

Employment Adjustment and Labor Utilization*

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June 26, 2016

Abstract

Standard models of labor adjustment assume that firms can change only the size of their workforce (the extensive margin) and not the number of hours of their existing employees (the intensive margin) in response to shocks. I propose a general equilibrium search model that allows for adjustment on both of these margins. The model includes on-the-job search that generates different vacancy filling and attrition rates across firms. I calibrate the model to a unique matched employer-employee panel of Danish firms and simulate two labor market policies aimed at promoting job creation: hiring subsidies and a reduction in the official workweek.

*Manuscript received April 2015; revised September 2015.

¹I am grateful to my late PhD advisor, Dale Mortensen, for his extensive help and guidance throughout this project. I thank the editor, Guido Menzio, and three anonymous referees for very helpful comments and suggestions. I also thank Rasmus Lentz, Christopher Taber, Eva Nagypal, Gadi Barlevy, Eric French, Pieter Gautier, and seminar participants at Chicago Fed, Northwestern University, Pennsylvania State University, Purdue University, Einaudi Institute for Economics and Finance, Uppsala University, Tinbergen Institute, Aarhus University, University of Bristol, Essex University, Oxford University, and NBER Summer Institute. This research was supported by the Labour Market Dynamics and Growth (LMDG) project at Aarhus University. Finally, I want to express my special thanks to Henning Bunzel for sharing his expertise on the data - this project would not exist without his invaluable help. Please address correspondence to: Ija Trapeznikova, Department of Economics, Royal Holloway University of London, Egham Hill, Egham, Surrey, TW20 0EX, UK. E-mail: ija.trapeznikova@rhul.ac.uk.

1 Introduction

The variation in employment depends on the ability of firms to adjust their labor demand in response to shocks. In the short run, firms can respond to productivity fluctuations by varying the hours of work of their existing employees. Yet, standard economic models of labor adjustment allow firms to change only the number of workers they employ (the extensive margin) and not the amount that each worker works (the intensive margin). The goal of this paper is to relax this assumption and to propose and calibrate a dynamic model that includes both margins of adjustment.

I start by documenting the importance of hours adjustment for firms' labor demand policies. The main reason why many of existing labor adjustment models abstract from changes in labor utilization is the scarcity of high-frequency micro data on work hours.² This paper is using a unique matched employer-employee panel of Danish administrative firm data that contains all private firms in the economy for the period of 1999-2006. This dataset includes firm-level information on employment and work hours on a quarterly basis. Based on these data, I show that firms use variation in hours to economize on changes in the number of workers. In particular, the growth rate of hours per worker and employment growth are negatively correlated at the firm level; while lagged changes in hours are positively correlated with changes in employment.³ An adjustment cost model provides a natural framework for explaining these empirical facts.

In this paper, I develop a general equilibrium model of joint dynamics of the number of workers and hours per worker. I extend a standard random search framework (see Mortensen and Pissarides, 1994) to include multi-worker firms with a decreasing returns to scale revenue function. The driving force of the model is idiosyncratic profitability shocks, which firms can accommodate by changing the work hours of their existing employees, instead of (or jointly with) varying the size of their labor force. Hours of work and compensation in the model are determined as an outcome of a cooperative game according to the firm's and workers' Shapley values. The presence of frictions in the labor market means that matching workers with vacant jobs takes time and uses resources. On the other hand, raising hours can be done immediately, although at a cost of higher wages. Hence, the firm faces a trade-off between these two channels of adjustment.

An important feature of the model is on-the-job (OTJ) search. Firstly, job-to-job transitions have important implications for firm-level employment dynamics: in this setup, not only does a firm post more

²The existing empirical studies are either limited to industry-level data (see for instance, Hamermesh and Pfann, 1996) or firm-level data that are more than three decades old (for example, Cooper, Haltiwanger and Willis, 2007 use Longitudinal Research Database 1972-1980).

³These facts have been documented also for the US labor market by Cooper et al. (2007).

vacancies in the event of a positive shock, it also finds it easier to fill vacancies and to retain its current employees, which increases the speed of employment adjustment. Secondly, on-the-job search is a necessary component that explains why there exists a negative correlation between hours and employment growth rates at contracting firms. That is, firms that are hit by a negative profitability shock face an increase in the quit rate of their existing workers. As the number of workers keeps falling due to higher attrition, the average work hours start rising. Finally, I find that on-the-job search enables the model to capture most of the features of the data regarding worker flows.

The model is calibrated to fit Danish firm data and is successful in capturing the overall features of the data. Given the calibrated parameter values, I find that the average cost of hiring a new worker is equal to about two weeks of wages. This value is low compared to the estimates found for other European countries (see among others Rota, 2004, Goux, Maurin and Pauchet, 2001, and Kramarz and Michaud, 2004), reflecting the fact that the Danish labor market is very mobile with worker flow rates averaging around 8 percent per month. In the next step, I simulate two types of policy experiments aimed at fostering job creation: (i) introducing a hiring subsidy and (ii) imposing an upper limit on the work hours. Using this model, I can assess which of the two policies is more effective in reducing the unemployment rate and at what cost. I find that a hiring subsidy reduces unemployment, while a shorter workweek increases it, although the effects are quantitatively small in both cases. I find considerably larger effects (or even qualitatively different) in a 'partial equilibrium' version of the model, in which the vacancy filling rates and the quit rates are kept unchanged. These results suggest that a partial equilibrium model significantly overestimates the effect of the adjustment costs on aggregate employment. Moreover, I show that excluding OTJ search from the model generates counterintuitive results in the above policy experiments, thus implying that endogenous offer acceptance and quit rates are key not only for matching main features of the data, but also for predicting a negative effect of hiring subsidies on unemployment.

There are two strands of literature that this work is based on. First, there is an extensive body of research that examines the effect of adjustment costs on employment within a neoclassical framework (see Hamermesh and Pfann, 1996 for a comprehensive survey).⁴ Previous work that accounts for labor utilization in adjustment cost models includes Caballero, Engel and Haltiwanger (1997) and Cooper and Willis (2009). Most of these papers are set within a partial equilibrium framework where firms optimize their labor demand in isolation from decisions of other firms or workers. The predictions of these models are often very different from those derived from a general equilibrium analysis.⁵ In contrast, my model accounts for

⁴The literature on labor adjustment costs is also closely related to the investment literature (see for example Caballero and Engel, 1991). Bond and Van Reenen (2007) survey econometric research on adjustment processes for both capital and labor using micro data.

⁵For example, Bentolila and Bertola (1990) show that higher dismissal costs increase aggregate employment in a partial equilib-

equilibrium interactions between firms and workers through a matching function and shows that hiring externalities across firms are quantitatively very important.

Secondly, this paper is linked to standard random search models (see for instance Mortensen and Pissarides, 1994) and more recent work that introduces a theory of multi-worker firms into search models (see for instance Coles and Mortensen, 2016, Acemoglu and Hawkins, 2014, and Moscarini and Postel-Vinay, 2013).⁶ This paper is most closely related to Cooper et al. (2007), which examines the variation in hours, employment, and vacancies using the US data. Given that the focus of their paper is primarily to examine the differences in labor dynamics at the firm level and in the macro data, they simplify labor adjustment and wage determination processes.⁷ Instead, I allow for richer and more realistic worker and hours dynamics across firms by extending their model to introduce endogenous quits and joint determination of hours and compensation.

The paper proceeds as follows. Section 2 presents empirical evidence on employment and hours adjustment using Danish firm data. Section 3 introduces and describes the model. Section 4 shows the calibration of the model and its fit to the data. Section 5 proceeds to demonstrate the impact of policy experiments on aggregate employment and output. Section 6 summarizes the findings. The appendix provides details on the data sources used in this paper, on the Shapley Values, as well as on the numerical simulation procedure.

2 Data

The Danish labor market is known for its so-called ‘flexicurity’ (flexibility+security) model featuring a minimum set of regulations and a generous unemployment insurance scheme. Despite a high level of unionization in Denmark, recent trends towards decentralization allow for more flexibility in determining both wages and working time because many conditions can now be negotiated at the firm level. The empirical analysis in this paper is based on administrative Danish data that contains all private firms in the economy for the period of 1999-2006 (a detailed description of the data sources and the Danish labor market is provided in Appendix A). Employment information is drawn from a monthly matched employer-

rium framework, while Hopenhayn and Rogerson (1993) find that this prediction is actually reversed in their general equilibrium model of heterogeneous firms with endogenous entry and exit.

⁶Kaas and Kircher (2015) develop a multi-worker firms model within a competitive search framework that has many similar implications for employment growth at the firm level.

⁷Cooper et al. (2007) simplify the wage setting mechanism by assuming that firms are paying workers their outside option so that workers are indifferent between being employed or unemployed. In that case, job-to-job transitions become irrelevant as workers get the same wage at all firms. Moreover, their setup implies that firms cannot adjust wages in the case of a negative profitability shock.

employee panel, while hours data come from two sources.

The first dataset is based on the Earnings survey that collects individual-level hours information on an annual basis.⁸ The trade-off between the number of workers and hours exists primarily in the short run as firms eventually adjust their employment to its optimal level. Therefore, to investigate the firm-level hours dynamics we need to observe changes in hours on a more frequent basis. To this end, I use the second dataset drawn from firms' mandatory pension contributions. In Denmark, the amount of pension contributions that a firm pays for each of its employees depends on which of the four intervals her weekly work hours fall into: 0-9, 9-18, 18-27 or more than 27 hours of week. Each firm reports the total amount of its contributions paid for all employees over a quarter. Based on this information and the number of workers for each firm, I compute a quarterly measure of the average hours per worker to capture the short-term variation in the intensive margin of employment.⁹

Bear in mind that this measure of labor utilization may mask some of the variation in hours as a firm's pension contribution amount will only change when at least one of its employees moves to a different 9-hour interval (for instance, if a part-time worker starts working full time). In order to check whether this measure captures the variation in labor resources well enough, I compare it to the Earnings survey: I find that at the annual level the two hours series move closely together, especially for the growth regions below 50 percent in absolute value (see Appendix A2 for details).

2.1 Hours and employment

Two key questions that this paper addresses are (i) to what extent firms vary work hours of their employees, and (ii) is the observed dynamic interaction between hours and employment consistent with a model of adjustment costs? Table 1 presents summary statistics on the cross-sectional variation in the quarterly growth rates of hours per worker and employment, and the relationship between them. I use \tilde{h} to denote the measure of hours per worker derived from the pension contributions data. First, firms exhibit a significant variation in employment growth: only one third of Danish firms employ the same number of workers in two consecutive quarters. Much lower magnitudes of employment changes have been reported for other European countries: Varejao and Portugal (2007), for instance, find that employment remains unaltered over the course of a quarter for 75 percent of establishments in a representative sample of Portuguese firms.

⁸This survey covers all private firms with more than 10 full-time employees, with the exception of agriculture and fishery.

⁹In particular, I use the left boundary of each 9-hour interval to obtain the lower bound measure of hours. Alternatively, I construct an upper bound measure using the right boundary of each 9-hour interval. These two measures behave very similarly, hence I only report the results based on the lower bound measure. Appendix A2 provides a detailed discussion on the construction of both of these variables.

Table 1: Variation in the firm-level growth rates of hours per worker and employment.

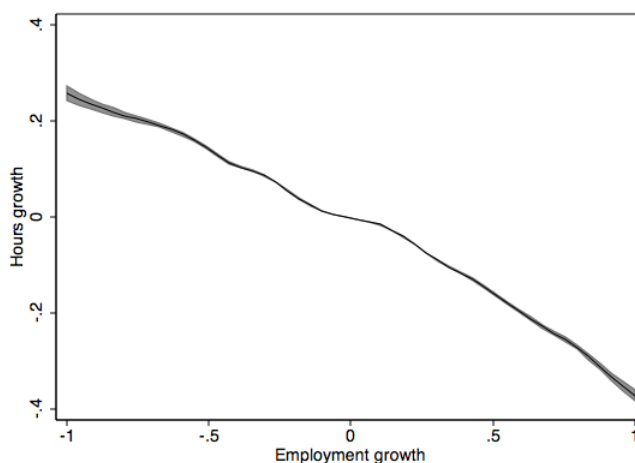
	Non-weighted	Employment-weighted	Employment-weighted, no time effects
Std. dev($\Delta \log N_t$)	0.277	0.237	0.236
Std. dev($\Delta \log h_t$)	0.285	0.232	0.231
Corr($\Delta \log N_t, \Delta \log \tilde{h}_t$)	-0.300	-0.427	-0.432
Corr($\Delta \log N_t, \Delta \log \tilde{h}_{t-1}$)	0.087	0.101	0.108

Source: Author's calculations from Danish firm data, 1999-2006. Note: \tilde{h} denotes the average hours per worker constructed using the interval measure of hours (see Appendix A2 for details).

This finding suggests that the Danish labor market, in contrast to other continental European countries, is characterized by relatively low adjustment costs.

