Policy-Motivated Candidates, Noisy Platforms, and Non-Robustness

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Abstract

This paper develops a model of a two-candidate election in which the candidates are mainly office-motivated but also to some (arbitrarily small) extent policy-motivated, and their chosen platforms are to some (arbitrarily small) extent noisy. The platforms' being noisy means that if a candidate has chosen a particular platform, the voters' perception is that she has, with positive probability, actually chosen some other platform. It is shown that (i) an equilibrium in which the candidates play pure exists whether or not there is a Condorcet winner among the policy alternatives, and (ii) in this equilibrium the candidates choose their own favorite platforms, which means that the platforms do not converge.

JEL classification: D64; D82

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"The strong convergence results in one dimension have proven rather robust; equally robust have been the instability results in multiple policy dimensions." Gary M. Miller (1997: 1185)

1 Introduction

Since the classic contributions of Hotelling (1929) and Downs (1957), a large body of literature on two-candidate elections has developed. Two main themes in this literature have been the questions: (i) Where are the equilibrium platforms of the candidates located? and (ii) Under what conditions do equilibria (in pure strategies) exist in settings that are more general than the Hotelling-Downs model? Concerning the first question, the famous answer is the so-called full-convergence result: in equilibrium, the candidates choose the same platform, namely the favorite policy of the median voter. As for the second question, a common conclusion in the literature is that, if the policy space has more than one dimension, an equilibrium in which the candidates do not randomize in their platform choices exists only under conditions that are very stringent and which fail to hold in many natural settings.[1]

This paper sheds light on the question whether these results are *robust*, that is, whether the full-convergence result and the result that pure-strategy equilibria typically do not exist in settings with more than one dimension (or when there is no Condorcet winner among the policy alternatives) still hold true *if one alters the model only slightly*. There are two things that I simultaneously add to the standard setting, although both are needed only in an arbitrarily small amount for my results go through. First, I assume that the candidates do not only care about winning the election; they also care about policy per se. Second, I assume that the candidates' platforms are noisy in the eyes of the voters; that is, if a candidate has chosen a particular platform, the voters' perception is that she has, with positive probability, actually chosen some other platform. One may think of this assumption as representing (exogenous) errors or misunderstandings in the transmission of information about the true platforms from candidates via mass media to voters.[2]

[1] See, for example, Plott (1967) and McKelvey (1979) or the survey in Austen-Smith and Banks (1999).

[2] While revising this paper I came across a new (now published) book manuscript by Gross-

In the literature, the most commonly used approach for obtaining non-convergence of equilibrium platforms is to assume that (i) the candidates are policy-motivated (as well as office-motivated) and (ii) they are uncertain about the outcome of the election. Papers that make these assumptions include Calvert (1985), Roemer (1994), and Wittman (1983). The degree to which the platforms diverge in these papers, however, varies continuously with the amount of uncertainty and with the relative weight the candidates put on policy. In particular, as either the amount of uncertainty or the weight on policy goes to zero, the equilibrium platforms approach each other. Hence, with regard to these two assumptions, the full convergence result is robust. As Calvert (1985: 70) argues, this is an important result:

> Such robustness against departures from the basic assumptions is vital to the development of any positive theory. As a model of general process, the basic multidimensional voting model serves mainly as a tractable guide to our thinking about electoral competition (and to more detailed modeling), and not necessarily as a direct generator of hypotheses about real-world elections. It must assume away many features of the real world. If the model's conclusions are robust against complications of that abstract picture, then it has captured the essence of electoral competition; it is a useful abstraction.

In the present paper, however, I show that such a robustness result fails to hold if we allow the candidates' platforms to be noisy in the sense described above. More precisely, if the candidates are to some (arbitrarily small) extent policy-motivated *and* if the platforms are to some (arbitrarily small) extent noisy, then there is an equilibrium in which both candidates choose their favorite policies. This means that the platforms do *not* converge, and this non-convergence result is obtained by altering the original Hotelling-Downs model only slightly.[3] Although the literature has identified many alternative ways of

man and Helpman (2001). In Chapter 3.2 of their book, these authors study an electoral-competition game in which platforms are noisy in the same sense as here (although they assume that the platforms are noisy in the eyes of only a fraction of the electorate; the others observe the platforms perfectly). In the setting of Grossman and Helpman, however, the candidates are not policy-motivated at all, which means that the phenomena studied in the present paper do not arise. Instead the authors investigate how the location of the equilibrium platforms change as the fraction of informed voters change.

[3] In the first model that I develop, where the only features that are added to the stan-

obtaining non-convergence, there is, to the best of my knowledge, no other result of non-robustness.

The result that the candidates choose their favorite policies is close in spirit to that of Alesina (1988). He points out that if the candidates care about policy to some small extent and if they are *not* able to precommit to policy platforms, then the candidates will, once they have been elected, implement their own favorite policies. In the present paper, however, it is assumed that the candidates *can* commit. Still, with the additional assumption that the platforms are to some small extent noisy, it turns out that the candidates' favorite policies have a very strong drawing power. As a result, the equilibrium outcome is the same as in Alesina's model.

The second main contribution of this paper is to show that the typical non-existence of pure-strategy equilibria in a setting where there is no Condorcet winner among the policy alternatives can be remedied by, again, assuming that the candidates are to some (arbitrarily small) extent policy-motivated and the platforms are to some (arbitrarily small) extent noisy. I show that, if we slightly alter the standard model in this fashion, there is an equilibrium in which the candidates play pure; in particular, in this equilibrium, the candidates choose their own favorite policies.

In order to facilitate an explanation of the results and the intuition behind them, the paper starts out in Sections 2 and 3 by considering two simple examples. The example in Section 2 concerns the full-convergence result whereas the example in Section 3 is about the non-existence of pure-strategy equilibria in a setting where there is no Condorcet winner among the policy alternatives. In Section 4 a more general model is analyzed, and it is shown that the main results from the two examples still hold true. This section can be skipped by those readers who are only interested in the economic intuition behind the results. The results and the insights in this paper draw heavily on an important and thought-provoking paper by Bagwell (1995) on the robustness of the

dard Hotelling-Downs setting are policy preferences and noisy platforms, there are also full-convergence equilibria which co-exist with the non-convergence equilibria. I argue, however, that the equilibria with full convergence are fragile. The reason why they can exist is that, in spite of the fact that it does not have a strict incentive to do so, the electorate may in a particular way make its voting behavior contingent on the noisy signal that it observes. In a plausible extension of the model, in which the electorate to some small extent also cares about the identity of the winning candidate, I show that only non-convergence can be an equilibrium.

first-mover-advantage result in the industrial organization literature. Section 5 reviews this paper and some others that extend and criticize Bagwell's analysis. That section also provides a concluding discussion. An appendix contains proofs of those results that are not proven in the main body of the paper.

2 Non-convergence of equilibrium platforms

Let us study the following very stylized electoral-competition game played between two candidates and one voter. The candidates first, simultaneously and independently, choose one electoral platform each; candidate i's (for $i \in \{1, 2\}$) chosen platform is denoted x_i. The voter then casts his vote for one of the candidates. The candidate who wins the election (i.e., the one who is voted for by the single voter) must implement her previously chosen platform.[4] There are three policy alternatives, Left (L), Center (C), and Right (R). In order to make the example as simple as possible, however, the candidates' choice sets are restricted so that candidate 1 is constrained to choose her platform x_1 from the set $\{L, C\}$, and candidate 2 must choose her platform x_2 from the set $\{C, R\}$.[5]

I will carry out the analysis under two alternative assumptions about the informational structure of the game. The first assumption says that the voter, prior to making his voting decision, can observe the candidates' chosen platforms perfectly. The second assumption says that the platforms are *noisy*; that is, the voter can observe only an (exogenous) signal about the candidates' platforms. The signal is denoted $s = (s_1, s_2)$, where $s_1 \in \{L, C\}$ and $s_2 \in \{C, R\}$. The signal technology works as follows. When a candidate chooses a particular platform, the voter will observe a signal specifying that same platform with probability $1 - \varepsilon$; if the signal does not specify the right platform, then the other possible platform is specified (see Table 1). The realizations of s_1 and s_2 are independent. The noise parameter ε is strictly positive, but it should be thought of as being small; in particular it is assumed that $\varepsilon \in \left(0, \frac{1}{3}\right]$.

[4] The assumption that electoral promises are binding will be discussed in Section 5.

[5] All three players will be allowed to randomize between their available pure actions. The notation needed for such mixed strategies will be introduced later.

$\Pr(s_1 = \widetilde{s} \mid x_1 = \widetilde{x})$	$\widetilde{s} = L$	$\widetilde{s} = C$	$\Pr(s_2 = \widetilde{s} \mid x_2 = \widetilde{x})$	$\widetilde{s} = C$	$\widetilde{s} = R$
$\widetilde{x} = L$	$1 - \varepsilon$	ε	$\widetilde{x} = C$	$1 - \varepsilon$	ε
$\widetilde{x} = C$	ε	$1 - \varepsilon$	$\widetilde{x} = R$	ε	$1 - \varepsilon$

Table 1: *The signal technology.* The probability that each "sub-signal" s_i is correct equals $1 - \varepsilon$, and the two sub-signals s_1 and s_2 are independent. It is assumed that $\varepsilon \in \left(0, \frac{1}{3}\right]$.

