

## Thermopower oscillations in a normal ring with one superconducting contact

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The thermopower of a normal ring with one superconducting contact has been found to oscillate in a magnetic field with a period corresponding to the magnetic flux quantum through the area of the ring. The oscillation symmetry is the same as that of the magnetoresistance. The absolute value of thermopower is of the order of 2 nV/K. These two facts suggest that the observed thermopower is due to quasiparticle thermoelectric currents in the ring rather than the giant thermopower in the superconductor-normal loops reported earlier.

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### I. INTRODUCTION

During the past two decades there has been a remarkable progress in understanding the superconducting proximity effect in mesoscopic normal metal–superconductor ( $N$ - $S$ ) structures both experimentally and theoretically (see Ref. 1 for a review and references therein). Recently, thermoelectric properties of  $N$ - $S$  systems have become a focus of experimental and theoretical investigations. After theoretical prediction of giant thermopower in  $N$ - $S$  structures,<sup>2</sup> thermopower oscillations in the geometry of the Andreev interferometer (AI) have been observed experimentally.<sup>3</sup> The Andreev interferometer is a device where a normal part is connected to a superconducting loop. The phase-coherent part of the conductance of the AI oscillates with magnetic field with a period corresponding to the magnetic flux quantum  $\Phi_0 = hc/2e$  through the area of the loop. The absolute value of the thermopower reported in various  $N$ - $S$  systems was found to exceed that in normal metals at low temperatures by more than two orders in magnitude.<sup>3–7</sup> The theory explains this giant thermopower as a result of temperature-dependent Josephson currents in the normal part of the AI induced by the applied magnetic field.<sup>8–11</sup> The second maximum in thermopower oscillations observed in Ref. 9 is explained by a long-range proximity effect similar to that in the conductance.<sup>9,11</sup> References 10 and 11 also explain the difference in the thermopower measured between  $N$  and  $S$  electrodes in the AI and that between two  $N$  electrodes reported in Ref. 6. The symmetry of these effects is that of  $\sin \phi$ , where  $\phi$  is the superconducting phase difference between the two  $N$ - $S$  contacts, so that the thermopower oscillations are  $\pi/2$  shifted with respect to the resistance oscillations which follow a  $\cos \phi$  law. However, giant thermopower oscillations of  $\cos \phi$  symmetry have been reported as well,<sup>3,7</sup> which are not explained by the theory. For a comprehensive review of the thermopower in Andreev interferometers, which includes all recent results, see Ref. 12.

In this Brief Report we introduce the geometry of a  $N$ - $S$  structure that has not yet been investigated either experimentally or theoretically, namely, a normal ring with only one superconducting contact. We observe  $\cos \phi$  thermopower oscillations and estimate the absolute value of the thermopower. The origin of the observed thermopower is discussed.

### II. EXPERIMENT

The samples were fabricated using  $e$ -beam lithography and standard processing. The geometry of the structures is shown in Fig. 1. The superconductor was placed on the extension of the  $N$  wire attached to one side of the ring. The ring diameter was 1.2  $\mu\text{m}$ . The superconductor was 60-nm-thick Al and the normal metal was 45-nm-thick Ag. The resistivity  $\rho$  of the Ag film was  $\rho = 1.3 \mu\Omega \text{ cm}$  and the diffusion constant  $D = 136 \text{ cm}^2/\text{s}$ . To obtain clean interfaces between the layers, the contact area was  $\text{Ar}^+$  plasma etched followed by the deposition of the second layer without breaking the vacuum. Contacts are numbered as follows: 1 and 6, heater wire, 2, superconducting contact, 3, normal reservoir, and 4 and 5, normal contacts.

The measurements were carried out in a  $^3\text{He}$  cryostat in the temperature range 0.28–1.5 K in magnetic fields up to 0.1 T applied perpendicular to the substrate. A four-point Wheatstone bridge was used with lock-in amplifier at the frequency 17.7 Hz to measure the magnetoresistance of the samples. For thermopower measurements a temperature gradient was created in the normal wire between the heater and

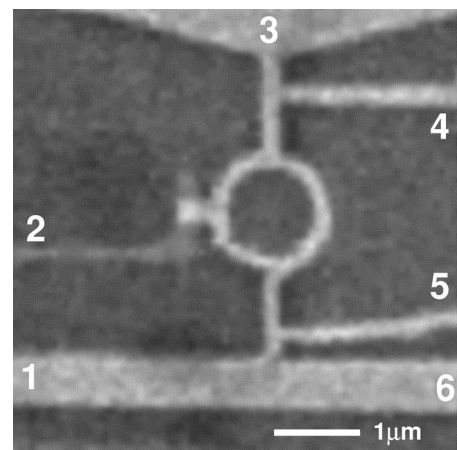


FIG. 1. Scanned electron microscope picture of a measured sample. Contacts numbered are 1 and 6, heater; 2, superconductor; 3, normal reservoir; 4 and 5, normal contacts. Superconductor, 60 nm Al; normal metal, 45 nm Ag.

