A thesis submitted for the degree of Doctor of Philosophy

Essays in Financial Economics

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Declaration of Authorship for Co-authored Work

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I confirm that the thesis I am presenting has been entirely my own work.

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I declare that the first chapter of my thesis is not co-authored and it is entirely my own work. I downloaded the balance sheets, income statements, and cash flow statements data from the Iran Banking Institute and CODAL platform, and have conducted all of the analyses independently.

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Shahabeddin Gharaati

Abstract

The first chapter indicates that the recent banking crisis in Iran is rooted in the real estate cycles. The first chapter uses the micro-data of banking in Iran to argue that the intervention of the government in the credit and housing market affected the loan-to-value ratio (LTV) on the housing market. The government extended the credit through the banking system. It forced banks to reduce the nominal lending rate in an economy with high inflation, which increased LTV. Since land, houses, and commercial buildings can be used as collateral to get loans, increasing LTV led to an increase in house prices and overinvestment in the housing sector. In contrast, when the government increased the nominal lending rate, LTV decreased and caused a decrease in house prices. When house prices decreased, borrowers defaulted on their loans, which increased banks' non-performing loans (NPL). This chapter uses a dynamic panel model to examine determinants of the NPLs in private and partly private banks in Iran. The results propose that the main reason for increased NPLs in banks is the recession in the housing and construction sector. However, the sanction also increases NPLs through credit cycles.

In the second chapter, I ask whether removing tax incentives to obtain a mortgage would improve welfare in the economy. I develop a two-period general equilibrium model with default and collateral constraints to examine the welfare effect of taxation. Tax incentives to purchase a house are common in many countries because they are supposed to relax credit constraints to help first-time buyers. However, the model shows that tax incentives can reduce total welfare. Taxes on purchasing houses shift the house's demand downwards because the income effect dominates the substitution effect in the first period. The government transfer in the second period offset the negative income effect of taxes for borrowers. As a result, lenders benefit from the decline in the house price and purchase more houses. The new equilibrium is Pareto-dominates the old equilibrium.

The third chapter shows in a model with collateral constraints that if the government taxes the issuance of some security in the first period and redistributes the tax revenue as a lump-sum transfer in the second period, social welfare increases. To illustrate the taxation effect, the third chapter provides a simple example when the durable good used as a collateral is traded on a spot

market in the second period. The example shows that when the tax increases, the borrower's demand for the durable good decreases. Furthermore, because substitution and income effect cancel each other out, the borrowers' consumption of durable and perishable goods does not change in the second period. However, since the price of the durable good declines, lenders purchase more of the durable good in the second period.

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Chapter 1

Housing Market Effects on the Evolution of Nonperforming Loans in the Banking System of Iran

Abstract

The current chapter indicates that the recent banking crisis in Iran is rooted in the real estate cycles. The paper argues that the intervention of the government in the credit and housing market affected the loan-to-value ratio (LTV) on the housing market. The government extended the credit through the banking system. It forced banks to reduce the nominal lending rate in an economy with high inflation, which increased LTV. Since land, houses, and commercial buildings can be used as collateral to get loans, increasing LTV led to an increase in house prices and overinvestment in the housing sector. In contrast, when the government increased the nominal lending rate, LTV decreased and caused a decrease in house prices. When house prices decreased, borrowers defaulted on their loans, which increased banks' non-performing loans (NPL). The chapter uses a dynamic panel model to examine determinants of the NPLs in private and partly private banks in Iran. The results propose that the main reason for increased NPLs in banks is the recession in the housing and construction sector. However, the sanction also increases NPLs through credit cycles. Finally, the chapter recommends using time-varying and calibrated loan-to-the-value (LTV), debt-to-income (DTI), and load-to-deposit (LTD) ratios to reduce the systemic risk induced by the real estate sector.

Keywords: Collateral equilibrium, Non-performing loans, Real estate cycle, Credit cycles, Macroeconomics determinants, Bank specific determinants

1.1 Introduction

Iran's banking system has faced challenges over the past couple of years that are a legacy of government intervention in the credit and housing market. Most banks have suffered from high nonperforming loans (NPLs), poor capitalization, and weak profitability. Increasing government arrears also affected NPLs and government debt to banks, and consequently led to liquidity pressures on banks. A shortage of liquidity resulted in competition between banks for sources of funding. State-owned, specialist and partly private banks depend mostly on central bank credit lines to comply with government credit directives. Private banks mainly relied on deposits and the interbank lending market. Consequently, overdraft from the central banks increased and led to high inflation between 2018 and 2019, and banks have been pushing interest rate up, and aggressively competed for more deposits.

This paper shows that the current banking crisis in Iran is rooted in the real estate cycle. In particular, cycle drove the loan-to-value ratio on the housing market. When the loan-to-value ratio (LTV) increased, the demand for houses and house prices rose. When the LTV decreased, then house prices plummet. An intervention of the government in the credit market is usually accompanied by extending credit through the banking system and forcing banks to reduce the nominal lending rate. In an economy with high inflation, decreasing the lending rate may lead to a negative real interest rate, which increases LTV.

In contrast, when an increase of deposit rates is accompanied by a cap on credit, which is a second direct instrument of monetary policy in Iran, inflation temporarily goes down. The increase in the nominal lending rate imposed by the central bank of Iran (CBI) and the decrease in inflation leads to a further rise of the real lending rate. Then the LTV of loans falls, and the demand and the price of collateral assets decrease. The cycle mentioned above appeared twice in the Iranian economy throughout 2005-2014 because of government projects such as extending credit to small and medium enterprises (SMEs) and the Mehr housing project. The crisis starts with bad news which affect asset prices. Then, the rise of disagreement among the lenders, followed by an increase of down payments, leads to further downward pressure on asset prices. When collateral values fall below the present value of the loans, borrowers begin to default and that increases non-performing loans (NPL) in banks. Investment of private banks in real estate and extending loans to their subsidiaries in the housing sector worsen the banking crisis.

In the second part, the paper uses a dynamic panel regression method to examine the determinants of nonperforming loans of private and partly private banks from 2010 to 2018 in Iran. The findings show that the primary reason for increased NPLs in banks is the recession in the housing and construction sector. However, fundamental macro variables and a lack of monitoring also increased NPLs.

The rest of the paper is organized as follows. Section 2 presents the mechanism of the real estate cycle in Iran, then presents background information and government policies and projects that led to the crisis. Section 3 provides an overview of the theoretical and empirical literature on the determinants of nonperforming loans and formulates the hypotheses relating bank-specific variables to NPLs. Section 4 describes the econometric methodology, while Section 5 presents the results of the empirical analysis. Section 6 suggests some macroprudential policy to help reducing the systemic risk that caused by the asset price and credit booms and bust cycle. Finally, Section 7 contains concluding remarks.

1.2 Distortive credit policies

The paper shows that the distortive credit policies that the government imposed on banks caused a cycle in the housing market. Furthermore, that cycle in the real estate market is a primary factor that influenced NPLs in Iranian banks. However, other factors, like macroeconomic conditions and mismanagement, also affected NPLs. This section explains distortive policy of the government and external shocks affected the housing market.

An understanding of current distress in Iranian banks requires knowledge of some facts about the capital market and role of the deposit rates in policymaking in Iran. First, since the capital market in Iran is very small, the banking system is the primary source of funding enterprises. Private sector financing through primary capital markets was less than 5% of GDP in 2014. However, advances and loans extended to different sectors were more than 30% of GDP in the same year. The primary source of these loans were household deposits which accounted for more than 40% of GDP.

Second, controlling banks' interest rates is the primary direct instrument of monetary policy of the central bank of Iran (CBI). Since CBI activity is under Usury-free banking law (Interest-free banking), there is no implementation of a conventional open market operation by selling and buying treasury bonds. The Money and Credit Council (MCC) determines the expected deposit and lending rates based on a partnership contract with a fixed return between banks and borrowers (depositors). Every year the MCC decides about bank deposit rates, banks lending rates, reserve requirements , and specifies the mechanisms for the use of funds and determines the ceiling of credit in each sector. Consequently, parliament and the government may intervene on deposits and lending rates. They may also oblige banks to allocate credits to a specific sector.

1.2.1 Real estate cycles

The collateral equilibrium model of Geanakoplos and Zame (2014) can explain the real estate cycles in Iran. One of the primary implications of the collateral equilibrium model is that when the loan-to-value ratio (LTV) goes up, asset prices go up, and when the LTV goes down, asset prices go down. For assets like houses, lands and commercial buildings, some buyers may pay more for the asset than the rest of the public. Buyers may think that house value always goes up, and therefore buying a house is a good investment. Buyers with a higher risk tolerance may believe the asset has liquidity, which means they can get their hand on more money through borrowing by putting up the asset as collateral. So, they may spend their money on the assets and drive asset prices up. If the buyer loses money, they will not be able to buy the asset, and the prices go down.

Increasing LTV in the Iranian economy is a consequence of a coincidence of rising the oil prices and a good economics outlook. For example, the economic outlook for Iran with 7% annual economic growth, 10% yearly inflation, and 30% industrial production growth. With a stable market and good economic conditions, the government, as the primary lender, decided to increase LTV by reducing lending rates to provide more cash for borrowers. The government extended loans to small and medium enterprises (SMEs) and the Mehr housing project, which will be explained in detail in the next section. The government extended loans to borrowers by the state-owned and specialized banks that are governed by the state. Banks allocated most of the loans through Islamic contracts called partnership contract. A partnership contract is a financial contract through which the bank provides funds for the borrower, who is an entrepreneur or owner of a business, and both sides agree on the interest rate of repayment. The borrower agrees to pay back the principal and interest of the loan through instalment payments, and the bank may require collateral from the borrower for insuring the debt. Enforcing a low lending rate through an executive by-law in high inflation increased LTV because the real interest rates were negative. The safe economic environment encouraged the government to increase LTV, which caused massive borrowing and led to and increase of the asset price.

Similarly, bad news decrease asset prices. As Fostel and Geanakoplos (2012a) show, declining asset prices increases volatility. Increasing volatility accompanies with more uncertainty and disagreement between the lenders and borrowers. Those banks that are not governed by the government stop lending and the others start overdraft from the CBI. Moreover, the reduction of oil price and sanctions were bad news that caused an increase in inflation. So, the CBI increased both deposit rates and lending rates to control inflation. Measures of the CBI increased real interest rates that led to a decline of the LTV. Bad news themselves reduced asset prices. But, when bad news are accompanied by a reduction of the LTV, asset prices decrease even more, and we may enter into a crisis stage.

The crisis started with bad news. The bad news caused asset prices to decline because of worse fundamentals. Here the most optimistic buyers lost, such as those private banks that invested in real estate or housing developers who are the most leveraged. They are forced to sell assets to meet their obligations. The forced sales reduced the asset prices further, and leveraged buyers lost more. When asset prices became less than the value of the loan, the borrower defaulted and houses were seized as collateral. When assets of banks deteriorated, banks were forced to sell the assets, and the price of the asset reduced even more, and the loss spiral continued to the point that some banks became insolvent.

When the financial situation of the intermediaries became worse, they tried to restrict their lending because of limited capital. For banks to exert sufficient efforts in monitoring, they must have a high stake in their business. If the net worth of the banks' stake declines, then moral hazard may occur, and banks may decrease their monitoring efforts that may force the market to fall back to direct credit extending without monitoring (Holmstrom and Tirole, 1997). The mechanism mentioned above explains how unlicensed financial institutions (UFI) arose and made the banking crisis even worse. Finally, CBI closed several bankrupt UFIs merged them with commercial banks, and transferred their balance sheets to licensed credit institutions. Additionally, CBI lent to those commercial banks, including paying depositors of merged UFIs, which increased the monetary base.

Another mechanism that can be seen was precautionary hoarding of liquidity among banks. Precautionary hoarding occurs when lenders are afraid that they might face with an interim shock, and that they will need liquidity for their projects. When the likelihood of an interim shocks increases, and it is hard to obtain outside liquidity, precautionary hoarding may increases. The recession in the real estate market, sanctions, and the Iranian embezzlement scandal 2012, which affected off-balance-sheet activities of some banks, are examples of such interim shocks. After those events, the bank's uncertainty about their funding needs increased and caused sharp spikes in interbank market interest rate between 2012 and 2013. Also, the interbank market interest rate was increasing until reach the hghiest point at 2015. (see Figure 1.1)

1.2.2 The small and medium enterprises project

Distortive credit policies of the government that were implemented through the 4th Five Year Development Plan between 2005 to 2010 (FYDP) in Iran had a crucial role in the challenges to the banking system. The main target of the 4th FYDP plan was to reduce the unemployment from 12.1% in 2006 to expected 8.4% in 2010. However, there was an agreement between members of parliament that reducing unemployment needs stable conditions in the economy, such as low inflation. Therefore, they decided to pass a law that fixed prices. The Islamic Consultative Assembly passed Article 3 as a part of the 4th FYDP of the Islamic Republic of Iran in 2004. Based on this law, the sales prices of oil and petroleum products, electricity, water, and services such as a telecommunication and post services are still the same as those in September 2004. Furthermore, any change in prices of goods and services need to have a valid

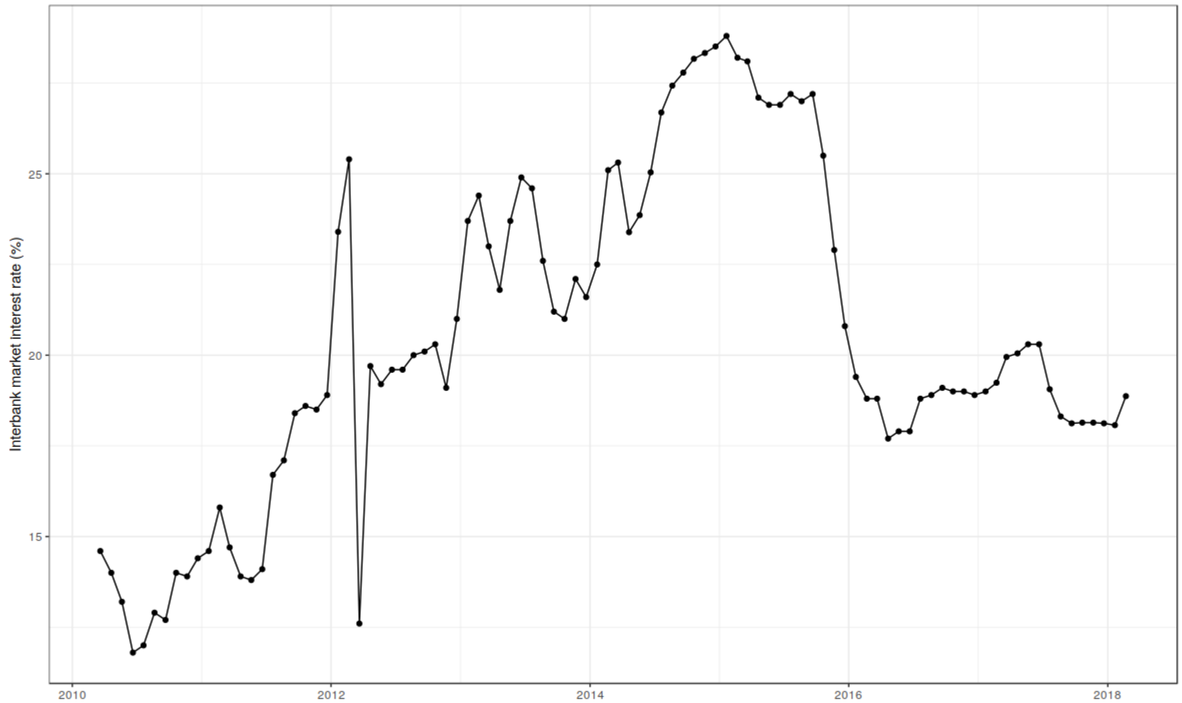

Figure 1.1: The graph shows the interbank market interest rate

economic reason and have to be proposed through a bill to parliament for approval.

Members of parliament argued that Article 3 can reduce inflation because the primary source of the increase in prices is the cost of oil products such as petrol and diesel. Members of parliament also believed that banks need to reduce deposit rates to induce customers to remove their deposits from banks. The policy aims to encourage entrepreneurs to start small enterprises and businesses to increase non-oil production and exports, strengthening economic activities and entrepreneurship, and creating job opportunities. As a result, they forced the central bank to reduce deposit rates. An important point here is that the Iranian banking system does not have a market-based interest rate. The primary direct instrument for monetary policy is controlling deposit and lending rates. The money and credit council (MMC) of the central bank of Iran (CBI) sets and enforces deposit and lending rates by executive instructions each year. However, the CBI must implement all the laws passed by the parliament, such as credit policies or budget laws.

One of Mahmoud Ahmadinejad's economic plans, after a victory in the presidential election

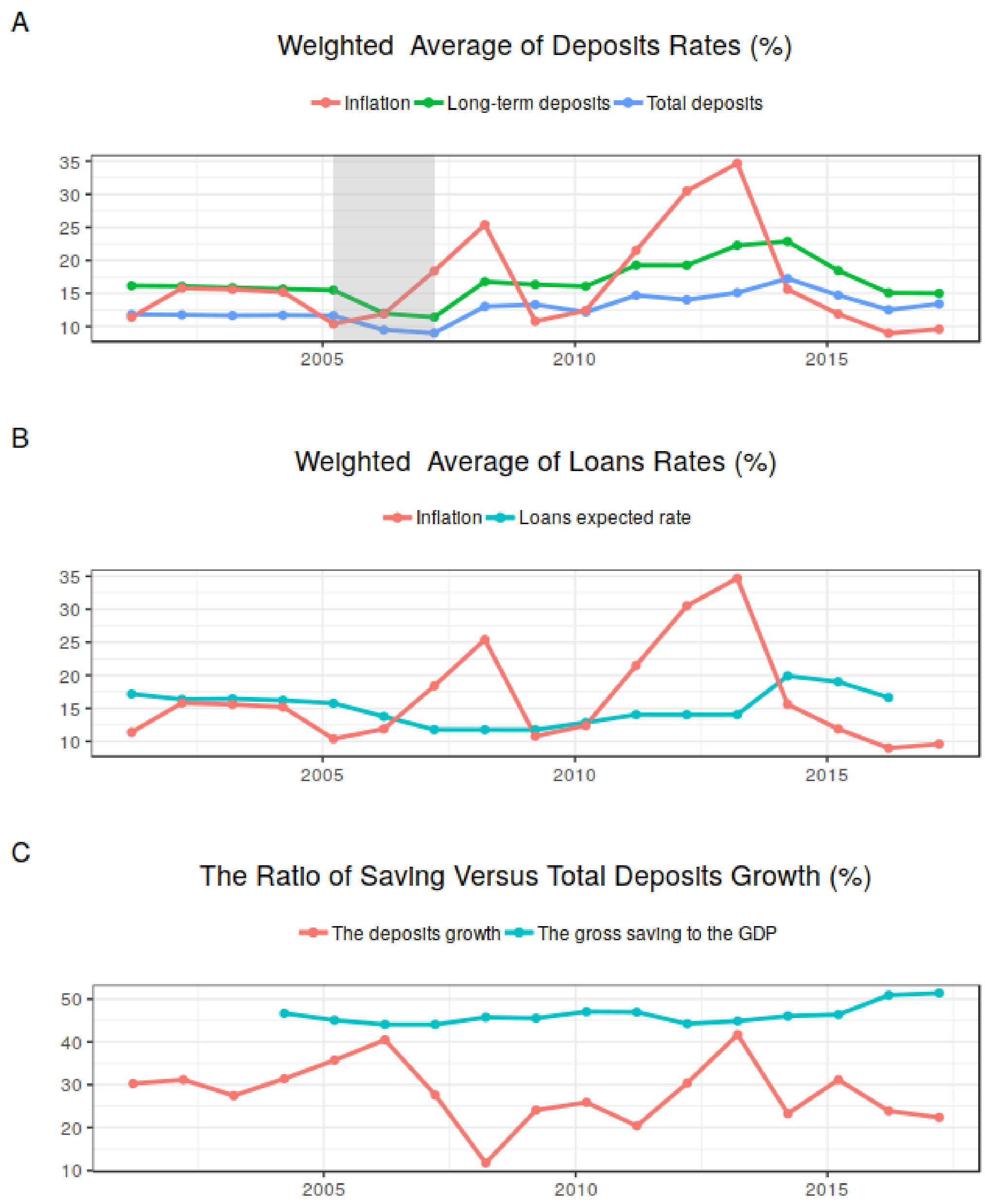

Figure 1.2: The graphs (A) and (B) show weighted average of deposit rates and lening rates respectively. The diagrams (C) illustrates the ratio of the gross saving to the GDP and the total deposit growth.

of Iran in June 2005, was to increase and to facilitate access to loans and advances for small and medium enterprises (SMEs). The SMEs project is one of the measures of the new government for creating job opportunities. The parliament approved an executive by-law for the expansion of small and medium enterprises (SMEs) in November 2005. Small enterprises are firms with less than ten employees and medium enterprises are firms with less than 50 employees that produce goods and services.

Another goal of this scheme was distributing credit among poor regions, to give them equal opportunities as compared to big cities. Most of the ministries and other governmental organizations had to provide regular reports about measures that they adopted to reduce the unemployment rate. At the same time, banks were required to facilitate access to loans for households. Banks extended loans to SMEs in the form of the Modarabah and the Civil-partnership contracts.

A Modarabah is an Islamic contract by which the capital owner entrusts capital to an agent who owns an enterprise or an idea and then returns to the investor the principal plus a previously agreed share of the profits. In a Modarabah, if the business is unsuccessful, then the investor bears all the losses of the project. A Civil-partnership or Mosharakah is another Islamic contract by which two or more agent combine either their capital or their workforce. They both have a share in profits and losses, and they have similar rights and liabilities.

The MCC reduced deposit rates and lending rates by an executive by-law to implement the policies of 4th FYDP in 2005. As expected, the fixed price law of FYDP did not reduce inflation, and after a year the inflation rate started to rise sharply. Consequently, deposit rates became less than the inflation rate and the incentive to hold deposits in banks decreased. That led to a fall of deposit growth between 2006 and 2008 (see Figure 1.2). So, banks lost household deposits as one of the primary sources for funding. Since the capital market was thin in Iran (less than 3% of the GDP), and the banking system was the primary source of capitalization of businesses (60% of the GDP), low-interest credit increased the demand for loans.

On the one hand, banks were exposed to pressure because growth of deposits declined. On the other hand, government leniency in extending credit and the negative real rate on loans increased the incentive for borrowing. A large number of loans were allocated based on personal connections without adequate monitoring. However, banks required collateral for loans to reduce the risks of default of the SMEs project. There were some debates between bankers and regulator about whether or not collateral tranching for different projects should be allowed. However, land, houses and commercial buildings were the most attractive collateral for banks because they assumed that real estate prices would rise. Moreover, borrowers were allowed to use one piece of collateral for different projects at the same times.

The structure of loan contracts, the continually low nominal lending rate, and the rise of inflation resulted in a negative real lending rate. The present value of loans exceeded the future discounted installments, and as mentioned in the section 1.1 the demand for collateral increased. Therefor, asset prices increased. Increasing house prices gave rise to more investment in housing and construction sector. The number of construction permits issued by the municipalities in Tehran and other big cities rose between 2005 and 2007 (see Figure 1.3). The former labor minister of Iran Ali Rabiei claimed in an interview in September 2013 that most of the credits that were extended for funding the SMEs project were spent in housing. The SME project could not fulfill the aim of increasing manufacturing and exports.

The central bank had to limit loans that are allocated to the SMEs in 2009, to prevent overdraft of banks from the central bank. The CBI explained that the stabilization measures, which the government had imposed to reign in inflation, and high banks credit arrears were the reasons for freezing SMEs' facilities. A temporary decline in inflation and contractionary credit policy of the government led to a decrease in house prices.