The three players have preferences over the policy alternatives L, C, and R, although the two candidates primarily care about being in office (a candidate "is in office" if she has won the election). In particular, each candidate receives an incremental payoff of 1 if being in office and an incremental payoff of 0 otherwise. In addition, each candidate receives an incremental payoff of $a \in \left(0, \frac{1}{4}\right)$ if her favorite policy is implemented; candidate 1's favorite policy is L and candidate 2's favorite policy is R. The voter gets a payoff of 1 if his favorite policy C is implemented and a payoff of 0 if either policy L or policy R is implemented. In sum, the players' payoffs are described in Table 2.

	candidate 1	candidate 2	voter
$1L$	$1 + a$	0	0
$1C$	1	0	1
$2C$	0	1	1
$2R$	0	$1 + a$	0

Table 2: *The players' payoffs.* The abbreviation $1C$ (respectively, $2C$) means that candidate 1 (respectively, 2) is in office and chooses policy C, and similarly with the abbreviations $1L$ and $2R$. It is assumed that $a \in \left(0, \frac{1}{4}\right)$.

Below the two versions of the model will be analyzed in turn. Before doing that, however, it will be useful to describe formally the strategies that are available to the players; at the same time some necessary notation can be introduced. First consider the game with perfect observability. Here, a strategy for candidate 1 is a number $p_O \in [0, 1]$, where p_O is the probability with which candidate 1 chooses her favorite policy L. (The subscript O is short for "observable.") Similarly, a strategy for candidate 2 is a number $q_O \in [0, 1]$, where q_O is the probability with which candidate 2 chooses her favorite policy R. The voter's strategy is a *function* that specifies with what probability the voter casts his vote for each candidate given a particular platform configuration. We can thus describe the voter's strategy by a vector with four components, $r_O = \left(r_O^{LC}, r_O^{LR}, r_O^{CC}, r_O^{CR}\right)$, where each component $r_O^{jk} \in [0,1]$ is the probability with which the voter elects candidate 1 if the platform configuration is $(x_1, x_2) = (j, k)$.

Now consider the version of the model where the platforms are noisy. Here the candidates' strategies have the same form as in the case with observable platforms. That is, a strategy for candidate 1 is a number $p_N \in [0,1]$, where p_N is the probability with which candidate 1 chooses her favorite policy L. (The subscript N is short for "noisy.") Similarly, a strategy for candidate 2 is a number $q_N \in [0,1]$, where q_N is the probability with which candidate 2 chooses her favorite policy R. The voter's strategy is, again, a function, although here it is not a function of the actual platforms (since these are unobservable) but of the signal s. Hence, the voter's strategy can be described by a vector with four components, $r_N = \left(r_N^{LC}, r_N^{LR}, r_N^{CC}, r_N^{CR}\right)$, where each component $r_N^{jk} \in [0,1]$ is the probability with which the voter elects candidate 1 if the *signal* is $s = (j, k)$.

2.1 Observable platforms

Let us first analyze the version of the model where the platforms are perfectly observable. The solution concept that will be used here is that of subgame perfect equilibrium. We can therefore solve the model by backward induction. Hence, let us begin by considering the voter's decision for whom to vote, given that he has observed some platforms x_1 and x_2. Clearly, if candidate 1 has chosen C and candidate 2 has chosen R, then the voter will elect candidate 1 ($r_O^{CR} = 1$); and if candidate 1 has chosen L and candidate 2 has chosen C, then the voter will elect candidate 2 ($r_O^{LC} = 0$). For the two remaining possible platform configurations, the voter is indifferent between the candidates; this means that the probabilities r_O^{CC} and r_O^{LR} can take on any value between zero and one.

We can substitute this optimal behavior on the part of the voter into the expressions for the candidates' expected payoffs. Doing this yields the following

2×2 game matrix:

	$x_2 = C$	$x_2 = R$
$x_1 = L$	$0, 1$	$r_O^{LR}(1+a), \left(1 - r_O^{LR}\right)(1+a)$
$x_1 = C$	$r_O^{CC}, 1 - r_O^{CC}$	$1, 0.$

(1)

The first expression in each cell of the matrix is candidate 1's expected payoff and the second one is player 2's expected payoff. Now look at the special case where the candidates get elected with equal probability whenever the voter is indifferent between them: $r_O^{CC} = r_O^{LR} = 1/2$. The game matrix above then simplifies to the following.

	$x_2 = C$	$x_2 = R$
$x_1 = L$	$0, 1$	$\frac{1+a}{2}, \frac{1+a}{2}$
$x_1 = C$	$\frac{1}{2}, \frac{1}{2}$	$1, 0$

(2)

Clearly, since $a < 1$, this game between the two candidates has a unique Nash equilibrium where both of them, with probability one, choose platform C.

For the general case where the probabilities r_O^{CC} and r_O^{LR} can take on any value between zero and one, the analysis is a bit more involved, and it is therefore relegated to the Appendix. The result, however, is in line with the above: in any equilibrium, the candidate who wins the election chooses platform C with probability one. Indeed, in those equilibria in which $r_O^{CC} \in (0,1)$, *both* candidates choose platform C with probability one. We have the following.

Proposition 1. *Consider the version of the model where the platforms are perfectly observable. This model has a continuum of subgame perfect equilibria. In any one of these equilibria, either:*

a) $\left(p_O, q_O, r_O^{LC}, r_O^{LR}, r_O^{CC}, r_O^{CR}\right) = \left(0, 0, 0, r_O^{LR}, r_O^{CC}, 1\right)$, *for any* $r_O^{CC} \in (0,1)$ *and* $r_O^{LR} \in [0,1]$*; or*

b) $\left(p_O, q_O, r_O^{LC}, r_O^{LR}, r_O^{CC}, r_O^{CR}\right) = \left(p_O, 0, 0, r_O^{LR}, 0, 1\right)$, *for any* p_O *and* r_O^{LR} *such that* $p_O\left(1 - r_O^{LR}\right)(1+a) \leq 1$*; or*

c) $\left(p_O, q_O, r_O^{LC}, r_O^{LR}, r_O^{CC}, r_O^{CR}\right) = \left(0, q_O, 0, r_O^{LR}, 1, 1\right)$, *for any* q_O *and* r_O^{LR} *such that* $q_O r_O^{LR}(1+a) \leq 1$.

This is an example of the full-convergence result. The candidates primarily care about winning the election, and the voter will elect a candidate who chooses the voter's favorite policy C (if anyone of the candidates does this).

Hence, there is a pressure on the candidates to choose the voter's favorite policy instead of their own ones. Although there always exist equilibria in which one of the candidates chooses her own favorite policy with positive probability (and sometimes even with probability one), such a candidate must lose the election for sure.

2.2 Noisy platforms

Let us now turn to the version of the model where the platforms are noisy. This game is quite a bit more complex than the game with observable platforms. Indeed, only writing down an expression for each candidate's expected payoff will require a fair amount of space. We can write such an expression somewhat more succinctly, however, by using matrix notation. To this end, let us define the following two matrices:

$$\Pi \equiv \begin{pmatrix} r_N^{LC} & r_N^{LR} \\ r_N^{CC} & r_N^{CR} \end{pmatrix}, \qquad E \equiv \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

We can now write candidate 1's expected payoff, given her opponent's strategy q_N and the voter's strategy r_N, as

$$\begin{aligned} EU_1\left(p_N, q_N, r_N\right) &= p_N\left(1+a\right)\begin{pmatrix} 1-\varepsilon & \varepsilon \end{pmatrix}\Pi\begin{pmatrix} \left(1-q_N\right)\left(1-\varepsilon\right)+q_N\varepsilon \\ q_N\left(1-\varepsilon\right)+\left(1-q_N\right)\varepsilon \end{pmatrix} \\ &\quad +\left(1-p_N\right)\begin{pmatrix} \varepsilon & 1-\varepsilon \end{pmatrix}\Pi\begin{pmatrix} \left(1-q_N\right)\left(1-\varepsilon\right)+q_N\varepsilon \\ q_N\left(1-\varepsilon\right)+\left(1-q_N\right)\varepsilon \end{pmatrix}. \end{aligned} \tag{3}$$

In order to understand this expression, it is instructive to first look at the case where candidate 1 chooses her own favorite policy with probability one, $p_N = 1$. Here the second term of (3) vanishes, and candidate 1's expected payoff equals $1 + a$ (which is her payoff if winning with her favorite platform) times the ex ante probability that she will win the election when $p_N = 1$, the latter being expressed as a product of three matrices. This ex ante probability thus equals a weighted average of the probabilities r_N^{jk}, where the weights are the probabilities that the signal $s = (j, k)$ will realize. In particular, the first row of Π (which consists of the probabilities r_N^{LC} and r_N^{LR}) will be selected with probability $1-\varepsilon$ whereas the second row (which consists of the probabilities r_N^{CC} and r_N^{CR}) will be selected with probability ε. The columns of Π will be selected with probabilities that depend on the noise parameter ε and the probability that candidate 2 chooses her favorite policy, q_N.

If candidate 1 chooses C instead of L (i.e., if $p_N = 0$), then two things change. First, if she wins the election, then her payoff equals 1 instead of $1 + a$, since she here will win with a policy that is not her favorite one. Second, now the first row of Π is selected with probability ε and the second row with probability $1 - \varepsilon$, which means that here the probabilities r_N^{CC} and r_N^{CR} are the most likely ones to be selected.

The expression for candidate 2's expected payoff can be written as

$$\begin{aligned} EU_2 (p_N, q_N, r_N) &= \begin{pmatrix} p_N (1-\varepsilon) + (1-p_N)\varepsilon \\ (1-p_N)(1-\varepsilon) + p_N\varepsilon \end{pmatrix}^t (E - \Pi) \begin{pmatrix} \varepsilon \\ 1-\varepsilon \end{pmatrix} (1+a) q_N \\ &+ \begin{pmatrix} p_N (1-\varepsilon) + (1-p_N)\varepsilon \\ (1-p_N)(1-\varepsilon) + p_N\varepsilon \end{pmatrix}^t (E - \Pi) \begin{pmatrix} 1-\varepsilon \\ \varepsilon \end{pmatrix} (1-q_N), \end{aligned} \tag{4}$$

where the superscript t denotes the transpose of a matrix. The logic of this expression is the same as the one for candidate 1's expected payoff, although here one must use $E - \Pi$ instead of Π since for each realization of the signal candidate 2 will win with probability $1 - r_N^{jk}$.

