Mixed up? That's good for motivation*

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Abstract

An essential ingredient in models of career concerns is ex ante uncertainty about an agent's type. This paper shows how career concerns can arise even in the absence of any such ex ante uncertainty, if the unobservable actions that an agent takes influence his future productivity. By implementing effort in mixed strategies the principal can endogenously induce uncertainty about the agent's ex post productivity and generate reputational incentives. Our main result is that creating such ambiguity can be optimal for the principal, even though this exposes the agent to additional risk and reduces output. This finding demonstrates the importance of mixed strategies in contracting environments with imperfect commitment, which contrasts with standard agency models where implementing mixed strategy actions typically is not optimal if pure strategies are also implementable.

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1 Introduction

To a large extent incentives in organizations arise both from workers' career concerns and from explicit incentive schemes. The seminal paper of Holmström (1982/99) shows that, in the absence of any explicit monetary rewards, a worker faced with a labor market that tries to infer his ability from observations of his past performance has an incentive to provide some effort to influence this updating process. An essential ingredient in extant models of such career concerns is the existence of some ex ante uncertainty about a worker's type (e.g., see Borland (1992)). Gibbons and Murphy (1992) show for such a setting that the combination of career concerns and explicit contracting does not affect the insurance versus incentives trade-off from pure explicit incentives models. Implicit incentives from career concerns are simply a substitute for explicit incentives. Therefore, as in pure explicit incentives models, the firm can only increase expected output at the cost of exposing the worker to more risk and thereby increasing the effort implementation cost. In other words, the intuitions gained from the analysis of single-agent incentive problems seem to carry over to dynamic settings with career concerns.

This paper shows that the above conclusion may not hold in situations with on-the-job human capital acquisition: the firm may optimally choose to strengthen incentives by exposing a risk averse worker to additional risk from uncertainty about his ability and thereby *reduce* his expected output. Moreover, we demonstrate that because the worker's actions affect his future productivity through learning by doing, reputational incentives can arise endogenously: starting from a situation with no ex ante uncertainty about the worker's ability, the firm may optimally design the explicit incentive contract to deliberately create such uncertainty ex post. The key difference to the setting with ex ante uncertainty about a worker's ability is that the market uses output to infer what action the worker has taken (the action affects future productivity through learning by doing and output is not directly informative about productive human capital) rather than using output as a signal of ability (from which the signal-jamming effect of effort needs to be filtered out).

We derive our results in a simple two-period model of human capital acquisition that can serve as a building block in richer settings. In the first period, a principal contracts with an agent, whose productivity is common knowledge, to induce unobservable effort through a spot contract. The agent's effort not only stochastically increases observable output but, through learning by doing, also deterministically raises the agent's unobservable human capital at the end of the first period. For simplicity, in the second period there is no effort decision and production depends only on the agent's human capital. The principal competes with other potential employers, who all update their estimates about the agent's human capital based on the contract signed and the output produced in the prior contractual relation.

The main argument in the paper can be summarized as follows. Suppose first that the principal's contract induces pure strategy effort in equilibrium. Then the agent acquires high human capital through learning by doing with probability one. The market observes the contract of the agent and therefore is certain that he has high human capital, regardless of the first-period output. This leads to a high second-period wage which does not depend on first-period output. Suppose now that the principal's contract induces the agent to exert effort only with probability smaller than one. High first-period output is evidence that the agent actually exerted effort and acquired high human capital, leading to a high second-period wage. In contrast, when observing low first-period output the market cannot be sure whether the agent did or did not exert effort and it assigns some probability to the agent having low human capital. Hence, Bayesian updating by the market leads to a low second-period wage. This creates a wedge between the second-period wage of a successful and of an unsuccessful agent, which provides reputational incentives in the first period. As a consequence, the first-period explicit compensation for high output can be lower than when pure strategy effort is implemented. First, less effort is demanded. Second, the agent has an additional incentive to exert effort to receive a higher second-period wage. Implementing mixed strategy effort is optimal for the principal if the gain from reduced pay outweighs the expected loss in output due to the lower probability of effort provision.

We show that, for the case of a risk-neutral or risk-averse agent, limited liability of the agent is a necessary condition for mixed strategy implementation. Explicit incentives work through the wedge in first-period compensation following high or low output. Since limited liability puts a downward constraint on the level of compensation, the agent earns a rent from the effect of learning by doing on general human capital. Mixed-strategy implementation removes part of this rent in the event of low first-period output and creates reputational incentives which decrease the firm's implementation cost. Absent limited liability the principal can directly lower the level of compensation when implementing pure strategy effort instead of distorting the effort decision. This is optimal with a risk-neutral agent – maximizing output net of the effort cost maximizes the joint surplus – and with a risk-averse agent – the agent is exposed to less risk and expected output is increased. Therefore, limited liability has an effect on incentive provision quite different from that of risk-aversion – which contrasts with standard principal-agent models (e.g., see Laffont and Martimort (2002)).

