Valued Authorization Policy Existence Problem

Jason Crampton
Royal Holloway, University of London
Egham, United Kingdom
jason.crampton@rhul.ac.uk

Eduard Eiben
Royal Holloway, University of London
Egham, United Kingdom
eduard.eiben@rhul.ac.uk

Gregory Gutin
Royal Holloway, University of London
Egham, United Kingdom
g.gutin@rhul.ac.uk

Daniel Karapetyan
University of Nottingham
Nottingham, United Kingdom
Daniel.Karapetyan@nottingham.ac.uk

Diptapriyo Majumdar
Royal Holloway, University of London
Egham, United Kingdom
diptapriyo.majumdar@rhul.ac.uk

ABSTRACT

Problems of satisfiability and resiliency in workflows have been widely studied in the last decade. Recent work has shown that many such problems may be viewed as special cases of the authorization policy existence problem (APEP), which returns an authorization policy if one exists and "No" otherwise. A solution may not exist because of the restrictions imposed by the base authorization relation and constraints that form part of the input to APEP.

However, in many practical settings it would be more useful to obtain a "least bad" policy than just a "No", where "least bad" is characterized by some numerical value associated with the policy indicating the extent to which the policy violates the base authorization relation and constraints. Accordingly, we introduce the Valued APEP, which returns an authorization policy of minimum weight, where the (non-negative) weight is determined by the constraints violated by the returned solution (and is 0 if all constraints are satisfied).

We then establish a number of results concerning the parameterized complexity of Valued APEP. We prove that the problem is fixed-parameter tractable if the set of constraints satisfies two restrictions, but is intractable if only one of these restrictions holds. (Most constraints known to be of practical use satisfy these restrictions.) We introduce the notion of a user profile for a weighted constraint, which enables us to prove a powerful result, a corollary of which improves on known complexity results for APEP. Finally, we consider Valued APEP when restricted to particular sub-classes of constraints and show that instances of such problems can be reduced to the valued workflow satisfiability problem, enabling us to exploit known algorithms to solve these particular instances.

CCS CONCEPTS

• Security and privacy → Formal security models; **• Theory of computation → Problems, reductions and completeness**; **Parameterized complexity and exact algorithms**.

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1 INTRODUCTION

Access control is a fundamental aspect of the security of any multi-user computing system. Access control is typically implemented by specifying an authorization policy and implementing a system to enforce the policy. Such a policy identifies which interactions between users and resources are to be allowed (and denied) by the access control system.

Over the years, authorization policies have become more complex, not least because of the introduction of constraints – often motivated by business requirements such as "Chinese walls" – which further refine an authorization policy. A separation-of-duty constraint (also known as the "two-man rule" or "four-eyes policy") may, for example, require that no single user is authorized for some particularly sensitive group of resources. Such a constraint is typically used to prevent misuse of the system by a single user.

The use of authorization policies and constraints, by design, limits which users may access resources. Nevertheless, the ability to perform one's duties will usually require access to particular resources, and overly prescriptive policies and constraints may mean that some resources are unavailable to users that need access to them. In other words, there may be some conflict between authorization policies and operational demands: a policy that is insufficiently restrictive may suit operational requirements but lead to security violations; conversely, too restrictive a policy may compromise an organization's ability to meet its business objectives.

Recent work on workflow satisfiability and access control resiliency recognized the importance of being able to determine whether or not security policies prevent an organization from achieving its objectives [6, 7, 18, 20, 24]. Bergé *et al.* introduced the Authorization Policy Existence Problem (APEP) [1], which generalizes many of the existing satisfiability and resiliency problems in access control. Informally, the APEP seeks to find an authorization policy, subject to restrictions on individual authorizations (defined by a base authorization relation) and restrictions on collective authorizations (defined by a set of authorization constraints).

APEP may be viewed as a decision or search problem. An algorithm to solve either version of the problem returns "no" if no authorization policy exists, given the base authorization relation and the constraints that form part of the input to the instance. Such a response is not particularly useful in practice: from an operational perspective, an administrator would presumably find it more useful if an algorithm to solve APEP returned some policy, even if that policy could lead to security violations, provided the risk of deploying that policy could be quantified in some way.

Hence, in this paper, we introduce a variant of APEP, which we call Valued APEP, where every policy is associated with a non-negative weight. A solution to Valued APEP is a policy of minimum weight; a policy of zero weight satisfies the base authorization relation and all the constraints.

We establish the complexity of Valued APEP for certain types of constraints, using multi-variate complexity analysis. We prove that APEP is fixed-parameter intractable, even if all the constraints are user-independent, a class of constraints for which WSP – a special case of APEP – is fixed-parameter tractable. However, we subsequently show that Valued APEP is fixed-parameter tractable when all weighted constraints are user-independent and the set of constraints is t-weight-bounded (t-wbounded). Informally, the identities of the users are irrelevant to the solution and there exists a solution of size no greater than t. We show that sets of user-independent constraints that contain only particular kinds of widely used constraints are t-wbounded. Bergé *et al.* [1] introduced and used a notion of a bounded constraint. Bounded and wbounded constraints have some similarities, but wbounded constraints are more refined and allow for more precise complexity analysis. In particular, the notion of a bounded constraint cannot be used for Valued APEP and we are able to derive improved complexity results for APEP using wbounded constraints.

A significant innovation of the paper is to introduce the notion of a user profile for a weighted constraint. Counting user profiles provides a powerful means of analyzing the complexity of (Valued) APEP, somewhat analogous to the use of patterns in the analysis of workflow satisfiability problems. This enables us to (i) derive the complexity of Valued APEP when all constraints are t-bounded and user-independent, (ii) establish the complexity of Valued APEP for the most common types of user-independent constraints, and (iii) improve on existing results for the complexity of APEP obtained by Bergé *et al.* [1]. We also prove that our result for the complexity of APEP with t-wbounded, user-independent constraints cannot be improved, unless a well-known and widely accepted hypothesis in parameterized complexity theory is false. Finally, we show that certain sub-classes of Valued APEP can be reduced to the Valued Workflow Satisfiability Problem (WSP) [8] with user-independent constraints, thereby establishing that these sub-classes are fixed-parameter tractable.

In the next section, we summarize relevant background material. We introduce the Valued APEP in Section 3 and define weighted user-independent constraints. We also show that Valued WSP is a special case of Valued APEP and describe particular types of weighted user-independent constraints for APEP. We then introduce the notion of a t-wbounded constraint and establish the complexity of Valued APEP when all constraints are t-wbounded. We prove the problem is intractable for arbitrary sets of t-wbounded constraints or user-independent constraints, but fixed-parameter tractable for t-wbounded, user-independent constraints. In the following two sections, we establish the complexity of other subclasses of Valued APEP. We discuss related work in Section 7, and conclude the paper with a summary of our contributions and some ideas for future work.

2 BACKGROUND

APEP is defined in the context of a set of *users* U, a set of *resources* R, a *base authorization relation* $\hat{A} \subseteq U \times R$, and a set of *constraints* C. Informally, APEP asks whether it is possible to find an authorization relation A that satisfies all the constraints and is a subset of $\hat{A}$.

For an arbitrary authorization relation $A \subseteq U \times R$ and an arbitrary resource $r \in R$, we write $A(r)$ to denote the set of users authorized for resource r by A. More formally, $A(r) = \{u \in U \mid (u,r) \in A\}$; for a subset $T \subseteq R$, we define $A(T) = \bigcup_{r \in T} A(r)$ to denote the set of users authorized for some resource $r \in T$. For a user u, $A(u) = \{r \in R \mid (u,r) \in A\}$; and for $V \subseteq U$, $A(V) = \{r \in R \mid (u,r) \in A, u \in V\}$.

An authorization relation $A \subseteq U \times R$ is

- *authorized* (with respect to $\hat{A}$) if $A \subseteq \hat{A}$,
- *complete* if for all $r \in R$, $A(r) \neq \emptyset$,
- *eligible with respect to* C if it satisfies all $c \in C$,
- *valid with respect to* $\hat{A}$ *and* C if A is authorized, complete, and eligible with respect to C.

An instance of APEP is *satisfiable* if it admits a valid authorization relation A.

2.1 APEP constraints

In general, there are no restrictions on the constraints that can appear in an APEP instance, although the use of arbitrary constraints has a significant impact on the computational complexity of APEP (see Section 2.3). Accordingly, Bergé *et al.* [1] defined several standard types of constraints for APEP, summarized in Table 1, generalizing existing constraints in the access control literature.

2.2 WSP as a special case of APEP

Consider an instance of APEP which contains any of the constraints defined in Section 2.1, and includes the set of cardinality constraints $\{(r, \leq, 1) \mid r \in R\}$. Any solution A to such an APEP instance requires $|A(r)| = 1$ for all $r \in R$ (since completeness requires $|A(r)| > 0$). Thus A may be regarded as a function $A : R \to U$. Since $|A(r)| = 1$, there is no distinction between existential and universal constraints (whether they are separation-of-duty or binding-of-duty): specifically, A satisfies the constraint $(r, r', \circ, \exists)$ iff A satisfies $(r, r', \circ, \forall)$ (for $\circ \in \{\updownarrow, \leftrightarrow\}$).

In other words, an APEP instance of this form is equivalent to an instance of WSP [7, 24], with separation-of-duty, binding-of-duty and cardinality constraints: resources correspond to workflow steps, the base authorization relation to the authorization policy, and an APEP solution to a plan. Accordingly, strong connections exist between APEP and WSP, not least because certain instances of APEP can be reduced to WSP [1]. In WSP, the set of resources is the set of *steps*, denoted by S.

Table 1: Standard APEP constraints: $r, r' \in R$, $t \in \mathbb{N}$

Description	Notation	Satisfaction criterion	Constraint family
Universal binding-of-duty	$(r, r', \leftrightarrow, \forall)$	$A(r) = A(r')$	$\mathsf{BoD_U}$
Universal separation-of-duty	$(r, r', \updownarrow, \forall)$	$A(r) \cap A(r') = \emptyset$	$\mathsf{SoD_U}$
Existential binding-of-duty	$(r, r', \leftrightarrow, \exists)$	$A(r) \cap A(r') \neq \emptyset$	$\mathsf{BoD_E}$
Existential separation-of-duty	$(r, r', \updownarrow, \exists)$	$A(r) \neq A(r')$	$\mathsf{SoD_E}$
Cardinality	$(r, \leq, t)$	$\lvert A(r) \rvert \leq t$	Card

2.3 Complexity of WSP and APEP

In the context of WSP, the authorization policy (the base authorization relation in APEP) specifies which users are authorized for which steps in the workflow. A solution to WSP is a plan π that assigns a single user to each step in the workflow. In general, WSP is NP-complete [24].

Let $k = |S|$ and $n = |U|$. Then there are n^k plans, and the validity of each plan can be established in polynomial time (in the size of the input). Thus WSP can be solved in polynomial time if k is constant. It is easy to establish that APEP is harder than WSP in general.