The voter's expected payoff at the stage where he has observed a signal $s = (j, k)$ can be written as

$$EU_3^{jk} \left(p_N, q_N, r_N^{jk}\right) = r_N^{jk} \Pr (x_1 = C \mid s_1 = j) + \left(1 - r_N^{jk}\right) \Pr (x_2 = C \mid s_2 = k). \tag{5}$$

In words, the voter's expected payoff is equal to the probability that the candidate that he casts his vote for has chosen platform C. This is because the voter's payoff is unity if the winning candidate has chosen C and zero otherwise. The probabilities $\Pr (x_i = C \mid s_i = j)$ can be calculated by the voter with the help of Bayes' rule. In particular one has

$$\Pr (x_1 = C \mid s_1 = C) = \frac{(1-p_N)(1-\varepsilon)}{p_N\varepsilon + (1-p_N)(1-\varepsilon)}, \tag{6}$$

$$\Pr (x_1 = C \mid s_1 = L) = \frac{(1-p_N)\varepsilon}{p_N (1-\varepsilon) + (1-p_N)\varepsilon}; \tag{7}$$

the corresponding expressions for candidate 2 is the same as above except that q_N is substituted for p_N.

In the version of the model with noisy platforms, subgame perfection does not have any bite. This is simply because here there are no subgames (except for the subgame that consists of the whole game). It is therefore natural to focus attention on the set of all Nash equilibria, which is what I will do. A strategy profile (p_N^*, q_N^*, r_N^*) is a Nash equilibrium if and only if

$$EU_1(p_N^*, q_N^*, r_N^*) \geq EU_1(p_N, q_N^*, r_N^*) \quad \text{for all } p_N, \tag{N1}$$

$$EU_2(p_N^*, q_N^*, r_N^*) \geq EU_2(p_N^*, q_N, r_N^*) \quad \text{for all } q_N, \tag{N2}$$

$$EU_3^{jk}\left(p_N^*, q_N^*, r_N^{jk*}\right) \geq EU_3^{jk}\left(p_N^*, q_N^*, r_N^{jk}\right) \quad \text{for all } r_N^{jk}, \tag{N3}$$

where (N3) must hold for all four possible signals $s = (j,k)$ and where $r_N^* \equiv \left(r_N^{LC*}, r_N^{LR*}, r_N^{CC*}, r_N^{CR*}\right)$. In words, for a particular strategy profile to be a Nash equilibrium, the strategy of each player must be a best response to the strategies of the other two players.

To start with, let us look for a Nash equilibrium in which each candidate chooses her own favorite platform with probability one, i.e., where $(p_N^*, q_N^*) = (1,1)$. Surprisingly, it turns out that such an equilibrium indeed exists. To see this first notice that, given the specified behavior on the part of the candidates, the four equilibrium conditions in (N3) will be satisfied for any r_N^*. This is simply because if both candidates choose their own favorite platforms with probability one, the voter's payoff is zero regardless of which candidate he elects; as a consequence, no matter which signal he has observed, the voter will be indifferent between the candidates.[6] It remains to check conditions (N1) and (N2). These will be satisfied if and only if $EU_1(1,1,r_N^*) \geq EU_1(0,1,r_N^*)$ and $EU_2(1,1,r_N^*) \geq EU_2(1,0,r_N^*)$. Using (3) and (4) together with $(p_N^*, q_N^*) = (1,1)$, these inequalities can after some straightforward algebra be written as

$$\frac{r_N^{LC}\varepsilon + r_N^{LR}(1-\varepsilon)}{1-\varepsilon-\varepsilon(1+a)} \geq \frac{r_N^{CC}\varepsilon + (1-\varepsilon)r_N^{CR}}{(1-\varepsilon)(1+a)-\varepsilon}, \tag{8}$$

$$\frac{1-r_N^{LR} + \left(r_N^{LR} - r_N^{CR}\right)\varepsilon}{1-\varepsilon-\varepsilon(1+a)} \geq \frac{1-r_N^{LC} + \left(r_N^{LC} - r_N^{CC}\right)\varepsilon}{(1-\varepsilon)(1+a)-\varepsilon}. \tag{9}$$

[6] One can verify this formally by plugging $p_N = 1$ into the expressions for $\Pr(x_1 = C \mid s_1 = j)$ in (6) and (7), which yields zero. Similarly, $q_N = 1$ implies $\Pr(x_2 = C \mid s_2 = j) = 0$. Hence, from (5) one has that $EU_3^{jk}\left(1,1,r_N^{jk}\right) = 0$ for all r_N^{jk}.

Let Ω_{LR} be the set of all $r_N \in [0,1]^4$ such that inequalities (8) and (9) are satisfied. One can readily verify that any vector (k,k,k,k), where $k \in [0,1]$, belongs to Ω_{LR}; that is, for example, if the voter always randomizes fifty-fifty or always votes for one of the candidates, then r_N belongs to Ω_{LR}.

Hence, in the version of the model where the platforms are noisy, there exist Nash equilibria in which the candidates' platforms do not converge. This result stands in sharp contrast to Proposition 1 and the full-convergence result in general. For the present result to hold, the candidates must to some extent care about policy ($a > 0$) and the platforms must to some extent be noisy ($\varepsilon > 0$). However, the result holds for *any* small a and ε greater than zero. The basic reason why this kind of equilibrium can exist is that although the voter cannot observe the platforms directly, he (correctly) believes that the candidates have chosen their own favorite platforms with probability one. Moreover, the voter rationally infers that if his beliefs are correct, a signal indicating that a candidate has chosen C must be wrong. Hence, the voter may just as well ignore the signal and, for example, regardless of which signal he has observed vote for each candidate with equal probability. If so, however, a candidate who were to choose platform C rather than her favorite platform would not be rewarded with a higher probability of winning. As a consequence, the candidates will be better off by indeed choosing their own favorite platforms, which confirms the voter's beliefs about their behavior. I will shortly return to a further discussion of this kind of equilibrium.

In this model, the equilibria with non-convergence are not the only ones that exist. There are also equilibria with full convergence, that is, where $(p_N^*, q_N^*) = (0,0)$. To see this first notice that, here again, the equilibrium condition (N3) will be satisfied for any r_N^*. For if both candidates choose platform C with probability one, the voter's payoff is unity regardless of which candidate he elects. This means that full convergence will be an equilibrium if and only if $EU_1(0,0,r_N^*) \geq EU_1(1,0,r_N^*)$ and $EU_2(0,0,r_N^*) \geq EU_2(0,1,r_N^*)$. Using (3) and (4) together with $(p_N^*, q_N^*) = (0,0)$, these inequalities can be written as

$$\frac{r_N^{LR}\varepsilon + r_N^{LC}(1-\varepsilon)}{1-\varepsilon-\varepsilon(1+a)} \leq \frac{r_N^{CR}\varepsilon + r_N^{CC}(1-\varepsilon)}{(1-\varepsilon)(1+a)-\varepsilon}, \tag{10}$$

$$\frac{1-r_N^{CR}+\varepsilon\left(r_N^{CR}-r_N^{LR}\right)}{1-\varepsilon-\varepsilon\left(1+a\right)}\leq\frac{1-r_N^{CC}+\varepsilon\left(r_N^{CC}-r_N^{LC}\right)}{\left(1-\varepsilon\right)\left(1+a\right)-\varepsilon}. \tag{11}$$

Let Ω_{CC} be the set of all $r_N \in [0,1]^4$ such that inequalities (10) and (11) are satisfied. For example, the vector $\left(r_N^{LC}, r_N^{LR}, r_N^{CC}, r_N^{CR}\right) = \left(0, \frac{1}{2}, \frac{1}{2}, 1\right)$ belongs to Ω_{CC}. That is, if the voting behavior is such that candidate 1 (respectively, candidate 2) is punished by getting elected with zero probability if the voter observes a signal $s = (L, C)$ (respectively, a signal $s = (C, R)$), then both candidates' choosing platform C can be part of an equilibrium.

One may wonder whether there are other kinds of equilibria, for example in which one or both candidates randomize between the platforms, that co-exist with the ones that are discussed above. The following lemma tells us that this is not the case.

Lemma 1. *Consider the version of the model with noisy platforms. If* (p_N^*, q_N^*) *is part of a Nash equilibrium of this model, then either* $(p_N^*, q_N^*) = (1,1)$ *or* $(p_N^*, q_N^*) = (0,0)$.

The proof of Lemma 1, which is relegated to the Appendix, proceeds in two steps. First it is shown that in any Nash equilibrium one must have $p_N^* = q_N^*$. The reason for this is that if, say, $p_N^* < q_N^*$, the voter would, for most signal realizations, prefer candidate 1 to candidate 2; the only chance for candidate 2 to win the election with positive probability would be if the voter observed the signal $s = (L, C)$. This creates an incentive for candidate 2 to choose platform C with probability one, which contradicts the assumption that $p_N^* < q_N^*$. The second step of the proof is to make use of the fact that if indeed $p_N^* = q_N^* \in (0,1)$, then both candidates must be indifferent between their pure actions. But if candidate 1 is indifferent between her pure actions, then this again will create an incentive for candidate 2 to choose platform C with probability one, which contradicts the assumption that $q_N^* \in (0,1)$.[7]

The following proposition, which follows from Lemma 1 and the arguments in the text above, summarizes the results.