The second observation is that the standard deviation of hours and employment growth is about the same, suggesting that firms use both margins of adjustment. Table 1 also shows that there is a negative association between hours and employment growth rates at the firm level. This relationship is monotone and the negative correlation between the two series is observed for virtually all values of employment growth (see Figure 1). Moreover, the relationship between employment growth at period t and hours at $t - 1$ is positive. This empirical evidence is consistent with the labor adjustment costs hypothesis: Suppose that hiring is impeded by search frictions and a firm is hit by a positive profitability shock. In response to the shock, the average work hours overshoot their optimal level and start falling as the firm's labor force builds up to its new desired level. As a result, hours per worker and employment move in the opposite directions and changes in hours lead changes in employment.

Figure 1: Non-parametric regression of hours growth on employment growth



Note: Nadaraya-Watson estimator using a Gaussian kernel with a bandwidth of 0.08. Shaded areas are 90 percent point-wise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on Danish firm data, 1999-2006.

Table 2: Average monthly job flow and worker flow rates

	Non-weighted	Employment-weighted	Employment-weighted, continuing firms
Hires	0.193	0.095	0.077
Separations	0.184	0.093	0.075
Job Creation	0.171	0.058	0.039
Job Destruction	0.162	0.056	0.037
Net employment change	0.002	0.002	0.002
Churning	0.044	0.073	0.074

Note: Sample includes all private firms and contains more than 10 million firm-month observations. Source: Author's calculations from the Danish firm data, 1999-2006.

2.2 Job and worker flows

In this subsection, I examine net and gross employment changes at the firm level. Most of the existing models that are used to assess the effect of labor adjustment costs on employment dynamics focus on net employment changes and do not explicitly distinguish between worker and job flows. However, this distinction matters for firms' labor demand: whether an increase in employment is obtained by hiring new workers or lowering attrition rates, or likewise whether a reduction in the workforce is achieved through quits or layoffs, has different implications in terms of adjustment costs.

Using matched employer-employee structure of the dataset, I construct monthly hires and separations (i.e. worker flows) at the firm level. Job flows are defined as the number of jobs created in growing firms (job creation) and the number of jobs destroyed in contracting firms (job destruction) within a month. The corresponding rates are expressed in flows divided by the average employment over two months. The data at hand indicate that there is a fair amount of job and worker mobility in the Danish labor market: monthly (size-weighted) hiring and separation rates average about 8 percent of employment (see Table 2). Job destruction and job creation rates are about 5-6 percent of employment (4 percent for continuing firms), more than twice the rates in the US labor market (see Davis, Faberman and Haltiwanger, 2006). That is, one of every 20 jobs on average gets destroyed every month.

To highlight the difference between job flows and worker flows, I construct a worker churning rate that refers to worker flows in excess of job flows.¹⁰ The fact that firms churn workers indicates that contracting businesses still hire workers and workers leave growing firms. The churning rate is high in Denmark, averaging 7 percent per month. On average over the period of 1999-2006, job creation constitutes just 32.2 percent of all (size-weighted) hires and only 30.6 percent of all separations are associated with job

¹⁰The churning rate is defined as the sum of the hiring and separation rates less the absolute value of the net growth rate in employment (see Burgess, Lane and Stevens, 2001 for more details on this measure).

destruction.¹¹

To sum up, the empirical evidence presented in this section shows that firms vary their labor input on both the extensive and intensive margin. Movements in hours and employment are inversely related, supporting the idea of adjustment costs causing a fast response of hours and a sluggish response of employment. I also show that worker flows and job flows are quite distinct - in fact, only about a third of monthly hires and separations arise in connection with job creation and job destruction, respectively. Different implications of net and gross employment changes in terms of adjustment costs call for a theory that explicitly models hiring and separation decisions of firms. The following section develops a labor adjustment model that incorporates the above features of the data.

3 Model

3.1 Setup

The model is a continuous time matching model of multi-worker firms with heterogeneous profitability. A final good Y is produced by a continuum of intermediate inputs x and is sold by many suppliers in a competitive output market at price P . Let the final good be determined by the (Dixit-Stiglitz) CES production function:

$$(1) \quad Y = \left[\int_0^K x(j)^{\frac{\rho-1}{\rho}} dj \right]^{\frac{\rho}{\rho-1}}, \quad \rho > 0,$$

where $x(j)$ is the quantity of product j , ρ represents the elasticity of substitution between any two intermediate goods, and K is the total measure of inputs available. The final good is produced by many competitive suppliers; therefore, a profit-maximizing amount of each input is given by

$$(2) \quad x(j) = \left(\frac{P}{p(j)} \right)^{\rho} Y, \quad j \in K,$$

where P is the price of the final good, and $p(j)$ is the price of input j . Let the final good be a numeraire with $P = 1$.

The intermediate good is produced using a linear technology, i.e. $x = qhn$, where x is the number of units supplied, q is firm's productivity, and hn is total labor input, the product of the number of workers n

¹¹Another sign of flexibility of the Danish labor market is its relatively frequent use of temporary workers and temporary layoffs. For example, I find that about 30 percent of monthly hires are recalls, i.e. workers who have been employed at the same firm during the last three months.

and the average work hours h . Then, a firm's revenue function is given by

$$(3) \quad R_n(h, q) = Y^{\frac{1}{\rho}} (qhn)^{\frac{\rho-1}{\rho}},$$

where R is revenues and Y is aggregate demand for the final good. Although the production technology is linear in the number of workers, the fact that each firm faces downward-sloping demand leads to a decreasing marginal revenue product.

Firms differ in their productivity level q , which at any given point in time is subject to a shock that arrives at Poisson rate μ . In the event of a shock, a new productivity level is drawn from distribution $\Phi(\cdot)$, independently of the current productivity level. The corresponding density function is denoted by $\phi(\cdot)$ and is defined on support $[q, \bar{q}]$. Existing firms are subject to the exogenous destruction risk and die at rate δ . At the same time, new firms enter the market at exogenous rate η , so that the measure of firms is stationary and equal to $K = \eta/\delta$.¹²

Firms and workers are brought together pairwise through a sequential and random matching process. To recruit, firms post vacancies v at cost $c(v)$ per unit of time, where $c(\cdot)$ is a strictly increasing and convex function. Reflecting search frictions, the offer arrival rate and the vacancy filling rate are exogenous to workers and firms but are determined in equilibrium. A job separation occurs if a worker quits or is laid off. Firing a worker is assumed to be costless.¹³

There is a continuum of infinitely lived identical workers, with a mass normalized to one, that supply labor to intermediate product firms. Individuals derive utility from aggregate good consumption and incur disutility from working. Worker's utility function, ω , is assumed to be separable in consumption and work hours:¹⁴

$$(4) \quad \omega(y, h) = y - g(h),$$

where y is the amount of the aggregate good consumed, h is the number of hours the individual is working, and $g(\cdot)$ is a strictly increasing convex function which takes the following form:

$$(5) \quad g(h) = \chi h^{\xi}, \quad \xi > 1, \chi > 0.$$

¹²In the online appendix, I consider a version of the model, in which entry rate η is endogenous, and report the results of policy experiments when the number of firms is allowed to vary.

¹³The firing costs in Denmark are virtually non-existent (see Appendix A1 for details); therefore, I focus on labor adjustment costs that are associated with hiring frictions only.

¹⁴Linear utility in consumption implies risk neutrality; therefore, there is no savings motive in workers' decisions.

Worker's consumption equals the real wage, w , when employed and equals to b when unemployed. Here, b can be viewed as unemployment insurance benefits that are indexed by the aggregate price level, or as the value of home production less utility costs arising from producing it. Finally, workers search while employed and unemployed. The work hours and compensation are determined as an outcome of a cooperative game between a firm and its workers. In particular, I use the Shapley values to divide the surplus of the match between the firm and its workers.¹⁵

3.2 Worker's decision problem

When unemployed, the worker obtains consumption flow b by means of home production, and she has an option of finding a job. Hence, the value of unemployment expressed in terms of final output, U , solves the following continuous time Bellman equation:

$$(6) \quad rU = b + \lambda(\theta) \int (\max\{W, U\} - U) dF(W),$$

where r is the common firms' and workers' discount rate, $\lambda(\theta)$ is the job arrival rate, and $F(W)$ is the cumulative distribution function of job vacancies posted by firms that provide workers with the value of employment of at most W .

The job arrival rate λ is derived from a matching function that is assumed to be increasing, concave, and homogenous of degree one in both arguments, vacancies and job seekers.¹⁶ Given the matching function properties, $\lambda(\theta)$ is increasing and concave in market tightness θ , which is the ratio of the aggregate number of vacancies posted to individuals searching for a job, the variable that is determined endogenously in equilibrium. Note that although the value of unemployment depends on aggregate objects, such as labor market tightness and the distribution of offers across firm types, they are not listed as arguments for

¹⁵A number of papers refer to Stole and Zwiebel bargaining mechanism, in which firms engage in pairwise negotiations with their workers, to determine wages in multi-worker firms (see, for example, Stole and Zwiebel, 1996, Smith, 1999, Cahuc and Wasmer, 2001, and Ebell and Haefke, 2009). Brügemann, Gautier and Menzio (2015) show, however, that the bargaining game as described in Stole and Zwiebel (1996) does not lead to the Shapley values as its solution. Instead they propose a modified bargaining game, referred to as the Rolodex game, that results in the Shapley values and hence offers a game-theoretic foundation to the standard cooperative solution. Hence, the workers' compensation determination process in this paper can be thought of as arising from the Rolodex game. An alternative wage determination structure has been proposed by Hawkins (2015), where firms commit to wage contracts so that wages of existing employees are not affected by new hires. The implication of this wage setting mechanism is wage dispersion both within and across firms (unless firms are allowed to pay different lump-sum signing bonuses to its workers). Given that I consider an economic environment with shocks to productivity and on-the-job search, allowing for wages to vary within a given firm (and hence for quit rates to vary across workers) would make the wage setting problem insolvable.

¹⁶See, for example, Petrongolo and Pissarides (2001) for details on the concept of a matching function.

notational simplicity.

The value of employment at a firm with n workers and productivity q , $W_n(q)$, satisfies the following Bellman equation:

$$(7) \quad rW_n(q) = \left\{ \begin{array}{l} \omega_n(q) + (\delta + s_0)(U - W_n(q)) + \lambda(\theta)\kappa \int (\max\{W', W_n(q)\} - W_n(q)) dF(W') \\ + H_n(q)(W_{n+1}(q) - W_n(q)) + s_n(q)(n-1)(W_{n-1}(q) - W_n(q)) \\ + \mu \int_{\underline{q}}^{\bar{q}} \left(\mathbf{1}_{[n > \bar{n}(q')]} \left(\frac{n - \bar{n}(q')}{n} U + \frac{\bar{n}(q')}{n} W_{\bar{n}(q')}(q') \right) \right. \\ \left. + \mathbf{1}_{[n \leq \bar{n}(q')]} W_n(q') - W_n(q) \right) \phi(q') dq' \end{array} \right\},$$

where $\omega_n(q)$ is the utility flow expressed in terms of final output as defined in equation (4). The worker becomes unemployed at constant Poisson rate $\delta + s_0$, where s_0 represents the exogenous component of the quit rate and δ refers to the destruction shock. The worker receives an alternative job offer at rate $\lambda(\theta)\kappa$, where $\kappa \geq 0$ represents relative search intensity of employed workers (if $\kappa = 1$ then workers search with the same intensity regardless of their employment status; $\kappa = 0$ implies no OTJ search). Hence, the next term on the RHS of equation (7) is attributed to the option value of moving to a better employment position.

The following two terms are related to the expected change in the value of employment when the firm adjusts its labor force. In particular, at rate $H_n(q)$ the firm hires another worker, and at rate $s_n(q)(n-1)$ one of the other $(n-1)$ workers separates from the firm. These rates are determined endogenously in equilibrium and are defined in the firm's problem below. The last term on the RHS reflects the expected change in the value due to a shock to firm's productivity q that arrives at rate μ . A new productivity is drawn from distribution $\Phi(\cdot)$, with corresponding density $\phi(\cdot)$. Let $\bar{n}(q)$ be the maximum labor force size that the firm and its workers are willing to sustain given current productivity level q . If the firm's labor force exceeds its maximum size, i.e. if $n > \bar{n}(q)$ then the worker gets unemployed with probability $\frac{n - \bar{n}(q)}{n}$. Separations are bilaterally efficient in this model; hence, there is no distinction between layoffs and voluntary quits.

The standard Blackwell's sufficient conditions for the Contraction Mapping Theorem apply and a unique solution for the value of employment $W_n(q)$ and the value of unemployment U exists (see Stokey and Lucas, 1989).

3.3 Firm's decision problem

The optimization problem of a firm is to choose optimal vacancy posting and the maximum level of employment. For the firm's problem, it is useful to write the hiring and separation rates explicitly. The rate, at which each worker separates from a firm with productivity q and employment n , is the sum of the

exogenous quit rate into unemployment and the job-to-job transition rate, i.e.

$$(8) \quad s_n(q) = s_0 + \lambda(\theta)\kappa[1 - F(W_n(q))],$$

where $F(W_n(q))$ is the fraction of vacancies posted by firms that provide workers with the value of employment of at most $W_n(q)$.