1.2.3 The Mehr housing project

Following the increase of house prices in 2008, Ahmadinejad's government introduced the Mehr housing project. The Mehr housing project aimed to provide 2 million cheap residential units for low-income households to mitigate the shortage of housing and to make the price of a home affordable for poor people. Under the plan, the government grants free land to real estate developer for the purpose of building residential units. Buyers are then allowed to lease the house on a 99-year lease contract. These facilities are adopted in Article 6, Budget Law for

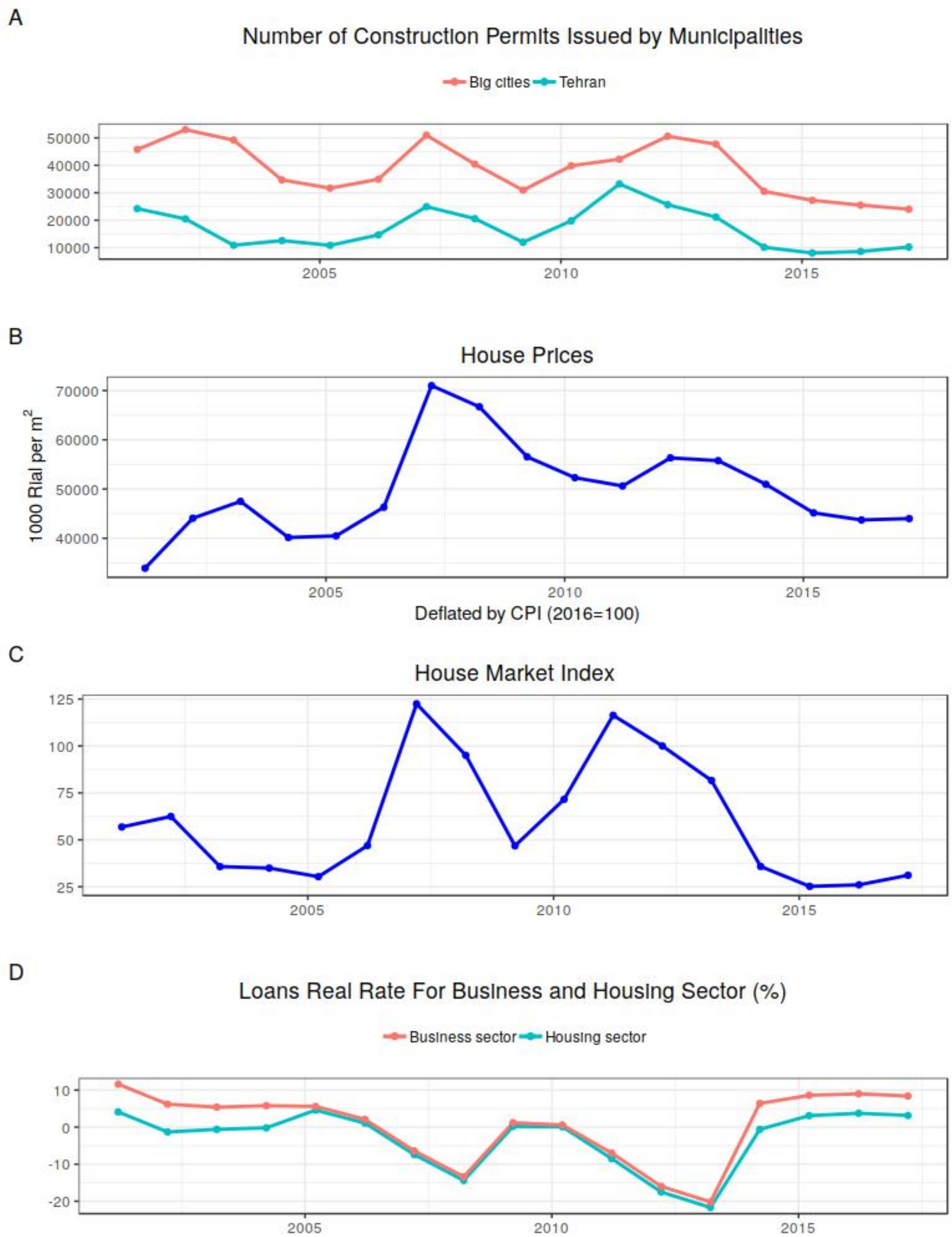

Figure 1.3: Graph (A) shows the number of construction permits issued by the municipalities in urban areas for Tehran and Big cities. Graph (B) shows the house prices per square meter. Graph (C) illustrates house market index, which is multiplication of the number of the construction permits and house price. Graph(D) shows the real lending rates in business sector and housing sector.

2007-2008, and the Law on Organization and Support for Production and Supply of Housing. During those years a significant increase of oil income accompanied the expansionary fiscal policy. The CBI proposed credit lines for the Mehr housing project every financial year, and the MCC approved those proposals. For example, a credit line facility of 35 trillion Rial was made available for the year 2009, since the CBI estimated that each house developer would receive up to 250 million Rial, and a maximum of 180 million Rial can be transferred to final purchasers under an instalment sale contracts.

The government forced some state-owned banks to allocate loans to the Mehr housing project, in particular the Mskan Bank, which is a specialized bank in the construction sector. Besides, under the Supervisory-Policy Package of the CBI for 2009-2010, other banks and financial institutions were banned from providing to their subsidiaries credit facilities for housing purchases, as this would encourage speculative behavior in the housing market. The government paid around 11 billion dollar advances through the banking system to finance this project up to March 2013. Figure 1.4 shows that loans and facilities allocated to the housing sector had more growth than all the other loans between 2008 and 2012. The Maskan bank allocated most of the advances to housing developers during the same period. Moreover, in the third year of the implementation of the Mehr housing project, the loans that were allocated to the project had a 130% growth. The demand for loans and advances increase the demand for houses and lands. As a result, the Mehr housing project increased the prices of houses and the number of construction permits issued by municipalities in urban areas.

The European Union joined international sanctions against Iran in 2012, which imposed restrictions on cooperation with Iran in financial services, the energy sector and technologies. The EU also agreed to an oil embargo against Iran, and to freeze the central bank assets. The sanctions were followed by decreasing oil prices, which meant that the expected income of the Iranian government fell. As a consequence, government arrears to banks increased. Private banks decided to invest in the housing sector directly or give loans and advances to their subsidiaries to invest in the housing sector. As Figure 1.4 shows, private banks and non-bank financial institution continued to increase loans to the housing sector after 2014.

After the victory of Hasan Rouhani in the 11th presidential election in 2013, the stabilization of the economy and a decrease of inflation became the primary goal of the new government. The policy package proposed for controlling inflation contained increases in both the lending rate and the deposit rate. With inflation reduced, the real rates rose, and the housing market fell into recession in 2013. The recession in house prices increased the number of defaulted borrowers. Consequently, banks seized collateral of defaulted borrowers, that decreased interest income of the banks, and made assets held by banks illiquid. Banks were forced to sell some of the assets and that decreased the price of the houses even more. A further fall of house prices was followed by defaults of more debtors, which led to an increase of NPLs. Figure 1.5 shows the NPLs of the banking system between 2001 and 2018. Overall NPLs in recent years decreased, because some of the unlicensed financial institutions (UFIs) are brought under CBI supervision and other problematic UFIs have been closed or merged with banks, and IFRS reporting standards have been implemented. Moreover, the annual budget laws in 2011, 2012 and 2013 forced banks to restructure bad loans.

1.2.4 Non-Performing Loans

There are four types of banks in the banking sector of Iran, state-owned commercial banks, state-owned specialized banks, private banks, and partly private banks. This paper is restricted to private and partly private banks, because the quarterly data for other types of banks is not available publicly. There are nineteen private banks and four partly private banks in the banking system of Iran. The term "partly private" refers to four former state-owned commercial banks that issued shares between 2008 and 2009 for the first time. The privatization of banks was a part of the implementation of the amendment of Article 44 of the constitutional law of the Islamic Republic of Iran. The amendment of Article 44 declares that state-owned companies will be privatized to decrease the government size and strengthen the private sector's role in the national economy. The private banks balanced panel contains Day, Eghtesad Novin, Parsian, Pasargad, Karafarin, Ansar, Sina, and Sarmaye, which accounts for 64 per cent of the total assets of all private banks on average. The partly private banks balanced panel includes Mellat, Saderat, and Tejarat, which accounts for 91 per cent of the total assets of all privatized banks

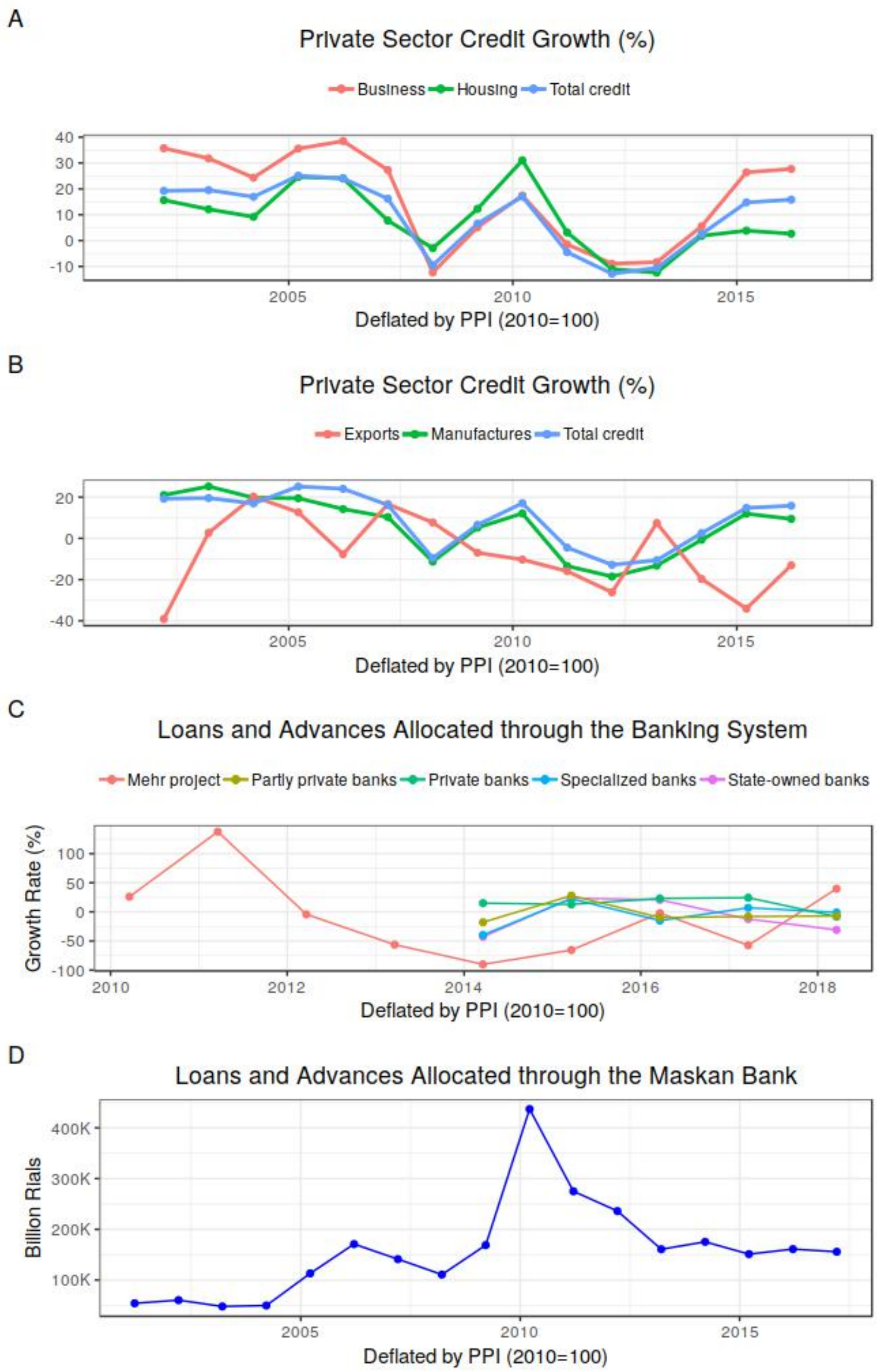

Figure 1.4: The graphs (A) and (B) show the growth of credits that allocated to different private sectors. The diagram (C) shows the growth loans and advances that are given to the Mehr housing project. Additionally, the graph (C) show the construction and housing credit sector growth in different type of banks.The graph (D) show loans that are allocated to the construction sector by the Maskan bank.

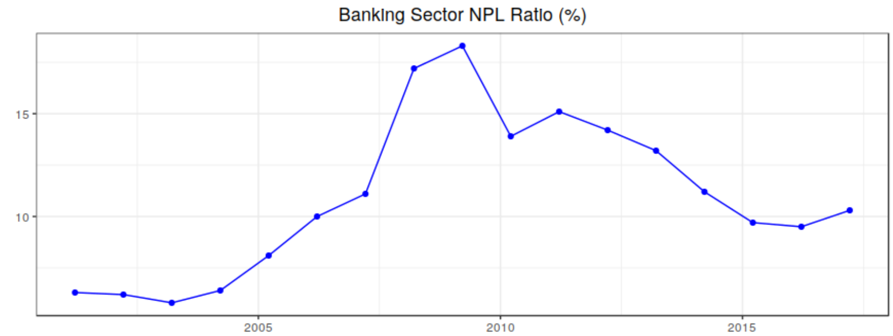

Figure 1.5: The graph shows the ratio of NPLs to total loans for the banking system.

on average. The elimination of other banks is inevitable, since some of them did not present their quarterly data, and some of the private banks emerged between 2012 and 2013. The new banks are excluded from the study, because the financial indicators for them are different from the others.

In 2006 the Money and Credit Council of the central bank of Iran passed a regulation and an administrative instruction for categorizing nonperforming loans. There was no single definition and guidance to classifying and reporting bad loans before that. Based on the regulation, the NPLs have five different categories: out due loans, deferred loans, doubtfull loans, uncollectible loans and bad loans. Banks only reported the first three categories on their balance sheets. According to the administrative instruction, there are three criteria for considering a loan as a bad loan. The first criterion is the delay in repayments of the loan. If the debtor does not pay installment between two and six months, then the debt is considered as an out due loan. In the case that a borrower does not pay his debt between six and eighteen months, the principle and interest of his debt are recognized as a deferred loan. The debt is doubtful if the borrower suspends repayment for more than eighteen months. The second criterion is financial indicators such as profitability, liquidity, and the possibility of default.

Figure 1.6 (A) and (B) depict the share of the different components of NPLs in the total loans. The private bank NPLs is 21% of all loans on average between 2011 and 2018, and 36% of the NPLs are doubtful loans on average. For partly private bank NPLs are 20% of all loans between 2011 and 2018, and 63% of the NPLs are doubtful loans. The NPLs decrease between 2012 and 2016. The executive instruction of the annual budget law passed in parliament every

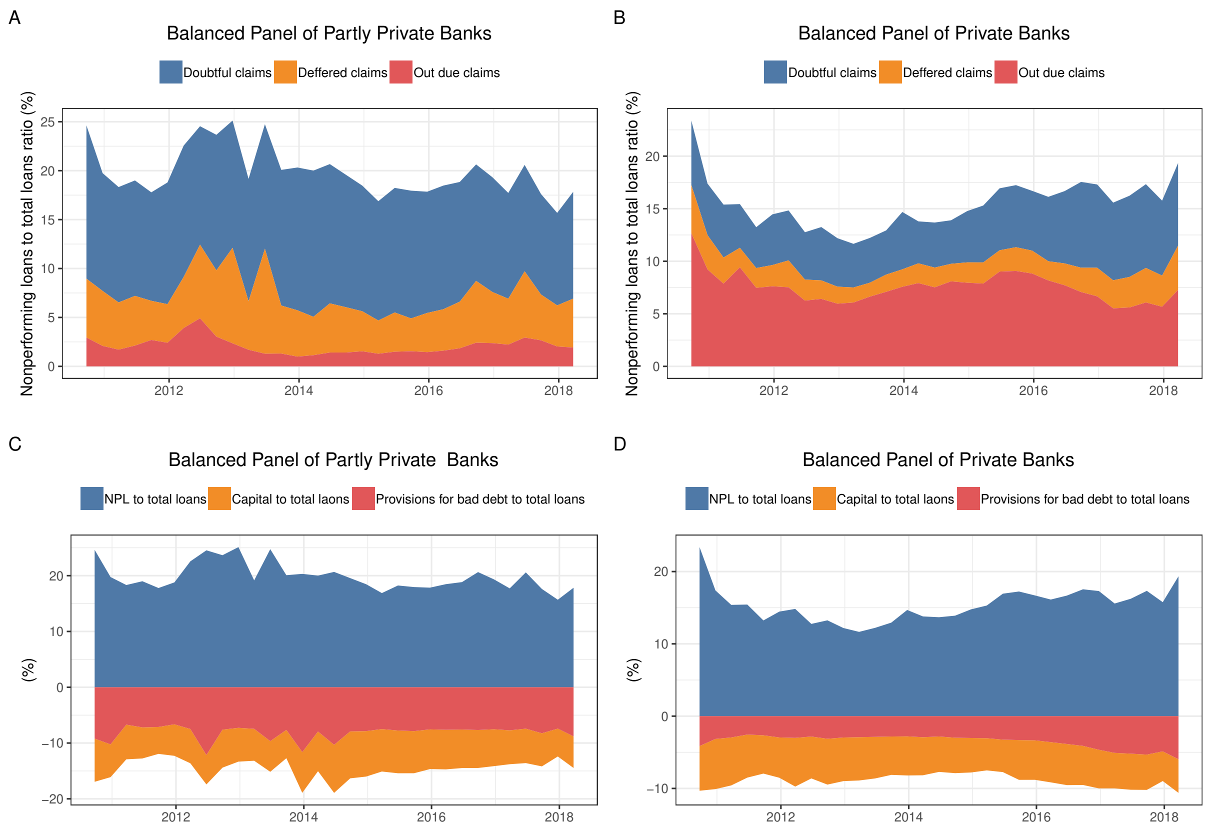

Figure 1.6: The graphs (A) and (B) show the share of different type of NPL to total loans for the banks. The diagrams (C) and (D) demonstrate the risk of the holding NPL inside of the balance sheet of the banks by showing the amount of provision and capital owned by the bank to cover the risk.

year may explain the decline of NPLs. The annual budget law in 2011, 2012 and 2013 forced banks to restructure bad loans. The budget law and the CBI did not give a clear procedure for restructuring of bad loans. The law gave the authority to the banks to restructure bad loans based on their economic conditions. It seems that most of the banks gave an extension to their customers for repaying the installments.

Figure 1.6 (C) and (D) show that the capital owned by banks and provisions for the bad loans do not cover nonperforming loans. The general provision for bad loans required by the regulator is 1.5% of total loans. The specific provisions for bad loans are between 10% and 100% depending on the type of debt. Usually, the specific provisions for out due loans are 10% of total out due loans, and for deferred loans they are 20% of total deferred loans. The specific provision for doubtful loans is not clear. The central bank instruction declares the provisions for doubtful loans should be between 50% to 100% of total doubtful loans. The banks should evaluate doubtful loans and make sure that they have sufficient provisions. The low amount of the provisions indicates a high risk of insolvency.

The red color in both diagrams of Figure 1.6 (C) and (D) depicts the total provisions for bad loans of private and partly private banks. There are two reasons for the low provisions of private banks in comparison with partly private banks. The Iranian National Tax Administration does not accept the provisions as cost of banking activities.

Figure 7 shows the collateral seized from bad loans for both private and partly private banks including houses, land, and commercial construction. Both graphs illustrate that the number of defaulted loans rapidly grew during the entire period.

1.3 Determinant factors of nonperforming loans

1.3.1 Fundamental macroeconomic factors

To consider the economic environment on NPLs, it is reasonable to hypothesize that the business cycle is connected with the quality of loans. When the economy expands, banks usually have relatively low NPL, so households and firms have a sufficient stream of income to settle their debt. However, the expansionary credit policy in Iran may have resulted in extending

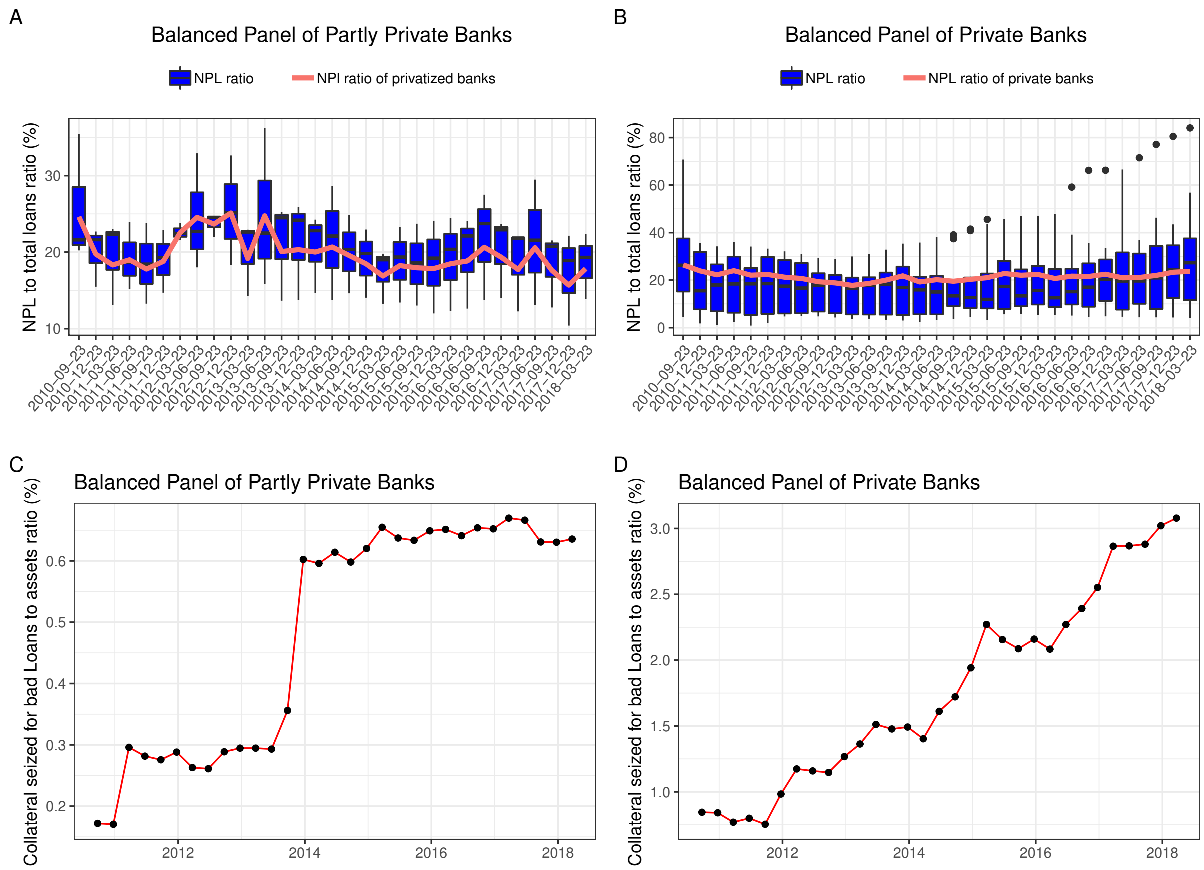

Figure 1.7: Graph (A) and (B) show Box plot of nonperforming loans to total loans ratio for both private and partly private banks. The red line shows the NPL ratio for aggregate balance sheets. Graph (C) and (D) shows collateral seized for bad loans as the primary sources of other assets growth.

loans to low-quality borrowers, since the government has full authority on the governance of state-owned, specialized banks. Troubled banks could borrow from the central banks. Additionally, the government holds 40% shares of partly private banks, which means that it also has a certain authority over partly private banks. Most bank managers in public banks are enlisted by the government, which means that they have to enforce the CBI executive by-laws and government policies. As mentioned in the previous section, when oil income increased, the government obliged the CBI and banks to facilitate extending credit to households, SMEs and the Mehr housing project. Consequently, credit expansion leads to an increase in the number of nonperforming loans in recession.

Several papers confirm the relationship between business cycles and NPLs. Quagliariello (2007) argues that the business cycle influences the NPLs of Italian banks between 1985 and 2002. Cifter et al. (2009) show that the lag of industrial production affects the number of NPLs in Turkey between 2001 and 2007. Salas and Saurina (2002) find that contemporaneous GDP growth has a negative effect on NPLs. Lawrence (1995) uses a life-cycle consumption model to show that debtors with low incomes have a higher risk of being unemployed. Because they may not be able to settle their debt, they have higher default rates. This paper use GDP growth and the unemployment rate as the main fundamental macroeconomics factor affecting NPLs.

1.3.2 Housing and construction sector

The paper aims to show that the main cause of increasing NPLs in private and partly private banks is the boom and recession of the housing and construction market over the period 2010-2018. Based on the aforementioned historical background, the government obliged state-owned banks to extend loans to the Mehr housing project. The price of housing and the number of construction permits issued by municipalities in urban areas increased. The expansionary credit policy of the government to support SMEs was another reason for increasing the prices in the construction sector. Banks accepted land, houses, and commercial construction as collaterals for loans. Consequently, increasing the demand for houses and land for borrowing led to a rise in prices.