Proposition 2.1. APEP *is* NP*-complete even when there is a single resource.*

The proof uses a simple reduction from Monotone 1-in-3 SAT [23] to an instance of APEP in which there is a single resource r: the set of variables corresponds to the set of users; $(x, r) \in A$ corresponds to assigning the value True to variable x; and every clause corresponds to a constraint comprising three "users", which is satisfied provided exactly one user is assigned to the resource.

Wang and Li [24] introduced parameterization[1] of WSP by parameter k. This parameterization is natural because for many practical instances of WSP, $k = |S| \ll n = |U|$ and k is relatively small. Wang and Li proved that WSP is intractable, even from the parameterized point of view. However, Wang and Li proved that WSP becomes computationally tractable from the parameterized point of view (i.e., fixed-parameter tractable) when the constraints are restricted to some generalizations of binary separation-of-duty (SoD) and binding-of-duty (BoD) constraints.

Similarly, for APEP, we denote $k = |R|$ and $n = |U|$. In the rest of the paper, we assume that k is relatively small and thus consider it as the parameter. While the assumption that k is small is not necessarily correct in some applications, our approach is useful where k is indeed small, for example in special cases such as WSP. Also, there are situations where strict controls are placed on the utilization of and access to (some small subset of system) resources by users.

2.4 User-independent constraints

Wang and Li's result has been extended to the much larger family of user-independent constraints, which includes the aforementioned SoD and BoD constraints and most other constraints that arise in practice [4, 16]. Informally, a constraint is called user-independent if its satisfaction does not depend on the identities of the users assigned to steps. (For example, it is sufficient to assign steps in a separation of duty constraint to different users in order to satisfy the constraint.)

The concept of a user-independent constraint for WSP can be extended formally to the APEP setting in the following way [1]. Let $\sigma : U \to U$ be a permutation on the user set. Then, given an authorization relation $A \subseteq U \times R$, we write $\sigma(A) = \{(\sigma(u), r) | (u, r) \in A\}$. A constraint c is said to be *user-independent* if for every authorization relation A that satisfies c and every permutation $\sigma : U \to U$, $\sigma(A)$ also satisfies c. It is not hard to see that the sets of constraints defined in Section 2.1 are user-independent [1], since their satisfaction is independent of the specific users that belong to $A(r)$ and $A(r')$.

Bergé *et al.* established a number of FPT results for APEP (restricted to t-bounded, user-independent constraints). We introduce a definition of user-independence and t-boundedness for weighted constraints in Sections 3 and 4, respectively, and show that we can improve on existing complexity results.

2.5 Parameterized complexity

An instance of a parameterized problem Π is a pair (I, κ) where I is the *main part* and κ is the *parameter*; the latter is usually a non-negative integer. A parameterized problem is *fixed-parameter tractable* (FPT) if there exists a computable function f such that any instance (I, κ) can be solved in time $O(f(\kappa)|I|^c)$, where $|I|$ denotes the size of I and c is an absolute constant. The class of all fixed-parameter tractable decision problems is called FPT and algorithms which run in the time specified above are called FPT algorithms. As in other literature on FPT algorithms, we will often omit the polynomial factor in $O(f(\kappa)|I|^c)$ and write $O^*(f(\kappa))$ instead.

Consider two parameterized problems Π and Π'. We say that Π has a *parameterized reduction* to Π' if there are functions g and h from $\mathbb{N}$ to $\mathbb{N}$ and a function $(I, \kappa) \mapsto (I', \kappa')$ from Π to Π' such that

- there is an algorithm of running time $h(\kappa) \cdot (|I| + \kappa)^{O(1)}$ which for input (I, κ) outputs (I', κ'), where $\kappa' \leq g(\kappa)$; and
- (I, κ) is a yes-instance of Π if and only if (I', κ') is a yes-instance of Π'.

While FPT is a parameterized complexity analog of P in classical complexity theory, there are many parameterized hardness classes, forming a nested sequence of which FPT is the first member: FPT$\subseteq$ W[1]$\subseteq$ W[2] $\subseteq \dots$. The *Exponential Time Hypothesis* (ETH) is a well-known and plausible conjecture that there is no algorithm solving 3-CNF Satisfiability in time $2^{o(n)}$, where n is the number of variables [14]. It is well known that if the ETH holds then FPT $\neq$ W[1]. Hence, W[1] is generally viewed as a parameterized intractability class, which is an analog of NP in classical complexity.

[1] We provide a brief introduction to parameterized complexity in Section 2.5.

A well-known example of a W[1]-complete problem is the CLIQUE problem parameterized by κ: given a graph G and a natural number κ, decide whether G has a complete subgraph on κ vertices. A well-known example of a W[2]-complete problem is the DOMINATING SET problem parameterized by κ: given a graph $G = (V, E)$ and a natural number κ, decide whether G has a set S of κ vertices such that every vertex in $V \setminus S$ is adjacent to some vertex in S. Thus, every W[1]-hard problem Π_1 is at least as hard as CLIQUE (i.e., CLIQUE has a parameterized reduction to Π_1); similarly, every W[2]-hard problem Π_2 is at least as hard as DOMINATING SET.

More information on parameterized algorithms and complexity can be found in recent books [10, 12].

3 VALUED APEP

As we noted in the introduction, we believe that it is more valuable, in practice, for APEP to return some authorization relation, even if that relation is not valid (in the sense defined in Section 2). Clearly, the authorization relation that is returned must be the best one, in some appropriate sense. Inspired by VALUED WSP, we introduce VALUED APEP, where every authorization relation is associated with a "cost" (more formally, a *weight*) and the solution to a VALUED APEP instance is an authorization relation of minimum weight.

3.1 Problem definition

We first introduce the notions of a *weighted constraint* and a *weighted user authorization function*. Let $A \subseteq U \times R$ be an authorization relation. A weighted constraint is a function $w_c : 2^{U\times R} \to \mathbb{N}$ such that $w_c(A) = 0$ iff A satisfies the constraint. (By definition, $w_c(A) > 0$ if the constraint is violated.) The intuition is that $w_c(A)$ represents the cost incurred by A, in terms of constraint violation. For example, a weighted constraint w_c such that $w_c(A) = 0$ iff $A(r) \cap A(r') = \emptyset$ and $w_c(A)$ increases monotonically with the size of $A(r) \cap A(r')$ encodes the usual APEP constraint $(r, r', \updownarrow, \forall)$. (We describe other weighted constraints in Section 3.2.)

A weighted user authorization function $\omega : U \times 2^R \to \mathbb{N}$ has the following properties:

$$\omega(u, T) = 0 \quad \text{if } u \text{ is authorized for each resource in } T \tag{1}$$

$$T' \subseteq T \quad \text{implies} \quad \omega(u, T') \leq \omega(u, T). \tag{2}$$

Then $\omega(u, T) > 0$ if u is not authorized for some resource in T and, vacuously, we have $\omega(u, \emptyset) = 0$ for all $u \in U$. The weighted user authorization function is used to represent the cost of assigning unauthorized users to resources.

Then we define the weighted authorization function $\Omega : 2^{U\times R} \to \mathbb{N}$, weighted constraint function $w_C : 2^{U\times R} \to \mathbb{N}$, and weight function $w : 2^{U\times R} \to \mathbb{N}$ as follows:

$$\Omega(A) = \sum_{u\in U} \omega(u, A(u)), \tag{3}$$

$$w_C(A) = \sum_{c\in C} w_c(A), \tag{4}$$

$$w(A) = \Omega(A) + w_C(A). \tag{5}$$

A relation A is *optimal* if $w(A) \leq w(A')$ for all $A' \subseteq U \times R$. We now formally define VALUED APEP.

VALUED APEP
Input: A set of resources R, a set of users U, a set of weighted constraints C, a weighted user authorization function ω
Parameter: $|R| = k$
Output: A complete, optimal authorization relation

Remark 3.1. *A base authorization relation $\hat{A}$ is implicitly defined in a* VALUED APEP *instance: specifically, $(u, r) \in \hat{A}$ iff $\omega(u, r) = 0$. An instance of* VALUED APEP *is defined by a tuple (R, U, C, ω), where C is a set of weighted constraints; we may, when convenient, refer to $\hat{A}$, as defined by ω.*

3.2 Valued APEP constraints

We now provide some examples of weighted constraints, extending the examples introduced in Section 2.1. First, let $f_c : \mathbb{Z} \to \mathbb{N}$ be a monotonically increasing function (i.e., $f_c(z) \leq f_c(z + 1)$ for all $z \in \mathbb{Z}$), where $f_c(z) = 0$ iff $z \leq 0$, and let ℓ_c be some constant. Define $\operatorname{maxdiff}(A, r, r')$ to be $\max\{|A(r) \setminus A(r')|, |A(r') \setminus A(r)|\}$. Then the equations below demonstrate how an unweighted APEP constraint c may be extended to a weighted constraint w_c using f_c.

Unweighted	Weighted	
$(r, \leq, t)$	$w_c(A) = f_c(\lvert A(r)\rvert - t)$	(6)
$(r, r', \updownarrow, \forall)$	$w_c(A) = f_c(\lvert A(r) \cap A(r')\rvert)$	(7)
$(r, r', \leftrightarrow, \forall)$	$w_c(A) = f_c(\operatorname{maxdiff}(A, r, r'))$	(8)
$(r, r', \updownarrow, \exists)$	$w_c(A) = \begin{cases} 0 & \text{if } A(r) \neq A(r'), \\ \ell_c & \text{otherwise.} \end{cases}$	(9)
$(r, r', \leftrightarrow, \exists)$	$w_c(A) = \begin{cases} 0 & \text{if } A(r) \cap A(r') \neq \emptyset, \\ \ell_c & \text{otherwise.} \end{cases}$	(10)

For example, the weighted cardinality constraint (6) evaluates to 0 if A assigns no more than t users to r, and some non-zero value determined by f_c and $|A(r)|$ otherwise. The specific choice of function f_c and the constant ℓ_c will vary, depending on the particular application and particular constraint that is being encoded. For notational convenience, we may refer to binding-of-duty and separation-of-duty constraints of the form $(r, r', \updownarrow, \forall)$, $(r, r', \leftrightarrow, \forall)$, $(r, r', \updownarrow, \exists)$ and $(r, r', \leftrightarrow, \exists)$. However, when doing so, we mean the relevant weighted constraint as defined in equations (7), (8), (9) and (10), respectively.

Given an authorization relation $A \subseteq U \times R$, we say a weighted constraint w_c is *user-independent* if, for every permutation σ of U, $w_c(A) = w_c(\sigma(A))$. We have already observed that the APEP constraints in Section 2.1 are user-independent. It is easy to see that the weighted constraints defined above for VALUED APEP are also user-independent.

In the remaining sections of this paper, we consider the fixed-parameter tractability of VALUED APEP. We will write, for example, APEP$\langle \mathsf{BoD_E} \rangle$ to denote the set of instances of APEP in which the set of constraints C contains only $\mathsf{BoD_E}$ constraints.