The probability that any offer is acceptable to a randomly contacted worker is

$$(9) \quad a_n(q) = \begin{cases} \frac{u+(1-u)\kappa G(W_n(q))}{u+(1-u)\kappa}, & \text{if } W_n(q) \geq U \\ 0, & \text{otherwise} \end{cases}$$

where u is the fraction of unemployed workers, and $G(W_n(q))$ is the fraction of employed workers who gain the value of employment of at most $W_n(q)$. Employed job seekers are weighted by their search intensity, κ . If the worker's participation constraint is binding then no worker will accept the firm's offer. The hiring rate is equal to $H_n(q) = a_{n+1}(q)v_n(q)v(\theta)$, where $v(\theta)$ is the rate, at which vacancies are matched with workers, and $v_n(q)$ is the number of vacancies posted by a firm with n employees and productivity level q . The rate $v(\theta)$ is exogenous to the firm and is derived in equilibrium from the matching function.

The value of a firm with productivity q and employment n , $V_n(q)$, expressed in final output terms, solves the following Bellman equation:

$$(10) \quad (r + \delta)V_n(q) = \max \left\{ \left(\begin{aligned} & \pi_n(q) + \max_{v \geq 0} \left\{ a_{n+1}(q)v(\theta)v(V_{n+1}(q) - V_n(q)) - c(v) \right\} + \\ & s_n(q)n(V_{n-1}(q) - V_n(q)) + \mu \int_{\underline{q}}^{\bar{q}} (V_n(q') - V_n(q))\phi(q')dq' \end{aligned} \right), (r + \delta)V_{n-1}(q) \right\},$$

under the assumption that firing a worker is costless. The first term on the RHS is the firm's profit flow equal to $\pi_n(q) = R_n(h_n(q), q) - w_n(q)n$. The second term refers to the capital gain obtained from the possibility of hiring an additional worker, given the optimally chosen vacancy posting decision. The third term is the expected capital loss related to the possibility that any worker quits. The last term accounts for the expected change in the value of the firm caused by a shock to firm's productivity q . As before, for notational simplicity the aggregate variables are omitted from the arguments in the firm's value function, as well as in the hiring and separation rates. Given the assumptions on vacancy cost function $c(\cdot)$ and profits being bounded from above, $V_n(q)$ has a unique solution that is a fixed point of the contraction mapping.

3.4 Wage and hours determination

Hours of work and compensation are determined as an outcome of a cooperative game between the firm and its employees. I use the Shapley values to divide the surplus of the match between the firm and its workers. The Shapley value, originally proposed by Shapley (1953), is a commonly used way to divide the total worth of the grand coalition in a cooperative game with transferable utilities. It represents the average marginal contribution of each player i and is often considered to be a fair way to split the total value of a match.

Let $\Omega_n(q) = V_n(q) + nW_n(q)$ represent the total value of a match at a firm with n workers and productivity q . Combining equations (7) and (10), the total value solves the following Bellman equation:

$$(11) \quad (r + \delta)\Omega_n(q) = \max_{h \geq 0} \{R_n(h, q) - g(h)n\} + \delta nU \\ + H_n(q) \left(\Omega_{n+1}(q) - W_{n+1}(q) - \Omega_n(q) \right) - c(v_n(q)) \\ + s_n(q)n \left(\Omega_{n-1}(q) - \Omega_n(q) \right) + s_0 nU + \lambda(\theta)\kappa n \int_{W_n(q)} W' dF(W') \\ + \mu \int_q^{\bar{q}} \left(\mathbf{1}_{[n \leq \bar{n}(q')]}\Omega_n(q') + \mathbf{1}_{[n > \bar{n}(q')]}\left((n - \bar{n}(q'))U + \Omega_{\bar{n}(q')}(q')\right) - \Omega_n(q) \right) \phi(q') dq'.$$

The flow value of the total worth of the coalition consists of the total revenue less the utility costs from working, given the optimally chosen number of hours. The firm gets destroyed at rate δ , in which case its workers receive the value of unemployment. The next term is the expected capital gain when the firm hires a new worker at rate $H_n(q)$ less the hiring costs. Note that the incumbents receive the value $\Omega_{n+1}(q) - W_{n+1}(q)$ as $W_{n+1}(q)$ is the value that is paid to a newly hired worker. Similarly, the expected capital loss when a worker leaves the firm is given by the change in the value for the remaining coalition $\Omega_{n-1}(q) - \Omega_n(q)$ plus U if the worker quits into unemployment or, if the worker moves to a different job, the expected value of a new job, $\int_{W_n(q)} W' dF(W')$. The last term on the RHS of equation (11) reflects the expected change in the total value attributable to a productivity shock. If a new value of the match is low enough then it may be optimal to reduce the firm's workforce, in which case excess workers become unemployed.

Denote the surplus of the match by $Z_n(q) = \Omega_n(q) - nU - V_{n-1}(q)$. Then, the Shapley value of a worker employed at a firm with n workers and productivity q is equal to (Appendix B provides explains how I solve for the Shapley values in detail):

$$(12) \quad W_n(q) = U + \frac{1}{n+1} \left(\Omega_n(q) - nU - V_{n-1}(q) \right) = U + \frac{1}{n+1} Z_n(q),$$

while the Shapley value of a firm is

$$(13) \quad V_n(q) = V_{n-1}(q) + \frac{1}{n+1} \left(\Omega_n(q) - nU - V_{n-1}(q) \right) = V_{n-1} + \frac{1}{n+1} Z_n(q).$$

This means that the firm and its workers split the match surplus $Z_n(q)$ equally among themselves. When a worker joins or leaves the firm, the size of the coalition changes, hence wages are renegotiated with all workers. Finally, if a worker gets an offer that yields a higher value of employment under the current wage scheme she will take the offer and leave her current employer.¹⁷

To see that there exists an upper bound on employment, let $\Psi_n(q) = \Omega_n(q) - nU$ and use equations (6) and (11) to obtain the following:

$$(14) \quad \begin{aligned} (r + \delta)\Psi_n(q) = & R_n(h_n(q), q) - g(h_n(q))n - bn - \lambda n \int (W - U) dF(W) \\ & + H_n(q) \left(\Psi_{n+1}(q) - \Psi_n(q) \right) - H_n(q)(W_{n+1}(q) - U) - c(v_n(q)) \\ & + s_n(q)n \left(\Psi_{n-1}(q) - \Psi_n(q) \right) + \lambda \kappa n \int_{W_n(q)} (W' - U) dF(W') \\ & + \mu \int_{\underline{q}}^{\bar{q}} \left(\mathbf{1}_{[n \leq \bar{n}(q')] } \Psi_n(q') + \mathbf{1}_{[n > \bar{n}(q')] } \Psi_{\bar{n}(q')}(q') - \Psi_n(q) \right) \phi(q') dq' \end{aligned}$$

Since $R_n(h_n(q), q) - g(h_n(q))n - bn$ falls without a bound as the firm's labor force increases, there exists an upper limit on employment, $\bar{n}(q)$, beyond which the surplus $Z_n(q) = \Psi_n(q) - V_{n-1}(q)$ becomes negative. Moreover, since the firm and its workers share the match surplus, separations are bilaterally efficient, i.e. $W_n = U$ and $V_n = V_{\bar{n}(q)}$ for all $n > \bar{n}(q)$.

Wages serve as linear utility transfers and are used to split the surplus according to the corresponding Shapley value of each player. Rewriting equation (13) as $\Psi_n(q) - V_n(q) = \frac{n}{n+1} Z_n(q)$ and subtracting equation (10) from equation (14), I can solve for wages:

$$(15) \quad \begin{aligned} w_n(q) = & g(h_n(q)) + b + \lambda \int (W - U) dF(W) - \lambda \kappa \int_{W_n(q)} (W' - U) dF(W') \\ & + (r + \delta + H_n(q) + s_n(q)n + \mu) \frac{Z_n(q)}{n+1} - H_n(q) \frac{Z_{n+1}(q)}{n+2} - s_n(q)(n-1) \frac{Z_{n-1}(q)}{n} \\ & - \mu \int_{\underline{q}}^{\bar{q}} \left(\mathbf{1}_{[n > \bar{n}(q')] } \frac{\bar{n}(q')}{n} \frac{Z_{\bar{n}(q')}(q)}{\bar{n}(q') + 1} + \mathbf{1}_{[n \leq \bar{n}(q')] } \frac{Z_n(q')}{n+1} \right) \phi(q') dq'. \end{aligned}$$

The resulting wage function is quite complex and cannot be easily interpreted. However, the important

¹⁷I assume that a worker cannot use an outside offer to renegotiate her wage at her current job. Similarly, if a worker switches coalitions she will not be able to use her previous coalition as an outside option.

feature of this function is its direct dependence on the utility costs $g(h)$. This means that the adjustment on the hours margin is costly for firms as they need to compensate their employees for working longer hours.

The optimal hours can be determined outside of the employment optimization problem by maximizing the production surplus: $h_n(q) = \arg \max_{h \geq 0} \{R_n(h, q) - g(h)n\}$. Assuming an interior solution, the optimal number of hours satisfies the following first order condition:

$$(16) \quad h_n(q) = \left[\left(\frac{\rho - 1}{\chi \xi \rho} \right)^\rho Y \frac{q^{\rho-1}}{n} \right]^{\frac{1}{(\xi-1)\rho+1}},$$

which is increasing in productivity q and decreasing in the number of employees n .

3.5 Steady state conditions

To close the model, I need to solve for the steady state distribution of firms across types, aggregate employment, output and market tightness. Denote by $K_n(q)$ the aggregate number of products supplied by the set of firms of type q with employment n . Then the steady state mass of firms conditional on the firm's type is derived by equating inflows into and outflows from $K_n(q)$. First, for all $n \in [1, \bar{n}(q) - 1]$ the following relationship must hold:

$$(17) \quad \begin{aligned} H_{n-1}(q)K_{n-1}(q) + s_{n+1}(q)(n+1)K_{n+1}(q) + \phi(q)\mu \int_{\underline{q}}^{\bar{q}} K_n(q')dq' \\ = H_n(q)K_n(q) + s_n(q)nK_n(q) + \delta K_n(q) + \mu K_n(q), \end{aligned}$$

where the LHS represents the inflow consisting of the expected hires and separations, as well as the average fraction of firms with n workers that are hit by an idiosyncratic shock and that become q -type firms. The outflow reflects the transition flows of firms with n workers to firms with $n + 1$ workers due to new hires and to $n - 1$ workers due to quits, the firm destruction $\delta K_n(q)$, and a change in productivity $\mu K_n(q)$.

For $n = 0$, equation (17) includes an additional term that accounts for entry of new firms of type q , $\eta\phi(q)$. Here, I assume that productivity of entrants follows the same distribution function, $\Phi(\cdot)$. Thus, the steady state relationship for $n = 0$ reads

$$(18) \quad s_1(q)K_1(q) + \phi(q)\mu \int_{\underline{q}}^{\bar{q}} K_0(q')dq' + \eta\phi(q) = H_0(q)K_0(q) + \delta K_0(q) + \mu K_0(q).$$

Finally, equation (17) has to be modified for $n = \bar{n}(q)$ to incorporate the possibility of a firm firing

workers in the event of an adverse productivity shock, i.e.

$$(19) \quad H_{\bar{n}(q)-1}(q)K_{\bar{n}(q)-1}(q) + \phi(q)\mu \int_q^{\bar{q}} \sum_{n=\bar{n}(q)}^{\bar{n}(q')} K_n(q')dq' = s_{\bar{n}(q)}(q)\bar{n}(q)K_{\bar{n}(q)}(q) + \delta K_{\bar{n}(q)}(q) + \mu K_{\bar{n}(q)}(q).$$

Note that the last equation uses the fact that $v_{\bar{n}(q)}(q) = 0$ and $K_n(q) = 0$ for all $n > \bar{n}(q)$ since there are no firms with employment that exceeds $\bar{n}(q)$. Therefore, $K_n(q) = 0$ for all $n > \bar{n}(q)$ serves as a boundary condition for a second order difference equation, defined in equations (17) - (19).