Extensions of the base model to a pure career concerns setting without explicit contracts and to a setting where the market does not observe contracts are considered. Moreover, we apply the model to analyze how much focus on specific tasks contracts should mandate. Because reputational incentives are affected by an agent's focus, it may be optimal for the principal to be vague on the type of task that the agent should pursue. In addition, we address the issue of optimal screening of job applicants. In a setting where a perfect screening technology is costless we derive conditions under which a principal optimally refrains from fully screening heterogeneous job applicants.

Related literature

In agency models it is typically not optimal for the principal to induce agents to play a mixed strategy if a pure strategy can be implemented.[1] However, several papers on contracting under *asymmetric information* demonstrate that mixed strategies can be optimal when a contractual incompleteness prevents the dynamic contracting problem from collapsing to one that is essentially static. Laffont and Tirole (1988) produce such a result in a setting where a principal with limited commitment power repeatedly contracts with the same agent to create incentives in a moral hazard problem. Contracting is complicated by a ratchet effect[2], and this induces the principal to implement mixed strategies for agents rather than fully revealing pure strategies. Bester and Strausz (2001) extend the revelation mechanism to account for imperfect commitment powers of the mechanism designer and show that, under an optimal mechanism, the agent does not reveal his type with certainty. Our results demonstrate that, even when principal and agent contract under *symmetric information* about the type of the agent, implementation of a mixed rather than a pure strategy can be optimal.

Our paper is also related to Dewatripont, Jewitt, and Tirole (1999a) who show that, in the context of Holmström (1982/99)'s model of pure career concerns, reputational incentives can increase as the signal structure becomes coarser. Similar results obtain when the design of

[1] There exists several papers in which the principal can only implement mixed strategies (e.g., Fudenberg and Tirole (1990) or Khalil (1997)).

[2] If the agent reveals that he is a low cost type he will face a tougher incentive scheme in the next period than if he reveals to be a high cost type.

explicit incentives interact with career concerns (Koch and Peyrache (2005a, 2005b)) or with ratchet effects (Meyer and Vickers 1997, Auriol, Friebel, and Pechlivanos 2002). A new contribution of our paper is that it shows how a principal can design explicit incentive schemes to create reputational incentives through ambiguity about an agent's future productive value, even when agents are ex ante homogeneous. The picture that emerges from this literature is that explicit and implicit incentives often affect each other in more complicated ways than just being substitutes.

The paper is organized as follows. Section 2 introduces the model. The limited liability case is then analyzed in Section 3. Section 4 treats the case with unlimited liability. In Section 5 we discuss the robustness of our findings and illustrate how the base model can be reinterpreted and applied to a multi-task setting as well as to determine the optimal amount of screening of job applicants. Section 6 concludes.

2 Model

We consider a two-period model. In the first period, an employee (the agent), whose ability is common knowledge, contracts with a risk-neutral firm (the principal) over the production of output $\tilde{y} \in \{\underline{y}, \bar{y}\}$. In the absence of effort (e=0), the agent produces $\underline{y}$ with probability one and accumulates human capital $\underline{H}$. By exerting unobservable effort e=1 at private cost ψ, he produces $\bar{y} \equiv \underline{y} + \Delta\, y$ with probability $\pi \in (0,1)$ and $\underline{y}$ with probability $1-\pi$. In addition, he then acquires human capital $\bar{H} \equiv \underline{H} + \Delta\, H$. A fraction α of this human capital is firm specific, whereas the rest is general. If he does not accept the job the agent faces an outside option that offers a life-time utility of zero.