3.3 Valued APEP and Valued WSP

We have already observed that WSP is a special case of APEP for certain choices of APEP constraints. Bergé *et al.* also proved that the complexity of some sub-classes of APEP can be reduced to WSP [1, Section 5].

The inputs to Valued WSP include a weighted authorization policy and weighted constraints, and the solution is a plan of minimum weight [8]. Similar arguments to those presented in Section 2.2 can be used to show that Valued WSP is a special case of Valued APEP. In this paper, we will show that some sub-classes of Valued APEP can be reduced to Valued WSP, thereby establishing, via the following result [8, Theorem 1], that those sub-classes of Valued APEP are FPT.

Theorem 3.2. Valued WSP*, when all weighted constraints are user-independent, can be solved in time* $O^*(2^{k \log k})$*, where* $k = |S|$.

4 t-BOUNDED CONSTRAINTS

In this section we consider instances of Valued APEP having an optimal solution A^* that is small; i.e., $|A^*| \ll |U \times R|$. We start by defining a natural restriction on weighted constraints that implies the existence of a small optimal solution for instances containing only constraints satisfying the restriction. Moreover, checking whether a constraint satisfies the restriction is often easier than checking for the existence of a small optimal solution. This restriction roughly says that if the size of an authorization relation is larger than t (i.e., $|A| > t$), then there are authorizations that are redundant, in the sense that they are not necessary to satisfy the given set of weighted constraints.

Definition 4.1 (t-wbounded)**.** *A set of weighted constraints C is t-wbounded if and only if for each complete authorization relation A such that $|A| > t$ there exists a complete authorization relation A' such that $A' \subseteq A$, $|A'| < |A|$, and $w_C(A') \leq w_C(A)$.*

We say that a weighted constraint w_c is t-wbounded if the set $\{w_c\}$ is t-wbounded. We remark that Bergé *et al.* [1] introduced the notion of $f(k,n)$-bounded user-independent constraints for APEP. While they introduced the notion only for the user-independent constraints it can be easily generalized for any APEP constraint as follows. For an authorization relation A and a user u let us denote by $A - u$ the authorization relation obtained from A by removing all the pairs that include the user u (i.e., the relation $A \setminus \{(u,r) \mid r \in R\}$).

Definition 4.2. *Given a set of resources R and a set of users U, a constraint c is $f(k,n)$-bounded if for each complete authorization relation A which satisfies c, there exists a set U' of size at most $f(k,n)$ such that for each user $u \in (U \setminus U')$, the authorization relation $A - u$ is complete and satisfies c.*

One way to generalize $f(k,n)$-bounded constraints to Valued APEP would be to say that a weighted constraint w_c is $f(k,n)$-bounded if for each complete authorization relations A there exists a set U' of at most $f(k,n)$ users such that for every user $u \in U \setminus U'$, the relation $A - u$ is complete and $w_c(A') \leq w_c(A)$. Given this we can show that our definition covers all constraints covered by Bergé et al.

Lemma 4.3. *If a weighted constraints w_c is $f(k,n)$-bounded, then w_c is $f(k,n) \cdot k$-wbounded. Moreover, if every $c \in C$ is user-independent and $f(k,n)$-bounded then C is $f(k,n) \cdot 2^k \cdot k$-wbounded.*

Proof. Let us consider a complete relation A. If $|A| > f(k,n) \cdot k$, then there are at least $f(k,n) + 1$ users authorized by A. It follows that there exists a user u such that $A(u) \neq \emptyset$ and the authorization relation $A' = A - u$ is complete and $w_c(A') \leq w_c(A)$. But $A' \subseteq A$ and $|A'| < |A|$. Hence w_c is $(f(k,n) \cdot k)$-wbounded. Now, if $|A| > f(k,n) \cdot 2^k \cdot k$, then for some $T \subseteq R$, $T \neq \emptyset$ there are at least $f(k,n) + 1$ users u such that $A(u) = T$. Since every $c \in C$ is user-independent and $f(k,n)$-bounded, it is not difficult to see that for a user u with $A(u) = T$ the authorization relation $A' = A - u$ is complete and $w_c(A') \leq w_c(A)$ for all $c \in C$. Therefore C is $f(k,n) \cdot 2^k \cdot k$-wbounded. □

We can now show that if the set of all constraints in an input instance is t-wbounded, then the size of some optimal solution is indeed bounded by t.

Lemma 4.4. *Let $\mathcal{I} = (R, U, C, \omega)$ be an instance of* Valued APEP *such that C is t-wbounded. Then there exists an optimal solution A^* of $\mathcal{I}$ such that $|A^*| \leq t$.*

Proof. Let A be an optimal solution of $\mathcal{I}$ that minimizes $|A|$. If $|A| \leq t$, then the result follows immediately. For the sake of contradiction, let us assume that $|A| > t$. Since A is a solution, it is complete. Hence by the definition of t-wboundedness, it follows that there exists a complete authorization relation $A' \subseteq A$ such that $A' \subseteq A$, $|A'| < |A|$, and $w_C(A') \leq w_C(A)$. Since A' is a complete authorization relation, it follows that A' is also a solution. Because $A' \subseteq A$, it follows that for all $u \in U$ we have $A'(u) \subseteq A(u)$ and by the monotonicity condition on ω and the definition of the function Ω, we have that $\Omega(A') \leq \Omega(A)$. Finally it follows that $w(A') \leq w(A)$ and A' is also an optimal solution. This however contradicts the choice of the optimal solution A to be an optimal solution that minimizes $|A|$. □

Recall that in WSP the solution is a plan that assigns each step to exactly one user. Hence, we can easily translate an instance of WSP into an APEP instance such that each constraint can be satisfied only if each resource is authorized by exactly one user. Let us call such constrains WSP constraints. It follows that if a relation $A \subseteq U \times R$ satisfies a WSP constraint c, then $|A| = k$ and there are at most k users authorized by A. It follows that in an instance of APEP obtained by a straightforward reduction from WSP we have that every constraint is k-bounded and the set of all constraints is k-wbounded. Therefore, the W[1]-hardness result for WSP established by Wang and Li [24] immediately translates to W[1]-hardness of APEP (and hence also Valued APEP) parameterized by the number of resources k even when the set of all constraints is k-wbounded.

Theorem 4.5. *APEP is W[1]-hard even when restricted to the instances such that C is k-wbounded and every constraint of C is k-bounded.*

Given the above hardness result, from now on we will consider only user-independent constraints. We first show that the user-independent constraints defined in Section 3.2 are t-wbounded.

Lemma 4.6. *Let C be any combination of weighted constraints that are $\mathsf{BoD_U}$, $\mathsf{BoD_E}$, $\mathsf{SoD_E}$, $\mathsf{SoD_U}$, or cardinality constraint with constraint weights as described in Section 3.2. Then C is $(2^k \cdot k)$-wbounded. Moreover, for all complete authorization relations A such that there exists a complete authorization relation A' such that $A' \subseteq A$, A' has at most 2^k users, and $w_C(A') \leq w_C(A)$.*

Proof. Let A be a complete authorization relation. If A has at most 2^k users, then $|A| \le 2^k \cdot k$ and there is nothing to prove. Else we show that we can safely remove one user from A to obtain A' such that $w_C(A') \le w_C(A)$. Repeating this argument until the number of users is at most 2^k completes the proof. If the number of users authorized for at least one resource by A is at least $2^k + 1$, then there exist two users $u_1, u_2 \in U$ such that $A(u_1) = A(u_2) \neq \emptyset$. Let $A' = A - u_2$. Clearly A' is complete, because A is complete, and for all $r \in R$ either $A'(r) = A(r)$ or $u_1 \in A'(r)$. Finally, it is not hard to show that for all $c \in C$, we have $w_c(A') \le w_c(A)$ and hence $w_C(A') \le w_C(A)$. Recall that to define each constraint in Section 3, we use either a non-decreasing function $f_c : \mathbb{Z} \to \mathbb{N}$, where $f_c(z) = 0$ iff $z \le 0$ or a constant $\ell_c > 0$.

- Consider a constraint $c = (r, \le t)$. By (6) in Section 3, $w_c(A) = f_c(|A(r)|-t)$. But $A'(r) \subseteq A(r)$ and hence $w_c(A') \le w_c(A)$.
- Consider a constraint $c = (r, r', \updownarrow, \forall)$. By (7) in Section 3, $w_c(A) = f_c(|A(r) \cap A(r')|)$. But $A'(r) \cap A'(r') \subseteq A(r) \cap A'(r')$ and hence $w_c(A') \le w_c(A)$.
- Consider a constraint $c = (r, r', \leftrightarrow, \forall)$. By (8) in Section 3, $w_c(A) = f_c(\max\{|A(r) \setminus A(r')|, |A(r') \setminus A(r)|\})$. But $A'(r) \setminus A'(r') = (A(r) \setminus A(r')) \setminus \{u_2\}$ and $A'(r') \setminus A'(r) = (A(r') \setminus A(r)) \setminus \{u_2\}$, hence $w_c(A') \le w_c(A)$.
- Consider a constraint $c = (r, r', \updownarrow, \exists)$. By (9) in Section 3, if $A(r) \neq A(r')$ then $w_c(A) = 0$; otherwise $w_c(A) = \ell_c > 0$. Because $A(u_1) = A(u_2)$, it is easy to see that $A(r) = A(r')$ if and only if $A'(r) = A'(r')$ and hence $w_c(A') = w_c(A)$.
- Consider a constraint $c = (r, r', \leftrightarrow, \exists)$. By (10) in Section 3, if $A(r) \cap A(r') \neq \emptyset$ then $w_c(A) = 0$; otherwise, $w_c(A) = \ell_c > 0$. Because $A(u_1) = A(u_2)$, we have $u_2 \in A(r) \cap A(r')$ if and only if $u_1 \in A'(r) \cap A'(r')$. So $A(r) \cap A(r') \neq \emptyset$ if and only if $A'(r) \cap A'(r') \neq \emptyset$ and $w_c(A') = w_c(A)$. □

Intuitively, if we have user-independent constraints, then we do not need to know which particular users are assigned to resources in order to determine the constraint weight of some authorization relation A. Instead, it suffices to know for each set $T \subset R$ how many users u are authorized by A precisely to the set T, i.e., the size of the set $\{u \in U \mid A(u) = T\}$. This leads us to the following definition of the *user profile of an authorization relation*. And Lemma 4.8 confirms the intuition behind the definition: if we have a user-independent constraint, then two authorization relations with the same user profile yield the same constraint weight.

Definition 4.7 (user profile). *For a set of resources R, a set of users U, and an authorization relation $A \subseteq U \times R$, we call the function $\mathrm{usr}_A : 2^R \to \mathbb{N}$ such that for every $T \subseteq R$, $\mathrm{usr}_A(T) = |\{u \in U \mid A(u) = T\}|$ the* user profile *of the authorization relation A.*

Note that $\mathrm{usr}_A(T)$ is not the same as $|A(T)|$. The integer $\mathrm{usr}_A(T)$ is the number of all users that are authorized for all resources in T and nothing else, while $A(T)$ is the set of users that are authorized to at least one resource in T.