Sanctions and the reduction of the oil price reduced the oil income of the government. Under conditions of sanction, the government may not pay its debt to the banking sector. Additionally, sanctions have a direct effect on trade and overseas activities. Businesses that have overseas activities may not be able to afford to pay back their debts. Finally, sanctions may increase unemployment, which has an indirect effect on NPLs. The poor state of the economy induced banks to extend loans to their subsidiaries to invest in housing and construction. The housing and construction market fell into recession in 2013, banks interest income decreased, and NPLs increased. After the fall in house prices in 2013, the number of defaults increased, which can be seen from the rise of collateral seized for bad loans for both types of banks (see Figure 1.7). The paper did not consider sovereign debt due to the fact that sovereign debt of Iran is less than 2% of the GDP. Therefore, the following hypotheses is posited:

Hypothesis 1: A decrease in house prices increases NPLs.

1.3.3 Bank specific factors

The factors that affect NPLs are not be restricted to macroeconomic variables. The policy choice of each bank to improve efficiency and to manage risk influences the evolution of NPLs. Therefore, a part of the literature on banking considers a casual relationship between bank-specific factors and NPLs.

Berger and DeYoung (1997) consider the causality among NPLs, capital of banks, and cost-efficiency. First, the paper explains the "bad management" hypothesis. Under the "bad management" hypothesis, "low measured cost efficiency is a signal of poor senior management practices, which apply to both day-to-day operations and to managing the loan portfolio. Sub par managers do not sufficiently monitor and control their operating expenses, which is reflected in low measured cost efficiency almost immediately. Managers in these banks also do not practice adequate loan underwriting, monitoring, and control. As bad managers, they may have poor skills in credit scoring and therefore choose a relatively high proportion of loans with low or negative net present values, be less than fully competent in appraising the value of collateral pledged against the loans, and have difficulty monitoring and controlling the borrowers after loans are issued to assure that covenants are obeyed."(Berger and DeYoung; 1997, p. 4)

The "bad management" hypothesis is not unrealistic for the case of Iran because of the following reasons. First, one of the measures that Ahmadinejad imposed, for what he called reducing corruption of the previous governments, was enlisting many inexperienced young people on management positions of different organizations. Second, before 2009, nobody needed permission from the central bank to enter the banking sector and register a bank holding group in the Administration of Registration of Enterprises. After 2009 the permission of those banks were not withdraw. Third, the police organization currently issue permissions for the establishment of Gharolhasaneh banks (non-profit Islamic banks), and many troubled unlicensed banks are currently Gharolhasaneh banks.

Under the "skimping" hypothesis, "the amount of resources allocated to underwriting and monitoring loans affects both loan quality and measured cost efficiency. Here, the critical decision of the bank lies in the trade off between short-term operating costs and future loan performance problems. A bank maximizing long-run profits may rationally choose to have lower costs in the short run by skimping on the resources devoted to underwriting and monitoring loans, but bear the consequences of greater loan performance problems and the possible costs of dealing with these problems in the future. The reduced effort devoted to screening loan customers, appraising collateral, and monitoring and controlling borrowers after loans are issued makes the bank appear to be cost efficient in the short run because fewer operating expenses can support the same quantity of loans and other outputs. The stock of nonperforming loans remains unaffected in the short run, but as time passes, a higher proportion of borrowers become delinquent on their loans and the inattention to the loan portfolio becomes apparent." (Berger and DeYoung; 1997, p. 5)

The "moral hazard" hypothesis is "the classical problem of excessive risk-taking when another party is bearing part of the risk and cannot easily charge for or prevent that risk-taking. Under this hypothesis, banks with relatively low capital respond to moral hazard incentives by increasing the riskiness of its loan portfolio, which results in higher nonperforming loans on average in the future." (Berger and DeYoung; 1997, p. 5) Berger and DeYoung (1997) use U.S commercial banks data between 1985 and 1994. They argue that bad management and the moral hazard caused increasing NPLs.

Diversification opportunities for banks may have an impact on the quality of the loan portfolio. In other words, an increase in diversification may decrease the number of NPLs, since the diversification reduces the risk of concentration on one sector. Two proxies for diversification for banks are bank size and non-interest income as a share of total income. Salas and Saurina (2002), Hu et al. (2004) and Rajan and Dhal (2003) find that bank size has a negative effect of NPLs, which means there is more opportunity for diversification for larger banks. The ratio of the non-interest income to total income shows whether the bank is diversified or not.

One of the policy concerns about the banking sector is too-big-to-fail. According to Stern and Feldman (2003), large banks may extend loans to lower quality customers by leveraging too much. In other words, creditors expect government protection when banking failure happens, since they do not expect to be subjected to market discipline. One proxy for testing TBTF is a leverage ratio. The TBTF hypothesis holds that large banks increase their leverage and take excessive risk, which results in more NPLs. Consequently, leverage should have a positive impact on NPLs conditional on size.

Variable	Defenition	Hypothesis tested
House price	$House_t$	Investment in housing (-)
Government debt	$Gdebt_{it}$	Government default (+)
Return on equity	$ROE_{it} = \frac{Profits_{it}}{\text{Total Equity}_{it}}$	"Procyclical credit policy" (+)
Solvency ratio	$SOLV_{it} = \frac{\text{Owned Capital}_{it}}{\text{Total Assets}_{it}}$	"Moral hazard" (-)
Inefficiency	$INEF_{it} = \frac{\text{Operating Expenses}_{it}}{\text{Operating Income}_{it}}$	"Bad Management"(+) "Skimping" (-)
Size	$SIZE_{it} = \frac{\text{Total Assets}_{it}}{\sum \text{Total Assets}_{it}}$	"Diversification" (-)
Non-interest income	$NII_{it} = \frac{\text{NonInterest Income}_{it}}{\text{Total Income}_{it}}$	"Diversification" (-)
Leverage ratio and size	$LEV_{it} = \frac{\text{Equity}_{it}}{\text{Total Assets}_{it}}, SIZE_{it}$	"Too-big-to-fail" (+)

Table 1.1: Definition of variables used to test the various hypotheses. All ratios are expressed in percentage points. The expected coefficient signs are shown in parenthesis.

The relationship between the lag of a performance measure of banks and NPLs is not clear. One can argue that low performance shows a lack of skills in lending activities. Hence, a negative relationship between past earning and increases in future NPLs shows bad management of banks. Rajan (1994) shows that short-term reputation of banks is an essential factor in determining credit policy. Therefore, some banks may inflate current earnings by extending loans to low-quality borrowers to prop up their current profitability. Consequently, the positive association of performance and increases of NPLs may indicate a liberal credit policy of banks. Table 1.1 exhibits the bank-specific variables used in the econometric model for robustness check.

1.4 Econometric methodology

One of the standard methods to investigate the determining factors of NPLs is to use a dynamic panel model (Louzis et al., 2012; Merkl and Stolz, 2006; Salas and Saurina, 2002). A dynamic panel model is useful because it accounts for persistence. It is given by:

$$y_{it} = \alpha y_{it-1} + \beta(L)X_{it} + \nu_i + \epsilon_{it}, |\alpha| < 1, i = 1, \ldots, N, t = 1, \ldots, T \tag{1.1}$$

Where the subscripts i and t indicate the cross-sectional dimension and time. Change in NPLs is denoted by y_{it}, $b(L)$ is an estimated lag operator polynomial, X_{it} denotes explanatory variables, ν_i are the unobserved individual bank-specific effects, and ϵ_{it} denotes the error term.

Arellano and Bond (1991) suggest using the Generalized Method of Moments (GMM) to estimate Eq.(1), and Arellano and Bover (1995) and Blundell and Bond (1998) generalize the method. The first step in using GMM is to eliminate the bank-specific effects by applying differences.

$$\Delta y_{it} = \alpha \Delta y_{it-1} + \beta(L)X_{it} + \Delta\epsilon_{it} \tag{1.2}$$

Here Δ is the first difference operator. In Eq. (2) correlation of lagged dependent variables, Δy_{it}, and the error term, $\Delta\epsilon_{it}$, leads to a biased estimator of the model. However, y_{it-2} does not have any correlation with $\Delta\epsilon_{it}$ for $t = 3, \ldots, T$, and can be used as an instrument to estimate Eq. (2). Assume that the errors ϵ_{it} are not serially correlated. Then, the following moment condition holds for a lag of two or higher of the dependent variable:

$$E[y_{it-s}\Delta\epsilon_{it}] = 0, for \quad t = 3, \ldots, T \qquad and \quad s \geq 2 \tag{1.3}$$

There is a potential endogeneity of the explanatory variables, so they may correlate with the error term. If the explanatory variable is strictly exogenous, then there is no correlation between all past and future values of the explanatory variable and the error term. In that case, the following conditions hold:

$$E[X_{it-s}\Delta\epsilon_{it}] = 0, for \quad t = 3, \ldots, T \qquad and \quad \forall s \tag{1.4}$$

As Louzis et al. (2012) assert when there is a reverse causality, the assumption of strict exogeneity is invalid i.e. $E[X_{is}\epsilon_{it}] \neq 0$ for $t < s$. Under the assumption of weakly exogenous explanatory variables current and lagged values of X_{it} are valid instruments. In that case, the following condition holds:

$$E[X_{it-s}\Delta\epsilon_{it}] = 0, for \quad t = 3, \ldots, T \qquad and \quad s \geq 2 \tag{1.5}$$

Based on Eqs. (3)-(5) the one-step GMM estimator can be used for estimating the above model. The one-step GMM estimates the parameters consistently based on homoscedasticity of both cross-sectional and time residuals (Louzis et al., 2012). Arellano and Bond (1991) introduced a two-step estimator that uses estimated residuals to create a consistent variance-covariance matrix of moment conditions (Louzis et al., 2012). However, using a two-step estimator may result in biased standard errors, and lead to unreliable asymptotic inference in data with a small number of cross-sections (Arellano and Bond, 1991; Blundell and Bond, 1998; Louzis et al., 2012).

The Sargan specification test examines the validity of the instruments. The null hypothesis heres valid moment conditions with asymptotic chi-square distribution (Arellano and Bond, 1991; Arellano and Bover, 1995; Blundell and Bond, 1998; Louzis et al., 2012). Moreover, the key assumption of the model is that the error terms, ϵ_{it}, are not serially correlated. The Second-order autocorrelation of $\Delta\epsilon_{it}$ is used to show that the GMM estimator is consistent (Louzis et al., 2012).

According to Eq. (1) the baseline model has the following from:

$$\Delta NPL_{it} = \alpha \Delta NPL_{it-1} + \beta_1 \Delta GDP_{it-1} + \beta_2 \Delta UNR_{it-1} + \beta_3 \Delta House_{it} + \beta_4 SANC_t + \nu_i + \epsilon_{it}, \quad (1.6)$$

$$|\alpha| < 1, i = 1, \ldots, N, t = 1, \ldots, T$$

In Eq. (6) ΔNPL is the first difference of the non-performing loans ratio to gross loans, ΔGDP is real GDP growth, ΔUNR is the change in the unemployment rate, $\Delta House$ is the change in real house prices, and SANC is a time dummy for international sanctions. The lag order of the variables are choosen by a "general to specific" procedure (Louzis et al., 2012).

Next, the bank-specific indicators are added to the baseline model of Eq. (6) in order to examine their explanatory power for robustness check. The limited number of cross sectional units may pose a limit on the number of instruments that can be used in the estimation (Louzis et al., 2012). Additionaly, the number of exogenous variables that can be added to Eq. (6) are limited (Louzis et al., 2012). Consequently, a restricted GMM procedure (Judson and Owen, 1999) is implemented here, i.e. only a limited number of lagged regressors are used as instruments. As Louzis et al. (2012), only one bank-specific variable is added at a time, total number of instruments does not exceed the number of cross sections. The Pearson correlation is used for controlling the moment conditions (3)-(5). For all baseline model, the existence of multicollinearity is checked. All the variables in all models have VIF value (Greene, 2003, p. 57) less than 2.

According to the Louzis et al. (2012) the hypotheses of Section 2 are tested (two-sided test) based on their coefficients, i.e.:

- $H_0 :\quad \beta_i > 0$ or < 0 depending on the hypothesis tested.
- $H_1 :\quad \beta_i = 0$

According to the Louzis et al. (2012) testing the TBTF hypothesis requires examining the effect of leverage conditional on size. The interaction term between the size and leverage helps to examine the marginal effect of leverage on NPLs conditional on banks' size. The null and alternative hypothesis are:

- $H_0: \quad \beta_{LEV} + \beta_{LEV \times SIZE} SIZE > 0$
- $H_1: \quad \beta_{LEV} + \beta_{LEV \times SIZE} SIZE = 0$

The macroeconomic factors and house prices are strictly exogenous in the model. However, strict exogeneity of bank-specific variables is a strong assumption. Banks may consider expected future levels of bad loans as their decisions. The model assumes banks specific-factors are weakly exogenous, which means bad loans may reversely cause bank-specific variables. But, there is no correlation between the future shocks in NPLs and the error terms. Eq. (4) and (5) introduce instruments to avoid the endogeneity above.

1.5 Emprical analysis

1.5.1 The data set

There are two different data set used for the analysis in this paper. The first data set is the quarterly data collected from the balance sheets, income statements, and cash flow statements of the banks reported on the online platform of the Securities and Exchange Organization of Iran (CODAL). The second data set consist of annual data from reports on the performance of the banking system of Iran published by the Iran Banking Institute (IBI), a nonprofit institute supported by the Central Bank of Iran. The data sets are balanced panels consisting of financial data from 2010 Q3 to 2018 Q1. All quarterly data are seasonally adjusted, and the Kalman filter is used for filling missing values.

Table 1.2 and 1.3 depict the descriptive statistics for the balanced panels of partly private banks and private banks. The differences of non-performing loans in private banks are more volatile than for partly private banks. Moreover, ROE, the share of non-interest income in total income, and the inefficiency ratio are very volatile in both private and partly private banks. The reason for the high volatility of these variables is that interest income gradually became negative, and in some cases, total income became lower than non-interest income. The negative

	Mean	st.dev	Min	Max	Skewness	Kurtosis	JB test	p-Values
ΔNPL	0.251	15.738	-52.833	57.956	-0.221	4.407	74.323	0
ΔGDP	0.423	8.129	-13.025	14.450	0.019	-1.210	1.350	0.509
ΔUNR	-0.671	8.314	-17.142	17.808	-0.041	-0.516	0.135	0.935
$\Delta House$	0.310	3.676	-6.383	7.619	0.164	-0.724	0.496	0.780
ΔGov	11.333	24.837	-53.919	154.278	2.412	13.251	736.022	0
ROE	-86.111	863.380	-7,879	510.895	-8.746	76.000	22,357	0
SIZE	33.333	6.798	24.278	47.887	0.421	-1.318	8.280	0.016
INEF	90.242	56.583	36.558	315.882	1.965	3.662	106.011	0
SOLV	3.556	1.088	2.041	6.829	1.037	0.807	18.426	0.00
NII	80.810	238.745	-133.938	1,920	6.267	42.402	7,112	0
LEV	5.246	2.295	-1.719	10.616	-0.274	0.667	3.094	0.213

Table 1.2: Descriptive statistics for the balanced panel of Partly private banks, which reports financial ratios and growth of macroeconomic variables in (%). JB denotes the Jarque–Bera normality test. The p-Value of JB test reports in last column.

	Mean	st.dev	Min	Max	Skewness	Kurtosis	JB test	p-Values
ΔNPL	4.770	22.350	-77.090	166.673	2.232	14.032	2,067	0
ΔGDP	0.423	8.129	-13.025	14.450	0.019	-1.210	1.350	0.509
ΔUNR	-0.671	8.314	-17.142	17.808	-0.041	-0.516	0.135	0.935
$\Delta House$	0.310	3.676	-6.383	7.619	0.164	-0.724	0.496	0.780
ROE	9.292	217.137	-3,170	227.745	-13.994	202.116	395,673	0
SIZE	12.500	8.345	0.772	33.872	0.828	-0.643	29.527	0.0
INEF	87.967	48.707	20.226	328.443	3.435	12.300	1,890	0
SOLV	5.512	4.185	2.173	33.073	3.488	16.591	3,084	0
NII	36.363	47.604	-17.838	431.843	5.522	40.648	16,874	0
LEV	7.186	10.433	-59.743	43.678	-2.218	14.207	2,111	0

Table 1.3: Descriptive statistics for the balanced panel of private banks, which reports financial ratios and growth of macroeconomic variables in (%). JB denotes the Jarque–Bera normality test. The p-Value of JB test reports in last column.

net-profit and retained earnings of some banks explain the volatility of ROE. The Jarque-Berra test shows that the normality assumption is not rejected for differenced macroeconomic variables and house prices.

1.5.2 Panel data estimations

Table 1.4 and Table 1.5 summarize the individual lag-one coefficient estimates for the balanced panel consisting of the eight private banks and the three partly private banks. A separate estimation for partly private banks is not possible due to the low cross-section dimension. Table 1.6 and Table 1.7 present the individual lag-one GMM coefficient estimates for private banks. All tables show the Sargan and the m_2 test results at the bottom of the tables. The model (1) of Table 1.4 and Table 1.6 are our baseline models with macroeconomic variables.

All models show that the coefficient of the lagged dependent variable is negative and statistically significant. Bad loans likely increased when they have declined in the last quarter. The result is not unexpected, as the decrease in NPLs was the result of the parliament law that forced banks to write-off or to postpone bad debts in 2011, 2012, and 2013. The write-off of debts was not transparent and based on personal connections. Thus, most banks decided to restructure out-due loans or deferred loans without clear criteria. In the presence of sanctions and the recession, most of the restructured loans did not pay back to banks and became a part of doubtful debts.

The findings from the baseline model of the mixed panel and the private banks panel show that rising unemployment significantly increases NPLs. Unemployment is a result of a reduction of workforce to reduce costs and to avoid bankruptcy. Unemployment is a primary factor that affects household incomes. Consequently, the decline in household income increases the default rate. The findings are in line with Lawrence (1995) and Rinaldi and Sanchis-Arellano (2006), who show that current income and the unemployment rate affect the probability of default. There is a difference between the mixed panel and the private banks panel regarding bank-specific factors. In most models of the mixed panel with bank-specific factors, the unemployment rate is statistically significant, but in the private banks panel with bank-specific factors the unemployment rate is insignificant in one of the models.

GDP does not have any effect on NPLs for all models for both the mixed panel and private banks panel. International sanctions has a no effect on bad loans in most of models. The sanctions variable is statistically significant in the base-line model of the mixed panel. As a trade barrier, sanctions have a negative effect on the net worth of companies, so that they have less borrowing power after the shock. Accordingly, a decrease of borrowing capacity in short term by companies may increase the probability of default in long-term. There are two reasons for contractionary credit policy during the sanctions. The first reason was, precautionary hoarding of banks which discussed in section 1.1. Second, the shock of sanction to the net-worth of companies decreased their borrowing power. So, companies couldn't obtain credit to invest in its activities. The firms and companies my reduce activities, which decreases output and increases unemployment. Therefore, sanctions have an indirect effect on the unemployment rate that increases the number of NPLs.

House prices are negatively associated with debt, and they have the strongest impact on NPLs in all models. There are three channels through which house prices affects the evolution of NPLs. First, loans that were given to small businesses increased the demand for land, houses, and commercial buildings that served as collateral. Second, credits that were allocated to the Mehr housing project increased the demand for loans and advances among real estate developers. Third, banks decided to invest in the construction and housing sector due to sanctions and the recession of 2012 in Iran. However, the housing and construction market fell into recession in 2013. As a result, banks interest income fell, and most of the banks did not consider lands, houses, and construction as valuable collateral. Thus, the share of the land, houses and constructions as part of banks' collateral decreased. Accordingly, the demand in the construction sector reduced, which led to a decline in house prices.

One of the results of the falling house prices was to increase the number of defaults, which can be seen from the rise of collateral seized for bad loans for both types of banks (see Figure 1.7). Geanakoplos (2009) and Geanakoplos and Zame (2014) argue that lower-quality borrower may not be able to repay their debt during a recession because of decreasing asset prices, which serve as the collateral.

The coefficient of the inefficiency index is negative and statistically significant for NPLs. This finding supports the skimping hypothesis, which means that extending loans without monitoring lead to an increase of NPLs. As mentioned above, most of the loans allocated to households were based on personal connections without proper monitoring of customers. Weak monitoring and the lack of a credit rating mechanism increased the probability of default.

The solvency ratio is positively associated with bad loans, and it is statistically significant. Therefore, banks' risk attitude does not explain the increase of NPLs, and the moral hazard hypothesis does not find support. One possible explanation for that is that instead of excluding bad loans, banks decided to provide resources through overdraft from the central bank.

The rejection of the diversification hypothesis depends on the proxy in the model. The size of banks is a proxy that weakly supports the diversification hypothesis in the mixed panels. The results show that the small size of some private banks does not let them diversify their investment. Results by Salas and Saurina (2002) suggest a negative relationship between bank size and NPLs. Hu et al. (2004) and Rajan and Dhal (2003) report the same result. The non-interest income ratio is another proxy for the diversification hypothesis. The results show that the coefficients of the non-interest income ratio is weakly significant in the private panel.

The findings on the TBTF effect are reported in Table 1.5 and Table 1.7, and Figure 1.8 illustrates the marginal effect of leverage conditional on the size of banks for both panels. In partly private banks of a size up to 15%, leverage has a positive and statistically significant effect on NPLs. Figure 1.8 shows the same result for private banks of a size up to 9%. In both regressions, the size of a bank is measured as the ratio of this bank's assets to total assets of all banks. According to Louzis et al. (2012) these findings show that leverage tends to increase bad loans, but increasing occurs only up to a certain size threshold. leverage does not affect bad loans after that threshold. Thus, there is no incentive for larger banks to leverage more. The results show that the performance of the banks in both panels does not have explanatory power for increasing NPLs. The second lag of the ROE indicator is statistically significant and positively associated with NPLs in the panel of private banks. However, the effect is small and negligible.

1.6 Macroprudential policies

The current section discusses implementing policies to dampen credit and asset price boom. Although Iran's economy needs major reforms in monetary and fiscal policy, macroprudential policies can be used as a complement to ensure financial stability. The macroprudential policy aims to reduce systemic risk, which has a high economic cost and strengthen the financial system's stability to shocks. (Borio and Drehmann, 2009) Besides the current banking crisis, there is another reason for implementing macroprudential policies. Since Iran is an oil-exporting country, policymakers face challenges in managing risks associated with changing conditions in global energy markets. During the period of high oil prices, government revenue increases. Increasing government revenue leads to credit expansion in the banking system. since the fixed-income market in Iran is small, and because of the undiversified nature of the Iran economy, real estate is used as an asset for investment and collateral in the banking system. Thus, the real estate boom and bust cycles raise systemic risk, and macroprudential policies play an important role in managing financial distress. This section briefly introduces macroprudential instruments to prevent credit expansion and asset price boom.

Institutional arrangement to mandate macroprudential policies differs across countries and depends on country-specific circumstances. However, there is general guidance for arranging adequate institutions to direct macroprudential policies. The central bank should have the authority to direct macroprudential policy, and the financial stability mandate should be back by law. However, the complex regulatory structure and overlap of the supervisory role of the different institutions may not effectively mitigate systemic risk. Therefore, different regulatory entities should enhance their cooperation. For example, some emerging market counties establish a financial stability committee to facilitate the coordination between various entities. (Lim et al., 2013)

Literature introduces a wide range of instruments for implementing prudential policies. The prudential instruments can be microprudential, which focuses on ensuring that individual financial institutions are resilient to economic shocks. The instruments can be macroprudential, taking a system-wide and time-varying approach toward the risk. Sometimes it is hard to distinguish between two approaches because some instruments have multiple purposes. The proper instruments of macroprudential policies are selected based on specific characteristics. Those characteristics should be able to limit the increase of systemic risk and offer limited arbitrage opportunities. In addition, those instruments should focus on the roots of the systemic risk. Furthermore, they have features that are the least distortionary to the economy. Finally, some of these macroprudential toolkits are designed to prevent increasing systemic risk to a dangerous level, and some instruments are designed to absorb shocks during financial stress. (Seal et al., 2013) There are four types of macroprudential policy tools: broad-based capital tools, sectoral capital and borrower-based tools, Liquidity-related tools and Structural tools. Using broad-based capital tools aims to address vulnerabilities associated with credit booms, such as dynamic provisioning requirements and leverage ratio caps. The purpose of using sectoral capital and borrower-based tools is to address vulnerabilities related to the specific sector and asset markets, such as caps on the LTV and debt-to-income ratios. Liquidity-related tools are used for managing liquidity and foreign-exchange risk associated with lending booms, such as reserve and liquidity coverage ratio (LCR) requirements. These are tools aimed at addressing vulnerabilities from interconnectedness and limiting contagion. Finally, structural tools aim to address interconnectedness and limit contagion effect vulnerabilities, like interbank exposure limits.(Lim et al., 2013)

The following paragraphs suggest some policies that dampen credit and asset price boom. First, the Iranian banking system should adopt the risk-based supervisory review process, such as the Pillar 2 framework. Enhancing the role of the Pillar 2 framework would help ensure robust NPL classification and provision practices. In addition, it would oblige banks to do certain practices such as credit evaluation and collateral valuations.