Proposition 2. *Consider the version of the model with noisy platforms. There are two kinds of Nash equilibria of this model: either both candidates*

[7] It is in the proof of Lemma 1 that the assumptions $\varepsilon \leq \frac{1}{3}$ and $a < \frac{1}{4}$ are used.

choose their own favorite platforms (i.e., $(p_N^*, q_N^*) = (1, 1)$*) and the voter votes according to* $r_N^* \in \Omega_{LR}$*; or both candidates choose platform* C *(i.e.,* $(p_N^*, q_N^*) = (0, 0)$*) and the voter votes according to* $r_N^* \in \Omega_{CC}$*.*

2.3 Discussion and an extension

Hence, in any Nash equilibrium the candidates either both choose platform C (full convergence) or they both choose their own favorite platforms (non-convergence). Which one of these kinds of equilibria should we expect to be played? Clearly, the voter's payoff in an equilibrium with full convergence is greater than in one with non-convergence. Thus, if the voter somehow could avoid the equilibria in which the candidates choose their own favorite platforms, he would have an incentive to do so. One may argue that the voter should indeed be able to avoid the non-convergence equilibria by voting according to, for example, $\left(r_N^{LC}, r_N^{LR}, r_N^{CC}, r_N^{CR}\right) = \left(0, \frac{1}{2}, \frac{1}{2}, 1\right)$. As a matter of fact, as long as the voter expects the candidates to behave symmetrically relative to each other (i.e., as long as $p_N = q_N$), then this kind of behavior is weakly optimal for him: although the voter does not have a strict incentive to vote like this, it does not cost him anything either.[8]

This argument, however, critically relies on the model feature that the voter is exactly indifferent between the candidates when they have (or are believed to have) chosen the same platform. In the real world, voters typically do not only care about candidates' policy platforms but also about their leadership abilities, their looks, etc. If the voter in our model, because of such a reason, strictly preferred one candidate to the other when their platforms are (or are believed to be) identical, then the voting behavior described above would not be optimal for him.

We can formalize this idea in a simple way by assuming that the voter has the following lexicographic preferences: primarily he cares about policy, and his policy preferences are exactly as described in Table 2. But if the voter is indifferent between the candidates as far as their platforms are concerned, then he strictly prefers one of the candidates. In order to retain the symmetry of the

[8] It is important to note that this does *not* mean that the voter uses a weakly dominated strategy in the equilibria with non-convergence. For if we had $p_N \neq q_N$, then choosing, for example, some $r_N^{LR} \in (0, 1)$ would be suboptimal for the voter.

model (which is desirable since we want it to remain tractable), let us suppose the identity of the favorite candidate is private information to the voter, and that both candidates assign the probability 1/2 to the event that the favorite is candidate 1. The rest of the model is exactly as before.

Let us first look at how the analysis of the version of the model where the platforms are perfectly observable is affected by this alternative assumption about the voter's preferences. In this case, the probabilities r_O^{CC} and r_O^{LR} will, from the perspective of the candidates, equal 1/2. Hence, the game matrix depicted in (1) always simplifies to the one in (2). As a consequence, in this version of the model there is a unique Nash equilibrium in which both candidates choose platform C.

Next, consider the version of the model where the platforms are noisy. It can be verified that the proof of Lemma 1 is still valid under the alternative assumption that the voter has the lexicographic preferences described above. Hence, in any equilibrium we must have either full convergence or non-convergence. Can full convergence be an equilibrium? The answer is No. For if both candidates are expected to choose platform C, the voter will, for all signals, be indifferent between the candidates as far as their platforms are concerned; hence, the voter will elect his favorite candidate with probability one. This means that, from the perspective of the candidates, $r_N^{LC} = r_N^{LR} = r_N^{CC} = r_N^{CR} = 1/2$. But if the candidates expect to be elected with the same probability for all signals, they will have an incentive to deviate to their own favorite platforms. As a consequence, full convergence cannot be part of an equilibrium. A similar reasoning shows that non-convergence (i.e., $(p_N^*, q_N^*) = (1,1)$) is indeed part of an equilibrium.[9]

[9] In order to get a clear understanding of why the voter is not able to avoid the non-convergence equilibria, the following observation may be helpful. If the voter could, at an ex ante stage, make a credible commitment to some voting behavior r_N, then he would have a *strict* incentive to commit to, for example, $r_N = \left(0, \frac{1}{2}, \frac{1}{2}, 1\right)$ rather than some $r_N \in \Omega_{LR}$ (since he strictly prefers full convergence to non-convergence). Yet at the time he actually is voting, the voter will — given that he has the specified lexicographic preferences — strictly prefer one candidate to the other, which means that voting according to $r_N = \left(0, \frac{1}{2}, \frac{1}{2}, 1\right)$ will not be ex post optimal for him. Rather, the r_N that is ex post optimal for the voter has all of its four components equal to zero or one; hence, from the perspective of the candidates, the voter's perceived ex post optimal behavior is given by $r_N = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$, which belongs to Ω_{LR} but not to Ω_{CC}. We may because of this reason think of the phenomenon that only equilibria with non-convergence can exist as being due to a time-inconsistency problem.

3 Existence of a pure-strategy equilibrium

Now consider the following simple electoral-competition game played between two candidates and three voters. The candidates are indexed by $i \in \{1, 2\}$ and the voters by $j \in \{1, 2, 3\}$. There are again three policy alternatives, Left (L), Center (C), and Right (R), although now both candidates are free to choose any policy. The sequence of events is also the same as in the previous section. First the two candidates simultaneously and independently choose platforms, $x_i \in \{L, C, R\}$. Then the three voters simultaneously and independently vote for one of the candidates. The candidate who receives a majority of votes wins office and must implement her previously chosen platform.

In a first version of the model all three voters can, prior to casting their votes, observe the candidates' chosen platforms perfectly. In a second version of the model, voter 2 can only observe a noisy signal $s \in \{L, C, R\}$ about candidate 2's platform; voter 2 can observe candidate 1's platform perfectly, and voters 1 and 3 can observe both candidates' platforms perfectly.[10] The signal technology is similar to the one in the previous section. More specifically, when candidate 2 chooses a particular platform, then voter 2 will observe a signal specifying that same platform with probability $1 - \varepsilon$; if the signal does not specify the right platform, then the two other platforms have an equal chance of being specified (see Table 3). The noise parameter ε is strictly positive, but it should be thought of as being small; in particular it is assumed that $\varepsilon \in \left(0, \frac{1}{2}\right)$.

$\Pr(s = \widetilde{s} \mid x_2 = \widetilde{x})$	$\widetilde{s} = L$	$\widetilde{s} = C$	$\widetilde{s} = R$
$\widetilde{x} = L$	$1 - \varepsilon$	$\varepsilon/2$	$\varepsilon/2$
$\widetilde{x} = C$	$\varepsilon/2$	$1 - \varepsilon$	$\varepsilon/2$
$\widetilde{x} = R$	$\varepsilon/2$	$\varepsilon/2$	$1 - \varepsilon$

Table 3: *The signal technology.* The probability that the signal s is correct equals $1 - \varepsilon$, where $\varepsilon \in \left(0, \frac{1}{2}\right)$.

The five players' payoffs are specified in Table 4. The voters' payoffs are such that voter 1 prefers L to R and R to C; voter 2 prefers C to L and L to

[10] That is, it is only the platform of *one* of the candidates that is noisy, and this platform is noisy only in the eyes of *one* of the three voters. This modeling choice serves two purposes. First, it shows that only a very small amount of noise is needed for the results to go through (but it is important both *which* candidate's platform is noisy and in the eyes of *which* voter). Second, and more importantly, it simplifies the analysis. The more general model in Section 4 will assume that *both* candidates' platforms are noisy in the eyes of *all* voters.

R; and voter 3 prefers R to C and C to L. These preferences are chosen to make sure that there is no policy alternative that is a Condorcet winner. That is, if the voters vote directly on policy alternatives (and if they vote for their favorite policies), then there is no alternative that can beat both the other ones in a pairwise comparison: $L \prec C \prec R \prec L$ (where "$\prec$" means "is beaten in a pairwise comparison by"). It is also assumed that although the voters primarily care about policy, they also to some extent care about the identity of the winning candidate, with candidate 1 being their favorite (this is captured by the term $\lambda \in (0,1)$).[11] The candidates have payoffs that are similar to the ones in the model of the previous section: although they are mainly office-motivated they also to some extent care about policy per se (which is captured by the term $a \in (0,1)$); candidate 1's favorite policy is L, and candidate 2's favorite policy is R.

	candidate 1	candidate 2	voter 1	voter 2	voter 3
$1L$	$1+a$	0	$1+\lambda$	λ	$-1+\lambda$
$2L$	a	1	1	0	-1
$1C$	1	0	$-1+\lambda$	$1+\lambda$	λ
$2C$	0	1	-1	1	0
$1R$	1	a	λ	$-1+\lambda$	$1+\lambda$
$2R$	0	$1+a$	0	-1	1

Table 4: *The players' payoffs.* It is assumed that $a, \lambda \in (0,1)$.

3.1 Observable platforms

To start with, let us consider the version of the model where the platforms are perfectly observable. We are looking for the subgame perfect equilibria of this game. I also impose the requirement that no voter uses a weakly dominated strategy. That is, even if a voter's vote does not change the outcome of the election, he votes for a candidate if he prefers that candidate to the other candidate.[12] From Table 4 it follows that candidate 2 will win the election in three cases, namely if the platforms are $(x_1, x_2) = (R, L)$, $(x_1, x_2) = (L, C)$, or

[11] The assumption that the voters are biased in favor of candidate 1 is made for the sake of tractability: it simplifies the analysis considerably since it breaks ties.