Lemma 4.8. *Let U be a set of users, R a set of resources, (c, w_c) a user-independent weighted constraint and $A_1, A_2 \subseteq U \times R$ two authorization relations such that $\mathrm{usr}_{A_1}(T) = \mathrm{usr}_{A_2}(T)$ for all $T \subseteq R$. Then $w_c(A_1) = w_c(A_2)$.*

Proof. We will define a permutation $\sigma : U \to U$ such that $\sigma(A_1) = A_2$. The lemma then immediately follows from the definition of user-independence. For $i \in \{1, 2\}$ and $T \subseteq R$, let U_T^i be the set of users that are assigned by A_i precisely to the resources in T and nothing else. That is $U_T^i = \{u \in U \mid A_i(u) = T\}$. Now, let us fix for each U_T^i an arbitrary ordering of the users in U_T^i and let $u_{T,j}^i$ for $j \in [|U_T^i|]$ denote the j-th user in U_T^i. Note that for all $T \subseteq R$, we have $\mathrm{usr}_{A_1}(T) = \mathrm{usr}_{A_2}(T)$ by the assumptions of the lemma and hence $|U_T^1| = |U_T^2|$. Moreover, each user in U is assigned exactly one (possibly empty) subset of resources in each of the authorization relations A_1 and A_2. Hence the sets $\bigcup_{T \subseteq R}\{U_T^1\}$ and $\bigcup_{T \subseteq R}\{U_T^2\}$ are both partitions of U. We are now ready to define the permutation σ as $\sigma(u_{T,j}^1) = u_{T,j}^2$ for all $T \subseteq R$, $j \in [|U_T^1|]$. It remains to show that $\sigma(A_1) = A_2$. By the definition of the users $u_{T,j}^1$ and $u_{T,j}^2$, we get that for all $T \subseteq R$, all $j \in [|U_T^1|]$, and all $r \in R$ we have $(u_{T,j}^1, r) \in A_1$ if and only if $r \in T$ if and only if $(u_{T,j}^2, r) \in A_2$ and the lemma follows. □

Lemma 4.9. *Let $\mathcal{I} = (R, U, C, \omega)$ be an instance of* Valued APEP *such that all constraints in C are user-independent and let $\mathrm{usr} : 2^R \to \mathbb{N}$ be a user profile. Then there exists an algorithm that finds a relation A which minimizes $w(A)$ among all relations with user profile* usr.

Proof. It follows from Lemma 4.8 and the fact that all constraints in C are user-independent that $w_C(A)$ only depends on the user profile of A and hence we only need to find an authorization relation A with user profile $\mathrm{usr}_A = \mathrm{usr}$ such that $\Omega(A)$ is minimized.

Note that if $\sum_{T \subseteq R} \mathrm{usr}(T) \neq |U|$, then there is no authorization relation with given user profile. This is because for every authorization relation A and every user u, the set $A(u)$ is defined as is a (possibly empty) subset of R. Hence from now on we assume that $\sum_{T \subseteq R} \mathrm{usr}(T) = |U|$. We start by creating a weighted complete bipartite graph $G = (V_1 \cup V_2, E)$, with parts V_1, V_2 such that $V_1 = U$ and V_2 contains for each $T \subseteq R$ a set of $\mathrm{usr}(T)$ vertices; let us denote these vertices $\{v_1^T, v_2^T, \ldots, v_{\mathrm{usr}(T)}^T\}$. For a user $u \in U$ and a vertex v_i^T, the weight of the edge uv_i^T is defined as $w(uv_i^T) = \omega(u, T)$. Since $\sum_{T \subseteq R} \mathrm{usr}(T) = |U|$, it follows that $|V_1| = |V_2|$. We show that there is a correspondence between perfect matchings of the graph G and authorization relations with user profile usr.

First, let A be an authorization relation such that $\mathrm{usr}_A = \mathrm{usr}$. Then, we can get a perfect matching M_A of G of weight $\Omega(A)$ as follows. Because, $\mathrm{usr}_A = \mathrm{usr}$, we have that for every $T \subseteq R$ there are exactly $\mathrm{usr}(T)$ many users $u \in U$ such that $A(u) = T$. Hence for every $T \subseteq R$ there is a perfect matching M_A^T between these users and vertices $\{v_1^T, v_2^T, \ldots, v_{\mathrm{usr}(T)}^T\}$. Moreover, the cost of an edge between a user $u \in U$ such that $A(u) = T$ and a vertex v_j^T, $j \in \mathrm{usr}(T)$, is $\omega(u, T)$ which is precisely the contribution of the user u to $\Omega(A)$. Hence the cost of the matching $M_A = \bigcup_{T \subseteq R} M_A^T$ is precisely $\Omega(A)$.

On the other hand if M is a perfect matching in G, then we can define an authorization relation A_M as $(u, r) \in A_M$, if and only if u is matched to a vertex v_i^T with $r \in T$. Clearly, every user u is then matched by M to a vertex v_i^T such that $A_M(u) = T$ and weight of the edge in M incident to u is precisely $\omega(u, A_M(u))$, which is the contribution of u to $\Omega(A)$.

It follows that G has a perfect matching of cost W if and only if there is an authorization relation A with user profile usr and $\Omega(A) = W$ and given a perfect matching of G, we can easily find such an authorization relation. Therefore, to finish the proof of the lemma we only need to compute a minimum cost perfect matching in the weighted bipartite graph G, which can be done using the well-known Hungarian method in $O(mn)$ time [17], where n is the number of vertices and m is the number of edges in G. □

The next ingredient required to prove our main result (Theorem 4.14) is the fact that the number of all possible user profiles for all authorization relations of size at most t is small and can be efficiently enumerated. Let $\mathcal{I} = (R, U, C, \omega)$ be an instance of Valued APEP and $A \subseteq U \times R$ an authorization relation. Then for the user profile usr_A of A we have that $\sum_{T \subseteq R} |T| \cdot \text{usr}_A(T) = |A|$ and $\sum_{T \subseteq R} \text{usr}_A(T) = |U|$. Moreover, if A is complete, then $t \geq k$. Note that the number of users in an optimal solution for a t-wbounded set of weighted constraints is at most t by Lemma 4.4. However, sometimes we are able to show that the number of users in an optimal solution is actually significantly smaller than the bound t such that the set of weighted constraints C is t-wbounded (see, e.g., Lemma 4.6). Moreover, if usr_A is a user profile of an authorization relation with at most ℓ users, then $\sum_{T \subseteq R, T \neq \emptyset} \text{usr}_A(T) \leq \ell$. The following lemma will be useful because we are only interested in complete authorization relations of size at most t that use at most $\ell \leq t$ users.

Lemma 4.10. *Let $\mathcal{I} = (R, U, C, \omega)$ be an instance of* Valued APEP *such that $|R| = k$ and let $\ell \in \mathbb{N}$. Then the number of possible user profiles,* $\text{usr} : 2^R \to \mathbb{N}$*, such that $\sum_{T \subseteq R, T \neq \emptyset} \text{usr}(T) \leq \ell$ is $\binom{\ell+2^k-1}{\ell}$. Moreover, we can enumerate all such functions in time $O^*(\binom{\ell+2^k-1}{\ell})$.*

Proof. It is well known that the number of weak compositions of a natural number q into p parts (the number of ways we can assign non-negative integers to the variables $x_1, x_2, \ldots, x_p$ such that $\sum_{i=1}^{p} x_i = q$) is precisely $\binom{p+q-1}{q-1} = \binom{p+q-1}{p-1}$ (see, e.g., [15]). Note that because $\sum_{T \subseteq R} \text{usr}(T) = |U|$, each user profile usr is determined by assigning $\text{usr}(T)$ for all $T \neq \emptyset$. It is not difficult to see that the number of ways in which we can assign $\text{usr}(T)$ for all $T \neq \emptyset$ such that $\sum_{T \subseteq R, T \neq \emptyset} \text{usr}(T) \leq \ell$ is the same as the number of weak partitions of ℓ into 2^k parts - each of the first $2^k - 1$ parts is identified with one of $2^k - 1$ sets $T \subseteq R$ such that $T \neq \emptyset$. The last part is then a "slack" part that allows $\sum_{T \subseteq R, T \neq \emptyset} \text{usr}(T)$ to be also smaller than ℓ. It follows that the number of possible user profiles is at most $\binom{\ell+2^k-1}{\ell}$. To enumerate them in $O^*(\binom{\ell+2^k-1}{\ell})$ time we can do the following branching algorithm: We fix some order $T_1, T_2, \ldots, T_{2^k-1}$ of the non-empty subsets of R. We first branch on $\ell + 1$ possibilities for $\text{usr}(T_1)$, then we branch on $\ell + 1 - \text{usr}(T_1)$ possibilities for $\text{usr}(T_2)$, and so on, until we branch on $\ell + 1 - \sum_{i \in [2^k-2]} \text{usr}(T_i)$ possibilities for $\text{usr}(T_{2^k-1})$. Afterwards, we compute $\text{usr}(\emptyset)$ from $\sum_{T \subseteq R} \text{usr}(T) = |U|$. Each leaf of the branching tree gives us a different possible user profile and we spend polynomial time in each branch. Hence the running time of the enumeration algorithm is $O^*(\binom{\ell+2^k-1}{\ell})$. □

Because the number of possible user profiles that authorize at most ℓ users appears in the running time of our algorithms, it will be useful to keep in mind the following two simple observations about the combinatorial number $\binom{\ell+2^k-1}{\ell}$.

Observation 4.11. $\binom{\ell+2^k-1}{\ell} \leq 2^{\ell+2^k-1} \leq 4^{\max(\ell, 2^k-1)}$.

Observation 4.12. *If $\ell \geq 4$, then* $\binom{\ell+2^k-1}{\ell} \leq \min(2^{\ell k}, \ell^{2^k-1}) + 1$.