The central bank should mandate using macroprudential instruments such as calibrated and time-varying debt-to-income (DTI) and LTV ratios. These toolkits are not use in Iran. The central bank should impose a cap on monthly repayment as a share of the borrower's monthly salary. Many oil-exporting countries use this measure to put a ceiling on personal loans. For example, the cap on DTI for Saudi Arabia is 33 percent (some reference), and for Bahrain in 50 percent. (Prasad et al., 2014)

Since boom and bust episodes of real estate have significant adverse effects on the financial system, using LTV ratios on real estate lending has a crucial role in protecting the banking system. In particular, it is essential to set caps on LTV ratios and adjust them to market conditions for lending to the housing sector. In addition, it is essential to put caps on the LTV ratios for commercial properties. Because using commercial buildings as collateral for loans during credit booms may increase overinvestment in this segment. In addition, time-varying Loan-to-Deposit ratios (LTDs) can be used to help manage liquidity risk and procyclicality problems. For example, Setting lower ratios during a boom can slow down credit expansion.

Using LTVs and DTIs ratios is insufficient to mitigate the systemic risk in the Iranian banking system because of the undiversified nature of the economy. Iran's limited range of fixed-income assets make houses and commercial building a major asset class in the economy and banking system. Therefore, the exposure to the housing sector and borrowers who are real estate developers should be limited. Thus, the authority should impose a cap on real estate exposure. Some oil-exporting countries, such as Saudi Arabia and Kuwait, limit exposure to the real estate sector. (Prasad et al., 2014) In addition, the Central Bank of Iran needs to define real estate exposure appropriately to encompass all finance activities related to buying and constructing houses and commercial buildings. This policy may restrict banks from lending to specific sectors, such as SMEs, where housing and commercial constructions are used as collateral.

1.7 Concluding remarks

This paper uses a dynamic panel regression method to examine the determinants of nonperforming loans of private and partly private banks from 2010 to 2018 in Iran. The findings suggest that the primary reason for increased NPLs in banks is the recession in the housing and construction sector. The allocation of loans to the small businesses, to the Mehr housing project, and direct investment of banks in the housing and construction sector are three channels that affected the housing market cycle. Consequently, the recession in the housing market lead to more defaults in loan repayments, which increased NPLs. International sanctions also affected the credit cycle by making it difficult for borrowers to find credit for their businesses. Most borrowers cannot afford to pay their previous debt, which increases NPLs. The other macroeconomic variables do not have a substantial impact on NPLs in most of the models.

Moreover, models with bank-specific variables indicate that a weakness of monitoring and credit rating is one of the reasons for the increase of nonperforming loans. Most of the small private banks have diversification issues. Furthermore, evidence for the presence of a TBTF effect is observed for banks up to a certain size.

The results of the paper have various implications in terms of regulation and policy. First of all, banks need an asset quality review to be able to estimate the risks of low-quality assets. There is evidence that lack of monitoring and credit rating of banks serve as factors for future bad loans. The existing banking law needs to change, and the central bank's supervisory, disciplinary, and resolution powers should increase. The central bank should be able to restrict banks' access to central bank reserves. The central bank should be able to force distressed banks to have liquidity management and a recovery plan. The government arrears are now estimated to have reached 50% of GDP by end-2016/17. Thus securitization of government arrears and low-quality assets of banks may be a measure to reduce banking crisis. In addition, using macroprudential policies such as time-varying and calibrating on LTV and DTI ratios. Imposing caps on LTV and DTI ratios helps mitigate systemic risk. Using a time-varying LTD ratio to manage liquidity risk and to slow down the credit expansion during the boom. Finally, it is crucial to limit exposure to the real estate sector.

	Dependent variable:				
	ΔNPl_{it}				
	(1)	(2)	(3)	(4)	(5)
ΔNPl_{it-1}	-0.291^{***}	-0.282^{***}	-0.320^{***}	-0.310^{***}	-0.308^{***}
	(0.075)	(0.083)	(0.098)	(0.082)	(0.091)
ΔGDP_{it-1}	0.128	-0.257	-0.212	-0.477	-0.140
	(0.268)	(0.237)	(0.227)	(0.300)	(0.304)
ΔUNR_{it-1}	0.079^{***}	0.077^{***}	0.073^{***}	0.085^{***}	0.058^{*}
	(0.027)	(0.026)	(0.026)	(0.025)	(0.035)
$\Delta House_{it}$	-1.557^{***}	-1.521^{***}	-1.436^{***}	-1.695^{***}	-1.779^{***}
	(0.461)	(0.403)	(0.529)	(0.349)	(0.351)
$SANC_t$	0.199^{**}	0.165	0.140^{*}	0.099	0.129
	(0.099)	(0.111)	(0.081)	(0.113)	(0.097)
ΔGov_{it-1}		0.017			
		(0.090)			
$SIZE_t$			-8.627^{*}		
			(4.597)		
ROE_{it-1}				1.089^{*}	
				(0.581)	
ROE_{it-2}				1.896^{*}	
				(0.990)	
$INEF_{it-1}$					-0.209
					(0.147)
$INEF_{it-2}$					-0.141^{***}
					(0.042)
sargan test	71	73	73	71	71
	[1.00]	[1.00]	[1.00]	[1.00]	[1.00]
m_2	1.45	1.11	1.54	0.81	0.53
	[0.14]	[0.26]	[0.12]	[0.41]	[0.59]

Table 1.4: GMM estimation results for balanced panel of private and partly private Banks NPLs. Notes: *, **, *** denote significance at 10%, 5%, and 1% respectively. Standard errors are reported in parenthesis. The p-values for the Sargan and the m_2 test are reported in brackets.

	Dependent variable:			
	ΔNPl_{it}			
	(6)	(7)	(8)	(9)
ΔNPl_{it-1}	-0.299^{***}	-0.395^{***}	-0.342^{***}	-0.452^{***}
	(0.085)	(0.119)	(0.121)	(0.106)
ΔGDP_{it}	-0.435	-0.256	-0.347	-0.054
	(0.311)	(0.186)	(0.223)	(0.471)
ΔUNR_{it}	0.084^{***}	0.057^{***}	0.057^{***}	0.055^{***}
	(0.025)	(0.017)	(0.019)	(0.019)
$\Delta House_{it}$	-1.851^{***}	-1.501^{***}	-1.699^{***}	-1.761^{**}
	(0.504)	(0.430)	(0.334)	(0.747)
$SANC_t$	0.124	0.172	0.182^{*}	0.186
	(0.113)	(0.115)	(0.101)	(0.127)
NII_{it-1}	-0.051			
	(0.059)			
NII_{it-2}	-0.015			
	(0.034)			
$SOLV_{it-1}$		2.784^{***}		
		(0.690)		
$SOLV_{it-2}$		-0.696		
		(0.485)		
LEV_{it-1}			6.754^{***}	5.518^{***}
			(1.525)	(0.756)
$LEVit-1$			-4.420^{***}	
			(1.527)	
$SIZE_t$				1.510
				(4.747)
$LEV_{it-1} \times SIZE_{it}$				-27.809^{*}
				(15.425)
sargan test	71	71	71	45
	[1.00]	[1.00]	[1.00]	[1.00]
m_2	0.89	0.87	0.87	1.12
	[0.37]	[0.38]	[0.38]	[0.25]

Table 1.5: GMM estimation results for balanced panel of private and partly private Banks NPLs.
Notes: *, **, *** denote significance at 10%, 5%, and 1% respectively. Standard errors are reported in parenthesis. The p-values for the Sargan and the m_2 test are reported in brackets.

	Dependent variable:			
	ΔNPl_{it}			
	(1)	(2)	(3)	(4)
ΔNPl_{it-1}	-0.312^{**}	-0.309^{**}	-0.356^{***}	-0.327^{***}
	(0.130)	(0.125)	(0.118)	(0.113)
$\Delta GDPit-1$	-0.399	-0.322	-0.622^{*}	-0.403
	(0.327)	(0.268)	(0.343)	(0.248)
$\Delta UNRit-1$	0.054^{***}	0.050^{***}	0.049	0.056^{***}
	(0.019)	(0.016)	(0.031)	(0.020)
$\Delta House_{it}$	-1.993^{***}	-1.727^{***}	-2.288^{***}	-1.928^{***}
	(0.754)	(0.669)	(0.614)	(0.659)
$SANC_t$	0.034	0.014	0.040	0.032
	(0.090)	(0.089)	(0.089)	(0.091)
$SIZE_{it}$		-4.575		
		(6.935)		
ROE_{it-1}			5.227^{*}	
			(2.718)	
ROE_{it-2}			1.928	
			(2.050)	
$INEF_{it-1}$				0.047
				(0.179)
$INEF_{it-2}$				-0.244^{**}
				(0.101)
sargan test	24	24	22	22
	[0.99]	[0.99]	[0.99]	[0.99]
m_2	0.53	0.63	0.06	0.59
	[0.58]	[0.52]	[0.95]	[0.54]

Table 1.6: GMM estimation results for private Banks NPLs.
Notes: *, **, *** denote significance at 10%, 5%, and 1% respectively. Standard errors are reported in parenthesis. The p-values for the Sargan and the m_2 test are reported in brackets.

	Dependent variable:			
	ΔNPl_{it}			
	(5)	(6)	(7)	(8)
ΔNPl_{it-1}	-0.317^{**}	-0.523^{***}	-0.374^{**}	-0.423^{***}
	(0.145)	(0.195)	(0.158)	(0.139)
ΔGDP_{it-1}	-0.490	-0.625	-0.403	-0.291
	(0.374)	(0.454)	(0.362)	(0.554)
ΔUNR_{it-1}	0.066^{***}	0.102^{**}	0.039^{*}	0.063^{**}
	(0.022)	(0.041)	(0.022)	(0.025)
$\Delta House_{it}$	-2.061^{**}	-5.609^{*}	-2.374^{**}	-2.512^{***}
	(0.941)	(2.946)	(0.976)	(0.704)
$SANC_t$	0.031	0.390	0.040	
	(0.100)	(0.256)	(0.094)	
NII_{it-1}	-0.071			
	(0.083)			
NII_{it-2}	-0.369^{*}			
	(0.208)			
$SOLV_{it-1}$		5.131		
		(4.206)		
$SOLV_{it-2}$		-5.023		
		(7.114)		
LEV_{it-1}			8.405^{***}	4.705^{***}
			(1.265)	(0.848)
LEV_{it-2}			-4.958^{***}	
			(1.858)	
$SIZE_t$				0.749
				(5.704)
$LEV_{it-1} \times SIZE_{it}$				-19.046
				(16.691)
sargan test	22	17	22	46
	[0.99]	[0.96]	[0.99]	[1.00]
m_2	-1.58	-0.57	0.32	1.18
	[0.11]	[0.56]	[0.74]	[0.23]

Table 1.7: GMM estimation results for Private Banks NPLs.

Notes: *, **, *** denote significance at 10%, 5%, and 1% respectively. Standard errors are reported in parenthesis. The p-values for the Sargan and the m_2 test are reported in brackets.

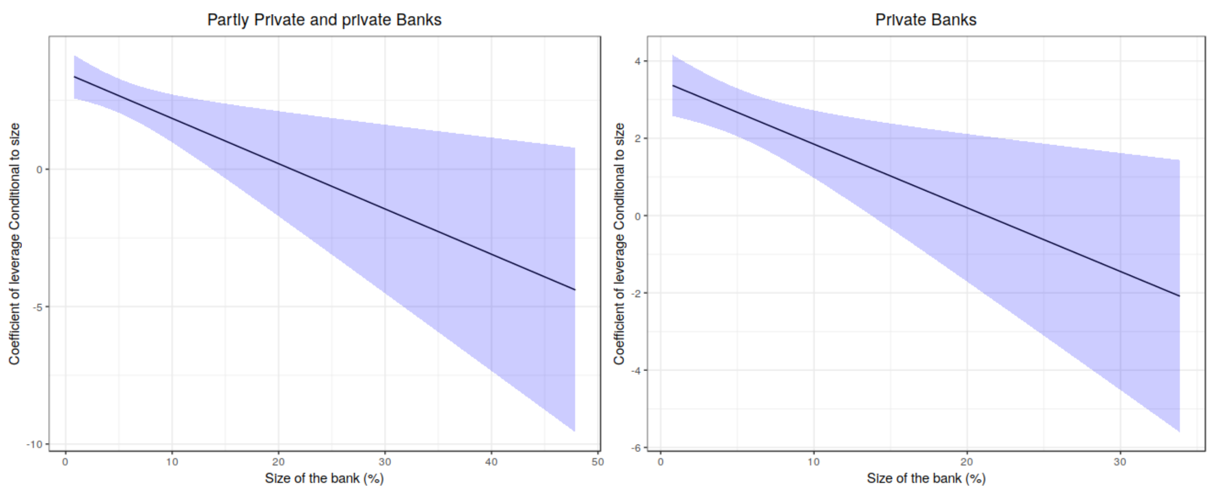

Figure 1.8: Marginal effect of leverage on banks NPLs. Mixed panels of private and partly private banks consider 11 banks. The panel of private banks includes eight banks.

Chapter 2

Welfare Improvement in Incomplete Market with Binding Collateral Constraints

Abstract

This chapter examines the welfare effect of taxation in the general equilibrium model, with default and collateral constraints. The chapter considers removing tax incentives to purchase a durable good in the first period and redistributing the tax revenue as a lump-sum government transfer in the second period. The income effect dominates the substitution effect in the first period. Thus, removing tax incentives limits agents' borrowing who sold promises in the first period to purchase a durable good. As a consequence, borrowers purchase less of the durable goods that serves as collateral. The lower demand for the durable good has a negative income effect, which is compensated by the government transfer in the second period. Hence, the expected utility of borrowers does not change. Lenders purchase more durable goods due to the substitution effect in the first period. Therefore, their consumption in the second period rises due to the income effect in this period. The new equilibrium, Pareto dominates the old equilibrium allocation. Prices of the promise and the durable good decrease because lenders' marginal rates of the substitutions decline.

Keywords: Collateral equilibrium, Incomplete market, General equilibrium theory

2.1 Introduction

The majority of debt in an economy is guaranteed by assets that are called collateral: residential mortgages are secured by the mortgaged properties, corporate bonds are secured by the plant and equipment of the firm, and collateralised debt obligations are secured by pools of the loans that are in turn secured by houses and other physical properties. In the classical finance model, agents always deliver on their promises, so collateral plays no role in those models. How both collateral and the interest rate are determined endogenously through equating supply and demand puzzled financial economists for a long time. Dubey et al. (2005) introduced default in terms of the utility penalty, and Geanakoplos and Zame (2014) introduced default and collateral into the two-period general equilibrium model with incomplete markets (GEI).

Dubey et al. (2005) shows that if the utility penalties from the default are large enough, then default does not occur. Geanakoplos and Zame (2014) incorporate default and collateral into the two-period GEI model. The most significant finding of Geanakoplos and Zame (2014) is that a collateral equilibrium (CE) always exists. As Hart (1975) showed, an equilibrium may not exist in the standard GEI model with real securities. In CE, the collateral constraints impose a bound on security sales and purchases, thus there is no discontinuity in demand, therefore equilibrium exists. Furthermore, Geanakoplos and Zame (2014) point out that if the collateralisable durable good is scarce, most securities are not traded in equilibrium, leading to the market's incompleteness. Another result of Geanakoplos and Zame (2014) is the *Constrained efficiency* theorem. The theorem states that if the relative price of the durable good to the perishable good does not change in period 1, any income redistribution that allows the market to clear in period 1 does not lead to a Pareto-improvement. Moreover, if a particular asset is banned from trading, the allocation does not Pareto dominate the old equilibrium allocation. Araújo et al. (2012) provides examples that show that regulations on a collateral requirement do not lead to a Pareto-improvement, and when equilibrium is constraint efficient, the regulation makes a majority of agents better off.

The current paper examines the welfare effect of taxation on the asset that sold in the first period and a lump-sum transfer of tax revenue in the second period. The paper shows that taxation leads to the Pareto-improvement, in a two-period GEI model with uncertainty in the second period. There is a single consumption good in the first period and in each state of the second period, with a durable good that serves as collateral, but it is not consumed. Assets are promises that agents make before the states of the economy is realized. The enforcement mechanism for payment of promises is that borrowers hold durable goods as collateral. The receiver of the promises makes a payment in the first period in return. Thus, assets in this model are characterised by collateral requirement (one unit of the durable good per unit of the sold asset) and promised payoffs in the second period. The realized payoffs of an asset is the minimum of the promise and the value of the associated collateral. Taxation of sold asset reduces the borrowing of agents who sold the asset.

The intuition for the welfare enhancing effect of taxation is as follows. The income effect in the first period dominates the substitutions effect, which shift demand downwards. The government transfer in the second period compensates negative income effect of having fewer durable goods, therefore the expected utility of borrowers does not change. The lender benefits from the decline of the durable good's price, so he purchases more durable goods and becomes better off.

The results imply that tax incentives to take a mortgage for purchasing a house can amplify the housing bubble. For example, Spain and Ireland are two countries that encountered a house price bubble between 2001 and 2007. One of the reasons for the bubble was mortgage interest tax deduction for owners who occupied the houses. Furthermore, there were tax breaks for primary residences who sold their houses. (Duca et al., 2021; Van den Noord, 2005; Muellbauer, 2005) The paper suggests that the removal of tax incentives decreases the houses' price and makes non-borrowers who purchase a house without using a mortgage better off.

The rest of the paper is organized as follows. Section 2 specifies the model and states all assumption. In section 3, the theorem about the effect of taxation on the welfare of agents is stated and proven. Section 4 concludes.

2.1.1 Relations to the literature

The first model of *collateral equilibrium* was introduced by Geanakoplos (1996), and the model was presented in a more general form by Geanakoplos and Zame (2014). Geanakoplos (1996) shows that introducing default and collateral to the general equilibrium model determines endogenously which securities are traded. A borrower needs to hold the durable good that serves as collateral to take a short-positions on the security. The inconvenience of holding collateral and a scarcity of the collateral good in the economy may hinder many securities to be traded actively. Araujo et al. (2002) used a collateral model to show the impossibility of a Ponzi schemes in an infinite horizon model, which implies that the model does not need the transversality conditions usually imposed in standard models. Araujo et al. (2005) allowed borrowers to choose their collateral bundles. They use a continuum of agents to overcome a non-convexity in the budget set.. Steinert and Torres-Martínez (2007) extend the model to incorporate security pools and tranching in the general equilibrium model.

The first model of the *leverage cycle* was introduced by Geanakoplos (2010). Geanakoplos (2010) used heterogeneity of the agents, collateral and default to explain the asset price fluctuations. The heterogeneity of agents is based on agents' beliefs about future asset prices. Optimistic agents borrow to buy the durable asset because they believe that asset price will rise in the future, and pessimist agents lend to optimists and accept the asset as collateral. There is an agent who is indifferent between borrowing and lending, the *marginal buyer*, whose belief determines the asset price. When leverage rises, a more optimistic agent becomes the marginal buyer, and the asset price rises. However, bad news can increase uncertainty about the future of the economy. The lenders now increase the loan's margin, and consequently a new marginal buyer is a more pessimist agent. So, deleveraging leads to a reduction of the asset price. Fostel and Geanakoplos (2008) introduce the concepts of *collateral value*, *liquidity wedge* and *liquidity value* in the leverage cycle and the collateral equilibrium models. Fostel and Geanakoplos (2012b) use the same framework to show that financial innovation such as tranches can cause bubbles, while Credit Default Swaps (CDS) cause crashes of the asset price.

Despite increased research into the leverage cycle and collateral equilibrium, few papers focus on how government intervention makes everybody better off. Geanakoplos and Zame (2014) argue that if agents have identical homothetic utility functions, regulation on a collateral requirement cannot lead to the Pareto-improvement. Araújo et al. (2012) highlight that when a large set of assets is available in the economy, the abundance of collateral goods results in the complete market and a Pareto efficient allocation. However, when collateral is scarce, few assets are traded, and the market is incomplete. They provide several examples which show that regulation on a collateral requirement cannot lead to a Pareto-improvement. Although, the majority of agents becomes better off. Gottardi and Kubler (2015) show that in a complete-markets economy, if the collateral is scarce, not only is the allocation not Pareto optimal, but it is also constraint inefficient. They show that restricting borrowing can make everybody better off. Kubler and Geanakoplos (2014) provides an example to show that in an incomplete-market economy, endogenous leverage is sub-optimal because of too much default, and limiting the borrowing can change future spot prices, which leads to Pareto-improvement. Araújo et al. (2015) consider the effects of asset purchases of risky assets in a model with collateral constraints. They argue that when collateral is sufficiently abundant so that for no agent the collateral constraints bind, central bank asset purchases does not have any welfare effect. However, when the collateral constraint binds, central bank asset purchases can be both Pareto-improving or sub-optimal depending on how collateral constraint binds.

There is an extensive literature on credit constraints in macroeconomics. The most relevant works are the papers by Kiyotaki and Moore (1997), Bernanke et al. (1996), and Caballero and Krishnamurthy (2001). These papers point out that if the loan-to-value (LTV) is fixed, then borrowing rises proportionally with the prices of collateralisable assets. Conversely, if bad news affect asset prices, then borrowing goes down proportionally. In all of those models, leverage is fixed. Hence, they produce *credit cycles* and leverage is counter-cyclical. Although a credit cycle is a feedback between borrowing and asset prices, a leverage cycle is a feedback between leverage and asset prices. leverage cycle produce credit cycles, but the converse is not true.

2.2 Model

The model has two periods. There is uncertainty about period 1. The S different states model the uncertainty. The individual decides about his consumption and the number of assets that he trades in period 0. In period 1, the dividend of assets and government transfer are paid to individuals to consume. There are two types of assets in the model: promises sold or purchased in the period 0 that pay in the next period, and a durable good (housing). Agents trade across states by making promises and receivers of promises provide a payment in the period 0 in return. A payment that is provided by receivers of a promise is price of the promise. The paper distinguishes between promises and durable goods. Thus, in the rest of the paper, assets refer to promises.

There are differences between the consumption and durable good. The durable good can be regarded as a home production technology that preserves the endowment that the agent holds in period 0 and transfers it to the next period. Thus, in period 1, the durable-good endowment appears in the budget set, and in each state, houses pay a dividend. The model has a collateral constraint. The collateral constraint says that an agent needs to pledge collateral to sell one unit of an asset. Here, the durable good can be used as collateral. One way of differentiating assets from each other is to charge different collateral requirement for selling assets, but these assets need to pay the same payoff. Another way to differentiate the assets is to set the collateral requirements to one unit of the durable good, but payoffs are different. This paper uses the latter approach.

There is one consumption good for each period. In period 0, the agent decides how much he consumes in both periods. Agents trade a portfolio of assets to smooth consumption. Thus, the budget set in period 0 contains the consumption, the durable good, and the portfolio of assets that the agent trades. In the period 1, the durable good and assets dividends are paid in consumption good units. An agent may default in payment of the asset payoff in some states. In that case, the lender seizes the collateral if the asset payoff is more than the dividend of durable goods in those states.

Consumption in the first period is none-negative. Thus, there is an income effect in period

1 when the government transfer changes consumption and welfare.

In the model, markets are incomplete. In general, there are two possible scenarios for incompleteness. First, the number of assets in the economy are less than the number of the states. The second scenario is scarcity of collateral. In the later case, even if a large set of assets are available, then the number of traded assets is less than the number of states in the economy. There is no perfect insurance in incomplete market economies. Since the paper is restricted to incomplete markets, the number of assets introduced in the economy is less than the number of states. Thus, here the paper uses the later approach. The rest of this section represents the model formally.

Consider a two periods exchange economy $t = 0, 1$ with uncertainty about the state of nature in period 1, states denoted by the subscript $s \in \mathcal{S} = \{1, \ldots, S\}$. The economy consists of a finite number of agents distinguished by the superscript $i \in \mathcal{I} = \{1, \ldots, I\}$. There is only one consumption good per state. There is one durable good in the economy, which is denoted by x^i_{0H}, and pays a dividend of consumption good denoted by a vector $D \in \mathbb{R}^s$ satisfies $D_s > 0$ for state s, and there is a finite number J of real assets denoted by subscript $j \in \{1, \ldots, J\}$. The assets are in zero-net supply. At date 0, individuals have the same prior probability for the occurrence of each states at date 1 but the state is not revealed, and assets are traded. At date 1, the state of the world is revealed to all individuals, and assets pay off a specified quantity of consumption goods. The payoff of assets may be different across the states, but the assets' supply is fixed.