[12] In this subsection, all five players will be allowed to randomize between their pure actions. (Notation for such mixed strategies will be introduced shortly.) In Section 3.2, however, I will only look for equilibria in which the candidates play pure (since the point with the analysis there is to show that there exists an equilibrium in which the candidates do not randomize).

$(x_1, x_2) = (C, R)$. For any other platform configuration candidate 1 will win the election.

Substituting this optimal behavior on the part of the voters into the expressions for the candidates' expected payoffs yields the following 3×3 game matrix:

	$x_2 = L$	$x_2 = C$	$x_2 = R$
$x_1 = L$	$1 + a, 0$	$0, 1$	$1 + a, 0$
$x_1 = C$	$1, 0$	$1, 0$	$0, 1 + a$
$x_1 = R$	$a, 1$	$1, a$	$1, a.$

(12)

As in the model in the previous section, candidate 1 is the row player and candidate 2 is the column player, and the first expression in each cell of the matrix is candidate 1's payoff and the second one is player 2's payoff. It can easily be verified that the game depicted in (12) does not have any pure-strategy equilibrium. Of course, however, there is a mixed-strategy equilibrium of this game. In fact, there is a unique equilibrium of the game in which both candidates are randomizing over all three policy alternatives. Let $\sigma_i(k)$ be the probability with which candidate i chooses platform $x_i = k$, for $k \in \{L, C, R\}$. The proof of the following proposition is fairly straightforward and is therefore omitted.

Proposition 3. *Consider the version of the model with perfect observability. This model has only one Nash equilibrium that is both subgame perfect and in which the voters do not use weakly dominated strategies. In this equilibrium the candidates behave as follows:*

$$\sigma_1(L) = \frac{1 - a^2}{3 - a^2}; \qquad \sigma_1(C) = \frac{1 - a}{3 - a^2}; \qquad \sigma_1(R) = \frac{1 + a}{3 - a^2};$$

$$\sigma_2(L) = \frac{1}{3 - a^2}; \qquad \sigma_2(C) = \frac{1 + a - a^2}{3 - a^2}; \qquad \sigma_2(R) = \frac{1 - a}{3 - a^2}.$$

We see that if a, the parameter that measures the strength of the candidates' policy preferences, is small, both candidates choose all three platforms with approximately the same probability (one third).

3.2 Noisy platforms

Let us now turn to the version of the model where candidate 2's platform is noisy in the eyes of voter 2. The solution concept that will be employed here is that of perfect Bayesian equilibrium: each one of the five players is required to make an optimal choice at any information set where he or she has a move.[13] I also impose the requirement that no voter uses a weakly dominated strategy. In the rest of this section and in the proof of Proposition 4, I will refer to such a perfect Bayesian equilibrium simply as an "equilibrium."

It turns out that in this model there is indeed an equilibrium in which none of the candidates is randomizing in their choice of platform. In particular, there is an equilibrium in which both candidates choose their own favorite policies. To see this, suppose that candidate 1 indeed chooses L and candidate 2 indeed chooses R. Voter 2 cannot observe candidate 2's choice directly, only the noisy signal; he correctly believes, however, that candidate 2 chooses R. This means that voter 2 will vote for candidate 1 no matter which signal s he has observed (remember that voter 2 prefers L to R; see Table 4). Moreover, voter 1 will vote for candidate 1 and voter 3 will vote for candidate 2. Hence, the winner is candidate 1. It remains to check that none of the candidates has an incentive to deviate. Candidate 1 certainly does not have such an incentive, since she is winning with her favorite policy. Nor does candidate 2 have a (strict) incentive to deviate since this would not make her win. The reason for this is that voter 2 cannot observe her chosen platform directly and rationally ignores the signal. Hence, if candidate 2 deviated to C, for example, voter 2 would not change his voting behavior. Voters 1 and 3 can indeed observe candidate 2's deviation; their preferences, however, are such that they still would not change their behavior (see Table 4). If deviating to L, candidate 2 certainly would not win.

There also exists another equilibrium where both candidates play pure, namely where both candidates choose L with probability one. In this equilibrium, too, candidate 2 loses the election with probability one. The following result is proven in the Appendix.

Proposition 4. *Consider the version of model with noisy platforms. In this model there is an equilibrium in which candidate 1 chooses L with probability one and candidate 2 chooses R with probability one. There is also an equilibrium in which both candidates choose L with probability one.*

[13] As in Section 2.2, subgame perfection does not have any bite, since there are no subgames. Moreover, Nash equilibrium is too weak because here voters 1 and 3 can observe the platforms perfectly, which means that some information sets may be off the equilibrium path.

There is no other equilibrium in which none of the candidates is randomizing in her choice of platform.

4 A general model

Consider a model of an electoral competition with two candidates, labeled $j \in \{1, 2\}$, and $n \geq 1$ voters, labeled $i \in \{1, \ldots, n\}$.[14] The sequence of events is as follows. First the candidates simultaneously commit to platforms $x_j \in X \subset \Re^d$ (for some positive integer d). Second, the voters observe the chosen platforms with some, possibly very small, noise. Formally, each voter i observes a signal $s_i = (s_{i,1}, s_{i,2}) \in S = X^2$, where each $s_{i,j}$ is independently drawn from a density function $f(\cdot \mid x_j)$ which has positive support on the whole of X. Finally, each voter casts his vote for one of the candidates. The candidate who receives the largest number of votes wins office and gets his chosen policy implemented. In case of a tie, each candidate wins with equal probability.

On the part of the candidates, only pure strategies will be considered. Hence, a strategy for candidate j is a vector x_j with d components. The voters, however, are allowed to randomize in their voting decisions. A strategy for voter i is a mapping r_i from the set of possible signals, S, to the one-dimensional unit simplex, $[0, 1]$. Let us make the interpretation that $r_i(s_i)$ is the probability that voter i votes for candidate 1 conditional on having observed a signal s_i. Denote a vector of voting decisions by $r = (r_1(s_1), \ldots, r_n(s_n))$.

Each voter has preferences over the policy space X and the set of candidates $\{1, 2\}$ that can be represented by the utility function $U^i(x, j)$, where x is the winning candidate's policy. The candidates care about being in office and about policy, although the latter is possibly of only slight importance. Candidate j's utility if policy x is implemented and she is (respectively, is not) in office is $V^j(x, 1)$ (respectively, $V^j(x, 0)$). It is assumed that there is an $\widehat{x}_j \in X$ such that, for $j \in \{1, 2\}$,

$$V^j(\widehat{x}_j, 1) > V^j(x, 1), \qquad \forall x \neq \widehat{x}_j,$$

[14] It is worthwile emphasizing that the model that will be presented here will in some respects *not* be more general than the examples analyzed in the previous sections. For example, in contrast to what we assumed in Section 2, mixed strategies on the part of the candidates will not be allowed. Moreover, in contrast to what we assumed in Section 3, neither the voters nor the candidates will be allowed to use a weakly dominated strategy.

and that

$$V^j(\widehat{x}_j, 1) > V^j(\widehat{x}_j, 0).$$

Hence, each candidate has a favorite policy $\widehat{x}_j$ and — at least if her favorite policy is implemented — a candidate prefers to be in office to not be in office.

From what is said above it follows that, at the stage where a candidate is to choose her platform, she does not know what signals the voters will observe. Nor does she know what the realizations of any mixed strategies on the part of the voters will be. Hence, the outcome of the election will be uncertain for her.[15] Given a vector of voting decisions r and a pair of platforms (x_1, x_2), let $P^j(x_1, x_2, r)$ be the probability that candidate j wins the election. The candidates and the voters are assumed to be expected utility maximizers. Candidate j's expected utility is denoted by EV^j, where

$$EV^j(x_1, x_2, r) = P^j(x_1, x_2, r)\, V^j(x_j, 1) + P^k(x_1, x_2, r)\, V^j(x_k, 0)$$

for $j, k \in \{1, 2\}$ and $k \neq j$. Voter i's expected utility at the stage where he makes his voting decision is denoted by EU^i. Since the candidates are constrained to use pure strategies and, in an equilibrium, the voters correctly anticipate the candidates' actions, the signals that the voters observe will not be informative about the chosen platforms. Hence, we can write voter i's expected utility as

$$EU^i(x_1, x_2, r) = P^1(x_1, x_2, r)\, U^i(x_1, 1) + P^2(x_1, x_2, r)\, U^i(x_2, 2).$$

The equilibrium concept employed is that of Nash equilibrium (conditions (E1) and (E3) below). In addition it is required that none of the players is in equilibrium using a strategy that is weakly dominated (conditions (E2) and (E4)). Formally, an *equilibrium* of the model described is a pair of platforms (x_1^*, x_2^*) and a list of voting decisions $r^* = (r_1^*(s_1), \ldots, r_n^*(s_n))$ such that: (E1) for all $j, k \in \{1, 2\}$ and $k \neq j$,

$$EV^j(x_1^*, x_2^*, r^*) \geq EV^j(x_j, x_k^*, r^*), \qquad \forall x_j;$$

(E2) there is no $x_j' \in X$ such that

$$EV^j\left(x_j', x_k, r\right) \geq EV^j(x_j^*, x_k, r), \qquad \forall (x_k, r),$$

[15] Yet another source of uncertainty is the above-mentioned fact that, in case of a tie, a lottery will decide which candidate wins.

for all $j, k \in \{1, 2\}$ and $k \neq j$, with the inequality holding strictly for some (x_k, r); (E3) for all $i \in \{1, \ldots, n\}$ and all $s_i \in S$,[16]

$$EU^i\left(x_1^*, x_2^*, r^*\right) \geq EU^i\left(x_1^*, x_2^*, r_i, r_{-i}^*\right), \qquad \forall r_i;$$

and (E4) there is no $r_i^{'}$ such that

$$EU^i\left(x_1, x_2, r_i^{'}, r_{-i}\right) \geq EU^i\left(x_1, x_2, r_i^*, r_{-i}\right), \qquad \forall\left(x_1, x_2, r_{-i}\right),$$

with the inequality holding strictly for some (x_1, x_2, r_{-i}).