Proof. If $k = 0$, then $\binom{\ell+2^k-1}{\ell} = \binom{\ell}{\ell} = 1 \leq \min(2^{\ell \cdot 0}, \ell^{2^0-1}) + 1$. If $k = 1$, then $\binom{\ell+2^k-1}{\ell} = \ell + 1 \leq \min(2^{\ell \cdot 1}, \ell^{2^1-1}) + 1$. If $k = 2$, then $\binom{\ell+2^k-1}{\ell} = \binom{\ell+3}{3} = \frac{\ell^3+6\ell^2+11\ell+1}{6}$ and since $\ell \geq 4$, it follows that $\binom{\ell+3}{3} \leq \ell^3 \leq 2^{2\ell}$. From now on, let us assume that $k \geq 3$ and $\ell \geq 4$. We distinguish between two cases depending on whether $\ell < 2^k$ or $\ell \geq 2^k$. Let us first consider $\ell \geq 2^k$. Note that in this case $\ell^{2^k-1} \leq 2^{\ell k}$ and because $k \geq 2$ we have $\binom{\ell+2^k-1}{\ell} = \binom{\ell+2^k-1}{2^k-1} \leq \frac{(\ell+2^k-1)^{2^k-1}}{(2^k-1)!} \leq \frac{2^{2^k-1}}{(2^k-1)!} \cdot \ell^{2^k-1} \leq \ell^{2^k-1}$. On the other hand, let us now assume $\ell \leq 2^k - 1$. Note that, because $k \geq 2$ and $\ell \geq 3$, it holds that $(2^k - 1)^\ell \leq \ell^{2^k-1}$. Furthermore, $(2^k - 1)^\ell < 2^{\ell k}$. Then $\binom{\ell+2^k-1}{\ell} \leq \frac{(\ell+2^k-1)^\ell}{\ell!} \leq \frac{2^\ell}{\ell!} \cdot (2^k - 1)^\ell \leq (2^k - 1)^\ell$. □

We are now ready to show the main lemma of this section, which establishes that there exists an FPT algorithm that finds the best solution among all solutions that authorize at most ℓ users. In particular, this algorithm finds an optimal solution for the case of user-independent t-wbounded constraints.

Lemma 4.13. *Let $\mathcal{I} = (R, U, C, \omega)$ be an instance of* Valued APEP *such that all weighted constraints in C are user-independent and let $\ell \in \mathbb{N}$. Then there exists an algorithm that in time $O^*(\binom{\ell+2^k-1}{\ell})$ computes a complete authorization relation A such that $w(A) \leq w(A')$ for every complete authorization relation $A' \subseteq U \times R$ that authorizes at most ℓ users for some resource in R.*

Proof. Note that it suffices to compute such an authorization relation A that also authorizes at most ℓ users. Let A^* be one such complete authorization relation that satisfies the statement of the theorem. Let $\text{usr}_{A^*} : 2^R \to \mathbb{N}$ be the user profile of A^*. Observe that $\sum_{T \subseteq R, T \neq \emptyset} \text{usr}_{A^*}(T) \leq \ell$. Moreover, observe that $\sum_{T \subseteq R} \text{usr}_{A^*}(T) = |U|$, as every user in U is assigned to precisely one subset of resources by A^*. Furthermore, notice that since A^* is complete, for all $r \in R$ we have that $\sum_{\{r\} \subseteq T \subseteq R} \text{usr}_{A^*}(T) \geq 1$ and we may restrict our attention to user profiles that also satisfy $\sum_{\{r\} \subseteq T \subseteq R} \text{usr}_{A^*}(T) \geq 1$ for all $r \in R$. By Lemma 4.10, there exist $\binom{\ell+2^k-1}{\ell}$ different functions (possible user profiles) $\text{usr} : 2^R \to \mathbb{N}$ such that $\sum_{T \subseteq R, T \neq \emptyset} \text{usr}(T) \leq \ell$ and $\sum_{T \subseteq R} \text{usr}(T) = |U|$. Moreover, we can enumerate all of them in time $O^*(\binom{\ell+2^k-1}{\ell})$.

Now, let $\mathcal{P}$ be the set of all such possible user profiles obtained by Lemma 4.10 that also satisfy $\sum_{\{r\} \subseteq T \subseteq R} \text{usr}_{A^*}(T) \geq 1$ for all $r \in R$. Since $\mathcal{P}$ is a subset of functions computed by Lemma 4.10, it follows that $|\mathcal{P}| \leq \binom{\ell+2^k-1}{\ell}$. Moreover, it is easy to see that $\text{usr}_{A^*} \in \mathcal{P}$. The algorithm then branches on all possible profiles in $\mathcal{P}$ and for a profile $\text{usr}_i \in \mathcal{P}$, $i \in [|\mathcal{P}|]$, it computes an authorization relation A_i such that $\text{usr}_{A_i} = \text{usr}_i$ and $w(A_i)$ is minimized, which can be done in polynomial time by Lemma 4.9. Finally, the algorithm outputs the authorization relation A_i for the user profile usr_i that minimizes

$w(A_i)$ among all $\mathrm{usr}_i \in \mathcal{P}$. The running time of the whole algorithm is $O^*(\binom{\ell+2^k-1}{\ell})$.

To establish correctness, first notice that for all $i \in [|\mathcal{P}|]$ we have $\sum_{\{r\}\subseteq T\subseteq R} \mathrm{usr}_i(T) \geq 1$ for all $r \in R$, so the authorization relation A_i is complete. Furthermore, recall that $\mathrm{usr}_{A^*} \in \mathcal{P}$. For $i \in [|\mathcal{P}|]$ such that $\mathrm{usr}_i = \mathrm{usr}_{A^*}$, we have that $w(A_i) \leq w(A^*) \leq w(A')$ for all complete authorization relations $A' \subseteq U \times R$ that authorizes at most ℓ users for some resource in R. □

Note that it follows from Lemma 4.4 that given an instance $\mathcal{I} = (R, U, C, \omega)$ of Valued APEP such that all weighted constraints in C are user-independent and C is t-wbounded there exists an optimal solution A^* of $\mathcal{I}$ such that $|A^*| \leq t$. Moreover, $|A^*| \leq t$ implies that A^* authorizes at most t users for some resource. Hence, in combination with Observation 4.12, we immediately obtain the main result of this section as a corollary, which establishes that there exists an FPT algorithm for the case of user-independent t-wbounded constraints.

Theorem 4.14. *Let $\mathcal{I} = (R, U, C, \omega)$ be an instance of* Valued APEP *such that all weighted constraints in C are user-independent and C is t-wbounded. Then there exists an algorithm solving $\mathcal{I}$ in time* $O^*(\binom{t+2^k-1}{t}) = O^*(\min(2^{tk}, t^{2^k-1})) = O^*(2^{\min(kt,(2^k-1)\log t)})$.

Combining Lemmas 4.13 and 4.6 and Observation 4.11, we obtain the following result for Valued APEP$\langle \mathrm{BoD_U}, \mathrm{BoD_E}, \mathrm{SoD_E}, \mathrm{SoD_U}, \mathrm{Card}\rangle$, where all constraints are $\mathrm{BoD_U}$, $\mathrm{BoD_E}$, $\mathrm{SoD_E}$, $\mathrm{SoD_U}$, or cardinality constraint with constraint weights as described in Section 3.

Corollary 4.15. Valued APEP$\langle \mathrm{BoD_U}, \mathrm{BoD_E}, \mathrm{SoD_E}, \mathrm{SoD_U}, \mathrm{Card}\rangle$ *is fixed-parameter tractable and can be solved in $O^*(4^{2^k})$ time.*

In Sections 5 and 6 we develop more specialized algorithms that solve Valued APEP more efficiently for instances where all constraints are from some specific subset of the above constraints. We conclude this section by showing that restricting attention to user-independent constraints is not sufficient to obtain an FPT algorithm parameterized by the number of resources k even for APEP. Because all weighted constraints are necessarily $(k \cdot |U|)$-wbounded, as a corollary of our conditional lower-bound, we will also show that, unless the Exponential Time Hypothesis (ETH) fails, the algorithm in Theorem 4.14 is in a sense best that we can hope for from an algorithm that can solve Valued APEP with arbitrary user-independent t-wbounded weighted constraints.

Because all the constraints we saw so far were 2^k-wbounded, we will need to introduce new, more restrictive, constraints to obtain our W[2]-hardness and ETH lower-bound. As we consider only user-independent constraints, it is natural for a constraint c to be a function of the user profile of A. To this end, we define a new type of user-independent constraint $c = (\tau, X, \bigvee)$, where $\tau \subseteq 2^R$ and $X \subseteq \mathbb{N}$. The constraint $(\tau, X, \bigvee)$ is satisfied if and only if there exist $T \in \tau$ and $x \in X$ such that $\mathrm{usr}_A(T) = x$. Less formally, c is satisfied if and only if some specified number of users (in X) is authorized for some specified set of resources (in τ). To obtain our ETH lower-bound we make use of the following well-known result in parameterized complexity:

Theorem 4.16 ([19]). *Assuming ETH, there is no $f(k)n^{o(k)}$-time algorithm for* Dominating Set, *where n is the number of the vertices of the input graph, k is the size of the output set, and f is an arbitrary computable function.*

Given the above theorem, we are ready to prove the main negative result of this section.

Theorem 4.17. *APEP is W[2]-hard and, assuming ETH, there is no $f(|R|) \cdot |\mathcal{I}|^{o(2^{|R|})}$-time algorithm solving APEP even when all constraints are user-independent and the base authorization relation is $U \times R$.*

Henceforth, we will write $[k]$ to denote $\{1, \ldots, k\}$. For a graph $G = (E, V)$ and vertex $x \in V$, $N_G(x) = \{y \in V \mid xy \in E\}$ is the set of vertices adjacent to x in G; for a set $S \subseteq V(G)$, $N(S) = \bigcup_{x\in S} N_G(x) \setminus S$.

Proof. To prove the theorem we give a reduction from the Dominating Set problem. Let (G, k) be an instance of the Dominating Set problem. Let $|V(G)| = n$ and let $V(G) = \{v_1, v_2, \ldots, v_n\}$; that is we fix some arbitrary ordering of the vertices in G and each vertex of G is uniquely identified by its position in this ordering (index of the vertex). For a vertex $v_i \in V(G)$ we let the set $X_i = \{i\} \cup \{j \mid v_j \in N_G(v_i)\}$. In other words, for a vertex $v_i \in V(G)$, the set X_i is the set of indices of the vertices in the closed neighbourhood of v_i. The aim of the Dominating Set problem is then to decide whether G has a set S of at most k vertices such that for all $i \in [n]$ the set X_i contains an index of some vertex in S.

Let $\mathcal{I} = (U, R, \hat{A}, C)$ be an instance of APEP such that

- $R = \{r_1, \ldots, r_\ell\}$ such that $2^{\ell-1} \leq k < 2^\ell$,
- $|U| = k \cdot n$,
- $C = \bigcup_{i\in[n]}\{(\tau, X_i, \bigvee)\}$, where $\tau \subset 2^R$ such that $\emptyset \notin \tau$ and $|\tau| = k$, and
- $\hat{A} = U \times R$.

We prove that (G, k) is a YES-instance of Dominating Set if and only if $\mathcal{I}$ is a YES-instance of APEP. Let $\tau = \{T_1, T_2, \ldots, T_k\}$. Observe that because $2^{\ell-1} \leq k$ and $T_i \neq T_j$ for $i \neq j$, it follows that every resource appears in T_i for some $i \in [k]$.