Vectors in the portfolio space $\mathbb{R}^J$ are asset prices q, the quantities of asset sales ψ^i, and the quantities of asset purchases φ^i. Their components are indexed by subscript $j \in \{1, \cdots, J\}$. The durable good price is denoted by $p_H \in \mathbb{R}$ and consumption serves as the numeraire in each state $s = 0, 1, \ldots, S$. The market here is assumed to be incomplete. Thus, the consumption space in period 1 is not spanned by the assets payoffs, $J < S$. There is just one consumption good for individuals at date 0, and in all future states. Vectors in the consumption space $\mathbb{R}^{S+1}$ are consumption x^i, and consumption good endowments e^i. Their components are indexed by states: x^i_s denotes consumption in state $s = 1, \ldots, S$, x^i_0 denotes period 0 consumption. The durable good for the agent i is denoted by $x^i_{0H} \in \mathbb{R}$, and the endowment of the durable good for the agent i is represented by e^i_{0H}.

The preference ordering of agent i is represented by a utility function $U^i : \mathbb{R}^{S+1}_+ \to \mathbb{R}$, defined over consumption $x^i = (x^i_0, \cdots, x^i_S) \in \mathbb{R}^{S+1}_+$. The characteristics of agents in the economy is described by utility functions and endowment vectors $\mathcal{U} = (U^i, e^i)_{i \in \mathcal{I}}$ satisfying the followings assumptions:

Assumption 1: (preferences)

U^i denotes the von Neumann-Morgenstern utility function for individual i. The agents have common prior probabilities of π_s for each state of the world. For each agent i,

A 1. U^i is quasi-linear separable across time and states: $U^i(x^i) = x^i_0 + \sum_s \beta^i \pi_s u^i(x^i_s)$ with $u^i : \mathbb{R}_+ \to \mathbb{R}$

A 2. $u^i : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable on $\mathbb{R}_+$.

A 3. $u^i : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, and strictly concave on $\mathbb{R}_+$.

The utility function is linear in period 0. The quasi-linear utility function is an assumption that helps to exclude income effect.

The asset payoff vectors $A_j \in \mathbb{R}^{\mathbb{S}}$ satisfy $A_j \gg 0$ for all assets j. The payoff vectors specify the consumption goods that are promised for delivery in each state. The payoff matrix is denoted by $A = (A_1, \cdots, A_J) \in \mathbb{R}^{S \times J}$ which is a linear transformation $A : \mathbb{R}^J \to \mathbb{R}^S$. The durable good dividend vector $D \in \mathbb{R}^S$ satisfies $D_s > 0$ for all states s. Consider the model in which all the assets have a collateral requirement. Furthermore, one unit of the durable good is used as collateral for asset j whose effective payoff per unit sold is $A_{sj} = \min\{\eta_{sj}, D_s\}$ for some constant face value $\eta_{sj} > 0$ in state s. So, the collateral requirement is one unit of durable good for all assets, and assets are differentiated by the different face values $\eta_{sj} > 0$. Agents default on their promises whenever the dividend of the durable good they hold as collateral is lower than their face value of their promise. It is useful to define the loan to value ratio for asset j as $LTV_j = \dfrac{q_j}{p_H}$, so the leverage ratio is $LEV_j = \dfrac{1}{1 - LTV_j}$.

Assumption 2: (No redundant assets)

$$rank(A) = J$$

This assumption rules out indeterminacy of portfolio holdings.

Assumption 3: (Endowments)

For each agent i,

1. $e_0^i > 0$.

2. $e_s^i > 0$.

3. The aggregate of durable good endowments is positive, $\sum_i e_{0H}^i > 0$.

Assumption 3 puts a lower bound on individual endowments. An individual's asset endowment may include short positions. If endowments are large enough, these bounds help the individuals to have a positive consumption in all states of the world. These bounds also rule out the minimum wealth problem and ensure continuity of demand.

Definition 1. *An economy with collateral,* $\mathcal{E}_{CE} = (U, e, (A, \mathbf{1}))$ *is tuple in* $\mathcal{U} \times \mathbb{R}^{(S+2)\cdot I} \times \mathbb{R}^{S\cdot J}$ *where agents utilities and endowments* $U = (U^1, \ldots, U^I)$*,* $e = (e^1, \ldots, e^I)$*, and the asset structure* $(A, \mathbf{1})$ *satisfy Assumptions 1,2, and 3.*

In definition above vector $\mathbf{1} \in \mathbb{R}^J$ denotes the collateral requirement for sale of assets. Inequality (2.1) below describes the first-period budget constraint. The term $g_0 = \tau q_1 \psi$ is a government transfer. If an agent trades one unit of asset $j = 1$ and sells it at the price q_1, then she needs to transfer the fraction $\tau \in (0, 1)$ of the revenue from the sale to the government.

$$x_0^i + p_{\mathrm{H}} x_{0\mathrm{H}}^i + q \cdot \varphi^i + g_0 \leq e_0^i + p_{\mathrm{H}} e_{0\mathrm{H}}^i + q \cdot \psi^i \tag{2.1}$$

Inequality (2.2) below specifies the collateral constraint. The collateral constraint puts a restriction on sale of assets. An agent cannot sell asset by more than the quantity of the durable good owned by himself in the first period. Thus, assets are differentiated by different payoffs. The collateral constraint can be written as follows:

$$\psi^i \cdot 1 \leq x^i_{0\mathrm{H}} \tag{2.2}$$

where $1 \in \mathbb{R}^J$ is a column vector of ones.

Inequality (2.3) below specifies the second-period budget constraints for the different states. T^i denotes the government transfer to the agent in the second period which is constant across all states. In this model, the price of the consumption good in each state is normalized to one ($p_s = 1$).

$$x^i_{\mathrm{s}} + \sum_j \psi^i_j \min\{\eta_{sj}, D_{\mathrm{s}}\} \leq e^i_{\mathrm{s}} + \sum_j \varphi^i_j \min\{\eta_{sj}, D_{\mathrm{s}}\} + D_{\mathrm{s}} x^i_{0\mathrm{H}} + T^i \tag{2.3}$$

In state s, the asset pays $\min\{\eta_{sj}, D_s\}$, the agent has endowment e^i_s and receives $D_s \cdot {x^i_{0H}}^i$ units from his durable good endowment in the first period. Therefore, the budget set can be written as in (2.4).

$$\mathcal{B}^i(p_H, q) = \Bigg\{ (x^i, x^i_{0H}, \varphi^i, \psi^i) \in \mathbb{R}^{S+1} \times \mathbb{R} \times \mathbb{R}^J \times \mathbb{R}^J \quad s.t. \tag{2.4}$$

$$x^i_0 + p_H x^i_{0H} + g^i_0 \leq e^i_0 + p_H e^i_{0H} + q \cdot (\psi^i - \varphi^i);$$

$$\psi^i \cdot 1 \leq x^i_{0\mathrm{H}};$$

$$x^i \leq e^i + \sum_j \min\{\eta_{sj}, D_{\mathrm{s}}\}(\varphi^i_j - \psi^i_j) + D x^i_{0H} + T^i \Bigg\}$$

The utility maximization problem of a single agent i in such an economy is $\max_{x^i, x^i_{0H}, \psi^i, \varphi^i} U^i(x^i)$ subject to the budget and collateral constraints.

$$(x^i, x^i_{0H}, \psi^i, \varphi^i) \in \mathcal{B}^i(p_H, q) \tag{2.5}$$

The government budget constraint is given below by equation (2.6), which states that the tax revenue in the first period is equal to the total government transfer in the second period.

$$\sum_{i=1}^{I} g_0^i = \sum_{i=1}^{I} T^i \tag{2.6}$$
$$\sum_{i=1}^{I} \tau q_1 \psi^i = \sum_{i=1}^{I} T^i$$

A social welfare function is used to compare the possible social outcomes. The utilitarian social welfare function is used as society's judgement on how agent utilities have to be compared to create an ordering of possible social outcomes. A utilitarian social welfare function is a function $\mathcal{W} : \mathbb{R}^I \to \mathbb{R}$ that assign a value to each vector of $(U^1, \cdots, U^I) \in \mathbb{R}^I$ by summing agents utilities with equal weights. The utilitarian social welfare function can be written as follows,

$$\mathcal{W}(U^1(x^1), \cdots, U^I(x^I)) = \sum_{i=1}^{I} U^i(x^i) \tag{2.7}$$

A collateral equilibrium for the economy $\mathcal{E}_{CE}$ is defined by optimal decisions of individuals and the market clearing conditions.

Definition 2. *A collateral equilibrium for the economy $\mathcal{E}_{CE}$ is a vector $\langle(\hat{x}, \hat{x}_{0H}, \hat{\varphi}, \hat{\psi}); (\hat{p_H}, \hat{q})\rangle$ with $(\hat{x}, \hat{x}_{0H}, \hat{\varphi}, \hat{\psi}) = (\hat{x}^i, \hat{x}^i_{0H}, \hat{\varphi}^i, \hat{\psi}^i)_{i \in \mathcal{I}}$ such that:*

(i) $(\hat{x}^i, \hat{x}^i_{0H}, \hat{\varphi}^i, \hat{\psi}^i)$ *solve the agents' problems.* $(U^i(x^i) \le U^i(\hat{x}^i)$ $\forall x^i \in \mathcal{B}(\hat{p_H}, \hat{q}))$

(ii) $\sum_{i=1}^{I}(\hat{x}^i_0 - e^i_0) = 0.$

(iii) $\sum_{i=1}^{I}(\hat{x}^i_{0H} - e^i_{0H}) = 0.$

(iv) $\sum_{i=1}^{I}(\hat{x}^i_s - e^i_s - D_s \hat{x}^i_{0H} - T^i) = \mathbf{0}.$

(v) $\sum_{i=1}^{I}(\hat{\varphi}^i - \hat{\psi}^i) = \mathbf{0}.$

There is a difference between general equilibrium with incomplete markets (GEI) and collateral equilibrium. The collateral constraint in collateral equilibrium may be binding for some agents. If the collateral constrain is not binding for all agents in Definition 2, then the equilibrium is as in GEI.

Theorem 1. *For any economy $\mathcal{E}_{CE}$ that satisfies Assumptions 1, 2, and 3 there exists a collateral equilibrium.*

A proof of Theorem 1 can be found in Geanakoplos and Zame (2014).

2.3 Welfare effect of taxation

Assume that all J assets are traded in the equilibrium. The paper aims to consider whether removing the tax incentive for purchasing a durable good can change consumption, prices, and welfare in the economy when agents sell an asset under a binding collateral constraint. The paper shows that if an agent pays tax for one unit of the asset that she sold in period 0, the lump-sum transfer from the government in period 1 may lead to a Pareto improvement.

Binding collateral constraints mean that the agent sold the asset by pledging his entire holding of the durable good in period 0 as collateral. Thus, there is an implicit assumption of scarcity of the durable good in the economy. Without that assumption the collateral constraint for the agent may not binding and the equilibrium is as in GEI and not a collateral equilibrium. In terms of mathematics, the collateral constraint is binding if (2.2) holds with equality, $\psi^i \cdot 1 = x^i_{0\mathrm{H}}$. The agent's maximization problem can be written as $\max_{x^i,\psi^i,\varphi^i} U^i(x^i)$ subject to the budget constraints.

$$x^i + \begin{bmatrix} p_H \mathbf{1}^T - q_\tau \\ \tilde{A} \end{bmatrix} \psi^i + \begin{bmatrix} q \\ -A \end{bmatrix} \varphi^i \leq e^i + T^i \begin{bmatrix} 0 \\ \mathbf{1} \end{bmatrix} \qquad \forall i \in \mathcal{I} \tag{2.8}$$

where q_τ is the asset price vector of sold assets after taxation of asset $j = 1$, $q_\tau = ((1 - \tau)q_1, q_2, \cdots, q_J)$, and q is the price vector of purchased assets, $q = (q_1, \cdots, q_J)$. J are the number of the assets that are traded by agent i. The matrix $\tilde{A}$ is the payoff matrix of sold asset after rewriting the durable good as a sum of sold assets, and the jth column of $\tilde{A}$ is

$$\tilde{A}_j = \begin{pmatrix} \min\{\eta_{sj}, D_1\} - D_1 \\ \vdots \\ \min\{\eta_{sj}, D_S\} - D_S \end{pmatrix}.$$ Thus, $\tilde{A}_j$ is non-positive. To demonstrate the existence of the Pareto improvement, the Schur-Complement from linear algebra is helpful, which is summarized in the following Lemma.

Lemma 1. *(Schur-Complement) Supposed the square matrix $\mathcal{M}$ is partitioned into blocks as follows*

$$\mathcal{M} = \begin{pmatrix} \overbrace{\mathcal{A}}^{\mathbb{R}^m} & \overbrace{\mathcal{B}}^{\mathbb{R}^n} \\ \mathcal{C} & \mathcal{D} \end{pmatrix} \begin{matrix} \}\ \mathbb{R}^m \\ \}\ \mathbb{R}^n \end{matrix} \tag{2.9}$$

If $\mathcal{A}$ is non-singular, the Schur-complement $\mathcal{M} \setminus \mathcal{A}$ of $\mathcal{M}$ with respect to $\mathcal{A}$ is defined by

$$\mathcal{M}\backslash\mathcal{A} = \mathcal{D} - \mathcal{C}\mathcal{A}^{-1}\mathcal{B} \tag{2.10}$$

Then $\det(\mathcal{M})$ *may be calculated as follows,*

$$\det(\mathcal{M}) = \det(\mathcal{A})\det(\mathcal{M}\backslash\mathcal{A}) = \det(\mathcal{A})\det(\mathcal{D} - \mathcal{C}\mathcal{A}^{-1}\mathcal{B}) \tag{2.11}$$

If $\mathcal{D}$ is non-singular, the Schur-complement $\mathcal{A} \setminus \mathcal{D}$ of $\mathcal{M}$ with respect to $\mathcal{D}$ is defined by

$$\mathcal{M}\backslash\mathcal{D} = \mathcal{A} - \mathcal{B}\mathcal{D}^{-1}\mathcal{C} \tag{2.12}$$

Then $\det(\mathcal{M})$ *may be calculated as follows,*

$$\det(\mathcal{M}) = \det(\mathcal{D})\det(\mathcal{M}\backslash\mathcal{D}) = \det(\mathcal{D})\det(\mathcal{A} - \mathcal{B}\mathcal{D}^{-1}\mathcal{C}) \tag{2.13}$$

Proof. Boyd and Vandenberghe (2004) p.650. ■

Theorem 2. *Assume that $\langle(x^i, x_H^i, \varphi^i, \psi^i); (p_H, q)\rangle$ is a collateral equilibrium. If the tax rate τ is increased on asset $j = 1$, the welfare of borrowers, who sell the asset, does not change, and* *the welfare of lenders may increase.*

The proof of the theorem consists of two parts. The first part picks an agent who sells taxed asset. Since collateral equilibrium exists, there is an allocation and prices in which first-order conditions for the agent hold. The asset payoff matrix of the agent contains two partitions. The first partition contains those assets that the agent sells them. The prices of these assets are more than their fundamental values. Another partition contains those assets that the agent purchases or the agents do not trade them at all. The prices of these assets are equal to their fundamental values.

The implicit function theorem and the chain rule are used to calculate the welfare effect of taxation. Hence, the inverse of the Jacobian matrix needs to be calculated. Here, the Schur-complement is used to show that the Jacobian matrix has full-rank. Then, the inverse of the matrix is calculated. Two cases are considered in the proof. In first case, agent does not consume at period 0. In second case, agent consume at period 0. The calculation shows that the expected utility of the agent does not change when the tax is introduced in both cases. The other important result is that the agent's demand for the durable good decreases.

The second part of the proof focuses on the expected utility of other agents who do not sell taxed asset. The market-clearing condition of the durable good is used to show that the durable good's demand for these agents increases. The intuition behind that is that the durable good price decreases, and the agent benefits from the increasing interest rate of the durable good.

Proof. First, consider an agent who sells asset $j = 1$ as a borrower. The Lagrangian of this agent's maximization problem in a collateral equilibrium can be written as:

$$\mathcal{L}^i(x^i, \psi^i, \varphi^i, \mu^i) = U^i(x^i) - \mu^i \left((x^i - e^i) + \begin{bmatrix} \hat{p} \\ \hat{A} \end{bmatrix} \hat{\psi}^i - T^i \begin{bmatrix} 0 \\ \mathbf{1} \end{bmatrix} \right) \tag{2.14}$$

where $\mu^i \in \mathbb{R}^{S+1}_{++}$ is a vector of Lagrange multipliers for the budget constraints. The following notation is used: $x^i = \begin{bmatrix} x^i_0 \\ \bar{x}^i \end{bmatrix} \in \mathbb{R}^{S+1}_{++}$, $e^i = \begin{bmatrix} e^i_0 \\ \bar{e}^i \end{bmatrix} \in \mathbb{R}^{S+1}_{++}$, $\hat{\psi}^i = \begin{bmatrix} \bar{\psi}^i \\ \bar{\varphi}^i \end{bmatrix} \in \mathbb{R}^J$, and $\begin{bmatrix} \hat{p} \\ \hat{A} \end{bmatrix} =$

$\begin{bmatrix} p_H \mathbf{1}^T - q^i_\tau & q_i \\ \tilde{A}_i & -A_i \end{bmatrix} \in \mathbb{R}^{S+1}_{++} \times \mathbb{R}^J_{++}$. Based on Theorem 2 of Geanakoplos and Zame (2014) agents do not purchase and sell assets at the same time. q^i_τ is the asset price vector of issued promises after taxation of asset $j = 1$, $q^i_\tau = ((1-\tau)q_1, q_2, \cdots, q_{J_i})$, and q_i is the price vector of other assets, $q = (q_{J_i+1}, \ldots, q_J)$. The payoff matrix of sold promises for agent i is denoted by $\tilde{A}_i$, and the payoff matrix of other assets is denoted by A_i. Thus, $\{1, \cdots, J_i\}$ are used for indexing assets that are sold, and $\{J_i + 1, \cdots, J\}$ are used for indexing asset that are purchased $\bar{\psi}^i = \begin{pmatrix} \psi^i_1 \\ \vdots \\ \psi^i_{J_i}, \end{pmatrix}$, and $\bar{\varphi}^i = \begin{pmatrix} \varphi^i_{J_i+1} \\ \vdots \\ \varphi^i_J, \end{pmatrix}$. Without lose of generality assume that the index of taxed asset for those who sell the asset is 1. They may re-index other assets arbitrarily. Here, the bar notation for asset denotes issued promises, and purchased promises by the agent i. Here, the bar notation for consumption, endowments and Lagrange multipliers denotes consumption, endowments and Lagrange multiplier vector excluding period 0, $\bar{x}^i = \begin{pmatrix} x^i_1 \\ \vdots \\ x^i_S \end{pmatrix}$, $\bar{e}^i = \begin{pmatrix} e^i_1 \\ \vdots \\ e^i_S \end{pmatrix}$, and $\bar{\mu}^i = (\mu^i_1, \cdots, \mu^i_S)$. The notation $\hat{\psi}^i$ is used to denote all promises that are successfully issued, which is useful for further representation. The transfer matrix $X = \begin{bmatrix} \hat{p} \\ \hat{A} \end{bmatrix}$ contains prices and payoffs of all assets in the economy. Here, $\hat{p}$ denotes the vector $(p_H \mathbf{1}^T - q^i_\tau, q_i)$, and $\hat{A}$ denotes the payoff matrix $\begin{pmatrix} \tilde{A}_i & -A_i \end{pmatrix}$. If the collateral constraint is binding, the agent sold $J_i < J$ assets $\psi^i_j > 0$ such that $\sum_{j=1}^{J_1} \psi^i_j = x^i_{0H}$.

The first order conditions of agent i's optimization problem can be written as follows:

$$
\begin{aligned}
\mathcal{D}_{\mu^i}\mathcal{L}^i = \mathbf{0} \Leftrightarrow &\quad (x^i - e^i) + \begin{bmatrix} p_H \mathbf{1}^T - q^i_\tau \\ \tilde{A}_i \end{bmatrix} \psi^i + \begin{bmatrix} q_i \\ -A_i \end{bmatrix} \varphi^i - T^i \begin{bmatrix} 0 \\ \mathbf{1} \end{bmatrix} = \mathbf{0} \\
\mathcal{D}_{\psi^i}\mathcal{L}^i = \mathbf{0} \Leftrightarrow &\quad \mu^i \begin{bmatrix} p_H \mathbf{1}^T - q^i_\tau \\ \tilde{A}_i \end{bmatrix} = \mathbf{0} \\
\mathcal{D}_{\varphi^i}\mathcal{L}^i = \mathbf{0} \Leftrightarrow &\quad \mu^i \begin{bmatrix} q_i \\ -A_i \end{bmatrix} = \mathbf{0} \\
\mathcal{D}_{x^i_0}\mathcal{L}^i \leq 0 \Leftrightarrow &\quad 1 - \mu^i_0 \leq 0, \qquad (1 - \mu^i_0) x^i_0 = 0 \\
\mathcal{D}_{\bar{x}^i}\mathcal{L}^i = \mathbf{0} \Leftrightarrow &\quad \nabla_{x_s} u^i(x^i_s) - \mu^i_s = \mathbf{0}
\end{aligned} \tag{2.15}
$$

Two cases need to be considered here. The first case, is that there is no consumption in the first period $x^i_0 = 0$. The second case, is that there is a positive consumption in the first period $x^i_0 > 0$.

Case 1: $(x^i_0 = 0)$

First consider the case when there is no consumption in the first period. If $x^i_0 = 0$ then the first order conditions can be written as follows:

$$\mathcal{D}_{\bar{\mu}^i}\mathcal{L}^i = \mathbf{0} \Leftrightarrow \quad (\bar{x}^i - e_T^i) + \hat{A}\hat{\psi}^i = \mathbf{0}$$

$$\mathcal{D}_{\mu_0^i}\mathcal{L}^i = \mathbf{0} \Leftrightarrow \quad -e_0^i - p_H e_H^i + \hat{p}\hat{\psi}^i = 0$$

$$\mathcal{D}_{\hat{\psi}^i}\mathcal{L}^i = \mathbf{0} \Leftrightarrow \quad \mu^i \begin{bmatrix} \hat{p} \\ \hat{A} \end{bmatrix} = \mathbf{0} \tag{2.16}$$

$$\mathcal{D}_{\bar{x}^i}\mathcal{L}^i = \mathbf{0} \Leftrightarrow \quad \nabla_{x_s} u^i(x_s^i) - \mu_s^i = \mathbf{0}$$

Now consider the open sets $\Phi = \mathbb{R}^S \times \mathbb{R} \times \mathbb{R}^J \times \mathbb{R}^S$, $\Gamma = \mathbb{R}^S \times \mathbb{R} \times \mathbb{R}^J \times \mathbb{R}^S$, and $\Theta = \mathbb{R}^{S+1} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}^{SJ}$, where Φ, and Θ denote the space of economies. Define the mapping $\mathcal{F}_i : \Phi \times \mathbb{R} \times \mathbb{R}^J \times \Theta \to \Gamma$ by

$$\mathcal{F}_i(\rho, \hat{p}, \theta) = \begin{pmatrix} \mathcal{D}_{\bar{\mu}^i}\mathcal{L}^i \\ \mathcal{D}_{\mu_0^i}\mathcal{L}^i \\ \mathcal{D}_{\hat{\psi}^i}\mathcal{L}^i \\ \mathcal{D}_{\bar{x}^i}\mathcal{L}^i \end{pmatrix} = \begin{pmatrix} (\bar{x}^i - e_T^i) + \hat{A}\hat{\psi}^i \\ e_0^i + p_H e_H^i + \hat{p}\hat{\psi}^i \\ \begin{bmatrix} \hat{p}^T & \hat{A}^T \end{bmatrix} (\mu^i)^T \\ \nabla_{x_s} u^i(x_s^i) - \bar{\mu}^i \end{pmatrix} \begin{matrix} \} \ \mathbb{R}^S \\ \} \ \mathbb{R} \\ \} \ \mathbb{R}^J \\ \} \ \mathbb{R}^S \end{matrix} \tag{2.17}$$

Consider the tuple of arguments $\rho = (\bar{x}^i, \mu_0^i, \hat{\psi}^i, \bar{\mu}^i)$, and the tuple of parameters $\theta = (e^i, e_H^i, \tau, \hat{A})$. Given parameters vector θ, if ρ is a solution to the agent maximization problem, then the first order conditions derived from (2.14) hold, $\mathcal{F}_i(\rho, \hat{p}, \theta) = 0$. The Jacobian of the agent's maximization problem is $\mathcal{D}_\rho \mathcal{F}_i$ when the array $\rho = (\bar{x}^i, \mu_0^i, \hat{\psi}^i, \bar{\mu}^i)$ has the following form:

$$\mathcal{D}_\rho \mathcal{F}_i = \begin{pmatrix} \overbrace{\mathbf{I}}^{\mathbb{R}^S} & \overbrace{\mathbf{0}}^{\mathbb{R}} & \overbrace{\hat{A}}^{\mathbb{R}^J} & \overbrace{\mathbf{0}}^{\mathbb{R}^S} \\ 0 & 0 & \hat{p} & 0 \\ \mathbf{0} & \hat{p}^T & \mathbf{0} & \hat{A}^T \\ \nabla^2 u^i(x_s^i) & 0 & 0 & -\mathbf{I} \end{pmatrix} \begin{matrix} \} \ \mathbb{R}^S \\ \} \ \mathbb{R} \\ \} \ \mathbb{R}^J \\ \} \ \mathbb{R}^S \end{matrix} \tag{2.18}$$

where $\mathbf{I}$ denotes the identity matrix. In order to use the implicit function theorem and the chain rule, $\mathcal{D}_\rho \mathcal{F}_i$ needs to be non-singular. If $\det(\mathcal{D}_\rho \mathcal{F}_i)$ is nonzero, then $\mathcal{D}_\rho \mathcal{F}_i$ is non-singular. Using elementary matrix operations $\mathcal{D}_\rho \mathcal{F}_i$ can be transformed as follows:

$$\overline{\mathcal{D}_\rho \mathcal{F}}_i = \begin{pmatrix} \overbrace{\mathbf{I}}^{\mathbb{R}^S} & \overbrace{\mathbf{0}}^{\mathbb{R}} & \overbrace{\hat{A}}^{\mathbb{R}^J} & \overbrace{\mathbf{0}}^{\mathbb{R}^S} \\ 0 & 0 & \hat{p} & 0 \\ \mathbf{0} & \hat{p}^T & \mathbf{0} & \hat{A}^T \\ 0 & 0 & (-\nabla^2 u^i(x_s^i))\hat{A} & -\mathbf{I} \end{pmatrix} \begin{matrix} \} \ \mathbb{R}^S \\ \} \ \mathbb{R} \\ \} \ \mathbb{R}^J \\ \} \ \mathbb{R}^S \end{matrix} \tag{2.19}$$

Here, $\overline{\mathcal{D}_\rho \mathcal{F}_i}$ is the matrix after implementing a number of elementary operations on $\mathcal{D}_\rho \mathcal{F}_i$ to eliminate $\nabla^2 u^i(x_s^i)$ from the first S columns of the matrix $\mathcal{D}_\rho \mathcal{F}_i$. The proof of Theorem 1 still not finished and Lemma 2 below is used for the rest of the proof.