The unemployment rate can be derived from the labor market clearing condition, which states that in equilibrium labor supplied to the market should be equal to total employment across all firms:

$$(20) \quad 1 - u = \int_q^{\bar{q}} \sum_{n=1}^{\bar{n}(q)} nK_n(q)dq.$$

Aggregate market tightness is defined as the ratio of vacancies to job seekers weighted by their search intensity, i.e. $\theta = \frac{v}{u+(1-u)\kappa}$, where the total number of vacancies posted by all firms is

$$(21) \quad v = \int_q^{\bar{q}} \sum_{n=0}^{\bar{n}(q)} v_n(q)K_n(q)dq.$$

Equilibrium in the goods market is achieved when total output produced by all intermediate firms is equal to the aggregate demand for the final good, i.e.

$$(22) \quad Y = \left[\int_q^{\bar{q}} \sum_{n=1}^{\bar{n}(q)} (qh_n(q)n)^{\frac{\rho-1}{\rho}} K_n(q)dq \right]^{\frac{\rho}{\rho-1}} = \left(\frac{\rho-1}{\chi\tilde{\zeta}\rho} \right)^{\frac{1}{\xi-1}} \left[\int_q^{\bar{q}} q^{\frac{\xi(\rho-1)}{(\xi-1)\rho+1}} \sum_{n=1}^{\bar{n}(q)} n^{\frac{(\xi-1)(\rho-1)}{(\xi-1)\rho+1}} K_n(q)dq \right]^{\frac{(\xi-1)\rho+1}{(\xi-1)(\rho-1)}}.$$

Finally, the job offer distribution $F(W)$ is defined as the fraction of aggregate vacancies that are posted by firms, which provide workers with the value of employment of W or less. Similarly, the steady state distribution of workers, $G(W)$, refers to the fraction of total workforce employed at jobs with the value of W or less. That is,

$$(23) \quad F(W) = \frac{\int_q^{\bar{q}} \sum_{n=0}^{\bar{n}(q)} \mathbf{1}[W_n(q) \leq W] v_n(q)K_n(q)dq}{v} \quad \text{and} \quad G(W) = \frac{\int_q^{\bar{q}} \sum_{n=1}^{\bar{n}(q)} \mathbf{1}[W_n(q) \leq W] nK_n(q)dq}{1-u}.$$

3.6 Equilibrium

Definition: A steady state market equilibrium is a set of numbers (θ, u, U, Y) , a set of distribution functions $(G(W), F(W)) : \mathbb{R}_+ \rightarrow [0, 1]$, a set of functions defined on a state space $(W_n(q), V_n(q), \Omega_n(q), \Psi_n(q), Z_n(q), h_n(q), v_n(q), w_n(q), K_n(q)) : [q, \bar{q}] \times \mathbb{I}_+ \rightarrow \mathbb{R}_+$, a set of functions defined on firm productivity types: $\bar{n}(q) : [q, \bar{q}] \rightarrow \mathbb{I}_+$, that satisfy equations (6) - (23).

The model is too complex to be solved analytically, hence I use numerical methods to solve for a steady state equilibrium. In particular, I look for a fixed point of the mapping where the worker's and firm's problems are solved for given aggregate market tightness, unemployment, aggregate demand, and distribution functions of vacancies and workers across firm types. Then, the aggregate variables and steady state distributions are updated using the optimal employment decisions of firms. Appendix C provides details on a steady state equilibrium solution algorithm.

3.7 Optimal labor demand policies

For illustration purposes, I present the optimal firm policies as a function of employment and productivity for a simulated version of the model. The driving force in the model is firm-specific productivity shocks that the firm can accommodate by adjusting its workforce and/or the number of hours that its employees work.¹⁸ The hours schedule $h_n(q)$, as defined in equation (16), guarantees that a positive q shock produces an increase in the average hours if there is no (or little) change in employment. As the number of workers starts growing, the work hours decline. Hence, the model can produce a negative relationship between hours and employment growth if the response of employment to shocks is slow enough.

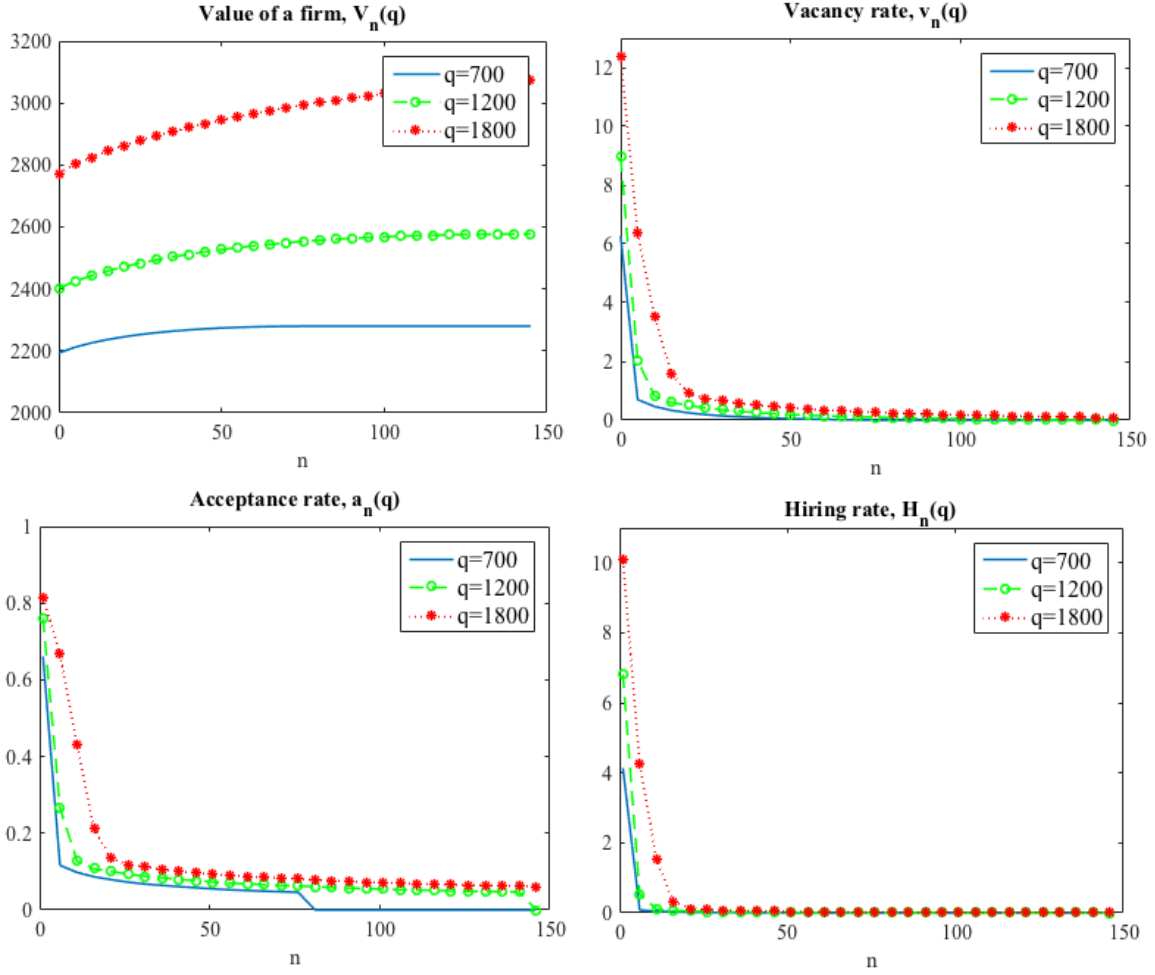
The employment level of a firm is a stochastic variable that is affected by the firm's recruiting efforts and the rate at which its existing workers quit to unemployment or move to a better job. To recruit a worker, the firm needs to post vacancies that are then randomly matched with job seekers. Two factors are at play here. First, the marginal benefit from hiring an extra worker, $V_n(q) - V_{n-1}(q) = \frac{1}{n+1}Z_n(q)$ is increasing in q and decreasing in n . The top left panel of Figure 2 shows that the value of a firm increases in employment but at a diminishing rate due to (i) a decreasing marginal revenue product, (ii) convex vacancy costs, and (iii) an increasing worker turnover described in detail below. Hence, the vacancy posting rate $v_n(q)$ rises

¹⁸Note that I refer to a shock to q as a productivity shock. However, it can be thought of as a firm-specific demand shock or, more generally, as a profitability shock. For instance, consider an alternative specification where the aggregate demand function is defined as

$$Y = \left[\int_0^K \alpha(j)x(j)^{\frac{\rho-1}{\rho}} dj \right]^{\frac{\rho}{\rho-1}},$$

where $\alpha(j)$ is a firm-specific demand shock, and production technology for the intermediate good is $x = hn$. This specification is equivalent to the current formulation of the production side of the market with $q = \alpha^{\frac{\rho}{\rho-1}}$.

Figure 2: The value of a firm, the vacancy rate, the acceptance rate, and the hiring rate.



with productivity and falls with the number of workers (the top right panel of Figure 2). The level of employment at which $V_n(q)$ flattens out, or equivalently, $v_n(q)$ reaches zero, is the maximum labor force $\bar{n}(q)$ that the firm is willing to sustain.

The second factor that affects a firm's vacancy posting decision is workers' acceptance rate, $a_n(q)$, that depends on the overall distribution of workers across the firm types, $G(W_n(q))$. Both wages and the value of a job to a worker, $W_n(q)$, decrease in employment and so does the acceptance rate reaching its minimum at $\bar{n}(q)$ when only unemployed workers accept the job (see the bottom left panel of Figure 2). Beyond $\bar{n}(q)$, the acceptance rate falls to zero as workers prefer to be unemployed instead. For this reason, the overall hiring rate $H_n(q) = a_{n+1}(q)v_n(q)v(\theta)$ falls even more rapidly than the vacancy rate as the firm hires more workers (the bottom right panel). Conversely, the separation rate as determined by job-to-job transitions is increasing in the number of workers and falling in the firm's productivity level. Together, the hiring and separation rates determine the steady state size distribution of firms, with a mass of firms of a given size

being roughly proportional to the ratio of the hiring rate to the separation rate.

4 Calibration

The model is simulated under the assumption that the economy is in steady state. I solve for the equilibrium hiring and separation rates, $H_n(q)$ and $s_n(q)$, respectively, as well as the maximum labor force size, $\bar{n}(q)$. Then, under the assumption of a Poisson arrival rate of firm-specific shocks, I simulate firms' employment histories (further details can be found in Appendix C). Worker flows are simulated at a monthly frequency; while hours and value added are aggregated into quarterly series to mimic the reporting frequency of the administrative data. Most importantly, to ensure consistency with the data, the simulated hours measure \tilde{h} is constructed using the pension contribution payment rule, in parallel with the data. I list all parameter values in Table 3 and discuss how I obtain them in the following subsections.

4.1 Pre-defined parameters

Solving for an equilibrium is computationally intensive, hence I choose to limit the estimated parameter space and set some parameter values outside of the main distance-minimization problem. Those are given in the top panel of Table 3.

The matching function is assumed to take a Cobb-Douglas form, where the aggregate number of matches is equal to $M(v, u + (1 - u)\kappa) = mv^\zeta(u + (1 - u)\kappa)^{1-\zeta}$, with $0 < \zeta < 1$ and a matching efficiency parameter $m > 0$. Without the data on vacancies, the matching function parameters m and ζ cannot be identified separately. For the purpose of this simulation, the elasticity of the matching function with respect to vacancies, ζ , is set to 0.5. The elasticity of substitution between intermediate goods, ρ , determines the degree of decreasing returns to scale of the revenue function that is equal to $\frac{\rho-1}{\rho}$. Following Gourio and Kashyap (2007), I choose $\rho = 2.5$, which yields the returns to scale parameter of 0.6.¹⁹

The curvature of a worker's disutility function determines the costs of adjusting work hours. Hence, a higher value of ζ , everything else equal, reduces the variation in hours that the model can generate. In order to fit the cross-sectional variation in firm-level hours, the model requires a relatively low value of the convexity parameter (below 2.0), which implies a Frisch elasticity of labor supply higher than one. I use $\zeta = 2$ as a conservative estimate that fits the standard deviation of hours reasonably well, while keeping the value of labor supply elasticity plausible.²⁰ A relatively low value of the convexity parameter implies

¹⁹As an alternative specification, I have tried to use $\rho = 4$ which implies the returns to scale parameter of 0.75. The results of policy experiments are not substantially different and are available in the online appendix.

²⁰Given the additive separability of income and the disutility costs in the utility function, the Frisch elasticity of labor supply in the

Table 3: Parameter values

Parameter	Value	Rationale
Pre-defined parameters		
r	0.4%	Yearly interest rate of 5%.
δ	0.52%	Average (size-weighted) monthly firm exit rate.
ζ	0.5	Curvature of matching. Assumed to be equal to 0.5.
ρ	2.5	Returns to scale parameter of 0.6 (Gourio and Kashyap, 2007).
ξ	2.0	Convexity of disutility from working. Implies a Frisch elasticity of 1.0.
Calibrated parameters		
b	4,820	Value of non-market time. Matches the unempl. rate of 4.8% (OECD Economic Outlook).
χ	0.65	A scaling parameter so that average hours are 30.8. Based on the Earnings survey.
c_0	1,130	Scale parameter of vacancy cost. Chosen to match a monthly job finding rate of 0.2.
c_1	2.0	Convexity of the vacancy cost. Chosen to match the std. dev of the hiring rate.
κ	0.85	Relative search intensity of employed workers. Set to match average worker flows.
s_0	0.004	Exogenous quit rate. Chosen to match the std. dev of the separation rate.
m	0.4	Matching efficiency. Governs the hours-employment trade-off.
μ	0.04	Shock arrival rate. Set to match the persistence of labor productivity.
η	5.7E-04	Entry rate. Determines the average firm size.
Generalized Pareto distribution $\Phi(q)$		
k	0.43	A shape parameter. Affects the skewness of the size distribution.
σ	436	A scale parameter. Affects the std. dev of the size distribution.
q	384	A location parameter. A normalisation to match average monthly wages of DKK 20,000.

low costs of adjusting hours, which is consistent with extremely flexible working time rules in Denmark.²¹

As one of the robustness checks below, I also try $\xi = 3$, corresponding to a Frisch elasticity of 0.5.