Let $S = \{v_{q_1}, v_{q_2}, \ldots, v_{q_k}\}$ be a dominating set of G of size k (note that if we have a dominating set of size at most k, then we have a dominating set of size exactly k). Let A be an authorization relation such that $\mathrm{usr}_A(T_i) = q_i$. Because $|U| = k \cdot n$ and $1 \leq q_i \leq n$ for all $i \in [k]$, it is easy to construct such an authorization relation. For each $i \in [k]$ we simply select q_i many fresh users u such that $A(u) = T_i$ and leave the remaining users not assigned to any resources ($A(u) = \emptyset$). Because $2^{\ell-1} \leq k$ and for all $T_i \in \tau$ we have $\mathrm{usr}_A(T_i) \geq 1$, it is easy to see that A is a complete authorization relation. Since $\hat{A} = U \times R$, A is authorized. It remains to show that A is eligible w.r.t. C. Consider the constraint $c_i = (\tau, X_i, \bigvee)$. Since S is a dominating set, the closed neighbourhood of v_i contains a vertex $v_{q_j} \in S$. But then $q_j \in X_i$, $T_j \in \tau$, and $\mathrm{usr}_A(T_j) = q_j$, hence c_i is satisfied.

On the other hand let A be valid w.r.t. $\hat{A}$. We obtain a dominating set of G of size at most k as follows. Without loss of generality, let as assume that if $i < j$, then $\mathrm{usr}_A(T_i) \geq \mathrm{usr}_A(T_j)$ and let $k' \in [k]$ be such that $\mathrm{usr}_A(T_{k'}) \geq 1$ and $\mathrm{usr}_A(T_{k'+1}) = 0$ (note that if $\mathrm{usr}_A(T_k) \geq 1$, then $k' = k$). We let $S = \bigcup_{i\in[k']}\{v_{\mathrm{usr}_A(T_i)}\}$. We claim that S is a dominating set (clearly $|S| = k' \leq k$). Let v_i be arbitrary vertex in $V(G) \setminus S$. Consider the constraint $c_i = (\tau, X_i, \bigvee)$.

Clearly c_i is satisfied and there exists $T_j \in \tau$ and $x \in X_i$ such that $\mathrm{usr}_A(T_j) = x$. But by definition of X_i, $x \geq 1$ and v_x is a neighbour of v_i. Moreover, by the definition of S, we have $v_x = v_{\mathrm{usr}_A(T_j)} \in S$. It follows that S is a dominating set.

Now for each $i \in [n]$, the set X_i has size at most n and τ has size $k \leq n$, so the size of the instance $\mathcal{I}$ is polynomial in n. Moreover $2^{\ell-1} \leq k < 2^{\ell}$, hence APEP is W[2]-hard parameterized by $|R|$ and an $f(|R|) \cdot |\mathcal{I}|^{o(2^{|R|})}$ time algorithm for APEP yields an $f(k)n^{o(k)}$ time algorithm for Dominating Set, and the result follows from Theorem 4.16. □

The set C is trivially $(|R| \cdot |U|)$-wbounded for every Valued APEP instance $\mathcal{I} = (R, U, C, \omega)$, so we obtain the following result, which asserts that the lower bound asymptotically matches the running time of the algorithm from Theorem 4.14.

Corollary 4.18. *Assuming ETH, there is no $t^{o(2^{|R|})} \cdot n^{O(1)}$ time algorithm that given an instance $\mathcal{I} = (R, U, C, \omega)$ of* Valued APEP *such that all constraints in C are user-independent and C is t-wbounded computes an optimal solution for $\mathcal{I}$.*

5 $\mathrm{SoD_U}$ AND $\mathrm{BoD_U}$ CONSTRAINTS

In this section, we will consider Valued APEP, where all constraints are only $\mathrm{BoD_U}$ and $\mathrm{SoD_U}$. We will show how to reduce it to Valued WSP with user-independent constraints, with the number of steps equal to the number k of resources in Valued APEP. As a result, we will be able to obtain an algorithm for Valued APEP with only $\mathrm{BoD_U}$ and $\mathrm{SoD_U}$ constraints of running time $O^*(2^{k \log k})$.

Let us start with Valued APEP$\langle \mathrm{SoD_U} \rangle$. Recall that the weighted version of an $\mathrm{SoD_U}$ constraint $(r, r', \updownarrow, \forall)$ is $w_c(A) = f_c(|A(r) \cap A(r')|)$ for some monotonically increasing function f_c.

The weight of a binary SoD constraint $c = (s', s'', \neq)$ in Valued WSP is 0 if and only if steps s' and s'' are assigned to different users. Valued WSP using only SoD constraints of this form will be denoted by Valued WSP($\neq$).

Lemma 5.1. *Let $\mathcal{I} = (R, U, C, \omega)$ be an instance of* Valued APEP$\langle \mathrm{SoD_U} \rangle$ *and let A^* be an optimal solution of $\mathcal{I}$. Let A' be arbitrary authorization relation such that $A' \subseteq A^*$ and $|A'(r)| = 1$ for every $r \in R$. Then A' is an optimal solution of $\mathcal{I}$. Moreover, in polynomial time $\mathcal{I}$ can be reduced to an instance $\mathcal{I}'$ of* Valued WSP($\neq$) *such that the weights of optimal solutions of $\mathcal{I}$ and $\mathcal{I}'$ are equal.*

Proof. Let A' be an arbitrary relation such that $A' \subseteq A^*$ and $|A'(r)| = 1$ for every $r \in R$. By definition, A' is complete. By (2) and (3), $A' \subseteq A^*$ implies $\Omega(A') \leq \Omega(A^*)$. Let $c = (r, r', \updownarrow, \forall) \in C$. Since $A'(r) \cap A'(r') \subseteq A^*(r) \cap A^*(r')$, $w_c(A) = f_c(|A(r) \cap A(r')|)$ and f_c is non-decreasing, we have $w_c(A') \leq w_c(A^*)$. Thus, $\Omega(A') + w_C(A') \leq \Omega(A^*) + w_C(A^*)$. Since A^* is optimal, A' is optimal, too.

Define an instance $\mathcal{I}'$ of Valued WSP($\neq$) as follows: the set of steps is R, the set of users is U, and $(r, r', \neq)$ is a constraint of $\mathcal{I}'$ if $(r, r', \updownarrow, \forall)$ is a constraint of $\mathcal{I}$. The weight of $(r, r', \neq)$ equals $w_c(A') = f_c(1)$ (recall that $|A'(r)| = 1$ for all r) and the weights of (u, T), $u \in U, T \subseteq R$, in both $\mathcal{I}$ and $\mathcal{I}'$ are equal. Observe that $\pi : R \to U$ defined by $\pi(r) = A'(r)$ is an optimal plan of $\mathcal{I}'$. Thus, the optimal solution of $\mathcal{I}$ has the same weight as that of $\mathcal{I}'$. □

We now will consider Valued APEP$\langle \mathrm{BoD_U}, \mathrm{SoD_U} \rangle$. Recall that the weighted version of a $\mathrm{BoD_U}$ constraint $(r, r', \leftrightarrow, \forall)$ is given by $f_c(\max\{|A(r) \setminus A(r')|, |A(r') \setminus A(r)|\})$ for some monotonically increasing function f_c.

The weight of binary BoD constraint $c = (s', s'', =)$ in Valued WSP is 0 if and only if steps s' and s'' are assigned the same user. Valued WSP(=) denotes Valued WSP containing only BoD constraints; Valued WSP(=, $\neq$) denotes Valued WSP containing only SoD and BoD constraints. In fact, Valued WSP(=) is already NP-hard which follows from Theorem 6.4 of [5]. This theorem, in particular, shows that Valued WSP(=) is NP-hard even if the weights are restricted as follows: $w_c(\pi) = 1$ if a plan π falsifies a constraint c, $\omega(u, r) = \infty$ if $(u, r) \notin \hat{A}$.

Lemma 5.2. *Let $\mathcal{I} = (R, U, C, \omega)$ be an instance of* Valued APEP$\langle \mathrm{BoD_U}, \mathrm{SoD_U} \rangle$ *and let A^* be an optimal solution of $\mathcal{I}$. There is an optimal solution A' of $\mathcal{I}$ such that $A' \subseteq A^*$ and $|A'(r)| = 1$ for every $r \in R$. Moreover, in polynomial time $\mathcal{I}$ can be reduced to an instance $\mathcal{I}'$ of* Valued WSP(=, $\neq$).

Proof. Consider an optimal solution A^* of $\mathcal{I}$ and define A' as follows. We first define an equivalence relation $\cong$ on R, where $r \cong r'$ if and only if $A^*(r) = A^*(r')$. This gives a partition $R = R_1 \uplus \ldots \uplus R_p$ such that $p \leq k$. For each R_i, we choose $u_i \in A^*(r)$, where $r \in R_i$. Then $A' = \cup_{i=1}^{p} \{(u_i, r) : r \in R_i\}$.

By definition, A' is complete. By (2) and (3), $A' \subseteq A^*$ implies $\Omega(A') \leq \Omega(A^*)$. Let $c = (r, r', \leftrightarrow, \forall) \in C$. If A^* satisfies c then A' also satisfies c. If c is falsified by A^* then

$$\max\{|A^*(r) \setminus A^*(r')|, |A^*(r') \setminus A^*(r)|\} \geq 1$$

but $\max\{|A'(r) \setminus A'(r')|, |A'(r') \setminus A'(r)|\} = 1$. Hence, $w_c(A^*) \geq f_c(1) = w_c(A')$. Now let $c = (r, r', \updownarrow, \forall) \in C$. By the proof of Lemma 5.1, we have $w_c(A^*) \geq w_c(A')$.

Thus, $\Omega(A') + w_C(A') \leq \Omega(A^*) + w_C(A^*)$. Since A^* is optimal, A' is optimal, too.

An instance of $\mathcal{I}'$ of Valued WSP(=, $\neq$) is defined as in Lemma 5.1, but the constraints $(r, r', =)$ correspond to constraints $(r, r', \leftrightarrow, \forall)$ in C. It is easy to see that the optimal solution of $\mathcal{I}$ has the same weight as that of $\mathcal{I}'$. □

We are now able to state the main result of this section. The result improves considerably on the running time for an algorithm that solves Valued APEP for arbitrary weighted t-bounded user-independent constraints (established in Theorem 4.14).

Theorem 5.3. Valued APEP$\langle \mathrm{BoD_U}, \mathrm{SoD_U} \rangle$ *is FPT and can be solved in time $O^*(2^{k \log k})$.*

Proof. Let $\mathcal{I}$ be an instance of Valued APEP$\langle \mathrm{BoD_U}, \mathrm{SoD_U} \rangle$. By Lemma 5.2, $\mathcal{I}$ can be reduced to an instance $\mathcal{I}'$ of Valued WSP(=, $\neq$). It remains to observe that $\mathcal{I}'$ can be solved in time $O^*(2^{k \log k})$ using the algorithm of Theorem 3.2, as $(r, r', =)$ and $(r, r', \neq)$ are user-independent constraints. □

6 $\mathrm{BoD_E}$ AND $\mathrm{SoD_U}$ CONSTRAINTS

In this section, we consider Valued APEP$\langle \mathrm{BoD_E}, \mathrm{SoD_U} \rangle$. We provide a construction that enables us to reduce an instance $\mathcal{I}$ of Valued APEP$\langle \mathrm{BoD_E}, \mathrm{SoD_U} \rangle$ with k resources to an instance $\mathcal{I}'$ of Valued WSP with only user-independent constraints containing at most $k(k-1)$ steps. Moreover, the construction yields a Valued WSP instance in which the weight of an optimal plan is equal to the weight of an optimal solution for the Valued APEP instance. Finally, we show that it is possible to construct the optimal solution for the Valued APEP instance from an optimal plan for the Valued WSP instance.