Neither the agent's effort nor his human capital are observed. However, the contract signed with the agent is publicly observable, and at the end of the first period all the parties (the agent, the principal and other potential employers) get to know the agent's level of output[3]. For simplicity, in the second period there is no effort decision and production depends only on the agent's human capital. As a consequence, in the second period, the first principal ("the incumbent" denoted by I) competes à la Bertrand with other principals for the agent under symmetric information. If the agent stays with principal I, he produces output equal to the level of total human capital H accumulated in the first period. In contrast, when switching

[3]For an analysis where the principal can decide on what type of performance information to reveal to the market and the repercussions that this has on the design of incentive schemes see Koch and Peyrache (2005b).

to a new principal, the agent can only produce $(1-\alpha)H$. Thus, it is straightforward that in the equilibrium of the second-period continuation game, the agent stays with principal I, who matches outside offers. That is, the agent earns a second-period wage t_2 equal to his expected general human capital conditional on the contract and on realized output in the first period:[4]

$$t_2(\tilde{y}) = (1-\alpha)\, E\left[H|\tilde{y}_1, contract\right]. \tag{1}$$

As is standard in models of career concerns, we assume that principal I can only offer the agent a contract that makes first-period transfers t_1 contingent on output in that period (spot contract).[5] That is, the contract offered is a wage pair $c = \left[t_1\left(\underline{y}\right), t_1\left(\bar{y}\right)\right]$.

The agent is assumed to be risk-neutral or risk averse with preferences represented by the following time-separable utility function over first- and second-period transfers:

$$U(e, \tilde{y}) = u(t_1(\tilde{y})) + u(t_2(\tilde{y})) - e \cdot \psi, \tag{2}$$

where $u : I\!R \rightarrow I\!R$, $u(0) = 0$, $u' > 0$ and $u'' \leq 0$. Finally, let $g \equiv u^{-1}$.

To summarize, the timing of the game is as follows. Principal I offers an (observable) contract $c = \left[t_1\left(\underline{y}\right), t_1\left(\bar{y}\right)\right]$. Each possible contract c induces a subgame in which the agent first chooses to accept or refuse the contract and then, if he accepts, selects a (mixed) strategy $p \in [0,1]$. Nature then realizes the (unobservable) effort $e \in \{0,1\}$ and the (observable) output $\tilde{y} \in \{\underline{y}, \bar{y}\}$. The second-period labor market ensues. The principal I and other principals simultaneously make wage offers among which the agent selects the maximum offer.

3 Limited liability

We first solve for perfect Bayesian equilibria assuming that the agent is protected by limited liability, i.e., $t_\tau \geq 0$, $\tau = 1, 2$. We start by considering candidate equilibria where the contract implements pure strategy effort and where the contract implements a mixed strategy $p \in (0,1)$. We then prove that each contractual offer by principal I induces a subgame with a unique continuation equilibrium. Therefore, the design of contract in the first stage reduces to a

[4]Our results go through under any formulation in which the second-period rent accruing to the agent is increasing in his expected human capital.

[5]Typically, in labor markets parties lack full pre-commitment power (e.g., workers cannot cede their right to revoke a contract because slavery is forbidden). If the parties had such powers the dynamic incentive problem would essentially collapse to a static one in which reputational incentives do not matter.

straightforward maximization problem using the payoffs from the candidate equilibria initially considered.

3.1 Implementing effort in pure strategies

Consider the contract c that implements effort at the lowest cost in the static incentive problem corresponding to the first-period in our model. It involves first-period wages that induce first-period utilities $\bar{u}_1^c \equiv u(t_1^c(\bar{y}))$ and $\underline{u}_1^c \equiv u(t_1^c(\underline{y}))$ which satisfy the following static incentive and individual rationality constraints:

$$\pi\, \bar{u}_1^c + (1-\pi)\, \underline{u}_1^c - \psi \;\; \geq \;\; \underline{u}_1^c, \tag{3}$$

$$\pi\, \bar{u}_1^c + (1-\pi)\, \underline{u}_1^c - \psi \;\; \geq \;\; 0. \tag{4}$$

The incentive constraint is binding. Moreover, the individual rationality constraint binds without violating limited liability since $u(0) = 0$. First-period utilities $\underline{u}_1^c = 0$ and $\bar{u}_1^c = \frac{\psi}{\pi}$ are achieved by transfers $\underline{t}^P = g\left(\underline{u}_1^c\right) = 0$ and $\bar{t}^P = g\left(\bar{u}_1^c\right) = g\left(\frac{\psi}{\pi}\right)$. The superscript P refers to pure strategy implementation, and is used later in comparisons with mixed strategy implementation, carrying superscript M.