4.2 Calibrated parameters

The remaining parameters in Table 3 are estimated simultaneously to match the calibration targets reported in Table 4 by minimizing the distance between empirical and simulated moments. While it is not possible to associate individual parameters with individual moments, the results of numerical simulations help to identify particular moments that play key roles in identifying structural parameters. Those are reported in Table 4 to serve as a guideline, while further discussion is provided below.

The value of non-market time b implies that the replacement ratio, defined as the ratio of unemployment benefits to the average monthly wage, is about 25 percent. Although this might seem very low given the relatively generous unemployment insurance system in Denmark, recall that b includes the disutility from searching, which is normalized to zero in the model. In fact, relative to the level of worker's utility

model – the elasticity of hours with respect to the wage holding fixed the marginal utility of wealth – is $\frac{1}{\xi-1}$. That can be shown from worker's utility function $\omega = wh - \chi h^\xi$, where w is the hourly wage rate. Then $\frac{d \ln h}{d \ln w} = \frac{1}{\xi-1}$. In the empirical literature, the micro estimates of labor supply elasticity range between zero and 0.7 for men and up to 2 or 3 for women (see, Keane (2011) for a survey).

²¹The evidence drawn from the Earnings survey suggests that the use of overtime hours in the data is infrequent and relatively cheap. For example, I find that overtime comprises only 2 percent of total annual work hours and 1.8 percent of total annual wage costs.

Table 4: Calibration targets

Statistics	Data	Model
Unemployment rate, u	0.048	0.048
job finding rate, λ	0.200	0.204
Average separation rate ^a , $E(SR_t)$	0.075	0.078
St. dev. of separation rate ^a , $sd(SR_t)$	0.150	0.125
St. dev. of hiring rate ^a , $sd(HR_t)$	0.150	0.127
Employment-hours growth relation, $corr(\Delta \log N_t, \Delta \log \tilde{h}_t)$	-0.300	-0.198
Employment-lagged hours growth relation, $corr(\Delta \log N_t, \Delta \log \tilde{h}_{t-1})$	0.089	0.085
Average weekly hours ^b , $E(h_t)$	30.8	30.7
Productivity autocorrelation ^b , $corr\left(\frac{R_{t-1}}{N_{t-1}}, \frac{R_t}{N_t}\right)$	0.77	0.65
Mean employment ^c , $E(N_t)$	9.6	8.8
Median employment ^c , $Med(N_t)$	4	4
Standard deviation of employment ^c , $sd(N_t)$	16	15.5

Notes: ^a - Continuing firms only, moments are weighted by employment share. ^b - Moments are weighted by employment share. ^c - Top one percent of firms (with more than 150 employees) are excluded from the sample.

$\omega_n(q) = w_n(q) - g(h_n(q))$, the flow value of unemployment is equal to about 55 percent, on average.²²

I use the following specification of the vacancy posting costs: $c(v) = c_0 v^{c_1}$, with $c_0 > 0$ and $c_1 > 1$. The curvature of the vacancy cost function, c_1 , reduces the variation in the hiring rate: a more convex cost function means that firms make smaller and slower adjustments to their workforce. I find that quadratic vacancy costs fit the data well.²³ The degree of convexity $c_1 = 2$ may seem low compared to some estimates found in previous studies. However, these values are sensitive to both time and cross-section aggregation. The observed labor variation patterns look smoother in the aggregate data and hence produce higher estimates of convexity (see, for example, Hamermesh, 1989 for aggregation over just seven plants; Bloom, 2009 for the effects of time and within-firm aggregation over different production units). Thus, while Mertz and Yashiv (2007) find that a cubic specification for adjustment costs fits the data well on a quarterly basis, $c_1 = 2$ is a reasonable value for monthly worker flows. I also try alternative values of c_1 as a robustness check and report the results below. Together c_0 and c_1 determine the aggregate vacancies in the market

²²Although, the Danish unemployment system is very generous with eligibility rules for claiming benefits lasting up to 4 years with the average replacement rate of approximately 60 percent, there are two reasons for why lower estimates of b are justifiable. First, unemployment insurance is voluntary in Denmark and a substantial fraction of workers (around 15 percent) are not insured. Second, the UI system has been reformed in the 1990s to increase the incentives to seek and accept jobs and take part in skills upgrading activities. After about a year of unemployment, the unemployed workers are required to partake in active labor market policies (ALMPs) such as accepting temporary jobs or participating in training courses. The ALMPs might have a significant negative effect on workers' utility because of stigmatizing or "taxing away" leisure. Supporting this hypothesis, Rosholm and Svarer (2008) find that a high perceived risk of future program participation reduces unemployment duration by two and a half weeks on average.

²³The estimated parameter is 2.15. However, rounding it to an integer significantly increases the speed of the numerical simulation and does not alter the results significantly. Hence, I choose to use the approximate estimate of 2.

and the overall job finding rate. I choose to match the job finding rate of 0.2, which corresponds to mean unemployment duration of 5 months.²⁴

Job separations in the model consist of job-to-job transitions and quit rates into unemployment. In steady state inflows into and outflows from the unemployment pool are equal, the model predicts that the separations to unemployment are small given a low level of unemployment and yet relatively long unemployment duration in the Danish labor market. Therefore, job-to-job transitions are required to fit the data: the relative search intensity κ is found to be close to one to match the average separation rate of about 8 percent per month. The exogenous quit rate parameter, s_0 , affects the standard deviation of the separation rate. That is, a larger share of exogenous quits in total separations reduces their responsiveness to productivity shocks and hence lowers the volatility of the separation rate.²⁵

I use the dynamic interaction between employment and hours to identify the matching efficiency parameter m . The hours-employment trade-off is stronger the slower is a firm's hiring process, which can be a result of either a higher c_0 value or a lower m value. In the simulation, I pin down vacancy posting costs to match the job finding rate and use the matching efficiency parameter m to make the hiring process more sluggish (i.e. the vacancy filling rate declines in m for a fixed job finding rate).

A well-established fact in the existing literature is that the size distribution of firms is highly skewed to the right with a very long right tail. That is, most of the firms in the data are small with a few firms that have much larger than average workforce. These features of the data put restrictions on the shape of the firm-specific productivity distribution $\Phi(\cdot)$ in the model, therefore I use a highly skewed distribution for q with its parameters set to match the observed size dispersion and its median. In particular, I assume that productivity follows Generalized Pareto Distribution with the density

$$\frac{1}{\sigma} \left(1 + k \frac{q - \bar{q}}{\sigma} \right)^{-\frac{1}{k} - 1},$$

²⁴The average distribution of unemployed workers by duration during the period of 1999-2006 was the following: 23.1% of workers were unemployed for less than one month; 18.4% - for 1 to 3 months; 19.5% - for 3 to 6 months; 17.6% - for 6 to 12 months, and 21.4% were unemployed for more than a year (OECD Economic Outlook 2007). Median duration is between 3 and 6 months. I choose mean duration of 5 months to be a reasonable target, which corresponds to the monthly job finding rate of 0.2.

²⁵Also, a higher value of s_0 increases the correlation between the separation rate and employment. Intuitively, under the assumption of an exogenous and constant quit rate, the separation rate is independent of the firm's employment (or positively related since the probability of layoffs rises with the firm's workforce). In the data, however, this relationship is slightly negative with the size-weighted correlation coefficient of -0.03. Thus, bringing endogenous quits into the model ensures that the correlation coefficient between the separation rate and employment is negative (-0.04) also in the simulated data.

where k is a shape parameter, σ is a scale parameter, and \underline{q} is a location parameter.²⁶ The ratio of the entry rate to the exit rate, η/δ , determines the total mass of firms, which in combination with the unemployment rate gives the average number of workers per firm. The scale parameter χ on the disutility from working is set to reproduce the average weekly work hours in the model.

Finally, a more persistent shock process strengthens the dynamic interaction between hours and the number of workers. The firm is more likely to respond to changes in profitability by adjusting its labor force size if shocks last longer. On the contrary, if shocks are white noise then the firm will be more likely to keep its workforce at the same level and adjust on the hours margin instead. In the model, the arrival rate of shocks, μ , controls the persistence of the productivity process and hence the autocorrelation of labor productivity (measured as value added per worker).²⁷

4.3 Model fit

4.3.1 Employment and hours distribution

A standard search theory typically models single-worker firms, or more generally, it assumes a constant returns to scale production function, and hence does not have a meaningful definition of firm size. In this paper, under the assumption of diminishing returns to labor, the model produces an endogenous steady state size distribution, which then can be compared to its empirical counterpart. The model is able to replicate the overall shape of the observed employment distribution (see Figure 3). It successfully captures the fact that there is a significant size dispersion and that the average firm employs about twice as many workers as the median firm (see Table 4).

Figure 4 shows the cross-sectional distribution of firm-level weekly hours generated by the model and the corresponding distribution in the data. Given that the model assumes that all workers are identical, the only reason for hours to vary across firms is the deviation of employment from its optimal level. That is, in the absence of search frictions each firm would employ workers for the same number of hours, regardless of the firm's productivity level q . Therefore, the model tends to produce less variation in hours of work than is observed in the data: the standard deviation of hours, after taking out industry and time fixed effects, is 6.4 in the data versus 5.0 in the model. Despite this, I find that it replicates the empirical distribution of

²⁶The mean of the distribution is $\underline{q} + \frac{\sigma}{1-k}$ for $k < 1$ and variance is $\frac{\sigma^2}{(1-k)^2(1-2k)}$ for $k < 1/2$. A higher value of the shape parameter, k , means that the size distribution has a thicker right tail. Also note that if the following parameters - \underline{q} , σ , b , c_0 , and χ - are all scaled up by the same factor then the resulting equilibrium does not change. I use the location parameter \underline{q} for normalization and scale up the above parameters to match average monthly wages in the data.

²⁷This Poisson arrival shock process is equivalent to a discrete time mean-reverting AR(1) process with autocorrelation coefficient of $e^{-\mu}$. Thus, a lower value of μ implies higher persistence of the underlying productivity process.

hours relatively well.

4.3.2 Hours and employment adjustment

Table 5 compares the summary statistics for employment and hours growth in the data and in the simulation. The model captures the variation in employment growth reasonably well, but underestimates the variation in hours growth. This result partly reflects the way in which hours are measured in the data. Recall that changes in actual hours are registered in the data when at least one worker moves between the 9-hour intervals. For the variation in working time to be reflected in the simulation, *all* workers at a given firm have to move to a different 9-hour interval because they are identical in the model. Therefore, worker heterogeneity might be one of the reasons why the variation in hours growth differs between the data and the model.

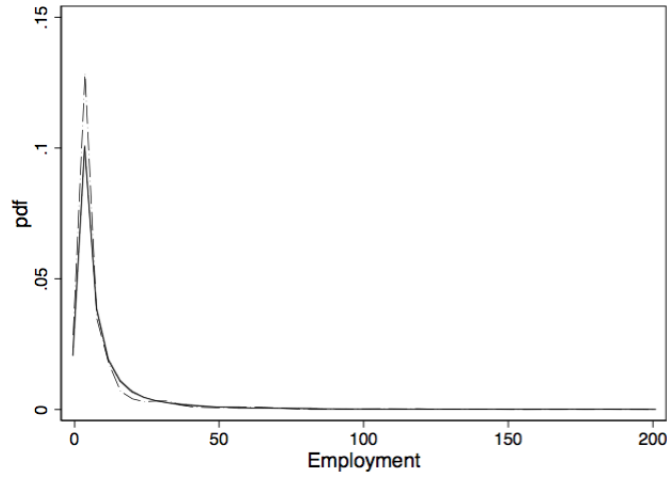
The model is capable of producing a negative relationship between the two margins of adjustment, although it falls short of matching the magnitude of it: the correlation coefficient between hours and workers growth is -0.198, compared to -0.300 in the data. The mechanism behind this trade-off is clear in the event of a positive productivity shock; however, it is less intuitive in the case of an adverse shock. That is, firms respond to a positive shock by increasing hours and posting more vacancies. Given search frictions in the labor market, it takes time to recruit new workers; therefore, as vacancies start filling up, hours of work begin to fall. In the event of a negative shock, the firm reduces work hours of its existing employees and, if productivity falls too low, dismisses some of its workers. The initial cut in employment happens immediately; however, the firm now faces a higher attrition rate so that its workforce continues to decline further down. The average hours per worker, on the contrary, start rising. Therefore, *endogenous quits are key in reproducing a negative co-movement of hours and employment for contracting firms.*

Note that a negative correlation between hours and employment growth can be generated in the model even in the absence of productivity shocks if the number of workers is stochastic (due to the decreasing returns to scale revenue function and convex disutility costs of hours). However, productivity shocks are crucial for matching the fact that changes in hours lead changes in employment. The model is successful in this respect: the correlation coefficient between employment growth and lagged hours growth is 0.085 compared to 0.089 in the data.