Let $\mathcal{I} = (R, U, C, \omega)$ be an instance of Valued APEP$\langle$BoD$_E$, SoD$_U\rangle$. (The weights of these types of constraints are defined by equations (10) and (7) in Section 2.1.) Let $R = \{r_1, \ldots, r_k\}$. Then we construct an instance $\mathcal{I}' = (S', U', C', \omega')$ of Valued WSP as follows.

- Set $U' = U$.
- For every $i \in [k]$, first initialize a set $\Gamma(r_i) = \emptyset$. Then, for every BoD$_E$ constraint $(r_i, r_j, \leftrightarrow, \exists) \in C$, add r_j to $\Gamma(r_i)$ and r_i to $\Gamma(r_j)$.
- For each resource $r_i \in R$, we create a set of steps $S' = \bigcup_{i=1}^{k} S^i$ where

$$S^i = \begin{cases} \{s^i\} & \text{if } \Gamma(r_i) = \emptyset, \\ \{s^i_j \mid r_j \in \Gamma(r_i)\} & \text{otherwise.} \end{cases}$$

Observe that $|S'| \le k(k-1)$. Given a plan $\pi : S' \to U'$, we write $\pi(S^i)$ to denote $\{\pi(s) \mid s \in S^i\}$. Let Π be the set of all possible complete plans from S' to U'.

We define the set of constraints C' and their weights $w'_{c'} : \Pi \to \mathbb{N}$ as follows.

- For each $c = (r_i, r_j, \leftrightarrow, \exists) \in C$, we add constraint $c' = (s^i_j, s^j_i, =)$ to C', and define

$$w'_{c'}(\pi) = \begin{cases} 0 & \text{if } \pi(s^i_j) = \pi(s^j_i), \\ \ell_c & \text{otherwise.} \end{cases}$$

  Note that c' is user-independent.
- For each $c = (r_i, r_j, \updownarrow, \forall) \in C$, we add constraint $c' = (S^i, S^j, \emptyset)$, where c' is satisfied iff $\pi(S^i) \cap \pi(S^j) = \emptyset$. Then define $w'_{c'}(\pi) = f_c(|\pi(S^i) \cap \pi(S^j)|)$, where f_c is the function associated with the weighted constraint $(r_i, r_j, \updownarrow, \forall)$. Observe that $(S^i, S^j, \emptyset)$ is a user-independent constraint.
- Let C' denote the set of all constraints in $\mathcal{I}'$ and define

$$w'_{C'}(\pi) = \sum_{c' \in C'} w'_{c'}(\pi).$$

We then define authorization weight function $\omega' : U' \times 2^{S'} \to \mathbb{N}$ as follows. Initialize $\omega'(u, \emptyset) = 0$. We set $\omega'(u, S^i) = \omega(u, \{r_i\})$. For a subset $T \subseteq S'$, let $R_T = \{r_i \in R \mid T \cap S^i \neq \emptyset\}$. We set $\omega'(u, T) = \omega(u, R_T)$. Given a plan $\pi : S' \to U'$, we denote $\sum_{u \in U'} \omega'(u, \pi^{-1}(u))$ by $\Omega'(\pi)$. Finally, define the weight of π to be $\Omega'(\pi) + w'_{C'}(\pi)$. See Example 1 for an illustration.

Based on the construction described above, we have the following lemma:

Lemma 6.1. *Let $\mathcal{I}$ be a* Valued APEP$\langle$BoD$_E$, SoD$_E\rangle$ *instance and $\mathcal{I}'$ be the* Valued WSP *instance obtained from $\mathcal{I}$ using the construction above. Then* $\mathrm{OPT}(\mathcal{I}) = \mathrm{OPT}(\mathcal{I}')$*, where* $\mathrm{OPT}(\mathcal{I})$ *and* $\mathrm{OPT}(\mathcal{I}')$ *denote the weights of optimal solutions of $\mathcal{I}$ and $\mathcal{I}'$ respectively. Furthermore, given an optimal plan for $\mathcal{I}'$, we can construct an optimal authorization relation for $\mathcal{I}$ in polynomial time.*

Proof. We first prove that $\mathrm{OPT}(\mathcal{I}) \le \mathrm{OPT}(\mathcal{I}')$. Let $\pi : S' \to U'$ be an optimal plan for the instance $\mathcal{I}'$. We construct A for the instance $\mathcal{I}$ as follows. For all $i \in [k]$, if $u \in \pi(S^i)$, then we put (u, r_i) into A. This completes the construction of A from π. Since π is complete, A is also complete. This can be implemented in polynomial time. Observe that $r_i \in A(u)$ if and only if there exists $s \in S^i$ such that $s \in \pi^{-1}(u)$. Equivalently, suppose that $T = \pi^{-1}(u)$. Then, $R_T = A(u)$. It implies that $\omega'(u, \pi^{-1}(u)) = \omega(u, A(u))$. Hence, we have

$$\Omega'(\pi) = \sum_{u \in U'} \omega'(u, \pi^{-1}(u)) = \sum_{u \in U} \omega(u, A(u)) = \Omega(A)$$

We now prove that $w_C(A) \le w'_{C'}(\pi)$. Consider a BoD$_E$ constraint $c = (r_i, r_j, \leftrightarrow, \exists) \in C$. Then, $r_j \in \Gamma(r_i)$, and $r_i \in \Gamma(r_j)$, and the corresponding constraint in $\mathcal{I}'$ is $c' = (s^i_j, s^j_i, =)$. By construction if $\pi(s^i_j) = \pi(s^j_i)$, then there exists $u \in A(r_i) \cap A(r_j)$. Hence, $w_c(A) \le w'_{c'}(\pi)$. Now consider an SoD$_U$ constraint $c = (r_i, r_j, \updownarrow, \forall)$. The corresponding constraint in $\mathcal{I}'$ is $c' = (S^i, S^j, \emptyset)$. Observe that by construction, if $\pi(S^i) \cap \pi(S^j) = \emptyset$, then $A(r_i) \cap A(r_j) = \emptyset$. Otherwise, if $|\pi(S^i) \cap \pi(S^j)| = t > 0$, then by construction $|A(r_i) \cap A(r_j)| = t$. Hence, $w'_{c'}(\pi) = w_c(A)$. We obtain an authorization relation A in polynomial time such that $w_C(A) + \Omega(A) \le w'_{C'}(\pi) + \Omega'(\pi) = \mathrm{OPT}(\mathcal{I}')$. Therefore, $\mathrm{OPT}(\mathcal{I}) \le \mathrm{OPT}(\mathcal{I}')$.

To complete the proof we prove that $\mathrm{OPT}(\mathcal{I}) \ge \mathrm{OPT}(\mathcal{I}')$. Let A be an optimal authorization relation for $\mathcal{I}$. We construct $\pi : S' \to U'$ as follows. If $\Gamma(r_i) = \emptyset$, we choose an arbitrary $u \in A(r_i)$ and set $\pi(s^i) = u$. Otherwise, $\Gamma(r_i) \neq \emptyset$, and two cases may arise.

- For $r_j \in \Gamma(r_i)$, $(r_i, r_j, \leftrightarrow, \exists)$ is satisfied by A. Then, we choose an arbitrary $u \in A(r_i) \cap A(r_j)$ and set $\pi(s^i_j) = \pi(s^j_i) = u$.
- For $r_j \in \Gamma(r_i)$, $(r_i, r_j, \leftrightarrow, \exists)$ is not satisfied by A. Then, we just choose arbitrary $u \in A(r_i), v \in A(r_j)$ and set $\pi(s^i_j) = u$, and $\pi(s^j_i) = v$.

This completes the construction of π. Note that π is complete.

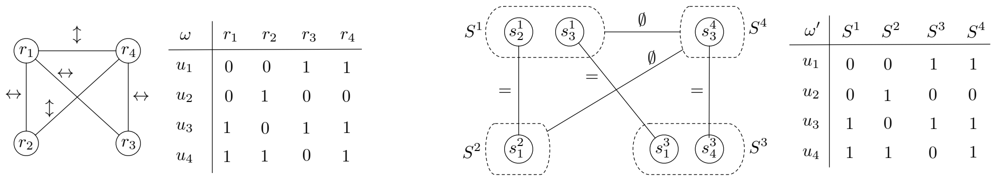
Let $T = \pi^{-1}(u)$. Observe that by construction, if $u \in \pi(S^i)$, then $u \in A(r_i)$. Equivalently, if there exists $i \in [k]$ such that $\pi^{-1}(u) \cap S^i \neq \emptyset$, then $r_i \in A(u)$. Therefore, $R_T \subseteq A(u)$. Using the monotonicity property of ω, we have that $\omega(u, R_T) \le \omega(u, A(u))$. This means that $\omega'(u, \pi^{-1}(u)) = \omega(u, R_T) \le \omega(u, A(u))$. Therefore, we have the following:

$$\Omega'(\pi) = \sum_{u \in U'} \omega'(u, \pi^{-1}(u)) \le \sum_{u \in U} \omega(u, A(u)) = \Omega(A)$$

Hence, $\Omega'(\pi) \le \Omega(A)$.