Suppose that by offering contract c the principal I induces a subgame where market beliefs are that the agent exerts effort with probability one in the first period, leading to second-period utilities

$$u_2^P \equiv u(t_2(\bar{y})) = u(t_2(\underline{y})) = u\left((1-\alpha)\,\bar{H}\right). \tag{5}$$

If the agent correctly anticipates this when taking his effort decision the incentive and individual rationality constraints are:

$$\pi\, \bar{u}_1^c + (1-\pi)\, \underline{u}_1^c - \psi + u_2^P \;\; \geq \;\; \underline{u}_1^c + u_2^P, \tag{IC:P}$$

$$\pi\, \bar{u}_1^c + (1-\pi)\, \underline{u}_1^c - \psi + u_2^P \;\; \geq \;\; 0, \tag{IR:P}$$

The second-period utility is constant so (IC:P) is equivalent to the static incentive constraint and binds. However, the individual rationality constraint (IR:P) is slack because $u_2^P > 0$. In fact, limited liability prevents reducing $t_1\left(\underline{y}\right)$ below zero, so (IR:P) cannot bind when (IC:P) is satisfied.

Under contract $c = \left[\underline{t}^P = 0, \bar{t}^P = g\left(\frac{\psi}{\pi}\right)\right]$ the principal's expected profit in the subgame equilibrium is:

$$\Pi^P = \underline{y} + \pi\left[\Delta\, y - g\left(\frac{\psi}{\pi}\right)\right] + \alpha \bar{H}. \tag{6}$$

Since pure strategy implementation will be our benchmark case we assume that the gain in output for the principal, Δy, exceeds the monetary transfer to the agent that is necessary to induce pure strategy effort, $g\left(\frac{\psi}{\pi}\right)$:

Assumption 1 $\qquad \Delta y > g\left(\frac{\psi}{\pi}\right).$

3.2 Implementing effort in mixed strategies

Suppose now that in the subgame induced by some contract c the market's beliefs are that the agent exerts effort with probability p and no effort otherwise. Then, the market's expectation of the agent's level of human capital becomes a function of the first-period output $\tilde{y}$. Correctly anticipating these beliefs, the agent faces one of the following second-period utilities, depending on the realized output:

$$\bar{u}_2^M \equiv u(t_2^M(\bar{y})) = u\left((1-\alpha)\,\bar{H}\right), \tag{7}$$

$$\underline{u}_2^M(p) \equiv u(t_2^M(\underline{y})) = u\left((1-\alpha)\left[\underline{H} + \frac{p\,(1-\pi)}{1-p\,\pi}\,\Delta H\right]\right). \tag{8}$$

Note that the dependence on p was dropped for the function that is constant in p to make the exposition clearer. Given these beliefs, to indeed implement mixed strategy p the contract c must be incentive compatible, i.e., satisfy

$$\pi\,\left(\bar{u}_1^c(p) + \bar{u}_2^M\right) + (1-\pi)\left(\underline{u}_1^c + \underline{u}_2^M(p)\right) - \psi = \underline{u}_1^c + \underline{u}_2^M(p). \tag{IC:M}$$

This incentive constraint implies that the individual rationality constraint,

$$\underline{u}_1^c + \underline{u}_2^M(p) \geq 0, \tag{IR:M}$$

is also satisfied. As before, the principal's contract can set $\underline{u}_1^c = 0$ so that the limited liability constraint binds and thus $\underline{t}^M = g(0) = 0$. From (IC:M) and the limited liability constraint we then obtain

$$\bar{u}_1^c(p) = \max\left\{\frac{\psi}{\pi} - \left(\bar{u}_2^M - \underline{u}_2^M(p)\right),\, 0\right\}. \tag{9}$$

The required monetary transfers are lower than with pure strategy implementation: for all $p \in (0,1)$ we have $\underline{t}^M = \underline{t}^P = 0$ and $0 \leq \bar{t}^M(p) = g\left(\bar{u}_1^c(p)\right) < \bar{t}^P$. For our argument it is sufficient to focus on the case where $\bar{t}^M(p) = g\left(\frac{\psi}{\pi} - \left(\bar{u}_2^M - \underline{u}_2^M(p)\right)\right)$ does not violate the limited liability constraint for any $p \in (0,1)$ and $\alpha \in (0,1)$.[6] Since $\bar{u}_2^M \leq u\left(\bar{H}\right)$ and

[6] Note that this is the least favorable case for mixed strategy implementation. Hence, if here mixed strategy implementation is optimal, a fortiori, it will be in the other case as well.