Proposition 5. *An equilibrium exists. Moreover, in any equilibrium both candidates choose their own favorite policies.*

PROOF: Recall from the model description that the candidates are constrained to play pure. That is, candidate j must choose $x_j = x_j^{'}$ with probability one for some $x_j^{'} \in X$. We need to show that (i) there exists such an equilibrium and (ii) in any such equilibrium $x_j^{'} = \widehat{x}_j$ for all $j \in \{1, 2\}$. Let us first prove (i). Suppose both candidates choose their own favorite policy with probability one, i.e., $x_j = \widehat{x}_j$. Moreover, suppose that none of the voters makes his voting choice contingent on the signal but votes for candidate j if $U^i(\widehat{x}_j, j) > U^i(\widehat{x}_k, k)$ for $k \neq j$; and if $U^i(\widehat{x}_1, 1) = U^i(\widehat{x}_2, 2)$, then voter i randomizes in some fashion that is not contingent on the signal. Let the corresponding vector of voting decisions be denoted by $\widehat{r}$; the probability that candidate j wins the election is thus $P^j\left(\widehat{x}_1, \widehat{x}_2, \widehat{r}\right)$. We must show that none of the candidates and none of the voters has an incentive to deviate unilaterally from this behavior, and that their strategies are not weakly dominated (i.e., that conditions (E1)-(E4) are satisfied). First, consider candidate 1. For this candidate not to have an incentive to deviate unilaterally, condition (E1) must be satisfied. This requires that

$$EV^1\left(\widehat{x}_1, \widehat{x}_2, \widehat{r}\right) \geq EV^1\left(x_1, \widehat{x}_2, \widehat{r}\right) \Leftrightarrow P^1\left(\widehat{x}_1, \widehat{x}_2, \widehat{r}\right)\left[V^1\left(\widehat{x}_1, 1\right) - V^1\left(x_1, 1\right)\right] \geq 0. \tag{13}$$

If $P^1\left(\widehat{x}_1, \widehat{x}_2, \widehat{r}\right) = 0$, then this inequality holds with equality. If $P^1\left(\widehat{x}_1, \widehat{x}_2, \widehat{r}\right) > 0$, then (13) simplifies to $V^1\left(\widehat{x}_1, 1\right) \geq V^1\left(x_1, 1\right)$, which is true by definition of

[16] I here use the standard notation $r_{-i}^* = \left(r_1^*\left(s_1\right), \ldots, r_{i-1}^*\left(s_{i-1}\right), r_{i+1}^*\left(s_{i+1}\right), \ldots, r_n^*\left(s_n\right)\right)$. A similar notation is also used below.

$\widehat{x}_1$. For candidate 1's strategy not to be weakly dominated, condition (E2) must hold. For condition (E2) *not* to hold, there must exist an $x_j^{'}$ such that

$$EV^1\left(x_1^{'},\widehat{x}_2,r\right) \geq EV^1\left(\widehat{x}_1,\widehat{x}_2,r\right) \Leftrightarrow P^1\left(\widehat{x}_1,\widehat{x}_2,r\right)\left[V^1\left(x_1^{'},1\right)-V^1\left(\widehat{x}_1,1\right)\right] \geq 0$$

for all r. This inequality, however, is not satisfied for any r such that $P^1\left(\widehat{x}_1,\widehat{x}_2,r\right) > 0$. Hence, condition (E2) must hold. For candidate 2 the arguments above are analogous. Now consider a voter. For a voter not to have a unilateral incentive to deviate, condition (E3) must be met. If voter i's vote cannot change the outcome of the election, then this condition clearly holds (with equality). Suppose that voter i's vote *can* change the outcome of the election. Then condition (E3) requires that he votes for his favorite candidate. This is consistent with what we postulated above about the voters' behavior. Finally, for a voter's strategy not to be weakly dominated, condition (E4) must hold. This condition is met if voter i always votes for his favorite candidate whenever $U^i(\widehat{x}_1,1) \neq U^i(\widehat{x}_2,2)$. This is also consistent with the above postulated behavior.

Let us now prove (ii). Suppose, per contra, that there is an equilibrium in which $x_1^{'} \neq \widehat{x}_1$ (the case $x_2^{'} \neq \widehat{x}_2$ follows the same logic). Now candidate 1's strategy, however, is weakly dominated; i.e., condition (E2) is violated. To see this one only needs to note that whenever r is such that $P^1\left(x_1,x_2,r\right) > 0$, candidate 1 can gain by deviating to $\widehat{x}$; and whenever r is such that $P^1\left(x_1,x_2,r\right) = 0$, candidate 1's choice of platform does not affect her payoff. □

5 Concluding discussion

As already mentioned in the Introduction, the results and the insights of this paper draw heavily on an article by Bagwell (1995). He develops a game in which one player moves first and then a second player, before making his own move, observes a noisy but possibly almost perfect signal about the first player's action. The players' payoffs are such that, if there were no noise at all, player 1 would benefit from his opportunity to move first. Bagwell's leading example is two oligopolists who compete in quantities — as is well known, the equilibrium profit of the leader in a Stackelberg duopoly is greater than a Cournot duopolist's equilibrium profit. Bagwell shows, however, that under a certain regularity condition,[17] "the set of pure-strategy Nash equilibrium outcomes for the noisy-leader game coincides exactly with the set of pure-strategy Nash equilibrium outcomes for the associated simultaneous-move game" (Bagwell, 1995: 272). In other words, when the leader's action is observed only with some (possibly very small) noise, then in any pure-strategy equilibrium the Cournot outcome is obtained instead of the Stackelberg outcome. Hence, Bagwell concludes, the first-mover advantage is eliminated.

Let us relate Bagwell's results to the ones in Section 2 of the present paper. In the model there, when the candidates' platforms are perfectly observable (Section 2.1) the unique equilibrium outcome involves full convergence, which corresponds to the Stackelberg outcome in Bagwell's game. If instead all three players made their moves simultaneously, we would get non-convergence[18] (the Cournot outcome in Bagwell's game). In the version of the model where the platforms are chosen before the voter's decision but are noisy (Section 2.2), we can, as in Bagwell's model, sustain the same outcome as in the simultaneous-move game, i.e., non-convergence. But in our electoral-competition game this is not the unique outcome: there also exist equilibria with full convergence. The reason for this is that in that game the regularity condition imposed by Bagwell is not satisfied. In the extension with lexicographic preferences (Section 2.3), however, we effectively imposed such a condition and, accordingly, only non-convergence could be part of an equilibrium.

In the literature following Bagwell's paper, two objections to his conclusion that the first-mover advantage is eliminated by the introduction of a small amount of noise have been raised.[19] First, some scholars have criticized Bagwell's focus on pure-strategy equilibria. For example, van Damme and Hurkens (1997) show that in Bagwell's game and under his regularity condition, there

[17] Namely that the player moving last has, given any action of the first player, a unique best-reply action.

[18] To see this, notice that any candidate who expected to get elected with positive probability would have a strict incentive to choose her own favorite platform instead of the voter's. The equilibrium outcome of such a game would thus be identical to the outcome of the non-convergence equilibria in the game where the platforms are chosen before the voter's decision but are noisy.

[19] In the electoral-competition game there is actually not a first-mover advantage but a first-mover disadvantage (since the candidates are worse off when their actions are observed). What is relevant for that game is the fact that the introduction of a small amount of noise may give rise to non-convergence (the Cournot outcome) instead of full convergence (the Stackelberg outcome).

always exists a mixed equilibrium that induces an outcome that is close to the equilibrium outcome of Bagwell's game without any noise.[20] They also suggest an equilibrium-selection theory that selects this mixed equilibrium.[21] Similarly, Güth, Kirchsteiger, and Ritzberger (1998) study an extension of Bagwell's game in which there are a set of leaders who all move simultaneously, whereupon another set of players, the followers, observe noisy signals about the leaders' actions and then choose their actions simultaneously. This extension is particularly interesting for us, since we can think of the set of leaders as candidates in an electoral competition and the set of followers as voters. The main result of Güth et al. is that, for almost all games they consider, there exists a subgame perfect equilibrium outcome of the game with no noise that is approximated by (possibly mixed) equilibrium outcomes of games with small noise.[22] This result, however, relies on generic payoffs, so it does not necessarily hold true in knife-edge cases.

Are the results of the present paper susceptible to the criticism that one should not confine attention to pure-strategy equilibria? The analysis in Section 3.2 does not at all consider the possibility of mixed-strategy equilibria. This neglect should be excused, however, since the challenge there was to find a remedy to the phenomenon that in the standard electoral-competition model there is no equilibrium in which the candidates do not randomize. The analysis in Section 2 indeed allowed for mixed strategies, and it was shown that a mixed equilibrium does not exist — on the other hand, in the noisy-platform model in Section 2.2 there exist pure-strategy equilibria with full convergence. In the

[20] The existence of such a mixed equilibrium was noted by Bagwell (1995), too, although in the context of a specific example.