Consider a BoD$_E$ constraint $c = (r_i, r_j, \leftrightarrow, \exists) \in C$. By construction, c is satisfied by A if and only if $c' = (s^i_j, s^j_i, =)$ is satisfied by π. Hence, $w_{c'}(\pi) = w_c(A)$. On the other hand, consider an SoD$_U$ constraint $c = (r_i, r_j, \updownarrow, \forall) \in C$. If c is satisfied by A, then by construction $c' = (S^i, S^j, \emptyset)$ is also satisfied by A. Finally, if c is violated by A, then let $t = |A(r_i) \cap A(r_j)| > 0$. By construction, $\pi(S^i) \cap \pi(S^j) \subseteq A(r_i) \cap A(r_j)$. Hence, $w'_{c'}(\pi) \le w_c(A)$, implying $w'_{C'}(\pi) \le w_C(A)$. Therefore, $\mathrm{OPT}(\mathcal{I}') \le \mathrm{OPT}(\mathcal{I})$. □

Example 1. We illustrate the construction of Valued WSP instance from Valued APEP instance, and the proof of Lemma 6.1 using Figure 1. In addition, given an optimal solution for the corresponding Valued WSP instance, we illustrate how to construct an optimal solution of the Valued APEP instance as described in Lemma 6.1. As per the figure, there are three BoD$_E$ constraints ($c_1 = (r_1, r_2, \leftrightarrow, \exists)$, $c_2 = (r_1, r_3, \leftrightarrow, \exists)$, and $c_3 = (r_3, r_4, \leftrightarrow, \exists)$) and



ω	r_1	r_2	r_3	r_4
u_1	0	0	1	1
u_2	0	1	0	0
u_3	1	0	1	1
u_4	1	1	0	1

(a) Valued APEP instance

ω'	S^1	S^2	S^3	S^4
u_1	0	0	1	1
u_2	0	1	0	0
u_3	1	0	1	1
u_4	1	1	0	1

(b) Valued WSP instance

Figure 1: An example illustrating the construction. Note that $R = \{r_1, r_2, r_3, r_4\}$ and $S' = S^1 \cup S^2 \cup S^3 \cup S^4$.

two SoD_U constraints ($c_4 = (r_1, r_4, \updownarrow, \forall)$, and $c_5 = (r_2, r_4, \updownarrow, \forall)$). We define their weights as follows. For an authorization relation $A \subseteq U \times R$ and $i \in \{1, 2, 3\}$, we set $w_{c_i}(A) = 0$ if A satisfies c_i, and $w_{c_i}(A) = 1$, otherwise. Let $w_{c_4}(A) = 0$ if A satisfies c_4; otherwise, $w_{c_4}(A) = |A(r_1) \cap A(r_4)|$. Similarly, $w_{c_5}(A) = 0$ if A satisfies c_5; otherwise, $w_{c_5}(A) = |A(r_2) \cap A(r_4)|$.

Observe that in this example there is no authorization relation A such that $w_C(A) + \Omega(A) = 0$. It means that if we look for an authorization relation A such that $w_C(A) = 0$, then we will have $\Omega(A) > 0$. Consider an authorization relation A^* such that $A^*(r_1) = \{u_1, u_2\}, A^*(r_2) = \{u_1, u_3\}, A^*(r_3) = u_2$, and $A^*(r_4) = \{u_4\}$. Observe that $w_C(A^*) = 0$ but $\Omega(A^*) = 1$. Conversely, if we look for an authorization relation A such that $\Omega(A) = 0$, then we will have $w_C(A) > 0$.

Consider the Valued WSP instance constructed in this example. Based on the construction, $c'_1 = (s^1_2, s^2_1, =), c'_2 = (s^1_3, s^3_1, =), c'_3 = (s^3_4, s^4_3, =), c'_4 = (S^1, S^4, \emptyset)$, and $c'_5 = (S^2, S^4, \emptyset)$. Observe that for a given plan $\pi : S' \to U'$, we have the following:

- $w_{c'_1}(\pi) = 0$ if π satisfies c'_1 and $w_{c'_1}(\pi) = 1$ otherwise,
- $w_{c'_2}(\pi) = 0$ if π satisfies c'_2 and $w_{c'_2}(\pi) = 1$ otherwise,
- $w_{c'_3}(\pi) = 0$ if π satisfies c'_3 and $w_{c'_3}(\pi) = 1$ otherwise,
- $w_{c'_4}(\pi) = 0$ if π satisfies c'_4 and $w_{c'_4}(\pi) = |\pi(S^1) \cap \pi(S^4)|$ otherwise, and
- $w_{c'_5}(\pi) = 0$ if π satisfies c'_5 and $w_{c'_5}(\pi) = |\pi(S^2) \cap \pi(S^4)|$ otherwise.

Consider an optimal plan $\pi : S' \to U'$ defined as follows: $\pi(s^1_2) = u_1, \pi(s^1_3) = u_2, \pi(s^2_1) = u_1, \pi(s^3_1) = u_2, \pi(s^3_4) = u_4$, and $\pi(s^4_3) = u_4$. Observe that $w'_{C'}(\pi) = 0$ as all constraints c'_1, c'_2, c'_3, c'_4, and c'_5 are satisfied. Then, $\Omega'(u_1, \{s^1_2, s^2_1\}) = \Omega(u_1, \{r_1, r_2\}) = 0$, $\Omega'(u_2, \{s^1_3, s^3_1\}) = \Omega(u_2, \{r_1, r_3\}) = 0$, and $\Omega'(u_4, \{s^3_4, s^4_3\}) = \Omega(u_4, \{r_3, r_4\}) = 1$. Finally, $\Omega'(u_3, \emptyset) = 0$. Thus, $\Omega'(\pi) = 1$.

We construct A from π as in the first part of the the proof of Lemma 6.1: $A(r_1) = \{u_1, u_2\}$, $A(r_2) = \{u_1\}$, $A(r_3) = \{u_2, u_4\}$, and $A(r_4) = \{u_4\}$. Observe that $w_C(A) = 0$ as all constraints $c_1, \ldots, c_5$ are satisfied by A and $\Omega(A) = 1$.

We can now state the main result of this section.

Theorem 6.2. Valued APEP$\langle \text{BoD}_\text{E}, \text{SoD}_\text{U} \rangle$ *is fixed-parameter tractable and can be solved in* $O^*(4^{k^2 \log k})$ *time.*

Proof. Let $\mathcal{I} = (R, U, C, \omega)$ be an instance of Valued APEP$\langle \text{BoD}_\text{E}, \text{SoD}_\text{U} \rangle$. We construct an instance $\mathcal{I}' = (S', U, C', \omega')$ of Valued WSP in polynomial time. We then invoke Theorem 3.2 to obtain an optimal plan $\pi : S' \to U'$. Finally, we invoke Lemma 6.1 to construct an optimal authorization relation A for $\mathcal{I}$ such that $\Omega(A) + w_C(A) = \text{OPT}(\mathcal{I}')$. The algorithm described in Theorem 3.2 runs in $O^*(2^{|S'| \log |S'|})$ time. Since $|S'| \leq k(k-1)$, the running time of this algorithm to solve Valued APEP$\langle \text{BoD}_\text{E}, \text{SoD}_\text{U} \rangle$ is $O^*(4^{k^2 \log k})$. □

7 RELATED WORK AND DISCUSSION

Valued APEP builds on a number of different strands of recent research in access control, including workflow satisfiability, workflow resiliency and risk-aware access control. Workflow satisfiability is concerned with finding an allocation of users to workflow steps such that every user is authorized for the steps to which they are assigned and all workflow constraints are satisfied. Work in this area began with the seminal paper by Bertino *et al.* [2]. Wang and Li initiated the use of parameterized complexity analysis to better understand workflow satisfiability [24], subsequently extended to include user-independent constraints [4] and the study of Valued WSP [8]. As we have seen APEP can be used to encode workflow satisfiability problems.

Workflow resiliency is concerned with ensuring business continuity in the event that some (authorized) users are unavailable to perform steps in a workflow [13, 18, 20]. Bergé *et al.* showed that APEP can be used to encode certain kinds of resiliency policies [1, Section 6].

Researchers in access control have recognized that it may be necessary to violate access control policies in certain, exceptional circumstances [21, 22], provided that those violations are controlled appropriately. One means of controlling violations is by assigning a cost to policy violations, usually defined in terms of risk [3, 11]. Thus, the formalization of problems such as Valued WSP and Valued APEP and the development of algorithms to solve these problems may be of use in developing risk-aware access control systems.

Thus, we believe that APEP and Valued APEP are interesting and relevant problems, and understanding the complexity of these problems and developing the most efficient algorithms possible to solve them is important. A considerable amount of work has been done on the complexity of WSP, showing that the problem is FPT for many important classes of constraints [4, 7, 16]. It is also known that Valued WSP is FPT and, for user-independent constraints, the complexity of the problem is identical to that for WSP (when polynomial terms in the sizes of the user set and constraint set are disregarded in the running time) [8]. Roughly speaking, this is because (weighted) user-independent constraints in the context of workflow satisfiability allow us to restrict our attention to partitions of the set of steps when searching for solutions, giving rise to the exponential term $2^{k \log k}$ in the running time of an algorithm to solve (Valued) WSP.

APEP, unsurprisingly, is known to be a more complex problem [1]. The complexity of APEP differs from WSP because it is not sufficient to consider partitions of the set of resources, in part because an arbitrary relation A is not a function. The results in this paper provide the first complexity results for Valued APEP, showing (in Corollary 4.15) that it is no more difficult than APEP for constraints in BoD_U, BoD_E, SoD_U and SoD_E (disregarding polynomial terms).

We believe the concept of a user profile and Theorem 4.14 are important contributions to the study of APEP as well as Valued APEP, providing a generic way of establishing complexity results for different classes of constraints. In particular, Corollary 4.15 of Theorem 4.14 actually shows how to improve existing results for APEP$\langle BoD_U, BoD_E, SoD_E, SoD_U \rangle$ due to Bergé *et al* [1]. Moreover, when an APEP instance is equivalent to a WSP instance (i.e, it contains a cardinality constraint $(r, \leq, 1)$ for each $r \in R$) then the instance is k-bounded, and a user profile is the characteristic function of some partition of R. Thus we essentially recover the known FPT result for Valued WSP, which is based on the enumeration of partitions of the set of workflow steps.

8 CONCLUDING REMARKS

We believe this paper makes two significant contributions. First, we introduce Valued APEP, a generalization of APEP, which, unlike APEP, always returns some authorization relation. The generalization is achieved by replacing the base authorization relation and constraints with functions that assign a non-negative weight to potential solutions, the weight being 0 iff the potential solution is authorized with respect to the base relation and the constraints. Thus a solution to Valued APEP is more useful than that provided by APEP: if there exists a valid authorization relation Valued APEP will return it; if not, Valued APEP returns a solution of minimum weight. This allows an administrator, for example, to decide whether to implement the solution for an instance of Valued APEP or adjust the base authorization relation and/or the constraints in the input in an attempt to find a more appropriate solution.

The second contribution is to advance the techniques available for solving APEP as well as Valued APEP. Specifically, the notion of a user profile plays a similar role in the development of algorithms to solve (Valued) APEP as patterns do in solving (Valued) WSP. The enumeration of user profiles is a powerful technique for analyzing the complexity of Valued APEP, yielding general results for the complexity of the problem (which are optimal assuming the Exponential Time Hypothesis holds) and improved results for APEP.

There are several opportunities for further work. Most importantly, we plan to encode Valued APEP in a form that can be solved using off-the-shelf solvers, as has already been done for WSP and Valued WSP [8, 9, 16].

We would like to improve the result in Corollary 4.15: we believe, based on results in Sections 5 and 6, that it should be possible to obtain time complexity $O(2^{p(k)})$, where p is polynomial. We also intend to investigate other (weighted) user-independent constraints for (Valued) APEP. First, we are interested in what other problems in access control can be encoded as APEP instances, apart from workflow satisfiability and resiliency problems. Second, we would like to consider appropriate weight functions for such encodings, which would have the effect of providing more useful (weighted) solutions for the original problems (rather a binary yes/no solution).

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