4.3.3 Job and worker flows

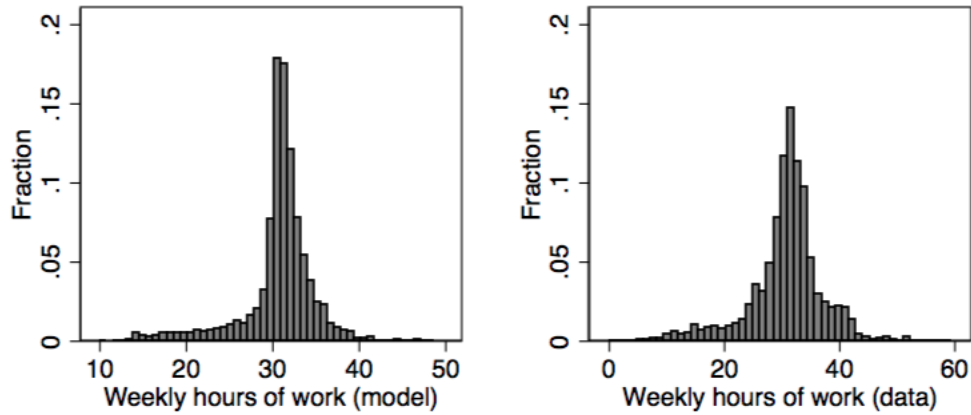
On-the-job search is a necessary component that enables the model to capture the characteristics of the data related to worker and job flows. Allowing for workers to search while employed means that both the quit rate and the offer acceptance rate depend on the firm's type: more productive firms face

Figure 3: Size distribution in the data (solid line) and in the model (dashed line)



Note: Density estimation is based on Gaussian kernel with a bandwidth of 1.5. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 1999-2006.

Figure 4: Hours distribution in the model (left panel) and data (right panel).



Note: Empirical distribution of weekly hours refers to the (size-weighted) firm-level work hours averaged over a year. Industry and year effects are taken out. Source: Author's calculations based on the Danish firm data, 2002-2006.

Table 5: Hours and employment growth rates in the model and in the data.

	Data	Model
Std.dev ($\Delta \log N_t$)	0.277	0.343
Std.dev ($\Delta \log \tilde{h}_t$)	0.285	0.150
Corr ($\Delta \log N_t, \Delta \log \tilde{h}_t$)	-0.300	-0.198
Corr ($\Delta \log N_t, \Delta \log \tilde{h}_{t-1}$)	0.089	0.085

Source: Author's tabulation from the Danish firm data over the period of 1999-2006 and the simulated data. Note: \tilde{h} denotes the average hours per worker constructed using the interval measure of hours in the model and in the data.

Table 6: Monthly job and worker flow rates in the data and in the model.

	Data	Model
Hires	0.077	0.078
Separations	0.075	0.078
Job Creation	0.040	0.048
Job Destruction	0.038	0.048
Net employment change	0.002	0.000
Churning	0.075	0.060

Note: These moments are size-weighted and refer to continuing firms only.

lower attrition rates and are able to attract workers faster than their less productive counterparts. These features of the model are consistent with empirical evidence reported in earlier studies: Davis, Faberman and Haltiwanger (2012), for instance, document that the vacancy yield (the number of hires per vacancy) increases in employment growth. In addition, Davis et al. (2006) find that quits account for a bigger portion of separations than layoffs for firms that shrink by less than 12 percent during the month. Similarly, my model predicts that a sizable workforce reduction can be brought about through quits in the case of an adverse profitability shock.

Table 6 summarizes empirical and simulated moments concerning worker and job flows. The average monthly job and worker flow rates in the data are matched closely by the model. In line with the data, worker flows are about twice the size of job creation and job destruction. The model can produce a relatively high churning rate, which means that also in the simulation contracting firms are still hiring workers, while growing firms lose workers.

Table 7 shows the relationship between monthly worker flows and net employment adjustment, size-weighted by employment share. Firms are split into five groups according to their net employment growth rate. Contracting firms reduce their labor force mostly through separations; while growing firms increase their employment mostly through hiring. However, even contracting firms are hiring at about a 5 percent rate. The model captures these employment growth patterns very well: in the model contracting firms still exhibit positive hiring rates, albeit lower than those observed in the data. In general, the model performs well matching firms with net employment adjustment between -10 and 10 percent, but underestimates worker turnover in firms that grow or contract by more than 10 percent.²⁸

²⁸A common claim in the literature is that non-convex adjustment costs are necessary to match a relatively large region of inaction observed in the data (see for instance Cooper and Willis, 2009). In the Danish labor market, 63% of all firms have zero monthly net employment change and they represent about one fifth of total employment. My model is capable of generating a significant share of firms with zero employment growth with convex adjustment costs: on average, 51% of simulated firms have zero net monthly employment change and they employ about 28% of total workforce. The reason for this is the presence of search frictions in the market – although firms post vacancies continuously, they may have zero hires if these vacancies are not matched with workers.

Table 7: Average monthly hiring and separation rates, by net employment growth rate.

Net Employment Growth	Data				Model			
	Hires	Sep.	Net	Empl. Share, %	Hires	Sep.	Net	Empl. Share, %
Less than -0.10	0.051	0.371	-0.320	8.9	0.015	0.270	-0.256	15.7
-0.10 to -0.025	0.041	0.094	-0.053	13.4	0.032	0.087	-0.055	12.7
-0.025 to 0.025	0.034	0.034	0.000	53.9	0.042	0.041	0.001	40.4
0.025 to 0.10	0.094	0.041	0.053	14.1	0.085	0.030	0.056	16.9
More than 0.10	0.361	0.049	0.312	9.7	0.285	0.021	0.264	14.3

Note: Data moments are based on continuous firms. Both data and simulated moments are weighted by employment.

4.4 Adjustment costs estimates

In general, it is not easy to obtain information on various sources and sizes of adjustment costs. Abowd and Kramarz (2003) and Kramarz and Michaud (2004) estimate employment adjustment costs directly based on survey data for a representative sample of French firms. They find considerable magnitudes of both hiring and separation costs, with the latter exceeding the former. Many of these costs, however, are implicit – such as forgone production when existing workers spend their time to train new hires – and thus are not reported. Meanwhile, estimating the adjustment costs indirectly from the firm-level labor dynamics captures all, including implicit, components of these costs.

The average cost of hiring a worker is computed as the flow cost of sustaining an open vacancy ($c_0 v^{c_1}$) multiplied by expected duration of that vacancy, or the inverse of the hiring rate. Given the vacancy cost parameters and the implied steady state worker meeting and offer acceptance rates, the average cost of hiring a new worker is found to be around 9,990 Danish kroner (DKK), which is equivalent to about half of monthly wages. Given how flexible the Danish labor market is, it is not surprising that the estimated costs are low. Other European labor markets are characterized by a higher degree of employment protection and thus tend to produce larger estimates of adjustment costs. For instance, Rota (2004), based on annual firm level data from the Italian manufacturing industry, reports an estimate of fixed adjustment costs of 15 months of labor costs. Mertz and Yashiv (2007) find that a marginal cost of hiring is roughly equivalent to two quarters of wage payments - about 12 times higher than my estimates. Note, however, that many of these estimates of labor adjustment costs pertain to net employment changes and therefore are likely to be higher than the estimates based on gross flows data.

5 Counterfactual experiments

Given the parameter values above, I perform two policy experiments – (i) an introduction of a hiring subsidy, and (ii) imposing an upper limit on work hours – and show their effects on aggregate employment

and output. I contrast the results of these experiments to those obtained in a partial equilibrium version of the model, in which the vacancy filling rates and quit rates are kept unchanged at the benchmark level. I also show the role of the hours margin and OTJ search explicitly by simulating the model without these components. Note that I do not consider transition dynamics after the changes are introduced, instead these experiments should be thought of as a comparison between two steady state economies - the baseline versus a counterfactual economy with alternative parameter values.

5.1 Hiring subsidy

One of the main concerns of policymakers during recessions is to find an effective way to stimulate job creation. Among proposed solutions is a new jobs tax credit. A recent example of this policy is the US Hiring Incentives to Restore Employment (HIRE) Act, enacted in March 2010, that provides tax incentives for businesses that hire previously unemployed workers. To examine the impact of such a policy, I simulate the model with an employment subsidy received by firms for every new hire (although without distinguishing whether this hire comes from the pool of employed or unemployed workers). To finance this policy, I assume that all firms pay a fixed lump-sum tax.²⁹ Unlike the HIRE Act, these policy changes are assumed to be permanent in the model and hence their effect is likely to be bigger than if the policy were temporary.

Panel A of Table 8 presents the results of the experiment in the baseline model. I consider three values of a hiring subsidy: 30, 50 and 80 percent of the average hiring costs, which corresponds to 3000, 5000 and 8000 Danish kroner in the baseline simulation. The introduction of a hiring subsidy reduces the unemployment rate by 1.8 to 4.5 percent.

Most of the existing models that analyze the impact of labor adjustment costs on labor demand use a partial equilibrium framework (see for instance Bentolila and Bertola, 1990). To mimic a partial equilibrium model, I keep the quit rates and the vacancy filling rates unchanged at the same level as in the benchmark model and simulate the introduction of hiring subsidies. The results of this experiment are presented in Panel B. The subsidies encourage firms to employ more workers and thus lead to a greater number of posted vacancies. In a general equilibrium model, newly created vacancies congest the market and thus lower the return on every single vacancy by reducing the probability of finding a worker. In the partial equilibrium version of the model these feedback effects are absent resulting in a much stronger effect of hiring subsidies on both aggregate employment and aggregate output. In the extreme case of an 80% hiring subsidy unemployment falls down by 80% (to about 1% unemployment rate) and output increases by about

²⁹There are other ways of financing this policy that I can consider, for example, a tax on sales. However, a lump-sum tax allows to identify the effect of hiring subsidies on the unemployment rate in a clean way, whereas the tax on sales would distort firms' hiring decisions.

Table 8: The effect of a hiring subsidy

	Hiring subsidy		
	30%	50%	80%
A. Baseline model:			
Percentage change in unemployment	-1.8	-3.0	-4.5
Percentage change in aggregate output	0.0	0.0	0.0
B. Partial equilibrium:			
Percentage change in unemployment	-28.6	-50.6	-79.5
Percentage change in aggregate output	1.4	2.8	4.6
C. Fixed hours:			
Percentage change in unemployment	-1.8	-3.3	-6.0
Percentage change in aggregate output	0.0	0.0	0.0
D. No OTJ search:			
Percentage change in unemployment	11.4	29.3	66.5
Percentage change in aggregate output	-0.5	-0.7	-1.9

5%. These results demonstrate that partial equilibrium models of labor demand significantly overstate the effect of adjustment costs on unemployment.³⁰

Next, I compare the results shown above to the model, in which the hours channel is shut down. That is, I impose a constant level of hours for all workers (so that the average hours are the same as in the benchmark) and re-calibrate the model to fit the remaining moments that do not include work hours. The hiring subsidy is again equal to 30, 50 and 80 percent of the average hiring costs, which are estimated to be DKK 17,600 in the model with fixed hours, almost 80 percent higher than in the benchmark. Compared to the baseline model, the resulting reduction in unemployment is similar for low levels of the subsidy and greater by a third for the top subsidy (see Panel C of Table 8). That is, a model without the intensive margin of adjustment leads to the overestimation of the effects of hiring subsidies, but not by a considerable amount.

Finally, I re-calibrate the model with no OTJ search, i.e. $\kappa = 0$.³¹ The average hiring costs in this case

³⁰This point has been previously made by Hopenhayn and Rogerson (1993) when evaluating the effects of dismissal costs on aggregate employment. They dispute the previous result of Bentolila and Bertola (1990) who showed that in a partial equilibrium framework higher dismissal costs increase employment. Instead, they find that in a general equilibrium model with endogenous entry and exit a tax on dismissals causes aggregate employment to fall. Allowing for endogenous entry in my model does not significantly alter the results presented in panel A of Table 8 since lump-sum taxes roughly cancel out the effects of a hiring subsidy on aggregate output and the overall firm value (see the online appendix for details).

³¹It is worth to mention that re-calibrating the model with $\kappa = 0$ proved difficult. First, it cannot match the magnitudes of the hiring and separation rates. Second, the model falls short of reproducing the trade-off between hours and employment dynamics. Third, the model overestimates the unemployment rate. The latter result is due to changes in the steady state distribution $K_n(q)$, which now features a higher mass of firms concentrated around the maximum employment $\bar{n}(q)$. Since these are the firms that lose workers when hit by a negative shock, the outflow into unemployment increases.

are estimated to be only 6 percent of the average monthly wages. This is due to the fact that without the OTJ search the acceptance rate of all offers above the reservation wage is equal to one, thus decreasing the expected duration of a vacancy.

The effect of a hiring subsidy on unemployment in the model with no OTJ search is now reversed, leading to a rise in the unemployment rate (see Panel D in Table 8). This result is counterintuitive and warrants a detailed explanation.³² In steady state equilibrium, the shape of employment distribution is determined primarily by the ratio of the hiring rate to the separation rate. In the model with the OTJ search, the hiring rate falls rapidly as n increases due to a steadily falling offer acceptance rate, thus making the size distribution to have a long right tail. On the contrary, if $\kappa = 0$, the employment distribution becomes more concentrated around the maximum employment level, $\bar{n}(q)$, for a given productivity level q .