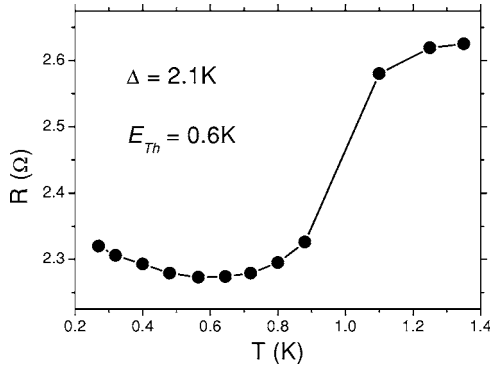


FIG. 2. Temperature dependence of the ring resistance at  $H=0$ . Current contacts are 3 and 5; potential, 2 and 4.

the reservoir by a heating current applied between contacts 1 and 6, which was a sum of dc,  $I_H$ , and ac,  $I_m$ , currents. The thermovoltage was measured between  $N$  and  $S$  contacts or between two normal contacts by the lock-in amplifier on the frequency of ac modulation.

### III. RESULTS

Figure 2 shows the temperature dependence of the ring resistance in zero magnetic field measured using current contacts 3 and 6 and potential contacts 4 and 5. The reentrance effect typical for mesoscopic proximity structures is seen.

The minimum in resistance is reached at a temperature of the order of the Thouless energy  $E_{Th} = \sqrt{\hbar}D/L^2$ , where  $L$  is the characteristic length of the diffusive transport. In our case  $E_{Th} \approx 0.6$  K corresponding to  $L=410$  nm, which is of the order of the electron phase breaking length. The superconducting gap energy obtained from Fig. 2 is  $\Delta = 1.7k_B T_c = 2.1$  K, where  $T_c$  is the superconducting transition temperature. Thus, at the base temperature of our experiment, the limit  $k_B T < E_{Th} < \Delta$  is realized.

Figure 3 shows the magnetoresistance of the ring recorded using the same contacts as for the data in Fig. 2. The oscillation period in the magnetic field  $\Delta H = 21.6$  G, corresponds to the magnetic flux quantum through the area of the loop. Measuring the dependence of the magnetoresistance oscillation as a function of temperature and the dc heating current,

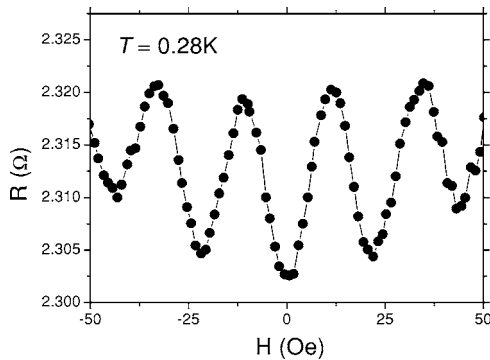


FIG. 3. Magnetoresistance of the ring at  $T=0.28$  K. Current contacts are 3 and 5, potential, 2 and 4.

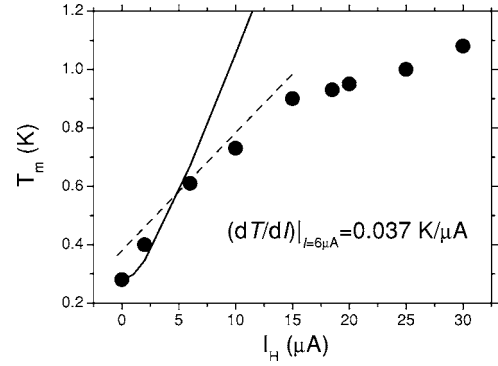


FIG. 4. Dots: effective temperature of the ring versus dc heating current contacts 1 and 6. Solid line: calculated dependence without electron-phonon coupling. Dashed line: the slope for  $I_H=6$   $\mu$ A.

the effective temperature of the ring can be obtained for every  $I_H$ , as shown in Fig. 4. Note that dependence close to that in Fig. 4 was obtained in Refs. 5 and 6 for Sb/Al samples of similar geometry.