[21] There are also a couple of other papers that — using different approaches and reaching different conclusions — investigate the problem of equilibrium selection in Bagwell's model. Oechssler and Schlag (2000) carry out an evolutionary-game-theory analysis of a simple version of Bagwell's model. They find that the pure-strategy equilibrium (i.e., the Cournot outcome) is selected by most evolutionary dynamics. Huck and Müller (2000) report on an experimental test of Bagwell's prediction. They do *not* find empirical support for his focus on the pure-strategy equilibrium.

[22] It is interesting to note that this result does not necessarily hold for *any* subgame perfect equilibrium of the game with no noise. This may be important because in an electoral-competition game with many voters who vote strategically there are typically many subgame perfect equilibria (some of them unreasonable). For in each subgame (i.e., given a pair of platforms) there are always many Nash equilibria; for example, all voters voting for one of the candidates is an equilibrium, because given this voting behavior no single voter can gain by a unilateral deviation. Such unreasonable behavior in the subgames can support equilibrium behavior on the part of the candidates that also is unreasonable. Hence, it might be, in principle, that only one of the unreasonable subgame perfect equilibrium outcomes can be approximated by (possibly mixed) equilibrium outcomes of games with small noise.

extension with lexicographic preferences in Section 2.3, however, we did not have full-convergence equilibria nor mixed ones. Still, this result is likely to be sensitive to the assumption that the voter's preferences over the identity of the winning candidate enter lexicographically. Suppose instead that the voter — as in the model in Section 3 — have more standard (in particular, continuous) preferences over the identity of the winning candidate. Then we should expect that even though a pure-strategy equilibrium with full convergence would not exist, a mixed equilibrium would; moreover, the outcome of this mixed equilibrium should be located close to the full-convergence outcome. This calls for some caution when drawing conclusions from the analysis: If one studied a model that allows for mixed equilibria and if the noise in the signal is small, then one may find a mixed equilibrium with almost full convergence besides the pure ones with non-convergence, and, if so, it is not clear which kind of equilibrium would be played.

The second objection that has been raised against Bagwell's conclusion, put forward by Maggi (1999),[23] is that it is sensitive to the introduction of uncertainty about the leader's type. Maggi develops a model that is similar to Bagwell's but in which the leader has private information about a parameter in her payoff function.[24] Under this assumption, the noisy signal that the follower observes is informative about the leader's action also when the leader is expected to play a pure strategy. Maggi shows that the extent to which there is a first-mover advantage in this model depends on the relative magnitude of the uncertainty about the leader's type and the noise in the signal; in particular, in a linear-normal version of his model, the equilibrium outcome moves smoothly from the Stackelberg outcome (when the ratio between the noise in the signal and the noise in the leader's type is small) to the Cournot outcome (when this ratio is large). This result suggests an interesting extension of the analysis of the present paper, namely an electoral-competition game where the candidates also have private information about their policy preferences and where the policy space is continuous. In such a setting, I conjecture that each candidate's equilibrium platform will be located in between this candidate's favorite policy

[23] See also Adolph (1996), who makes a related point.

[24] This parameter is not directly payoff relevant to the follower (e.g., it could be a cost parameter or, in our setting, a parameter measuring the strength of a candidate's policy preferences), which means that signaling is not an issue.

and the one of the median voter. Moreover, how close to the median voter's favorite policy and how close to each other the platforms will be located should depend on the relative magnitude of the noise in the signal and the uncertainty about the candidates' types.[25]

An important assumption that was made throughout the paper is that the candidates can commit to electoral platforms (even though which platform they have committed to is not perfectly observable). The usual justification for this assumption is that, although feasible in principle, deviating from an announced platform is prohibitively costly because of the candidates' concerns for their reputation. Such reputational concerns were formally modeled by Alesina (1988), within the framework of a repeated game with an infinite horizon; see also Dixit, Grossman, and Gul (2000) for a generalization of Alesina's work. In Alesina's as well as in Dixit et al.'s framework, a political outcome "in the middle" is sustained through punishment strategies on the part of the two candidates: each candidate punishes her opponent in case the opponent deviates. Alternatively, one could construct a similar model where the punishments were carried out by the voters. In either case, it is important that any deviation can be observed by the punisher.

This suggests that, also in a setting in which candidates cannot commit, the model feature that the candidates' choices (of actual policies or of announced policy platforms) can only be imperfectly observed may have important implications for the location of equilibrium policies. This should be particularly true if the signals that the players observe about the candidates' actions are private to the receiver of the signal: What is the credibility of, say, candidate 2's (or the voters') threat to punish particular actions if candidate 1 cannot observe candidate 2's (noisy) observation of her action? An interesting topic for future research would be to explore these questions in a formal model; for a related analysis within a seller-buyer framework, see Bhaskar and van Damme (2000). It is my hope that the insights from the analysis of the present paper, with its single-period setting and its commitment assumption, will constitute a first step

[25] A third limitation of Bagwell's argument has been pointed out by Fershtman and Kalai (1997). They show that his result does not hold for any kind of signal technology. In particular, if (i) the signal that the follower observes is correct with some positive probability and (ii) when this happens the follower *knows* that the signal is correct, then Bagwell's result as well as the ones in the present paper break down. Yet another paper that is closely related to Bagwell's is Levine and Martinelli (1998).

in understanding those more far-reaching questions.

Let me conclude by relating to a recent discussion in Persson and Tabellini (2000) on the role of the commitment assumption. They refer to models making this assumption as models of *preelection politics*. In such models, "[t]he essential political action [...] takes place in the electoral campaign, and the role of the election is to select a particular policy" (p. 11). In models of *postelection politics* (i.e., where electoral promises are *not* binding), "the role of elections is very different [...]. Rather than directly selecting policies, voters select politicians on the basis of their ideology, competence, or honesty, or more generally, their behavior as incumbents" (pp. 12-13). Persson and Tabellini are of the opinion that existing research has not produced a clear consensus concerning the question whether electoral promises are binding or unimportant, and they call for more work on this issue: "progress in the field depends on our finding a way of resolving some of these tensions, building a bridge between pre- and postelection politics" (p. 14).

The results of the present paper can hopefully constitute a small contribution to that research agenda by showing that, even if electoral promises are assumed to be binding, a small amount of noise in the transmission of these promises to the electorate can make the model look like one of post- rather than preelection politics: The candidates' own policy preferences have an even stronger drawing power than has been acknowledged in the previous literature.

Appendix

Proof of Proposition 1 Let us first look for Nash equilibria of the reduced-form game in (1) where $r_O^{CC} \in (0,1)$. It is immediate from the game matrix that $(p_O, q_O) = (0,0)$ is a Nash equilibrium of this game. One may also easily verify that neither $(p_O, q_O) = (0,1)$ nor $(p_O, q_O) = (1,0)$ is a Nash equilibrium. It turns out that under the assumption that $a < 1$, nor can $(p_O, q_O) = (1,1)$ be a Nash equilibrium. To see this, suppose that this is indeed a Nash equilibrium. Then for candidate 1 not to have an incentive to deviate, we must have $r_O^{LR}(1+a) \geq 1$, and for candidate 2 not to have an incentive to deviate, we must have $\left(1 - r_O^{LR}\right)(1+a) \geq 1$. These two conditions together imply that $a \geq 1$ — a contradiction. Hence, when $r_O^{CC} \in (0,1)$, the only pure-strategy Nash equilibrium of the game is the one where both candidates play C, i.e., where $(p_O, q_O) = (0, 0)$. Since there is only one pure-strategy Nash equilibrium, there can be no equilibrium in mixed strategies. This gives us the result stated under a). Next, let us look for Nash equilibria of the reduced-form game in matrix (1) where $r_O^{CC} = 0$. First, in any Nash equilibrium of this game, $q_O = 0$. To see this, suppose that $q_O > 0$. Then we must have $p_O \left(1 - r_O^{LR}\right)(1 + a) \geq 1$; otherwise candidate 2 will have an incentive to play C with probability 1. This condition implies $\left(1 - r_O^{LR}\right)(1 + a) \geq 1$ and $p_O > 0$. The condition $p_O > 0$ in turn implies that $q_O r_O^{LR}(1 + a) \geq q_O$; otherwise candidate 1 will have an incentive to play C with probability 1. This condition in turn implies that $r_O^{LR}(1 + a) \geq 1$, which together with the previously derived condition $\left(1 - r_O^{LR}\right)(1 + a) \geq 1$ imply that $a \geq 1$ — a contradiction. Given that $q_O = 0$, candidate 1 is indifferent between playing L and C. In order to have a Nash equilibrium where candidate 2 plays C with probability one and where candidate 1 randomizes with some probability p_O, we must have $p_O \left(1 - r_O^{LR}\right)(1 + a) \leq 1$. This gives us the result stated under b). It only remains to consider the possibility that $r_O^{CC} = 1$. This case, however, is analogous to the previous one; the result is stated under c). □

Proof of Lemma 1 The proof proceeds in two steps. First it will be shown that, in any Nash equilibrium, $p_N = q_N$. Next it is shown that, in any Nash equilibrium and given that $q_N = p_N \equiv p$, either $p = 0$ or $p = 1$.