While a hiring subsidy leads to firms posting more vacancies on average, its effect varies across firm types. That is, more productive firms find it easier to hire workers with the subsidy in place and thus their maximum employment $\bar{n}(q)$ increases. (In contrast, in the model with the OTJ search the acceptance rate falls rapidly as n increases, thus attenuating this effect.) The opposite is true for firms with low productivity and their maximum employment $\bar{n}(q)$ falls. This is because the outside option of workers increases following a rise in the fraction of vacancies posted by high productivity firms, thus making it more difficult for low productivity firms to hire workers. As a result, the steady state size distribution becomes more spread-out, which leads to a larger fraction of workers losing their jobs in the case of an adverse productivity shock. A higher layoff rate, in turn, raises inflows into unemployment and leads to an overall increase in the unemployment rate.

5.2 Limit on hours

In this subsection, I examine the effect of introducing a shorter workweek on aggregate employment and output. This change in working time regulations is often viewed as a cheaper alternative to a reduction in the hiring costs. The idea behind this policy is that firms would need to hire more workers to sustain their total labor input as they cannot increase work hours of their employees beyond a certain level. However, this policy comes at a loss of flexibility in firms' choices, which in turn lowers their profits and therefore might negatively affect firms' labor demand. The empirical evidence on the efficacy of this policy in fostering employment growth is mixed. Crepon and Kramarz (2002), for example, find that a reduction in the official workweek from 40 to 39 hours in 1982 in France led to employment losses of 2 to 4 percent. On the other hand, Chemin and Wasmer (2009) show that France's switch from a 39-hour to a 35-hour workweek

³²More details can be found in the online appendix.

Table 9: Introducing an upper limit on work hours.

	Maximum hours per week		
	$h_{max} = 37$	$h_{max} = 35$	$h_{max} = 33$
Percentage of workers employed for $h > h_{max}$			
Data	10.5	34.3	52.9
Baseline model	5.7	12.5	25.4
A. Baseline model:			
Percentage change in unemployment	0.9	1.3	2.1
Percentage change in aggregate output	-2.4	-4.2	-7.1
B. Partial Equilibrium:			
Percentage change in unemployment	-17.6	-14.7	-8.2
Percentage change in aggregate output	-1.4	-3.5	-6.7
C. No OTJ search:			
Percentage change in unemployment	4.0	4.1	4.4
Percentage change in aggregate output	-3.1	-5.0	-7.4

in 2000 had no significant impact on employment.

Here, I propose a similar experiment by imposing an upper limit on weekly work hours. In particular, I consider three different threshold values for h_{max} : 33, 35, and 37 hours a week. If the maximum hours constraint is binding – i.e. when the optimal number of hours as determined by equation (16) exceeds the upper limit – then the firm sets the work hours of its employees to h_{max} . The official workweek in Denmark is 37 hours a week. Note that hours distribution in the data is more skewed to the right than in the model, hence for the same mean the fraction of workers that work more than h_{max} is about twice as high in the data than in the model (see Table 9). As a consequence, the results of these simulations possibly underestimate the true effect on employment and output.³³

Panel A of Table 9 presents the results of this policy for the baseline model. It appears that the negative effect on profits dominates the work-sharing motive, on average, and thus the unemployment rate increases by 1-2 percent as the workweek becomes shorter. The effect on aggregate output is also negative, leading to a 7 percent drop in total production in the most restrictive case of 33 hours a week.

As before, I conduct the same experiment for a version of the model in which vacancy filling and attrition rates remain unchanged (see panel B of Table 9). The partial equilibrium model shows a reversal of the negative effect on employment - in fact, working time restrictions reduce the unemployment rate by up to 18 percent. As the workweek become shorter, a negative effect on profits becomes stronger and the gain in employment decreases also in a partial equilibrium. Finally, Panel C shows the impact of an upper limit on hours on aggregate employment and output in the model with no OTJ search. The effects are qualitatively

³³Moreover, allowing for endogenous firm entry would exacerbate the effects of the hours limit on employment and aggregate output even further (see the online appendix for details).

similar to the baseline scenario, albeit being of a larger magnitude.

5.3 Simulation robustness

The trade-off between changes in hours and employment comes from two sources: (i) the search frictions that halt the adjustment in the number of workers, and (ii) the costs of changing hours of work. If variations in hours were inexpensive then firms would make all the adjustment on the intensive margin alone. In the model, it is the convexity parameter of disutility of working, ζ , that generates the increasing marginal cost of employing a worker for an extra hour. In order to check how sensitive the results of the experiments are to this parameter, I recalibrate the model with $\zeta = 3$, which implies a Frisch elasticity of labor supply of 0.5. The model with a more convex disutility function leads to work hours being more concentrated around the mean. Hence, the upper limit on work hours is less binding for a longer workweek (e.g. in the baseline model 12.5 and 25.4 percent of workers have weekly hours above 35 and 33, respectively, while with $\zeta = 3$ the share of workers affected are 6.8 and 21.0 percent) and more binding as the workweek decreases (from 53.4 in the baseline to 61.1 percent in the re-calibrated version for 31-hour workweek). This is reflected in the left panel of Figure 5 - the effect on unemployment is smaller in the re-calibrated model when the workweek length is 39-33 hours and higher for 31 hours. The effects of a hiring subsidy on unemployment is virtually the same as in the baseline model (the right panel of Figure 5).

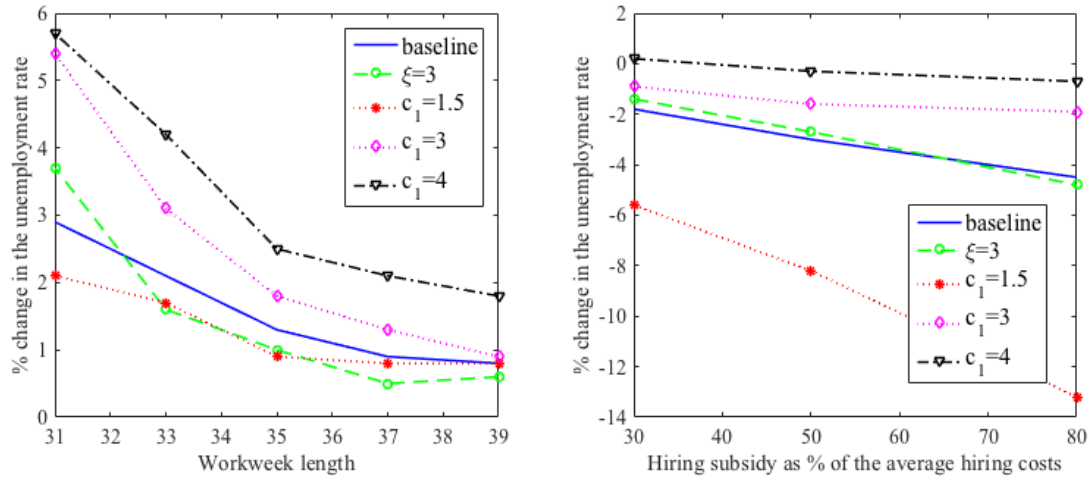
I also recalibrate the model with three alternative values of c_1 : 1.5, 3 and 4. As the vacancy costs convexity increases, employment adjustment becomes slower. As a result, the upper limit on hours leads to a more detrimental effect on employment for higher values of c_1 (see the left panel of Figure 5). Similarly, a hiring subsidy is more effective the lower is the convexity parameter, with the largest gain in aggregate employment found for $c_1 = 1.5$ (see the right panel of the same figure).

6 Conclusion

This study is motivated by the observation that firms use variation in work hours of their employees to adjust their labor demand in response to shocks. In particular, I use a matched employer-employee panel of Danish firms to document firm-level employment and hours growth patterns. I find a strong negative relationship between these two growth rates, which is consistent with a hypothesis that hours respond to shocks immediately, while changing firm's workforce takes time. These empirical facts call for a model that allows for both intensive and extensive margins of labor adjustment.

I build a general equilibrium theory of heterogeneous multi-worker firms that choose their hiring and firing policies optimally in an economy with search frictions. The driving force of the model is idiosyncratic

Figure 5: Robustness check



profitability shocks that firms can accommodate by varying work hours of their existing employees and/or adjusting their employment. Wages and hours are determined optimally based on the Shapley values. In addition, allowing for on-the-job search delivers a rich theory of quits that enables the model to capture most of the features of the data regarding employment dynamics.

The model is calibrated to assess its fit to the Danish firm data and appears to be successful in capturing the overall characteristics of the data. The numerical simulation does an outstanding job of reproducing employment variation at the firm level. It matches closely hiring and separation rates, job creation and job destruction rates, and the distribution of firms by net employment growth. In addition, the model is capable of generating a negative correlation between the growth rates of employment and hours per worker.

In the process of matching the model to the data, I obtain an indirect estimate of the average hiring costs. I find that to hire a new worker, the firm has to bear a cost in the amount of two weeks of wages, on average. This value is low compared to the estimates of the adjustment costs found in other European countries and it is more similar to the values reported for the US labor market. High magnitudes of worker flows found in the Danish labor market are at the heart of this result. I then use the model to simulate two policy experiments – introducing a hiring subsidy and imposing an upper limit on weekly work hours – and estimate their effects on aggregate employment and output. I show that introduction of a shorter workweek increases the unemployment rate, while a hiring subsidy reduces it. I then perform the same experiments in a partial equilibrium framework and find the results will be strikingly different (with a reduction in unemployment of up to 80%) if the quit rates and vacancy filling rates remain unchanged. Finally, I explicitly show the importance of the intensive margin and on-the-job search by simulating and comparing a version of the model without these components.

Appendix

A.1 Danish labor market

The Danish labor market is regulated mostly by collective bargaining agreements between trade unions and employer organizations: about 80 percent of all employees are unionized.³⁴ Collective agreements regulate wages and main issues concerning work conditions, such as overtime, paid leave, etc. The recent tendency in the labor market is for the unions to play the role of a coordinating institution, whereas wages are negotiated at the firm level.³⁵

There is no statutory protection against dismissals, as there is no statutory minimum wage. Collective agreements' rules on individual dismissals are particularly flexible, which makes the Danish labor market one of the least rigid by international standards.³⁶ Long-term employees receive an average of one month's wage compensation upon dismissal. For comparison, a severance pay in Portugal is three months of wages for short-term employees and up to twenty months for long-term employees, up to four months of wages in France, up to nine months of wages in Netherlands, and up to a year in Spain.³⁷ Regulations on dismissals may differ across industries. For instance, average notice periods vary from three days in construction to one month for industrial workers and up to three months for salaried workers depending on their seniority in the firm. The reason for short notices is that employees in turn have flexibility to switch jobs: workers are required to notify their employers eight days in advance if they want to quit. In addition, there has been an increase in the use of temporary contracts and there are no longer limitations on how often these temporary contracts can be renewed.

Working time has always been one of the primary issues in collective bargaining in Denmark; however, until the 1980s the total length of the workweek has been a dominant concern. In the mid-1990s the focus

³⁴See Danish Confederation of Trade Unions website (as downloaded on May 20, 2010):

<http://lo.dk/Englishversion/About%20LO/TheDanishLabourMarket.aspx>

³⁵Source: Andersen and Mailand (2005).

³⁶Here, I refer to measures of labor market flexibility developed by Botero, Djankov, Porta, de Silanes and Shleifer (2004). Their original data have been extended by the World Bank and are available at <http://www.doingbusiness.org/ExploreTopics/EmployingWorkers/>. Difficulty of firing index, which includes requirements for grounds for dismissal, dismissal procedures, severance pay and terms of notice, is 0 out of 100 in Denmark compared to, for instance, 30 in France, 40 in Italy, and an average of 22.6 for OECD countries. Overall rigidity of employment index, which refers to legal requirements concerning minimum pay, working time, paid holidays, use of part-time and fixed-time contracts, and dismissal procedures, is reported to be 7 out of 100 for Denmark compared to 26.4 OECD average (as downloaded on May 10, 2010).

³⁷Source: "The flexible labour market needs strong social partners. The European discussion on the Danish Labour Market: Flexicurity". Published by the Danish Confederation of Trade Unions on January 2008: <http://lo.dk/Englishversion/~ /media/LO/English/FinalFlexicurity.ashx> (as downloaded on May 10, 2010).

shifted to the variability of working time allowing for additional flexibility of work hours. For instance, in the manufacturing sector the collective agreement was introduced in 1998 that specified that the working time could vary over a twelve-month period as long as the average weekly hours amounted to 37 hours a week, provided that an agreement between management and a union representative is reached locally. Further changes were made in 2004, which stated that specific organization of the working time could be agreed directly with an individual employee or a group of employees.³⁸

To sum up, the Danish labor market is one of the most flexible in Europe permitting firms to adjust their workforce under a minimal set of regulations. Lax dismissal rules in combination with a generous unemployment insurance scheme result in a very mobile labor market.