To understand the shape of the  $T_m$  versus  $I_H$  curve in Fig. 4 it is instructive first to solve the heat transport equation neglecting electron-phonon interactions. We assume that local thermal equilibrium of the quasiparticle distribution is reached via electron-electron scattering.<sup>13</sup> This is the case when the inelastic scattering time is smaller than the diffusion time through the wire, i.e., the distance between the reservoirs is larger than electron phase breaking length. Then for the heater we have

$$\text{div}(q) = \rho_H j^2, \quad (1)$$

where  $q = -\kappa \nabla T$  is the thermal flux,  $\kappa$  is thermal conductivity,  $\rho_H$  is the resistivity of the heater, and  $j$  is the current density in the heater. The right-hand side of Eq. (1) is the heat generated by the current in unit volume per unit time. To integrate Eq. (1) we need the explicit expression for  $\kappa$ . At the lowest temperature of our experiment the Wiedemann-Franz law holds for electron-electron interactions, so that  $\kappa = L \times \sigma \times T$ , where  $\sigma$  is the electron electrical conductivity and  $L = 2.45 \times 10^{-8}$  V<sup>2</sup>/K<sup>2</sup> is the Lorentz number. The one-dimensional heat transport equation then becomes

$$-\frac{\partial}{\partial x} \left( T \frac{\partial T}{\partial x} \right) = \frac{\rho_H j^2}{L}. \quad (2)$$

Solving Eq. (2) with boundary conditions  $T=T_0$  at both ends of the heater wire, the temperature in the middle of the heater can be obtained as  $T_H = (T_0^2 + R_H^2 I_H^2 / 4L)^{1/2}$ , where  $R_H$  is the resistance of the heater between the reservoirs.

In the normal wire attached to the heater, the heat equation is the same as Eq. (2) but with zero right-hand side. The boundary conditions are  $T=T_H$  at the hot end and  $T=T_0$  at the  $N$  reservoir. The temperature in the middle of the interferometer,  $T_m$ , is equal to  $T_m = (T_0^2 + R_H^2 I_H^2 / 8L)^{1/2}$ . This dependence is shown in Fig. 4 as a solid line. One can see that this approximation is valid only for small heating currents. At higher currents electron-phonon interaction must be taken into account. This is done by adding a term proportional to  $T_e^5 - T_{ph}^5$  to the right-hand side of Eq. (2) describing heat

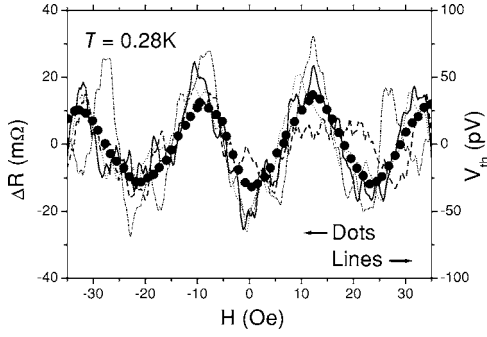


FIG. 5. Thermovoltage and magnetoresistance oscillations. Dots: amplitude of magnetoresistance oscillations. Lines: thermovoltage oscillations measured using heating current between 1 and 6, for various heating currents and potential contacts. Solid line,  $I_H=2.2 \mu\text{A}$ , potentials 2 and 5; dotted line,  $I_H=6.0 \mu\text{A}$ , potentials 2 and 5; dashed line,  $I_H=11.5 \mu\text{A}$ , potentials 2 and 5; dash-dotted line,  $I_H=6.0 \mu\text{A}$ , potentials 4 and 5.

transfer from the electron system at temperature  $T_e$  to the phonon system at  $T_{ph}$ .<sup>14</sup> As the heat dissipation into the phonon system increases it becomes more difficult to change the electron temperature by a heating current because the phonon temperature of the wire is coupled to that of the substrate, which works as a massive thermostat. For this reason it is difficult to reach temperatures above 1 K in our samples using this heating method.

The slope of the  $T_m$  versus  $I_H$  curve determines the effective modulation of the temperature difference  $(\Delta T)_m$  across the ring due to modulation of the heater current

$$(\Delta T)_m = \left( \frac{dT}{dI} \right)_{I=I_H} \times I_m. \quad (3)$$

For  $I_H=6 \mu\text{A}$ , which is used later for thermopower evaluation, the slope is equal to  $0.037 \text{ K}/\mu\text{A}$ . Substituting  $I_m=1 \mu\text{A}$  into Eq. (3) gives  $(\Delta T)_m=0.037 \text{ K}$ .