Lemma 2. *The Jacobian matrix $\mathcal{D}_\rho \mathcal{F}_i$ has an inverse.*

Proof. To calculate the determinant of $\overline{\mathcal{D}_\rho \mathcal{F}_i}$ the general formula for determinants of diagonal partition matrices can be used as follows:

$$\det(\overline{\mathcal{D}_\rho \mathcal{F}}_i) = \prod_{s=1}^{S} (-\beta^i \pi_s \partial^2 u^i(x_s^i)) \det(I) \det \begin{pmatrix} 0 & \hat{p} & 0 \\ \hat{p}^T & \mathbf{0} & \hat{A}^T \\ 0 & \hat{A} & (\nabla^2 u^i(x_s^i))^{-1} \end{pmatrix} \tag{2.20}$$

Using lemma 1 the determinant can be calculated as follows:

$$\det(\overline{\mathcal{D}_\rho \mathcal{F}}_i) = \prod_{s=1}^{S}(-\beta^i \pi_s \partial^2 u^i(x_s^i)) \det\left(\begin{pmatrix} 0 & \hat{p} \\ \hat{p}^T & \mathbf{0} \end{pmatrix} - \begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \hat{A}^T\big(\nabla^2 u^i(x_s^i)\big)\hat{A} \end{pmatrix}\right) \tag{2.21}$$

Then $\det(\overline{\mathcal{D}_\rho \mathcal{F}}_i)$ can be calculated as follows:

$$\det(\overline{\mathcal{D}_\rho \mathcal{F}}_i) = \prod_{s=1}^{S}(-\beta^i \pi_s \partial^2 u^i(x_s^i)) \det\begin{pmatrix} 0 & \hat{p} \\ \hat{p}^T & -\hat{A}^T\big(\nabla^2 u^i(x_s^i)\big)\hat{A} \end{pmatrix}$$

Using lemma 1 again the determinant can be calculated as follows:

$$\det(\overline{\mathcal{D}_\rho \mathcal{F}}_i) = \prod_{s=1}^{S}(-\beta^i \pi_s \partial^2 u^i(x_s^i)) \det(\hat{A}^T\big(\nabla^2 u^i(x_s^i)\big)\hat{A}) \det(\hat{p}\big(\hat{A}^T\big(\nabla^2 u^i(x_s^i)\big)\hat{A}\big)^{-1}\hat{p}^T) \tag{2.22}$$

Since $rank(\hat{A}) = J$, $\det(\overline{\mathcal{D}_\rho \mathcal{F}}_i)$ is positive, and $\det(\overline{\mathcal{D}_\rho \mathcal{F}}_i)$ is non-zero. Since the $\det(\mathcal{D}_\rho \mathcal{F}_i) = \det(\overline{\mathcal{D}_\rho \mathcal{F}}_i)$, $\mathcal{D}_\rho \mathcal{F}_i$ has an inverse. ■

The inverse matrix $(\overline{\mathcal{D}_\rho \mathcal{F}}_i)^{-1}$ is as follows:

$$(\overline{\mathcal{D}_\rho \mathcal{F}}_i)^{-1} = \begin{pmatrix} \overbrace{\mathbf{I}}^{\mathbb{R}^S} & \overbrace{-\hat{A}\hat{p}^T(\hat{p}\hat{p}^T)^{-1}}^{\mathbb{R}} & \overbrace{\mathbf{0}}^{\mathbb{R}^J} & \overbrace{\mathbf{0}}^{\mathbb{R}^S} \\ 0 & a & (\hat{p}\hat{p}^T)^{-1}\hat{p} & (\hat{p}\hat{p}^T)^{-1}\hat{p}\hat{A}^T \\ \mathbf{0} & \hat{p}^T(\hat{p}\hat{p}^T)^{-1} & \mathbf{0} & \mathbf{0} \\ 0 & -b & \mathbf{0} & -\mathbf{I} \end{pmatrix} \begin{matrix} \}\ \mathbb{R}^S \\ \}\ \mathbb{R} \\ \}\ \mathbb{R}^J \\ \}\ \mathbb{R}^S \end{matrix} \tag{2.23}$$

Here, the matrix elements are $a = -(\hat{p}\hat{p}^T)^{-1}\hat{p}\hat{A}^T(-\nabla^2 u^i(\bar{x}^i)I)\hat{A}\hat{p}^T(\hat{p}\hat{p}^T)^{-1}$, and $b = -(-\nabla^2 u^i(\bar{x}^i)I)\hat{A}\hat{p}^T(\hat{p}\hat{p}^T)^{-1}$

In order to do comparative statics analysis, $(\mathcal{D}_\rho \mathcal{F}_i)^{-1}$ needs to be calculated. According to the Hoffman and Kunze (1971) (p.26, Theorem 12) following equality holds:

$$\mathbf{I} = E_m \dots E_1 \mathcal{D}_\rho \mathcal{F}_i \tag{2.24}$$

where, $E_1,\dots,E_m$ are elementary matrices.[1] Assume that $E_1,\dots,E_k$ are those elementary matrices that construct $(\overline{\mathcal{D}_\rho \mathcal{F}_i})$. Then the following equality holds:

$$\overline{\mathcal{D}_\rho \mathcal{F}_i} = E_k \dots E_1 \mathcal{D}_\rho \mathcal{F}_i \tag{2.25}$$

Equities (2.24) and (2.25) imply following equality:

$$\mathbf{I} = E_m \dots E_{k+1} E_k \dots E_1 \mathcal{D}_\rho \mathcal{F}_i \tag{2.26}$$

Since, $(\overline{\mathcal{D}_\rho \mathcal{F}_i})^{-1} = E_m \dots E_{k+1}$, the inverse matrix $(\mathcal{D}_\rho \mathcal{F}_i)^{-1}$ is as follows:

$$(\mathcal{D}_\rho \mathcal{F}_i)^{-1} = (\overline{\mathcal{D}_\rho \mathcal{F}_i})^{-1} \begin{pmatrix} \overbrace{\mathbf{I}}^{\mathbb{R}^S} & \overbrace{\mathbf{0}}^{\mathbb{R}} & \overbrace{\mathbf{0}}^{\mathbb{R}^J} & \overbrace{\mathbf{0}}^{\mathbb{R}^S} \\ 0 & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -\nabla^2 u^i(\bar{x}^i)\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{matrix} \}\ \mathbb{R}^S \\ \}\ \mathbb{R} \\ \}\ \mathbb{R}^J \\ \}\ \mathbb{R}^S \end{matrix} \tag{2.27}$$

$$(\mathcal{D}_\rho \mathcal{F}_i)^{-1} = \begin{pmatrix} \overbrace{\mathbf{I}}^{\mathbb{R}^S} & \overbrace{\hat{A}\hat{p}^T(\hat{p}\hat{p}^T)^{-1}}^{\mathbb{R}} & \overbrace{\mathbf{0}}^{\mathbb{R}^J} & \overbrace{\mathbf{0}}^{\mathbb{R}^S} \\ -b^T & a & (\hat{p}\hat{p}^T)^{-1}\hat{p} & (\hat{p}\hat{p}^T)^{-1}\hat{p}\hat{A}^T \\ \mathbf{0} & \hat{p}^T(\hat{p}\hat{p}^T)^{-1} & \mathbf{0} & \mathbf{0} \\ \nabla^2 u^i(\bar{x}^i)I & -b & \mathbf{0} & -\mathbf{I} \end{pmatrix} \begin{matrix} \}\ \mathbb{R}^S \\ \}\ \mathbb{R} \\ \}\ \mathbb{R}^J \\ \}\ \mathbb{R}^S \end{matrix} \tag{2.28}$$

The effect of taxation of asset sales on consumption, durable goods, and multipliers are calculated by the implicit function theorem as follows:

[1]There are three kind of elementary matrix operations: Interchanging two rows, multiplying each elements in a row by non-zero number, and multiply a row by a non-zero number and adding the result to another row. The last matrix on the right hand side equation (2.27) is a result of the number of elementary operations, which is used to transform $(\mathcal{D}_\rho \mathcal{F}_i)$ to $(\overline{\mathcal{D}_\rho \mathcal{F}_i})$ and which do not change the determinant.

$$\mathcal{D}_\tau \rho = \begin{pmatrix} \mathcal{D}_\tau \bar{x}^i \\ \mathcal{D}_\tau \mu_0^i \\ \mathcal{D}_\tau \hat{\psi}^i \\ \mathcal{D}_\tau \bar{\mu}^i \end{pmatrix} = -(\mathcal{D}_\rho \mathcal{F}_i)^{-1} \mathcal{D}_\tau \mathcal{F}_i \tag{2.29}$$

where, $\mathcal{D}_\tau \mathcal{F}_i$ is the vector of derivatives of (2.17) with respect to τ, $\mathcal{D}_\tau \mathcal{F}_i = (\mathbf{0}, q_1\psi_1^i, \begin{bmatrix} q_1\mu_0^i \\ \mathbf{0} \end{bmatrix}, \mathbf{0})^T$. In order to calculate the welfare effect of taxation on borrowers, $\mathcal{D}_\tau U^i(x^i)$ needs to be calculated. If $\partial_\tau T^i = q_1\psi_1^i$ then $\mathcal{D}_\tau U^i(x^i)$ is calculated as follows:

$$\begin{aligned} \mathcal{D}_\tau U^i(x^i) &= \nabla_{x_s^i} u^i(x_s) \cdot \Big(\mathcal{D}_\tau \bar{x}^i + \mathcal{D}_{T^i} \bar{e}^i \partial_\tau T^i \Big) \\ &= \bar{\mu}^i \cdot \Big(- q_1\psi_1^i \mathbf{1} - q_1\psi_1^i \hat{A}\hat{p}^T (\hat{p}\hat{p}^T)^{-1} \Big) \end{aligned} \tag{2.30}$$

Multiplying both side of (2.30) by $\dfrac{(\hat{p}\hat{p}^T)}{\mu_0^i}$ the equation can be written as follows:

$$\begin{aligned} \frac{(\hat{p}\hat{p}^T)}{\mu_0^i} \mathcal{D}_\tau U^i(x^i) &= \frac{1}{\mu_0^i}\Big(- \sum_s \big(q_1\psi_1^i \mu_s^i (\hat{p}\hat{p}^T)\big) - q_1\psi_1^i (\bar{\mu}^i \hat{A})\hat{p}^T \Big) \\ &= \frac{1}{\mu_0^i}\Big(- \sum_s \big(q_1\psi_1^i \mu_s^i (\hat{p}\hat{p}^T)\big) - q_1\psi_1^i (\bar{\mu}^i \hat{A})\hat{p}^T \Big) \\ &= q_1\psi_1^i \Big(- \sum_s \big(\frac{\mu_s^i}{\mu_0^i} (\hat{p}\hat{p}^T)\big) + (\hat{p}\hat{p}^T) \Big) = 0 \end{aligned} \tag{2.31}$$

Since $\dfrac{(\hat{p}\hat{p}^T)}{\mu_0^i}$ is positive, the expected utility of the borrower does not change in response to taxation, $\mathcal{D}_\tau U^i(x^i) = 0$. The effect of taxation on the sale of assets is calculated as follows:

$$\mathcal{D}_\tau \hat{\psi}^i = -\hat{p}^T (\hat{p}\hat{p}^T)^{-1} q_1\psi_1^i \leq \mathbf{0} \tag{2.32}$$

The effect of taxation on the durable good (housing) is:

$$\partial_\tau x_H^i = \sum_{j=1}^{J} \partial_\tau \psi_j^i < 0 \tag{2.33}$$

Case 2: $(x_0^i > 0)$

Now consider the open sets $\Phi = \mathbb{R} \times \mathbb{R}^S \times \mathbb{R}^J \times \mathbb{R} \times \mathbb{R}^S$, $\Gamma = \mathbb{R} \times \mathbb{R}^S \times \mathbb{R}^J \times \mathbb{R} \times \mathbb{R}^S$, and $\Theta = \mathbb{R}^{S+1} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}^{SJ}$, which are Φ, and Θ denote the space of the economies. Define the mapping $\mathcal{F}_i : \Phi \times \mathbb{R} \times \mathbb{R}^J \times \Theta \to \Gamma$ by

$$\mathcal{F}_i(\rho, \hat{p}, \theta) = \begin{pmatrix} \mathcal{D}_{\mu_0^i} \mathcal{L}^i \\ \mathcal{D}_{\bar{\mu}^i} \mathcal{L}^i \\ \mathcal{D}_{\hat{\psi}^i} \mathcal{L}^i \\ \mathcal{D}_{x_0^i} \mathcal{L}^i \\ \mathcal{D}_{\bar{x}^i} \mathcal{L}^i \end{pmatrix} = \left(\begin{array}{c} x_0^i - e_0^i - p_H e_H^i + \hat{p}\hat{\psi}^i \\ (\bar{x}^i - e_T^i) + \hat{A}\hat{\psi}^i \\ \begin{bmatrix} \hat{p}^T & \hat{A}^T \end{bmatrix} (\mu^i)^T \\ 1 - \mu_0^i \\ \nabla_{x_s} u^i(x_s^i) - \bar{\mu}^i \end{array} \right) \begin{array}{l} \} \ \mathbb{R} \\ \} \ \mathbb{R} \\ \} \ \mathbb{R}^J \\ \} \ \mathbb{R} \\ \} \ \mathbb{R}^S \end{array} \tag{2.34}$$

Consider the tuple of arguments $\rho = (x_0^i, \bar{x}^i, \hat{\psi}^i, \mu_0^i, \bar{\mu}^i)$, and the tuple of parameters $\theta = (e^i, e_H^i, \tau, \hat{A})$. Given the parameter vector θ, if ρ is a solution to the agent's maximization problem, then the first order conditions derived from (2.14) hold, $\mathcal{F}_i(\rho, \hat{p}, \theta) = 0$. The Jacobian of the agent's maximization problem is $\mathcal{D}_\rho \mathcal{F}_i$, where $\rho = (x_0^i, \bar{x}^i, \hat{\psi}^i, \mu_0^i, \bar{\mu}^i)$, and it has the form

$$\mathcal{D}_\rho \mathcal{F}_i = \begin{array}{c} \begin{array}{ccccc} \overbrace{}^{\mathbb{R}} & \overbrace{}^{\mathbb{R}^S} & \overbrace{}^{\mathbb{R}^J} & \overbrace{}^{\mathbb{R}} & \overbrace{}^{\mathbb{R}^S} \end{array} \\ \begin{pmatrix} 1 & \mathbf{0} & \hat{p} & 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \hat{A} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \hat{p}^T & \hat{A}^T \\ 0 & \mathbf{0} & \mathbf{0} & -1 & \mathbf{0} \\ \mathbf{0} & \nabla^2 u^i(x_s^i) & \mathbf{0} & \mathbf{0} & -\mathbf{I} \end{pmatrix} \end{array} \begin{array}{l} \\ \} \ \mathbb{R} \\ \} \ \mathbb{R}^S \\ \} \ \mathbb{R}^J \\ \} \ \mathbb{R} \\ \} \ \mathbb{R}^S \end{array} \tag{2.35}$$

where $\mathbf{I}$ denotes the identity matrix. In order to use the implicit function theorem and the chain rule, $\mathcal{D}_\rho \mathcal{F}_i$ needs to be non-singular. If $\det(\mathcal{D}_\rho \mathcal{F}_i)$ is nonzero, then $\mathcal{D}_\rho \mathcal{F}_i$ is non-singular. Using elementary matrix operations the matrix of $\mathcal{D}_\rho \mathcal{F}_i$ can be transformed as follows:

$$\overline{\mathcal{D}_\rho \mathcal{F}}_i = \begin{pmatrix} \overbrace{1}^{\mathbb{R}} & \overbrace{\mathbf{0}}^{\mathbb{R}^S} & \overbrace{\hat{p}}^{\mathbb{R}^J} & \overbrace{0}^{\mathbb{R}} & \overbrace{\mathbf{0}}^{\mathbb{R}^S} \\ \mathbf{0} & \mathbf{I} & \hat{A} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \hat{p}^T & \hat{A}^T \\ 0 & \mathbf{0} & \mathbf{0} & -1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\nabla^2 u^i(x_s^i)\hat{A} & \mathbf{0} & -\mathbf{I} \end{pmatrix} \begin{matrix} \} \mathbb{R} \\ \} \mathbb{R}^S \\ \} \mathbb{R}^J \\ \} \mathbb{R} \\ \} \mathbb{R}^S \end{matrix} \tag{2.36}$$

The matrix $\overline{\mathcal{D}_\rho \mathcal{F}}_i$, which can be written as $\begin{pmatrix} \mathbf{I} & \tilde{\mathcal{M}}_1 \\ \mathbf{0} & \tilde{\mathcal{M}}_2 \end{pmatrix}$, has full rank. In fact, $\mathbf{I}$ and $\tilde{\mathcal{M}}_2$ have full rank. Thus, the matrix has an inverse, and the inverse of the matrix $\overline{\mathcal{D}_\rho \mathcal{F}}_i$ can be written as follows:

$$\big(\overline{\mathcal{D}_\rho \mathcal{F}}_i\big)^{-1} = \begin{pmatrix} \overbrace{1}^{\mathbb{R}} & \overbrace{\mathbf{0}}^{\mathbb{R}^S} & \overbrace{-\hat{p}\hat{a}}^{\mathbb{R}^J} & \overbrace{-\hat{p}\hat{a}\hat{p}^T}^{\mathbb{R}} & \overbrace{-\hat{p}\hat{b}}^{\mathbb{R}^S} \\ \mathbf{0} & \mathbf{I} & -\hat{A}\hat{a} & -\hat{A}\hat{a}\hat{p}^T & -\hat{A}\hat{b} \\ \mathbf{0} & \mathbf{0} & \hat{a} & \hat{a}\hat{p}^T & \hat{b} \\ 0 & \mathbf{0} & \mathbf{0} & -1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\nabla^2 u^i(x_s^i)\hat{A}\hat{a} & -\nabla^2 u^i(x_s^i)\hat{A}\hat{a}\hat{p}^T & \mathbf{0} \end{pmatrix} \begin{matrix} \} \mathbb{R} \\ \} \mathbb{R}^S \\ \} \mathbb{R}^J \\ \} \mathbb{R} \\ \} \mathbb{R}^S \end{matrix} \tag{2.37}$$

Here, the entries of the inverse matrix are $\hat{a} = (\hat{A}^T\hat{A})^{-1}\hat{A}^T\big(-\nabla^2 u^i(x_s^i)\big)^{-1}\hat{A}(\hat{A}^T\hat{A})^{-1}$, and $\hat{b} = (\hat{A}^T\hat{A})^{-1}\hat{A}^T\big(-\nabla^2 u^i(x_s^i)\big)^{-1}$. Thus, $\mathcal{D}_\rho \mathcal{F}_i^{-1}$ can be written as $\overline{\mathcal{D}_\rho \mathcal{F}}_i^{-1} \times \mathbf{E}$, where $\mathbf{E} = E_k \times \cdots \times E_1$ is multiplication of elementary matrix operation matrices and $\times$ denotes matrix multiplication. $\mathbf{E}$ has following form

$$
\mathbf{E} = \begin{pmatrix} \overbrace{1}^{\mathbb{R}} & \overbrace{\mathbf{0}}^{\mathbb{R}^S} & \overbrace{\mathbf{0}}^{\mathbb{R}^J} & \overbrace{\mathbf{0}}^{\mathbb{R}} & \overbrace{\mathbf{0}}^{\mathbb{R}^S} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & \mathbf{0} \\ \mathbf{0} & -\nabla^2 u^i(x_s^i)\hat{A} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{matrix} \}\ \mathbb{R} \\ \}\ \mathbb{R}^S \\ \}\ \mathbb{R}^J \\ \}\ \mathbb{R} \\ \}\ \mathbb{R}^S \end{matrix} \tag{2.38}
$$

The effect of taxation on consumption in different states, durable goods, and on multipliers is calculated by the implicit function theorem as follows:

$$
\mathcal{D}_\tau \rho = \begin{pmatrix} \mathcal{D}_\tau x_0^i \\ \mathcal{D}_\tau \bar{x}^i \\ \mathcal{D}_\tau \hat{\psi}^i \\ \mathcal{D}_\tau \mu_0^i \\ \mathcal{D}_\tau \bar{\mu}^i \end{pmatrix} = -(\mathcal{D}_\rho \mathcal{F}_i)^{-1} \mathcal{D}_\tau \mathcal{F}_i \tag{2.39}
$$

$$
\mathcal{D}_{T^i} \rho = \begin{pmatrix} \mathcal{D}_{T^i} x_0^i \\ \mathcal{D}_{T^i} \bar{x}^i \\ \mathcal{D}_{T^i} \hat{\psi}^i \\ \mathcal{D}_{T^i} \mu_0^i \\ \mathcal{D}_{T^i} \bar{\mu}^i \end{pmatrix} = -(\mathcal{D}_\rho \mathcal{F}_i)^{-1} \mathcal{D}_{T^i} \mathcal{F}_i \tag{2.40}
$$

Here, $\mathcal{D}_\tau \mathcal{F}_i = (q_1 \psi_1^i, \mathbf{0}, \begin{bmatrix} q_1 \mu_0^i \\ \mathbf{0} \end{bmatrix}, 0, \mathbf{0})^T$, $\mathcal{D}_{T^i} \mathcal{F}_i = (\mathbf{0}, -q_1 \psi_1^i \mathbf{1}, 0, \mathbf{0}, \mathbf{0})^T$, and $\partial_\tau T^i = q_1 \psi_1^i$. In order to calculate the welfare effect of taxation on a borrower, $\mathcal{D}_\tau U^i(x^i)$ needs to be calculated.

$$\begin{aligned}\mathcal{D}_\tau U^i(x^i) &= \nabla_{x^i} u^i(x) \cdot \Big(\mathcal{D}_\tau x^i + \mathcal{D}_{T^i} x^i \partial_\tau T^i\Big) \\ &= \mu_0^i \Big(q_1 \hat{\psi}_1^i - \hat{p}\hat{a} \begin{pmatrix} q_1 \mu_0^i \\ 0 \end{pmatrix} - \hat{p}\hat{b} \nabla^2 u^i(x_s^i) \hat{A} q_1 \psi_1^i\Big) + \\ \bar{\mu}^i &\Big(- q_1 \hat{\psi}_1^i \mathbf{1} - \hat{A}\hat{a} \begin{pmatrix} q_1 \mu_0^i \\ 0 \end{pmatrix} - \hat{A}\hat{b} \nabla^2 u^i(x_s^i) \hat{A} q_1 \psi_1^i\Big)\end{aligned} \tag{2.41}$$

The change in expected utility of borrowers can be written as follows:

$$\begin{aligned}\mathcal{D}_\tau U^i(x^i) &= \mu_0^i \Big(q_1 \hat{\psi}_1^i - \hat{p}\hat{a} \begin{pmatrix} q_1 \mu_0^i \\ 0 \end{pmatrix} - \hat{p}\hat{b} \nabla^2 u^i(x_s^i) \hat{A} q_1 \psi_1^i\Big) + \\ &\Big(- \sum_s \mu_s^i q_1 \hat{\psi}_1^i - \bar{\mu}^i A\hat{a} \begin{pmatrix} q_1 \mu_0^i \\ 0 \end{pmatrix} - \bar{\mu}^i \hat{A}\hat{b} \nabla^2 u^i(x_s^i) \hat{A} q_1 \psi_1^i\Big) \\ &= \mu_0^i \Big(q_1 \hat{\psi}_1^i - \hat{p}\hat{a} \begin{pmatrix} q_1 \mu_0^i \\ 0 \end{pmatrix} - \hat{p}\hat{b} \nabla^2 u^i(x_s^i) \hat{A} q_1 \psi_1^i\Big) + \\ &\Big(- \sum_s \mu_s^i q_1 \hat{\psi}_1^i + p\hat{a} \begin{pmatrix} q_1 \mu_0^i \\ 0 \end{pmatrix} + \hat{p}\hat{b} \nabla^2 u^i(x_s^i) \hat{A} q_1 \psi_1^i\Big) = 0\end{aligned}$$

Taxation does not change the welfare of the borrower. The effects of the tax on multipliers are as follows.

$$\begin{aligned}\partial_\tau \mu_0^i =& 0 \\ \mathcal{D}_\tau \bar{\mu}^i =& - \hat{A}(\hat{A}^T \hat{A})^{-1} \begin{pmatrix} q_1 \mu_0^i \\ 0 \end{pmatrix} \geq 0\end{aligned} \tag{2.42}$$

In order to consider the effect of taxation on the welfare of lenders two cases are considered. The first case is the one in which a borrower does not consume anything in period 0. In this case, the multipliers of a borrower change as follows:

$$\partial_\tau \bar{\mu}^i = -\big(- \nabla^2 u^i(x^i_s)\big) \hat{A}\hat{p}^T(\hat{p}\hat{p}^T) q_1 \hat{\psi}^i_1 \geq 0 \tag{2.43}$$

Thus, the consumption change for borrowers is non-positive in all sates. Since, the expected utility of a borrower is not change after taxation, the consumption change of a borrower does compensated by the government transfer in the second period. On the other hand, the market clearing conditions imply that the consumption of lenders does increase with increasing of τ. Therefore, the expected utility of a lender increases.

