter values. For example, it can be shown that there exists a non-degenerate range of parameter values for which $C_1 > C_5$ (e.g., if $P_L \in [0, P_H)$ and $P_H \in [P_L, 0.75)$). Moreover,

- $C_5 > C_{11}$ (see equation (31)).
- $C_5 - C_4 = \frac{P_L(1-P_H) + P_L(1-P_L)}{(P_H-P_L)(1-P_L)} > 0$.

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