A.2 Data sources

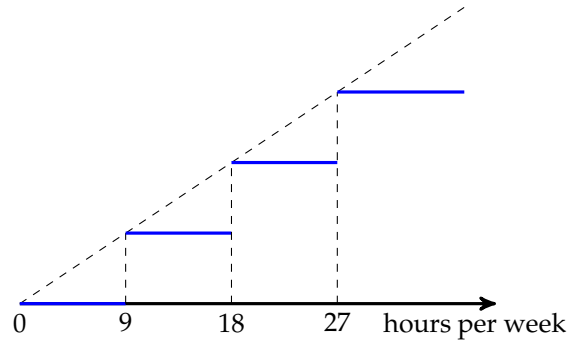
The empirical analysis in this paper is based on Danish firm data drawn from administrative records for 1999-2006. They come from four major sources. First, the detailed information on employment changes is obtained from a matched employer-employee panel that includes all individuals that have paid employment in a given month. Monthly employment is constructed as a head count of all individuals employed in a given firm. The number of employees per quarter is derived as the average of three months' employment for firms that have positive employment in all three months of a given quarter. The fact that this dataset has time-consistent identifiers for both firms and workers makes it possible to construct hires and separations series for each firm.

Second, I use the Earnings survey data for 2002-2006 to obtain the cross-sectional hours distribution across firms. It contains all firms in the private sector with more than 10 full-time employees, excluding agriculture and fishery. This survey collects information on paid hours (regular and overtime) for each employee on a yearly basis (or for the length of a job spell if it was shorter than a year). I construct a firm-level (employment-days weighted) hours per worker series and take the industry and time fixed effects (and their interactions) out.

Third, in order to see high frequency changes in work hours at the firm level, I utilize firms' mandatory pension contribution data that are collected on a quarterly basis. In Denmark, firms are required to pay pension contributions for each employee according to her weekly hours of work. The rule for the pension contribution (depicted in Figure 6) is as follows: (i) full amount of contribution (670.95 DKK in 1999-2005 and 731.70 DKK in 2006 per quarter) is paid for an employee working more than 27 hours a week; (ii) 2/3 is paid for an employee working between 18 and 27 hours a week; (iii) 1/3 is paid for an employee working

³⁸Source: Andersen and Mailand (2005).

Figure 6: Mandatory pension contribution scheme



between 9 and 18 hours a week; and (iv) zero contribution is paid for all employees working less than 9 hours a week.

The available data contain the *sum* of pension contributions paid by the firm for all of its employees in a given quarter. Then, a full-time equivalent (FTE) measure reported by the Danish Central Statistical Office is constructed as the total amount of quarterly pension contributions divided by the payment norm for a full-time employee (where full-time refers to working more than 27 hours per week). Given the proportionality of the schedule, the average hours per worker can be derived by dividing the total FTE measure, N^* , by the number of employees, N , and multiplying by 27 hours a week, i.e.

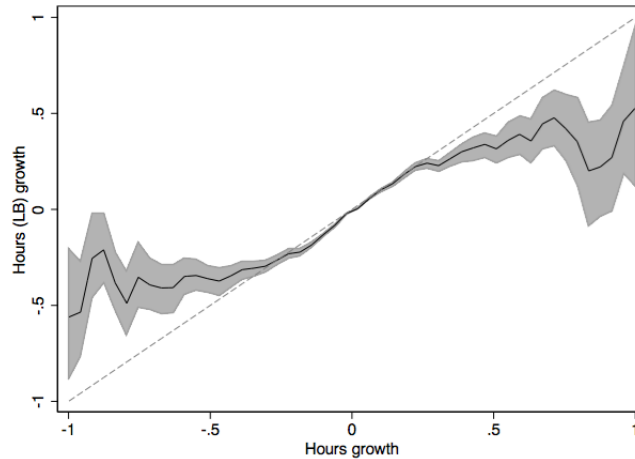
$$(LB) \quad h_{LB} = 27 \frac{N^*}{N}.$$

This approach implicitly assumes the left boundary of each 9-hour interval for all employees and therefore represents the lower bound on the weekly hours of work. Alternatively, I construct an upper bound measure of work hours, by taking the right boundary point for each of the 9-hour intervals in Figure 6. The right boundary of the last interval is assumed to be 36 hours a week. This assumption, albeit not very realistic, preserves the proportionality of the hours schedule. Also, note that the FTE measure N^* excludes employees that work less than 9 hours per week. Therefore, if the number of workers in a given firm is higher than the number of full-time employees, I allocate 9 hours of work to those extra workers. In sum, the upper bound on work hours per employee is defined as

$$(UB) \quad h_{UB} = \frac{36N^* + 9(N - N^*)\mathbf{1}[N > N^*]}{N}.$$

There are several concerns about using this hours measure that I try to address below. First, one can argue that the firm has an incentive to adjust hours only within (and not between) the 9-hour intervals in order to minimize its pension contributions. However, the level of pension contributions is relatively low

Figure 7: The relationship between the growth rates of actual and lower bound work hours.



Note: Estimates are based on a uniform kernel with a bandwidth of 0.05. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 2002 - 2006. Sample contains all private firms with more than 10 full-time employees, with the exception of agriculture and fishery.

compared to other labor costs, such as wages, income taxes and social security contributions: a pension contribution for a full-time employee amounts to about 1% of average wages. It is unlikely that firms have economically significant incentives to “bunch” workers at the right boundary point of each interval. Moreover, the Earnings survey data show no evidence of “bunching” (at least on the annual level) that would imply a higher mass of workers at the right boundary points of the pension scheme intervals.

Secondly, the LB measure of the work hours captures the variation in labor input at the firm level only if some of the firm's workers switch between the 9-hour intervals. Again, I can use the Earnings survey to check how well it reflects actual labor utilization at the annual level. Figure 7 below compares the annual growth rates of the two hours series: the hours measure based on the pension contributions and the hours series drawn from the Earnings survey. The average weekly work hours in the latter case are computed by dividing the total annual amount of hours by the number of employee-weeks in a given firm. In general, the two variables move closely together for the growth regions below 50% in absolute value.

The fourth dataset is drawn from the VAT statistics for the period of 2002-2006. It provides information on purchases and sales of all VAT-liable businesses on a quarterly basis. In Denmark a business enterprise must register for VAT if its annual turnover is expected to exceed 50,000 DKK. The VAT declaration frequency depends on the annual turnover: firms report monthly if their annual turnover exceeds 15 million DKK, quarterly if their turnover is between 1 million DKK and 15 million DKK, and semi-annually if it is below 1 million DKK. Hence, the empirical moments on value added and labor productivity in this paper refer to businesses with annual turnover above 1 million DKK. The empirical analysis is carried out based

on private firms data.³⁹ The resulting dataset has close to 3 million firm-quarter observations.

B. Solving for the Shapley values

This subsection draws heavily on Roth (1988). The Shapley value, originally proposed by Shapley (1953), is a commonly used way to divide the total worth of the grand coalition in a cooperative game with transferable utilities. It represents the average marginal contribution of player i , averaging over all different sequences according to which the grand coalition can be built up from the empty coalition:

$$(24) \quad \psi_i(z) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N| - |S| - 1)!}{|N|!} [z(S \cup \{i\}) - z(S)]$$

The summation is over all possible coalitions S out of the grand coalition N . The quantity $z(S \cup \{i\}) - z(S)$ represents the change in the value of coalition S when player i joins it. Multiply this quantity by the $|S|!$ different ways the set S could have been formed prior to player i 's addition and by the $(|N| - |S| - 1)!$ different ways the remaining agents could be added afterward, then divide by the total number of permutations of the $|N|$ players. Finally, sum over all possible sets S . Hence, $\psi(z)$ is the average marginal contribution of player i to the grand coalition if the players sequentially form this coalition in a random order. Note that the actual way the coalition was formed is irrelevant.

An alternative algorithm for computing the Shapley value was proposed by Maschler (1982) based upon the idea of building recursively a sequence of games, starting with the given game, by allocating in each step the worth of a coalition to the members of that coalition, until all coalitions have a zero worth. Then, the sum of allocations is proved to be equal to the Shapley Value.

To see how it works, define the unanimity game, u_S , for each coalition S as $u_S(T) = 1$ if $S \subseteq T$ and $u_S(T) = 0$ otherwise. The Shapley value for each unanimity game u_S is defined by $\psi(u_S) = \frac{1}{|S|}$ if $i \in S$ and $\psi(u_S) = 0$ otherwise. Intuitively, in the game u_S any coalition which contains S splits one unit between its members equally. Since players outside S do not contribute anything to the coalition they receive zero. Any characteristic function, z , can be represented uniquely as a weighted sum of the characteristic functions of the unanimity games $z = \sum_{S \subseteq N} c_S u_S$, for some appropriate constants c_S . To find c_S , start with $c_\emptyset = 0$, and define inductively on the number of elements in T , for all $T \subset N$, $c_T = z(T) - \sum_{S \subset T, S \neq T} c_S$.

I use the above method to find the Shapley value of each possible firm-workers coalition. For illustration purposes, consider a coalition consisting of three players, i.e. a firm that employs two workers. Let

³⁹In the data, I construct quarterly series of work hours only for those firms which have employment in all three months of a quarter to avoid a spurious negative correlation between the growth rates of hours and employment. In addition, I drop firms with negative value added. In total, 13.9% of firm-quarter observations are excluded due to missing quarterly information.

subscript F denote the firm and subscripts $W1$ and $W2$ denote workers 1 and 2, respectively. I first start with allocating to each player i her worth, that is $c_i = z_i - c_\emptyset$. A coalition that consists of a type q firm with no workers produces the value of $V_0(q)$, while a coalition consisting of an unmatched worker is worth the value of unemployment U . Hence, $c_{\{F\}} = V_0(q)$ and $c_{\{W1\}} = c_{\{W2\}} = U$. Then, I find the value of $c_{\{F,W1\}} = c_{\{F,W2\}} = \Omega_1(q) - V_0(q) - U$, where $\Omega_1(q)$ is the value of a match between a firm and one worker. Note that $c_{\{W1,W2\}} = 0$ as the total value of the coalition with two unemployed workers equals the sum of their outside options, U . Finally, $c_{\{F,W1,W2\}} = \Omega_2(q) - c_{\{F\}} - c_{\{W1\}} - c_{\{W2\}} - c_{\{F,W2\}} - c_{\{F,W1\}} = \Omega_2(q) - 2\Omega_1(q) + V_0(q)$. The value of the total match can be written as

$$(25) \quad z = c_{\{F\}}u_{\{F\}} + c_{\{W1\}}u_{\{W1\}} + c_{\{W2\}}u_{\{W2\}} + c_{\{F,W1\}}u_{\{F,W1\}} + c_{\{F,W2\}}u_{\{F,W2\}} + c_{\{F,W1,W2\}}u_{\{F,W1,W2\}}$$

and the corresponding Shapley values are

$$\begin{aligned} \psi_{\{F\}} &= c_{\{F\}} + \frac{1}{2}(c_{\{F,W1\}} + c_{\{F,W2\}}) + \frac{1}{3}c_{\{F,W1,W2\}}, \\ \psi_{\{W1\}} &= c_{\{W1\}} + \frac{1}{2}c_{\{F,W1\}} + \frac{1}{3}c_{\{F,W1,W2\}}, \\ \psi_{\{W2\}} &= c_{\{W2\}} + \frac{1}{2}c_{\{F,W2\}} + \frac{1}{3}c_{\{F,W1,W2\}}. \end{aligned}$$

I repeat this process for all possible values of n . The fact that all workers are identical simplifies the algebra and I end up with very simple and intuitive equations (12) and (13).

C. Simulation

I discretize the state space in terms of productivity and use Gaussian (Gauss-Laguerre) quadrature method to approximate the expected value of any function of q (see Judd (1998) for details). I use ten nodes for the productivity distribution. Here, I assume that productivity q follows Generalized Pareto Distribution. For a given draw of productivity, to obtain the corresponding hiring and separation rates, I use a linear interpolation between q nodes.

First, using the optimal hours given in equation (16), I compute workers' disutility and firm's revenue functions. Then, given an initial guess for distribution functions $F(W)$ and $G(W)$, market tightness θ , and the unemployment rate u , I construct the separation and offer acceptance rates. I apply the value function iteration procedure to find the value functions $V_0(q)$, $\Psi_n(q)$ and $Z_n(q)$. Using the surplus sharing rule, I then compute the value of employment $W_n(q)$, the value of a firm $V_n(q)$ and the value of unemployment U . The optimal vacancy posting rate $v_n(q)$ and the maximum labor force size $\bar{n}(q)$ derived from the firm's problem are then used to find a steady state distribution of products across types, $K_n(q)$. Using equations

(20) - (23), I update the initial guess for distribution functions $F(W)$ and $G(W)$, the unemployment rate u , and market tightness θ . I then repeat the procedure until the convergence of equilibrium objects is achieved.

The equilibrium hiring and separation rates, $H_n(q)$ and $s_n(q)$, as well as the maximum labor force size $\bar{n}(q)$, are the key variables that determine employment dynamics at the firm level. Given Poisson arrival rates, the waiting time until the next occurrence of any shock is distributed exponentially with parameter $\vartheta = \mu + \delta + H_n(q) + s_n(q)n$. Thus, I generate a time path for each of the simulated firms as a random draw from an exponential distribution. Whether it is a destruction shock, a productivity shock, a new hire, or a separation is decided according to the relative probability of each event.

I simulate 2000 firms for 120 months and discard first 30 months under the assumption that the economy will converge to a steady state equilibrium within the first 30 periods. The average hours series is constructed according to the pension contribution schedule to reproduce the lower bound (LB) measure reported in the data. Revenue and hours variables are aggregated over three months to generate the corresponding quarterly series.

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