In the second scenario the borrowers of asset 1 have positive consumption in period 0. In this case the borrower's consumption change in all states may be non-positive. By the market clearing conditions, the lender‘s consumption change in future states is then non-negative. When the tax increases, the consumption of a lender may increase. The buyer of asset 1 (lender) purchases more of the durable good in the period 0. Then, the higher quantity of the durable good has an income effect in the period 1 that increases consumption of goods in all future states. Thus, the welfare change is non-negative $\mathcal{D}_\tau \mathcal{W}(U^1, \cdots, U^I) \geq 0$, this finally complete proof of Theorem 2.

■

Proposition 1. *Assume* $\langle (x^i, x^i_H, \varphi^i, \psi^i); (p_H, q)\rangle$ *constitutes a collateral equilibrium. Assume that consumption in the first period is positive. If the tax* τ *increases the price of the taxed asset, then the price of the durable good decreases.*

Proof. According to Theorem 2, the consumption of an agent who purchases asset 1 increases. Increased consumption of the agent implies a decrease of the multipliers of the agent in future states. The first-order conditions for a lender implies $q_1 = \sum_s \mu^i_s \min\{\eta_1, D_s\}$. Thus, the price of the asset 1 decreases.

According to the first order conditions for an agent who sells asset 1, the price of the durable good is $p_H = q_1(1-\tau) + \sum_s \mu^i_s \big(D_s - \min\{\eta_1, D_s\}\big)$. Thus, the change of the durable good price can be written as follows

$$\partial_\tau p_H = \partial_\tau q_1(1-\tau) - q_1 - \mathcal{D}_\tau \bar{\mu}^i \hat{A}_1 \tag{2.44}$$

The Jacobian matrix calculated for the proof of Theorem 2 implies that

$$\mathcal{D}_\tau \bar{\mu}^i \hat{A} = -\big(\mu_0^i q_1 \cdots 0)(\hat{A}^T \hat{A})^{-1} \hat{A}^T \hat{A} = (-q_1 \ldots 0) \tag{2.45}$$

Thus, the change of the price of the durable good can be written as follows

$$\partial_\tau p_H = \partial_\tau q_1(1-\tau) \tag{2.46}$$

So the price of the durable good decreases.

■

2.3.1 A simple Example

Consider a two states economy with one consumption good and one durable asset which pays dividends $D_1 > D_2$ of consumption good. There are two agents $i = 1, 2$ characterized by utilities $U^i(x) = x_0^i + \sum_{s=1}^{2} \pi_s u^i(x_s)$, where $u^i(x_s) = ln(x_s)$. Agents do not discount the future, and they have an initial endowment of asset e_{0H}^i, $i = 1, 2$. They also have an endowment of consumption in each state e_0^i. All contract promises have a form of (η_1, η_2), and all are backed by one unit of durable good as collateral. Agents will never deliver a promise if the value of a promise is more than the value of the collateral.

Suppose endowments of the durable good for agents is $e_{0H}^1 = 0, \quad e_{0H}^2 = 1$. Suppose consumption goods endowments are given by $e^1 = (e_0^1, e_1^1, e_2^1) = (0.444, 0.1, 1)$ and $e^2 = (e_0^2, e_1^2, e_2^2) = (0.156, 0.7, 0.7)$. The parameters for utility function are $\pi_1 = 0.5$ and $\pi_2 = 0.5$. Promises for both states are $\eta_1 = \eta_2 = 0.2$. Finally, assets dividends are $D_1 = 1$ and $D_2 = 0.2$. According to the no-default theorem in Fostel and Geanakoplos (2008) the contract with $(\eta_1 = 0.2, \eta_2 = 0.2)$ is traded. The agent 1 purchases all the durable goods $x^1_{0H} = 1$. Furthermore, the agent sell $\psi^1 = 1$ promises. Thus, the equilibrium price for durable goods is $p = 0.666$, and the price of the promise is $q = 0.222$. The equilibrium allocation for agent 1 (borrower) is $x^1 = (x^1_0, x^1_1, x^1_2) = (0, 0.9, 1)$ and allocation for agent 2 (lender) is $x^2 = (x^2_0, x^2_1, x^1_2) = (0.5, 0.9, 0.9)$.[2]

If tax increases by $\tau = 0.1$, then the borrower purchases $x^1_{0H} = 0.97$ of the durable good. The price of the promises contract is $q = 0.219$, and the equilibrium price of the durable good is $p = 0.652$. The new equilibrium allocation for the agent 1 is $x^1 = (x^1_0, x^1_1, x^1_2) = (0, 0.879, 1.021)$. For the agent 2, the new allocation is $x^2 = (x^2_0, x^2_1, x^1_2) = (0.5, 0.924, 0.9)$. The amount of promises are traded is $\psi^1 = \phi^2 = 0.97$ As expected, the lender becomes better off, and the borrower's welfare does not change.

2.4 Concluding remarks

In collateral equilibrium, not only are allocations are not Pareto-efficient, they are not constraint efficient in some cases. Hence, the question that arises is whether a government intervention can lead to a Preto-improvement. In a collateral equilibrium model, how the government redistribute income in the first period is irrelevant if the market clearing prices do not change. In this instance, the allocation cannot Pareto dominate the old equilibrium allocation. The majority of the literature focuses on the restrictions in relation to borrowing and enforcing regulations on collateral requirements when the future spot prices change. With complete markets, limiting borrowing leads to a Pareto-improvement. However, restricting borrowing is sub-optimal with incomplete markets. If the central bank purchases risky assets, both Pareto-improving and sub-optimal equilibria can occur depending on how collateral constraints bind.

The present paper takes an alternative approach, introducing taxation on sold assets in the

[2] To find the above equilibrium, we solve for three variables in the three systems of the equations. The first equation is the first-order condition for purchasing a promise by the lender: $q = \pi_1 \frac{1}{x^2_1} D_2 + \pi_2 \frac{1}{x^2_2} D_2$. The second equation is the first-order condition of holding durable goods as an asset by the borrower: $p - q = \pi_1 \frac{1}{x^1_1}(D_1 - D_2)$. The third equation is the first-order condition of the lender to hold durable goods: $p = \pi_1 \frac{1}{x^2_1} D_1 + \pi_2 \frac{1}{x^2_2} D_2$. Finally, we can check if the borrower sells the promises, for this to be the case: $q > \pi_1 \frac{1}{x^1_1} D_2 + \pi_2 \frac{1}{x^1_2} D_2 +$.

first period and redistributing taxes as a lump-sum government transfer in the second period. The purpose is to, examine the welfare effect of the intervention. The current paper considers a two-period collateral equilibrium model with a single consumption good per state and a durable good serving as collateral. Taxation of asset sale limits the borrowing power of an agent who sells the asset. The demand for the collateral decreases, because the income effect in the first period dominates the substitutions effect. The lump-sum transfer from the government compensates for the welfare loss from the decline in the demand of the durable good. Thus, the expected utility of a borrower does not change. The lender benefits from the substitution effect in the first period and purchases more of the durable good. More durable good implies increased income in the second period, so the lender becomes better off. The paper shows that both prices of the asset and the durable good decrease.

As a policy implication, the paper suggests that the removal of tax incentives to purchase a house with a mortgage decreases the price of the house. The welfare of the borrowers who use credit to purchase houses does not change because of the government transfer. However, the other agents who do not use credit to buy houses benefit from decreasing house prices.

Chapter 3

Comment on Welfare Improvement in Incomplete Market with Binding Collateral Constraint

3.1 Introduction

Chapter 2 scrutinised the welfare effect of taxation in an economy with incomplete markets and binding collateral constraints. When markets are incomplete, the set of payoffs available via trade in the security markets cannot span the whole consumption space. Thus, no perfect insurance exist with incomplete markets. In addition, when the collateral constraints are binding and markets are incomplete, a market equilibrium is not Pareto efficient. Chapter 2 shows that collecting tax on issued security in the first period and redistributing tax revenue as a lump-sum transfer in the second period leads to a Pareto improvement. For issuers of the taxed security, the income effect in the first period dominates the substitution effect, so they borrow less. Thus, borrowers' demand for the durable good decreases. As such, in the second period, the borrowers hold less of the durable good used as collateral. The lump-sum transfer compensates for the negative income effect induced by purchasing less of the durable good. Therefore, the expected utility of borrowers does not change. However, other agents benefit from a decrease in the durable good price and instead purchase more of the durable good; increasing their consumption in the second period.

The model presented in Chapter 2 has a limitation as the durable good does not have a spot market in the second period. Suppose that the durable good is traded in the second period. Thus, introducing a tax on an issued security changes the relative price of the durable good. Consequently, the collateral value change. Thus, both substitution effect and income effect may change the welfare implication of Theorem 2 in Chapter 2.

The current paper provides an example to examine the welfare implication of introducing a tax when the durable good is traded on a spot market in the second period. Similar to Chapter 2, taxation on issued security decreases borrowing and shifts the durable good's demand to the left. However, consumption of both the durable good and the perishable good does not change, with the lump-sum transfer of government has a positive income effect. Although, the substitution effect and a changes in the relative price of the durable good cancel out, the positive income effect of the lump-sum transfer prevails. Therefore, the other agents benefit from the price decrease of the durable good, purchasing more of the durable good in the first period. Thus, their consumption in the second-period, and the value of a utilitarian social welfare function increases.

The paper is organised as follows: Section 2 describes the model and states all assumptions, Section 3 states and proves propositions about the effect of taxation on agents' welfare. Section 4 presents conclusion.

3.2 Model

This Section presents a two-period, two states model to consider the welfare effect of taxation on an economy with collateral. Most of the features in this model are similar to the model presented in Chapter 2, although there are three differences between two model discussed. First, agents consumes the durable good. The utility function of an agent is time and state separable. Thus, an agent's utility function takes into account of both perishable and durable goods in period 0 and each state of period 1. Second, each state has a spot market in period 1, and agents trade both perishable and durable goods in period 1. Therefore, both durable and perishable goods have spot prices in period 1.

Finally, borrowers deliver the minimum of the security payoff and the collateral value in each state of period 1. Two factors determine the collateral value: The collateral requirement for issuing one unit of the security, and the relative price of the durable good. Therefore, a tax changes the delivery function of the security. A formal specification of the model is presented below.

The model is two-period pure exchange general equilibrium model, $t = 0, 1$. There are two states in this economy, $S = 2$, which are denoted by $\mathcal{S} = \{U, D\}$, with a finite number of agents denoted by $\mathcal{I} = \{1, \ldots, I\}$. A total of two goods exist in the economy. Vector $x^i = \begin{pmatrix} x^i_0 \\ x^i_U \\ x^i_D \end{pmatrix} \in \mathbb{R}^6$ denotes the consumption vector for period 0, and all states of period 1, where $x^i_0 = \begin{pmatrix} x^i_{0F} \\ x^i_{0H} \end{pmatrix}$, and $x^i_s = \begin{pmatrix} x^i_{sF} \\ x^i_{sH} \end{pmatrix}$ for each state $s \in \mathcal{S}$. Subscript F indicates a perishable good and the subscript H indicates a durable good. A perishable good is a good that appears in only one period and one state. A durable good is a good that an agent consumes in period 0, but also appears in the two states in period 1. There is a storage technology represented by $Y_s \in \mathbb{R}^2$, that transforms one unit of the durable good in period 0 into $Y_s = (Y_{sF}, Y_{sH})$ units of the perishable and the durable good in state s. This paper assumes that for each state s, the storage technology will yield one unit of the durable good $Y_s = (0, 1)$. The above definition of a durable good is taken from Araújo et al. (2012). In Geanakoplos and Zame (2014) the durable good is considered as a production technology that acquires the durable good as an input in the first period and produces the durable good in some states in period 1 with a given rate of depreciation. Thus, the assumption of $Y_s = (0, 1)$ for all states can be interpreted as a durable good production technology without depreciation. In period 0, and each state of period 1 there are a spot markets for goods. Vector $p \in \mathbb{R}^6$ indicates spot prices of goods. The vector $e^i = \begin{pmatrix} e^i_0 \\ e^i_U \\ e^i_D \end{pmatrix} \in \mathbb{R}^6$ denotes endowments of agent i of both the durable good and the perishable good, where $e_0^i = \begin{pmatrix} e_{0F}^i \\ e_{0H}^i \end{pmatrix} \in \mathbb{R}^2$, and $e_s^i = \begin{pmatrix} e_{sF}^i \\ e_{sH}^i \end{pmatrix} \in \mathbb{R}^2$ for each state $s \in \mathcal{S}$. An agent i has a quasi-linear von Neumann-Morgenstern expected utility function, $U^i : \mathbb{R}^6 \to \mathbb{R}$, which can be written as:

$$U^i(x^i) = x_{0F}^i + x_{0H}^i + \pi_U u^i(x_{UF}^i, x_{UH}^i) + \pi_D u^i(x_{DF}^i, x_{DH}^i) \tag{3.1}$$

where, $x^i \in \mathbb{R}^6$ is the consumption vector for agent i. In this model, agents have the same prior probabilities of states. The prior probability of state s is denoted by π_s. An agent i satisfies the following assumptions:

i. $u^i : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, strictly concave, and twice continuously differentiable on $\mathbb{R}_+$.

ii. $\sum_{i \in \mathcal{I}} e_0^i > 0$.

iii. $\sum_{i \in \mathcal{I}} e_s^i + Y_s e_0^i > 0. \qquad \forall s \in \mathcal{S}$

Assume that one security is introduced into the economy. Thus, markets are incomplete. q denotes the price of the security in period 0. Assume that the security pays the price of one unit of the perishable goods for both states. $A = \begin{pmatrix} p_{DF} \\ p_{UF} \end{pmatrix} \in \mathbb{R}_+^2$ denotes the payoff of the security across states. By normalising prices of the perishable good the payoff is independent of the states, and the security pays one unite of the perishable good in each state. An agent has to hold collateral to issue the security. Here, the durable good can be used as collateral, and a borrower needs to hold $c > 0$ unit of the durable good to issue one unit of the security. Borrowers can default on their promise in state s whenever the market value of the collateral is less than the payoff of the security in that state. Thus, a delivery D_s for a state s is defined as the minimum of the payoff of the security and the value of the durable good in that state, $D_s = \min\{p_{sF}, p_{sH}c\}$. Furthermore, $\psi^i \in \mathbb{R}_+$ denotes the number of units of the security that are issued by an agent i, and $\varphi^i \in \mathbb{R}_+$ denotes the number of units of the security that are purchased by agent i.

The inequality (3.2) below specifies the period 0 budget constraint. The government transfer below is denoted by g_0^i, where $g_0^i = \tau q \psi^i$ and the tax rate is satisfies $0 \leq \tau \leq 1$.

$$x_{0\mathrm{F}}^i + p_{0\mathrm{H}} x_{0\mathrm{H}}^i + q\varphi^i + g_0^i \leq e_{0\mathrm{F}}^i + p_{0\mathrm{H}} e_{0\mathrm{H}}^i + q\psi^i \tag{3.2}$$

The inequality (3.3) below describes the collateral constraint. It stipulates that an agent needs to hold c unit of the durable good to issue one unit of the security.

$$c\psi^i \leq x_{0\mathrm{H}}^i \tag{3.3}$$

The inequality (3.4) below specifies the state s budget constraints in period 1. T^i denotes the government transfer for each states in period 1, and p_s is a vector of prices of the perishable good and the durable good. The price of the perishable good for both states is normalised, $p_{sF} = 1$.

$$p_s \cdot x_{\mathrm{s}}^i + \psi^i D_{\mathrm{s}} \leq p_s \cdot e_{\mathrm{s}}^i + \varphi^i D_{\mathrm{s}} + p_s \cdot Y_s x_0^i + T^i \tag{3.4}$$

The budget set for an agent i can be written as:

$$\mathcal{B}^i(p,q) = \Bigg\{ (x^i, \varphi^i, \psi^i) \in \mathbb{R}^{2(S+1)} \times \mathbb{R} \times \mathbb{R} \quad s.t. \tag{3.5}$$
$$x_{0F}^i + p_{0H} x_{0H}^i + g_0^i \leq e_{0F}^i + p_{0H} e_{0H}^i + q(\psi^i - \varphi^i);$$
$$c\psi^i \leq x_{0\mathrm{H}}^i;$$
$$p_s \cdot x_{\mathrm{s}}^i + \psi^i D_{\mathrm{s}} \leq p_s \cdot e_{\mathrm{s}}^i + \varphi^i D_{\mathrm{s}} + p_s \cdot Y_s x_0^i + T^i \Bigg\}$$

Given $p \in \mathbb{R}_{++}^6$ and $q \in \mathbb{R}_+$, agent i maximises his expected utility subject to the budget set, $\max_{(x^i,\psi^i,\varphi^i)\in\mathcal{B}^i} U^i(x^i)$. Utility functions of agents, $U = \big(U^i\big)_{i\in\mathcal{I}}$, endowments of agents, $e = (e^i)_{i\in\mathcal{I}}$, and the security structure, (A, c) constitute an economy with collateral $\mathcal{E}_{CE} = \big(U, e, (A, c)\big)$.

Assuming that all tax revenue collected by the government in period 0 is distributed as a government transfer in period 1, the government transfer can be written as:

$$\sum_i g_0^i = \sum_i T^i \tag{3.6}$$

The definition of a collateral equilibrium differs from Chapter 2. The durable good has a spot market in each state of period 1, and it is consumed by agents. Thus, there is a market clearing condition for the durable good for each state in period 1. Furthermore, tax revenue is transferred in units of the perishable good in period 1, thus, the government transfer appears in the market clearing conditions for the perishable good for each state in period 1.

Definition 3. *Given a collateral economy specified by* $\mathcal{E}_{CE} = \big(U, e, (A, c)\big)$, *an allocation* $(\hat{x}, \hat{\varphi}, \hat{\psi}) = (\hat{x}^i, \hat{\varphi}^i, \hat{\psi}^i)_{i\in\mathcal{I}}$ *and a price vector* $(\hat{p}, \hat{q})$ *constitute a collateral equilibrium if:*

(i) $(\hat{x}^i, \hat{\varphi}^i, \hat{\psi}^i)$ *solve the agents' problems.* $(U^i(x^i) \leq U^i(\hat{x}^i) \quad \forall x^i \in \mathcal{B}(\hat{p}, \hat{q}))$

(ii) $\sum_{i=1}^{I}(\hat{x}_0^i - e_0^i) = 0.$

(iii) $\sum_{i=1}^{I}(\hat{x}_{0H}^i - e_{0H}^i) = 0.$

(iv) $\sum_{i=1}^{I}(\hat{x}_s^i - e_s^i - Y_s e_0^i - \begin{bmatrix} T^i \\ 0 \end{bmatrix}) = \mathbf{0}. \qquad s = U, D$

(v) $\sum_{i=1}^{I}(\hat{\varphi}^i - \hat{\psi}^i) = \mathbf{0}.$

3.3 Taxation effect on welfare in binomial economy

Theorem 2 of Chapter 2 shows that raising the tax increases the consumption of some states and decreases the consumption of others. However, the expected utility of borrowers does not change. Proposition 1 in this section differs from Theorem 2 of Chapter 2. When the durable good is traded in the second period, taxation affects the relative prices of the durable good in all states. Changes in the relative prices can change the value of the collateral and the output of the delivery function. The substitution effect of the relative price changes may have welfare implications. Proposition 1 asserts that substitution and income effects cancel each other out in the second period if the tax increases. Thus, the borrower's consumption of durable and perishable goods does not change. On the other hand, lenders purchase more of the perishable good in the second period. As a result, increasing taxes increases social welfare. However, it is not clear that the new allocation is Pareto-dominates the old equilibrium allocation.

The proof of Proposition 1 is similar to Theorem 2 of Chapter 2. The Jacobian matrix of a borrower is derived from first order conditions. Then, the effects of taxation on a borrower's consumption is calculated by the chain rule. Finally, market clearing conditions are used to show that if the tax increases, then utilitarian social welfare increases. Lemma 1 and Lemma 2 below are used in the proof of Proposition 1. The proof of Lemma 2 can be found on p.650 of Boyd and Vandenberghe (2004).