Figure 5 shows thermovoltage oscillations measured at  $T=0.28 \text{ K}$  between contacts 2 and 5 (N-S) and 4 and 5 (N-N) at various values of the dc heating current. It is remarkable that measured traces for all potential electrode configurations and heating currents show that the thermovoltage oscillations are in phase with the magnetoresistance ones. Using the thermovoltage values we can estimate the absolute value of the oscillating thermopower  $S=V_{th}/\Delta T$ . Using  $V_{th} \approx 75 \text{ pV}$  for  $I_H=6 \mu\text{A}$  we obtain  $S=2.0 \text{ nV/K}$ .

#### IV. ANALYSIS AND DISCUSSION

The absolute value of the diffusive thermopower in pure Ag at low temperatures due to impurities and dislocations is of the order of  $S_D \approx 10T \text{ nV/K}^2$ , while that of the phonon drag is close to  $S_G \approx 0.5T^3 \text{ nV/K}^4$ .<sup>15</sup> At  $T=0.6 \text{ K}$  this gives  $S_D \approx 6 \text{ nV/K}$  and  $S_G \approx 0.1 \text{ nV/K}$ . Therefore, the contribution to thermopower from the phonon drag effect at the temperatures of our experiment can be neglected. The diffusive thermopower agrees well with the prediction of Mott's law,

where  $S_D$  is determined by a derivative of the logarithm of electrical conductivity  $\sigma$  with respect to energy  $\varepsilon$  taken at the Fermi level:<sup>16</sup>

$$S_D = \frac{\pi^2 k_B^2 T}{3 e} \left( \frac{\partial \ln \sigma}{\partial \varepsilon} \right)_{\varepsilon=\varepsilon_F}, \quad (4)$$

where  $k_B$  is the Boltzmann constant and  $e$  is the electron charge. In the absence of superconducting contacts to normal wires in metals with close to spherical Fermi surfaces, Eq. (4) leads to the following expression:

$$S_D = \frac{\pi^2 k_B k_B T}{3 e \varepsilon_F} \left( \frac{3}{2} + m \right), \quad (5)$$

where  $m$  is a constant depending on the energy dependence of the scattering time  $\tau \sim \varepsilon^m$ . Substituting values of  $m \approx -2.8$  (Ref. 15) and  $\varepsilon_F=6.4 \times 10^4 \text{ K}$  (Ref. 17) into Eq. (5) we get  $S_D=3.5 \text{ nV/K}$  which is in good agreement with the experiment.<sup>15</sup>

For mesoscopic N-S structures in the geometry of the Andreev interferometer, the theory predicts a giant thermo-emf due to Josephson currents circulating in the loop, induced by the applied magnetic flux. When  $\Phi=\Phi_0 n$ , where  $n$  is an integer, there is no supercurrent induced in the Andreev interferometer, so that the symmetry of thermopower oscillations as a function of magnetic field is always that of  $\sin \phi$ . This thermopower is not described by Mott's law and can be of the order of  $\varepsilon_F/k_B T$  larger than that predicted by Eq. (5). The geometry of the samples affects the absolute value of the thermopower and its temperature dependence but not the symmetry of the thermopower oscillations. This conclusion is also valid for different limits with respect to  $k_B T$ ,  $E_{Th}$ , and  $\Delta$ .<sup>12</sup>

When a normal metal is in contact with a superconductor its conductivity is modified by the proximity effect. In particular, there is a phase-dependent correction to the conductivity  $\sigma$ , which has the following form:

$$\sigma = \sigma(\varepsilon) + \Delta\sigma(\varepsilon)\cos \phi, \quad (6)$$

where  $\Delta\sigma(\varepsilon)$  is the energy-dependent oscillatory part of the correction to the conductivity due to the proximity effect. In our sample the relative correction  $\Delta\sigma/\sigma_N$  was about 1.1%, corresponding to a weak proximity effect,  $\sigma_N$  being the ring conductivity in the normal state. Upon substitution of Eq. (6) into Eq. (4),  $S_D$  will acquire an oscillatory part proportional to  $\cos \phi$ . We estimated the absolute value of the oscillatory thermopower at  $T=0.6 \text{ K}$  to be  $2 \text{ nV/K}$ , which is close to the classical diffusive thermopower discussed above. The symmetry of the thermopower oscillations in this case is that of the resistance oscillations.

In conclusion, we observed thermopower oscillations in a mesoscopic ring with one contact to a superconductor. The absolute value of the thermopower and the  $\cos \phi$  symmetry of the oscillations indicate that the observed oscillations are

due to a classical diffusive thermo-emf induced in the ring. We do not find the behavior reported for  $N$ - $S$  geometries with superconducting loops, where absolute values of the thermopower of the order of 100–1000 nV/K have been reported, and the oscillation symmetry is that of  $\sin \phi$ .

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