In order to prove that in any Nash equilibrium $p_N = q_N$, suppose, per contra, that $p_N \neq q_N$. Because of symmetry, we can without any further loss of generality assume that $p_N < q_N$. This implies $r_N^{CC} = r_N^{LR} = r_N^{CR} = 1$. To see this, let us calculate the voter's expected utility if voting for candidate 1 respectively candidate 2 upon observing the signal $s = (s_1, s_2) \in \{L, C\} \times \{C, R\}$. The voter uses Bayes' rule to update his beliefs about candidate i's platform upon observing the signal s_i:

$$\Pr(x_i = \widetilde{x}_i \mid s_i = \widetilde{s}_i) = \frac{\Pr(s_i = \widetilde{s}_i \mid x_i = \widetilde{x}_i)\Pr(x_i = \widetilde{x}_i)}{\sum_{\widetilde{x}_i \in \{L,C,R\}} \Pr(s_i = \widetilde{s}_i \mid x_i = \widetilde{x}_i)\Pr(x_i = \widetilde{x}_i)}. \quad (14)$$

By using the formula in (14), we get

$\Pr(x_1 = \widetilde{x}_1 \mid s_1 = \widetilde{s}_1)$	$\widetilde{s}_1 = L$	$\widetilde{s}_1 = C$
$\widetilde{x}_1 = L$	$\frac{(1-\varepsilon)p_N}{(1-\varepsilon)p_N+\varepsilon(1-p_N)}$	$\frac{\varepsilon p_N}{\varepsilon p_N+(1-\varepsilon)(1-p_N)}$
$\widetilde{x}_1 = C$	$\frac{\varepsilon(1-p_N)}{(1-\varepsilon)p_N+\varepsilon(1-p_N)}$	$\frac{(1-\varepsilon)(1-p_N)}{\varepsilon p_N+(1-\varepsilon)(1-p_N)}$

(15)

and

$\Pr\left(x_2 = \widetilde{x}_2 \mid s_2 = \widetilde{s}_2\right)$	$\widetilde{s}_2 = C$	$\widetilde{s}_2 = R$
$\widetilde{x}_2 = C$	$\frac{(1-\varepsilon)(1-q_N)}{(1-\varepsilon)(1-q_N)+\varepsilon q_N}$	$\frac{\varepsilon(1-q_N)}{\varepsilon(1-q_N)+(1-\varepsilon)q_N}$
$\widetilde{x}_2 = R$	$\frac{\varepsilon q}{(1-\varepsilon)(1-q_N)+\varepsilon q_N}$	$\frac{(1-\varepsilon)q_N}{\varepsilon(1-q_N)+(1-\varepsilon)q_N}$

(16)

Now, using the above tables and the payoff matrix in Table 2 in the main body of the paper, we get the following expressions for the voter's expected utility:

	$\widetilde{s}_i = L$	$\widetilde{s}_i = C$	$\widetilde{s}_i = R$
$EU\left(\text{vote for 1} \mid s_1 = \widetilde{s}_1\right)$	$\frac{\varepsilon(1-p_N)}{(1-\varepsilon)p_N+\varepsilon(1-p_N)}$	$\frac{(1-\varepsilon)(1-p_N)}{\varepsilon p+(1-\varepsilon)(1-p_N)}$	$---$
$EU\left(\text{vote for 2} \mid s_2 = \widetilde{s}_2\right)$	$---$	$\frac{(1-\varepsilon)(1-q_N)}{(1-\varepsilon)(1-q_N)+\varepsilon q_N}$	$\frac{\varepsilon(1-q_N)}{\varepsilon(1-q_N)+(1-\varepsilon)q_N}$.

(17)

Using the expressions in (17) it is easily verified that $p_N < q_N$ implies $EU\left(\text{vote for 2} \mid s_2 = C\right) < EU\left(\text{vote for 1} \mid s_1 = C\right)$, which in turn implies $r_N^{CC} = 1$. In a similar fashion one can check that $p_N < q_N$ also implies $r_N^{LR} = r_N^{CR} = 1$.

Next, note that $p_N < q_N$ and $r_N^{CC} = r_N^{LR} = r_N^{CR} = 1$ together imply $r_N^{LC} < 1$. This is because if $r_N^{LC} = 1$, candidate 1 would win with certainty no matter which platform she chose, which means that she should choose her favorite platform $x_1 = L$; but this in turn implies $p_N = 1 \geq q_N$.

Finally, let us show that the above implications in turn imply $q_N = 0$, which contradicts the assumption that $p_N < q_N$. The equality $q_N = 0$ holds if $EU_2\left(p_N, 0, r_N\right) > EU_2\left(p_N, 1, r_N\right)$ or, using (4) together with $r_N^{CC} = r_N^{LR} = r_N^{CR} = 1$,

$$\left[p_N\left(1-\varepsilon\right)+\left(1-p_N\right)\varepsilon\right]\left(1-r_N^{LC}\right)\left(1-\varepsilon\right) > \left[p_N\left(1-\varepsilon\right)+\left(1-p_N\right)\varepsilon\right]\left(1-r_N^{LC}\right)\left(1+a\right)\varepsilon, \tag{18}$$

which always holds since we know from above that $r_N^{LC} < 1$ and by assumption $\varepsilon \in \left(0, \frac{1}{3}\right]$ and $a \in \left(0, \frac{1}{4}\right)$.

We have thus established that in any Nash equilibrium, $p_N = q_N$. It thus remains to show that $p_N = q_N \in (0,1)$ cannot be part of a Nash equilibrium. Suppose, per contra, that there exists a Nash equilibrium in which $p_N = q_N \in (0,1)$. This implies that $r_N^{LC} = 0$ and $r_N^{CR} = 1$; see (17) above. It is also implied that both candidates must be indifferent between their two available actions. For candidate 1 we get the condition $U_1\left(1, q_N, r_N\right) = U_1\left(0, q_N, r_N\right)$. Using (3), $r_N^{LC} = 0$, $r_N^{CR} = 1$, $q_N = p_N$, and carrying out some algebra, this condition simplifies to

$$
\begin{aligned}
& r_N^{LR}\left[p_N(1-\varepsilon)+\left(1-p_N\right)\varepsilon\right][(1-\varepsilon)(1+a)-\varepsilon] \\
= & \left\{r_N^{CC}\left[\left(1-p_N\right)(1-\varepsilon)+p_N\varepsilon\right]+p_N(1-\varepsilon)+\left(1-p_N\right)\varepsilon\right\}[1-\varepsilon-\varepsilon(1+a)]
\end{aligned} \tag{19}
$$

or $r_N^{LR}=K_0+r_N^{CC}K_1$, where

$$
K_0=\frac{1-\varepsilon-\varepsilon(1+a)}{(1-\varepsilon)(1+a)-\varepsilon}, \qquad K_1=\frac{\left[\left(1-p_N\right)(1-\varepsilon)+p_N\varepsilon\right][1-\varepsilon-\varepsilon(1+a)]}{\left[p_N(1-\varepsilon)+\left(1-p_N\right)\varepsilon\right][(1-\varepsilon)(1+a)-\varepsilon]}.
$$

But this implies that candidate 2 will strictly prefer C to R. To see this, rewrite the condition $U_2(p_N,0,r_N)>U_2(p_N,1,r_N)$ as

$$
\begin{aligned}
& \left\{\left(1-r_N^{CC}\right)\left[\left(1-p_N\right)(1-\varepsilon)+p_N\varepsilon\right]+p_N(1-\varepsilon)+\left(1-p_N\right)\varepsilon\right\}[1-\varepsilon-\varepsilon(1+a)] \\
> & \left(1-r_N^{LR}\right)\left[p_N(1-\varepsilon)+\left(1-p_N\right)\varepsilon\right][(1-\varepsilon)(1+a)-\varepsilon],
\end{aligned} \tag{20}
$$

or $r_N^{LR}>r_N^{CC}K_1+1-K_1-K_0$. Using $r_N^{LR}=K_0+r_N^{CC}K_1$ from above, this inequality simplifies to $K_1>1-2K_0$. It can readily be verified that under the conditions $p_N\in(0,1)$, $\varepsilon\in\left(0,\frac{1}{3}\right]$, and $a\in\left(0,\frac{1}{4}\right)$, $K_1>0$ whereas $1-2K_0<0$. Hence, we have indeed $U_2(p_N,0,r_N)>U_2(p_N,1,r_N)$, implying $q_N=0$. This, however, contradicts the assumption that $q_N\in(0,1)$. □

Proof of Proposition 4 The existence of an equilibrium in which both candidates choose their favorite policies is already proven in the paragraphs preceding Proposition 4. Let us verify that there indeed exists an equilibrium in which both candidates choose L with probability 1. Clearly, for this platform configuration, candidate 1 will win the election (all three voters will vote for her). Hence, candidate 1 will not have an incentive to deviate. Nor does candidate 2 have a (strict) incentive to deviate to any one of the other platforms. If candidate 2 deviated to either R or C, then she would get voter 3's vote but not voter 1's, and voter 2 would not change his voting behavior since he cannot observe the deviation. Hence, candidate 2 cannot change the outcome of the election by deviating.

Let us finally check that there is no other equilibrium in which none of the candidates is randomizing in her choice of platform. There are seven remaining platform configurations to consider: (i) $(x_1,x_2)=(R,R)$, (ii) $(x_1,x_2)=$

(C, L), (iii) $(x_1, x_2) = (C, C)$, (iv) $(x_1, x_2) = (R, C)$, (v) $(x_1, x_2) = (C, R)$, (vi) $(x_1, x_2) = (R, L)$, and (vii) $(x_1, x_2) = (L, C)$. It is a straightforward exercise to verify the following. In case (i)-(iv) candidate 1 wins. In case (i) and (ii), however, candidate 1 has an incentive to deviate to L; in case (iii) and (iv), candidate 2 has an incentive to deviate to R respectively to L. In case (v)-(vii) candidate 2 wins. In case (v) and (vi), candidate 1 has an incentive to deviate to L; in case (vii), candidate 1 has an incentive to deviate to C or R. □

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