Lemma 3. *An agent does not issue and purchase a security at the same time.*

Proof. Assume there is an agent i, who issues and purchases a security at the same time. Thus, both $\psi^i > 0$, and $\phi^i > 0$ are strictly positive. The first-order conditions for an agent's optimization problem imply the following equations:

$$\mu_0^i(1-\tau)q - \sum_s \mu_s^i \min\{1, cp_{sH}\} = c\lambda^i, \qquad \psi^i > 0 \tag{3.7}$$

$$\mu_0^i q - \sum_s \mu_s^i \min\{1, cp_{sH}\} = 0 \qquad \phi^i > 0 \tag{3.8}$$

where, $\mu_0^i > 0$ is the multiplier of the period 0 budget constraint, and $\mu_s^i > 0$ is the multiplier of the state s budget constraint, and $\lambda^i > 0$ is the multiplier for the collateral constraint. Both equations (3.7), and (3.8) imply that $-\mu_0^i \tau q = c\lambda^i$, which is a contradiction. ∎

Lemma 4. *Assume that the square matrix $\mathcal{M}$ is partitioned into blocks as follows:*

$$\mathcal{M} = \overbrace{\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix}}^{\mathbb{R}^m \;\; \mathbb{R}^n} \begin{matrix} \}\, \mathbb{R}^m \\ \}\, \mathbb{R}^n \end{matrix} \tag{3.9}$$

If $\mathcal{D}$ is non-singular, the inverse of $\mathcal{M}$ in terms of Schur-complement of $\mathcal{D}$ in $\mathcal{M}$ (i.e., $\mathcal{M} \setminus \mathcal{D} =$

$\mathcal{A} - \mathcal{B}\mathcal{D}^{-1}\mathcal{C}$) can be written as follows:

$$\mathcal{M}^{-1} = \begin{pmatrix} \big(\mathcal{M} \setminus \mathcal{D}\big)^{-1} & -\big(\mathcal{M} \setminus \mathcal{D}\big)^{-1}\mathcal{B}\mathcal{D}^{-1} \\ -\mathcal{D}^{-1}\mathcal{C}\big(\mathcal{M} \setminus \mathcal{D}\big)^{-1} & \mathcal{D}^{-1} + \mathcal{D}^{-1}\mathcal{C}\big(\mathcal{M} \setminus \mathcal{D}\big)^{-1}\mathcal{B}\mathcal{D}^{-1} \end{pmatrix} \tag{3.10}$$

Proposition 2. *Suppose that $\langle(x^i, \varphi^i, \psi^i); (p, q)\rangle$ is a collateral equilibrium. If the tax rate on a security increases, utilitarian social welfare increases.*

Proof. Lemma 3 implies that in an equilibrium $\varphi^i = 0$ for an agent who issues the security. Thus, if ψ^i is replaced by $\dfrac{x^i_{0H}}{c}$, the Lagrangian of the borrower optimization problem can be written as follows:

$$\begin{aligned} \mathcal{L}^i(x^i, \mu^i) := U^i(x^i) - \mu^i_0\Big((x^i_{0F} - e^i_{0F}) - p_{0H}e^i_{0H} + (p_{0H} - \frac{q(1-\tau)}{c})x^i_{0H}\Big) \\ - \sum_{s\in\mathcal{S}} \mu^i_s\Big(p_s \cdot (x^i_s - e^i_s) + (\frac{\min\{1, cp_{sH}\}}{c} - p_{sH})x^i_{0H} - T^i\Big) \end{aligned} \tag{3.11}$$

where $\mu^i = (\mu^i_0, \mu^i_U, \mu^i_D)$ are the Lagrange multipliers for the borrower.

The first-order conditions of the borrower's maximization problem are as follows:

$$\begin{aligned} \mathcal{D}_{x^i_{sF}}\mathcal{L}^i = \mathbf{0} \Leftrightarrow &\quad \pi_s\partial_{x^i_{sF}}u^i(x^i_s) - \mu^i_s = 0 \\ \mathcal{D}_{x^i_{sH}}\mathcal{L}^i = \mathbf{0} \Leftrightarrow &\quad \pi_s\partial_{x^i_{sH}}u^i(x^i_s) - p_{sH}\mu^i_s = 0 \\ \mathcal{D}_{x^i_{0F}}\mathcal{L}^i \leq 0 \Leftrightarrow &\quad 1 - \mu^i_0 \leq 0, \qquad (1-\mu^i_0)x^i_{0F} = 0 \\ \mathcal{D}_{x^i_{0H}}\mathcal{L}^i = 0 \Leftrightarrow &\quad 1 - \mu^i_0\hat{q} + \textstyle\sum_{s\in\mathcal{S}}\mu^i_s\hat{A}_s = 0 \\ \mathcal{D}_{\mu^i_s}\mathcal{L}^i = \mathbf{0} \Leftrightarrow &\quad -\Big(p_s \cdot (x^i_s - e^i_s) - \hat{A}_s x^i_{0H} - T^i\Big) = 0 \end{aligned} \tag{3.12}$$

where $\hat{q} = \big(p_{0H} - \dfrac{q(1-\tau)}{c}\big)$, and $\hat{A}_s = \big(p_{sH} - \dfrac{\min\{1, cp_{sH}\}}{c}\big)$.

At a optimal solution to the maximization problem of a borrower i the marginal utility of consumption of a perishable good per unit of price is equal to the marginal utility of consumption of a durable good per unit of price in period 0. Thus, the following equation holds:

$$\mu_0 = \frac{\partial_{x^i_{0F}} U^i(x^i)}{p_{0F}} = \frac{\partial_{x^i_{0H}} U^i(x^i)}{\hat{q}} \tag{3.13}$$

Equation (3.13) and $p_{0F} = 1$ imply that $\mu^i_0 = 1$. Since, the multiplier of period 0 is constant, and the perishable good consumption can be derived by replacing the durable good in the period 0 budget constraint, the first-order conditions can be rewritten as follows:

$$\begin{aligned}
\mathcal{D}_{x^i_{sF}} \mathcal{L}^i = 0 \Leftrightarrow &\quad \partial_{x^i_{sF}} U^i(x^i) - \mu^i_s = 0 \\
\mathcal{D}_{x^i_{sH}} \mathcal{L}^i = 0 \Leftrightarrow &\quad \partial_{x^i_{sH}} U^i(x^i) - p_{sH}\mu^i_s = 0 \\
\mathcal{D}_{x^i_{0H}} \mathcal{L}^i = 0 \Leftrightarrow &\quad 1 - \hat{q} + \textstyle\sum_s \mu^i_s \hat{A}_s = 0 \\
\mathcal{D}_{\mu^i_s} \mathcal{L}^i = 0 \Leftrightarrow &\quad -\Big(p_s \cdot (x^i_s - e^i_s) - \hat{A}_s x^i_{0H} - T^i\Big) = 0
\end{aligned} \tag{3.14}$$

Define the mapping $\mathcal{F}_i : \Phi \times \mathbb{R}^4 \times \mathbb{R} \times \Theta \to \Gamma$ below to derive the Jacobian matrix for the first-order conditions, where $\Phi = \mathbb{R}^4 \times \mathbb{R} \times \mathbb{R}^2$, $\Gamma = \mathbb{R}^4 \times \mathbb{R} \times \mathbb{R}^2$, and $\Theta = \mathbb{R}^6 \times \mathbb{R} \times \mathbb{R}_{++}$ are open sets, and Θ denotes the space of economies.

$$\mathcal{F}_i(\rho,p,q,\theta) = \begin{pmatrix} \mathcal{D}_{x^i_{UF}}\mathcal{L}^i \\ \mathcal{D}_{x^i_{UH}}\mathcal{L}^i \\ \mathcal{D}_{x^i_{DF}}\mathcal{L}^i \\ \mathcal{D}_{x^i_{DH}}\mathcal{L}^i \\ \mathcal{D}_{x^i_{0H}}\mathcal{L}^i \\ \mathcal{D}_{\mu^i_U}\mathcal{L}^i \\ \mathcal{D}_{\mu^i_D}\mathcal{L}^i \end{pmatrix} = \begin{pmatrix} \partial_{x^i_{UF}}U^i(x^i) - \mu^i_U \\ \partial_{x^i_{UH}}U^i(x^i) - p_{UH}\mu^i_U \\ \partial_{x^i_{DF}}U^i(x^i) - \mu^i_D \\ \partial_{x^i_{DF}}U^i(x^i) - p_{DH}\mu^i_D \\ 1 - \hat{q} + \sum_s \mu^i_s \hat{A}_s \\ -\Big(p_U \cdot (x^i_U - e^i_U) - \hat{A}_U x^i_{0H} - T^i\Big) \\ -\Big(p_D \cdot (x^i_D - e^i_D) - \hat{A}_D x^i_{0H} - T^i\Big) \end{pmatrix} \tag{3.15}$$

In $\mathcal{F}_i(\rho,p,q,\theta)$ of Equation (3.15), $\rho = (x^i_U, x^i_D, x^i_{0H}, \mu^i_U, \mu^i_D)$ denotes the tuple of arguments, and $\theta = (e^i, \tau, c)$ denotes the tuple of parameters. Assume that the parameters vector θ is given. If ρ solve a borrower's problem, then the first-order conditions (3.14) hold, and consequently $\mathcal{F}_i(\rho,p,q,\theta) = 0$. The Jacobian matrix for a borrower i which is denoted by $\mathcal{D}_\rho\mathcal{F}_i$ has the following form:

$$\mathcal{D}_\rho\mathcal{F}_i = \begin{pmatrix} \partial^2_{x^i_{UF}}U^i & \partial^2_{x^i_{UF}x^i_{UH}}U^i & 0 & 0 & 0 & -1 & 0 \\ \partial^2_{x^i_{UF}x^i_{UH}}U^i & \partial^2_{x^i_{UH}}U^i & 0 & 0 & 0 & -p_{UH} & 0 \\ 0 & 0 & \partial^2_{x^i_{DF}}U^i & \partial^2_{x^i_{DF}x^i_{DH}}U^i & 0 & 0 & -1 \\ 0 & 0 & \partial^2_{x^i_{DF}x^i_{DH}}U^i & \partial^2_{x^i_{DH}}U^i & 0 & 0 & -p_{DH} \\ 0 & 0 & 0 & 0 & 0 & \hat{A}_U & \hat{A}_D \\ -1 & -p_{UH} & 0 & 0 & \hat{A}_U & 0 & 0 \\ 0 & 0 & -1 & -p_{DH} & \hat{A}_D & 0 & 0 \end{pmatrix} \tag{3.16}$$

The Jacobian matrix in (3.16) can be written as follows:

$$\mathcal{D}_\rho \mathcal{F}_i = \begin{array}{c} \overbrace{}^{\mathbb{R}^S} \quad \overbrace{}^{\mathbb{R}^S} \quad \overbrace{}^{\mathbb{R}} \quad \overbrace{}^{\mathbb{R}} \quad \overbrace{}^{\mathbb{R}} \\ \left(\begin{array}{ccccc} \mathcal{V}_U^i & \mathbf{0} & \mathbf{0} & -p_U^T & \mathbf{0} \\ \mathbf{0} & \mathcal{V}_D^i & \mathbf{0} & \mathbf{0} & -p_D^T \\ \mathbf{0} & \mathbf{0} & 0 & \hat{A}_U & \hat{A}_D \\ -p_U & \mathbf{0} & \hat{A}_U & 0 & 0 \\ \mathbf{0} & -p_D & \hat{A}_D & 0 & 0 \end{array} \right) \end{array} \begin{array}{l} \} \, \mathbb{R}^S \\ \} \, \mathbb{R}^S \\ \} \, \mathbb{R} \\ \} \, \mathbb{R} \\ \} \, \mathbb{R} \end{array} \tag{3.17}$$

where $\mathcal{V}_s^i = \begin{pmatrix} \partial^2_{x^i_{sF}} U^i & \partial^2_{x^i_{sF} x^i_{sH}} U^i \\ \partial^2_{x^i_{sF} x^i_{sH}} U^i & \partial^2_{x^i_{sH}} U^i \end{pmatrix}$, and $p_s = (1, p_{sH})$. In order to use the implicit function theorem and the chain rule, $\mathcal{D}_\rho \mathcal{F}_i$ needs to be non-singular. If $\mathcal{D}_\rho \mathcal{F}_i$ has full rank, then it is non-singular. Using elementary matrix operations $\mathcal{D}_\rho \mathcal{F}_i$ can be transformed as follows:

$$\overline{\mathcal{D}_\rho \mathcal{F}_i} = \begin{array}{c} \overbrace{}^{\mathbb{R}^S} \quad \overbrace{}^{\mathbb{R}^S} \quad \overbrace{}^{\mathbb{R}} \quad \overbrace{}^{\mathbb{R}} \quad \overbrace{}^{\mathbb{R}} \\ \left(\begin{array}{ccccc} \mathcal{V}_U^i & \mathbf{0} & \mathbf{0} & -p_U^T & \mathbf{0} \\ \mathbf{0} & \mathcal{V}_D^i & \mathbf{0} & \mathbf{0} & -p_D^T \\ \mathbf{0} & \mathbf{0} & 0 & \hat{A}_U & \hat{A}_D \\ \mathbf{0} & \mathbf{0} & \hat{A}_U & -\left(\partial^2_{x^i_{UF}} U^i\right)^{-1} & 0 \\ \mathbf{0} & \mathbf{0} & \hat{A}_D & 0 & -\left(\partial^2_{x^i_{DF}} U^i\right)^{-1} \end{array} \right) \end{array} \begin{array}{l} \} \, \mathbb{R}^S \\ \} \, \mathbb{R}^S \\ \} \, \mathbb{R} \\ \} \, \mathbb{R} \\ \} \, \mathbb{R} \end{array} \tag{3.18}$$

Here, the matrix $\overline{\mathcal{D}_\rho \mathcal{F}_i}$ was obtained by multiplying the first and the third row of the matrix $\mathcal{D}_\rho \mathcal{F}_i$ by $\left(\partial^2_{x^i_{sF}} U^i\right)^{-1}$, and adding them to the last two rows. The matrix $\overline{\mathcal{D}_\rho \mathcal{F}_i}$ has full rank, so the matrix $\mathcal{D}_\rho \mathcal{F}_i$ has full rank. Thus, both matrices have an inverse. Hoffman and Kunze (1971) (p.26, Theorem 12) state that if **E** is constructed by multiplication of two elementary operations, the inverse of the $\mathcal{D}_\rho \mathcal{F}_i$ can be calculated as follows:

$$(\mathcal{D}_\rho \mathcal{F}_i)^{-1} = (\overline{\mathcal{D}_\rho \mathcal{F}_i})^{-1} \times \mathbf{E} \tag{3.19}$$

$$\mathbf{E} = \begin{pmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 1 & 0 & 0 \\ \left(\partial^2_{x^i_{UF}} U^i\right)^{-1} \mathcal{V}^i_{U1} & \mathbf{0} & 0 & 1 & 0 \\ \mathbf{0} & \left(\partial^2_{x^i_{DF}} U^i\right)^{-1} \mathcal{V}^i_{D1} & 0 & 0 & 1 \end{pmatrix} \tag{3.20}$$

where $\mathcal{V}^i_{s1}$ is the first row of the matrix $\mathcal{V}^i_s$. The matrix $(\mathcal{D}_\rho \mathcal{F}_i)^{-1}$ can be obtained by using the Lemma (4). The matrix has the following form:

$$\left(\overline{\mathcal{D}_\rho \mathcal{F}_i}\right)^{-1} = \begin{pmatrix} \overbrace{(\mathcal{V}^i_U)^{-1}}^{\mathbb{R}^S} & \overbrace{\mathbf{0}}^{\mathbb{R}^S} & \overbrace{\mathcal{K}_1}^{\mathbb{R}} & \overbrace{\mathcal{K}_2}^{\mathbb{R}} & \overbrace{\mathcal{K}_3}^{\mathbb{R}} \\ \mathbf{0} & (\mathcal{V}^i_D)^{-1} & \mathcal{K}_4 & \mathcal{K}_5 & \mathcal{K}_6 \\ \mathbf{0} & \mathbf{0} & \frac{1}{\gamma} & \frac{\hat{A}_U \partial^2_{x^i_{UF}} U^i}{\gamma} & \frac{\hat{A}_D \partial^2_{x^i_{DF}} U^i}{\gamma} \\ \mathbf{0} & \mathbf{0} & \frac{\hat{A}_U \partial^2_{x^i_{UF}} U^i}{\gamma} & -\frac{a(\hat{A}_D)^2}{\gamma} & \frac{a\hat{A}_D \hat{A}_U}{\gamma} \\ \mathbf{0} & \mathbf{0} & \frac{\hat{A}_D \partial^2_{x^i_{DF}} U^i}{\gamma} & \frac{a\hat{A}_D \hat{A}_U}{\gamma} & -\frac{a(\hat{A}_U)^2}{\gamma} \end{pmatrix} \begin{matrix} \}\ \mathbb{R}^S \\ \}\ \mathbb{R}^S \\ \}\ \mathbb{R} \\ \}\ \mathbb{R} \\ \}\ \mathbb{R} \end{matrix} \tag{3.21}$$

where, $\gamma = \hat{A}_U^2 \partial^2_{x^i_{UF}} U^i + \hat{A}_D^2 \partial^2_{x^i_{DF}} U^i$, $a = \partial^2_{x^i_{UF}} U^i \partial^2_{x^i_{DF}} U^i$, $(\mathcal{V}^i_s)^{-1} = \frac{1}{\det(\mathcal{V}^i_s)} \begin{pmatrix} \partial^2_{x^i_{sH}} U^i & -\partial^2_{x^i_{sF} x^i_{sH}} U^i \\ -\partial^2_{x^i_{sF} x^i_{sH}} U^i & \partial^2_{x^i_{sF}} U^i \end{pmatrix}$, and $\mathbf{K} = \begin{pmatrix} \mathcal{K}_1 & \mathcal{K}_2 & \mathcal{K}_3 \\ \mathcal{K}_4 & \mathcal{K}_5 & \mathcal{K}_6 \end{pmatrix}$ is following matrix:

$$\mathbf{K} = -\begin{pmatrix} (\mathcal{V}_U^i)^{-1} & \mathbf{0} \\ \mathbf{0} & (\mathcal{V}_D^i)^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{0} & -p_U^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -p_D^T \end{pmatrix} \mathcal{M}^{-1} \tag{3.22}$$

where, the $\mathcal{M}^{-1}$ matrix is the following matrix:

$$\mathcal{M}^{-1} = \begin{pmatrix} \frac{1}{\gamma} & \frac{\hat{A}_U \partial^2_{x^i_{UF}} U^i}{\gamma} & \frac{\hat{A}_D \partial^2_{x^i_{DF}} U^i}{\gamma} \\ \frac{\hat{A}_U \partial^2_{x^i_{UF}} U^i}{\gamma} & -\frac{a(\hat{A}_D)^2}{\gamma} & \frac{a\hat{A}_D \hat{A}_U}{\gamma} \\ \frac{\hat{A}_D \partial^2_{x^i_{DF}} U^i}{\gamma} & \frac{a\hat{A}_D \hat{A}_U}{\gamma} & -\frac{a(\hat{A}_U)^2}{\gamma} \end{pmatrix} \tag{3.23}$$

In equations (3.24) below the implicit function theorem is used to calculate the effect of taxation on consumption of goods in different states, and consumption of the durable good in the first period:

$$\begin{aligned} \frac{dx^i_{0H}}{d\tau} &= \partial_\tau x^i_{0H} + \partial_{T^i} x^i_{0H} \partial_\tau T^i \\ \partial_\tau x^i_{0H} &= -(\mathcal{D}_{x^i_{0H}} \mathcal{F}_i)^{-1} \mathcal{D}_\tau \mathcal{F}_i \\ \partial_{T^i} x^i_{0H} &= -(\mathcal{D}_{x^i_{0H}} \mathcal{F}_i)^{-1} \mathcal{D}_{T^i} \mathcal{F}_i \end{aligned} \tag{3.24}$$

where, $\mathcal{D}_\tau \mathcal{F}_i = (\mathbf{0}, \mathbf{0}, -q\psi^i, 0, 0)^T$, $\mathcal{D}_{T^i} \mathcal{F}_i = (\mathbf{0}, \mathbf{0}, 0, 1, 1)^T$, $\partial_\tau T^i = q\psi^i$, and $\mathbf{0} \in \mathbb{R}^2$. Thus, $\frac{dx^i_{0H}}{d\tau}$ can be written as follows:

$$\frac{dx^i_{0H}}{d\tau} = -(\frac{-q\psi^i}{\gamma} + \frac{\hat{A}_U \partial^2_{x^i_{UF}} U^i}{\gamma} q\psi^i + \frac{\hat{A}_D \partial^2_{x^i_{DF}} U^i}{\gamma} q\psi^i) < 0 \tag{3.25}$$

The implicit function theorem can be used to calculate the consumption change with respect to taxation, $\left(\mathcal{D}_\tau x^i_s = -(\mathcal{D}_{x^i_s} \mathcal{F}_i)^{-1} (\mathcal{D}_\tau \mathcal{F}_i + \mathcal{D}_{T^i} \mathcal{F}_i q\psi^i)\right)$. Thus, the vectors of consumption changes in different states can be written as follows:

$$\begin{pmatrix} \mathcal{D}_\tau x^i_U \\ \mathcal{D}_\tau x^i_D \end{pmatrix} = \begin{pmatrix} (\mathcal{V}^i_U)^{-1} & \mathbf{0} \\ \mathbf{0} & (\mathcal{V}^i_D)^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{0} & -p_U^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -p_D^T \end{pmatrix} \mathcal{M}^{-1} \begin{pmatrix} -q\psi^i \\ q\psi^i \\ q\psi^i \end{pmatrix} \tag{3.26}$$

The first-order conditions imply that the marginal utility of the consumption of the durable good per unit of price is equal to the marginal utility of the perishable good per unit of its price, $(\partial_{x^i_{sH}} U^i = p_{sH} \partial_{x^i_{sF}} U^i)$. Therefore, $-(\mathcal{V}^i_s)^{-1} p_s^T$ is equal to $\mathbf{0}$, which implies that $\mathcal{D}_\tau x^i_s = \mathbf{0}$.

Thus, taxation reduces the amount of durable goods that a borrower purchases. However, because the durable good can be traded in the period 1, the substitution effect and the income effect cancel each other out and a borrower does not change his consumption.

Market clearing conditions implies that consumption of the perishable good for a lender increases in both states. However, consumption of the durable good does not change. The effect of taxation on welfare can be calculated as follows:

$$\mathcal{D}_\tau \mathcal{W}(U^1, \cdots, U^I) = \sum_{i \in \mathcal{I}} \nabla_{x^i} U^i(x^i) \cdot \mathcal{D}_\tau x^i = \tag{3.27}$$

$$\sum_{i \in \mathcal{I}} \partial_\tau x^i_{0F} + \partial_\tau x^i_{0H} + \sum_{s \in \mathcal{S}} \left(\partial_{x^i_{sF}} U^i(x^i) \partial_\tau x^i_{sF} + \partial_{x^i_{sH}} U^i(x^i) \partial_\tau x^i_{sH} \right) =$$

$$\sum_{i \in \mathcal{I}} \partial_\tau x^i_{0F} + \sum_{i \in \mathcal{I}} \partial_\tau x^i_{0H} + \sum_{i \in \mathcal{I}} \sum_{s \in \mathcal{S}} \left(\partial_{x^i_{sF}} U^i(x^i) \partial_\tau x^i_{sF} + \partial_{x^i_{sH}} U^i(x^i) \partial_\tau x^i_{sH} \right) = \tag{3.28}$$

$$\sum_{i \in \mathcal{I}} \sum_{s \in \mathcal{S}} \left(\partial_{x^i_{sF}} U^i(x^i) \partial_\tau x^i_{sF} + \partial_{x^i_{sH}} U^i(x^i) \partial_\tau x^i_{sH} \right) > 0$$

The first two expressions in (3.28) are 0 because of the market clearing conditions. The last expressions is positive since a lender consumes more perishable good in period 1, and a borrower does not change his consumption. Thus, welfare increases. ■

Proposition 3. *Suppose that $\langle (x^i, \varphi^i, \psi^i); (p, q) \rangle$ is a collateral equilibrium. If the tax rate on an issued security τ increases, then then prices of the security and the durable good in period* 0 *decrease.*

Proof. The first-order conditions for a lender show that the price of a security is equal to the fundamental value of the security, $\left(q = \sum_{s\in\mathcal{S}} \mu_s^i \min\{1, cp_{sH}\}\right)$. Proposition 1 states that if the tax rate on an issued security increases, a lender increases his consumption of the perishable good in period 1. The first-order conditions imply that $(\partial_{x^i_{sF}} U(x^i) = \mu_s^i)$ and $(\partial_{x^i_{sH}} U(x^i) = p_{sH}\mu_s^i)$. If the security default in all states, the price q does not change. Otherwise, the price of the security q decreases.

Equation (3.13) implies following equation:

$$1 = \frac{1 + \sum_{s\in S} \pi_s \partial_{x_{sH}} u^i(x_s^i)}{(p_{0H} - (1-\tau)q)} \tag{3.29}$$

Proposition 1 also states that borrower does not change his consumption of both durable and perishable goods in the period 1. Therefore, the following equation holds:

$$\frac{dp_{0H}}{d\tau} = -q + (1-\tau)\frac{dq}{d\tau} \tag{3.30}$$

Since, $(\frac{dq}{d\tau})$ is non-positive, the price of the durable good in the first period p_{0H} decreases.

■

3.4 Conclusion

The current paper presents an example to examine the welfare effect of taxation on issued securities in a collateral equilibrium model when the durable good is traded on a spot market in the second period. The government collects the tax on the issued security in the first period and redistributes the tax revenue in the second period as a lump-sum transfer. As a result, the tax security shifts the borrower demand of the durable good to the left in the first period. However, since substitution and income effects cancel each other out, borrowers do not change their consumption of both durable and perishable goods in the second period. On the other hand, lenders benefit from the fall the price of the durable good in the first period and buying more of the durable good, which increases their consumption in the second period. Therefore, the utilitarian social welfare increases. Yet, the new equilibrium does not necessarily Pareto-dominates the old equilibrium. In addition, the price of the security decreases due to diminishing lenders' marginal rate of the substitutions.

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