$\underline{u}_2^M(p) > u(\underline{H})$ for any $p \in (0,1)$ the following condition is sufficient to ensure that $\bar{t}^M(p) > 0$ for all possible values (α, p):

Assumption 2 $\quad \frac{\psi}{\pi} \geq u\left(\bar{H}\right) - u\left(\underline{H}\right).$

The principal's expected profit for such a contract implementing $p \in (0,1)$ is then given by:

$$\Pi^M(p) = \underline{y} + p\,\pi\,\left[\Delta\,y - g\left(\bar{u}_1^M(p)\right)\right] + \alpha\left[p\,\bar{H} + (1-p)\,\underline{H}\right]. \tag{10}$$

Our model incorporates on-the-job human capital acquisition. Therefore, whenever the market cannot perfectly infer the agent's effort level from the contract offered to the agent, it has to rely on the realized output as a signal for the effort actually exerted. A high level of output indicates that effort was exerted and that high human capital was acquired by the agent (cf. equation (7)). In contrast, a low level of output is bad news about human capital since the market is no longer certain that the agent exerted effort (cf. equation (8)). This creates a reputational wedge $\bar{u}_2^M - \underline{u}_2^M(p)$ that principal I can exploit to lower monetary transfers. Therefore, incentives for effort arise even under rather low-powered monetary incentive schemes.

3.3 The optimal contract

The preceding sections focused on particular subgame equilibria. To characterize perfect Bayesian equilibria of the overall game we proceed in two steps. First we show that each subgame induced by a contract choice has a unique equilibrium. This then allows working backwards to set up a well-defined maximization problem which determines the optimal contract choice in the equilibrium of the overall game.

Lemma 1

Each contract choice c induces a subgame with a unique equilibrium.

The proof is relegated to the appendix. The result allows us to work backwards by first determining the expected profit for principal I that each contract choice induces in the ensuing subgame and then finding the optimal contract by simply maximizing the expected profit. Among the contracts that induce pure strategy effort the one discussed in Section 3.1 offers the maximum expected profit: for any other contract satisfying (IC:P) and (IR:P) the limited liability constraint is slack. Similarly, for each mixed strategy $p \in (0,1)$ Section 3.2 characterizes the profit maximizing contract. Hence, the maximization problem of principal I simplifies to a comparison of these contracts only. Her gain from implementing mixed strategy $p \in (0,1)$ instead of pure strategy $p = 1$ is given by

$$G(p) \equiv \Pi^M(p) - \Pi^P = \underbrace{\pi \left[g\left(\frac{\psi}{\pi}\right) - p\, \bar{t}^M(p) \right]}_{\text{saved implementation cost}} - \underbrace{(1-p)\, [\pi\, \Delta\, y + \alpha \Delta\, H]}_{\text{loss in expected output}}. \tag{11}$$

The principal implements effort in mixed strategies whenever there exists a probability $p \in (0,1)$ such that the saved expected implementation cost outweighs the expected loss in output relative to pure strategy effort. The following result gives a sufficient condition under which a contract that implements a mixed strategy is optimal:

Proposition 1

Under limited liability of the agent, a sufficient condition for the equilibrium contract to involve a mixed strategy $p^* \in (0,1)$ is

$$\underbrace{\pi \left(\Delta\, y - g\left(\frac{\psi}{\pi}\right)\right) + \alpha \Delta H}_{\textit{marginal loss in output for } p \uparrow 1} < \underbrace{\pi (1-\alpha)\, Z \frac{\Delta H}{1-\pi}}_{\textit{marginal gain in cost reduction for } p \uparrow 1} \tag{12}$$

where $Z \equiv g'\left(\frac{\psi}{\pi}\right) u'\left((1-\alpha)\, \bar{H}\right)$.

Proof. Taking the derivative of $G(p)$ with respect to p, we get

$$\begin{aligned} \frac{d\, G(p)}{d\, p} &= \pi\, \Delta\, y + \alpha\, \Delta\, H \\ &- \pi\, g\left(\bar{u}_1^M(p)\right) - p\, \pi\, g'\left(\bar{u}_1^M(p)\right)\, u'\left((1-\alpha)\, \left[\underline{H} + \frac{p\,(1-\pi)}{1-p\,\pi}\, \Delta\, H\right]\right)\, (1-\alpha) \frac{1-\pi}{(1-p\,\pi)^2}\, \Delta\, H. \end{aligned} \tag{13}$$

Given that $G(0) = \pi \left(g\left(\frac{\psi}{\pi}\right) - \Delta\, y\right) - \alpha\, \Delta\, H < 0$ (by Assumption 1) and $G(1) = 0$, a sufficient condition for an interior solution is that $\left.\frac{d\, G(p)}{d\, p}\right|_{p=1} < 0$. ■

The result demonstrates that moving towards less powerful monetary incentives can be beneficial for the principal. The probability with which the agent is going to exert effort is determined by the trade-off between inducing provision of effort in the first period and extracting rents from the second period employment relation.

To understand the forces driving this trade-off, it is useful to first consider the comparative statics of condition (12). The larger the difference in first-period outputs $\Delta\, y$ and the firm-specific component of human capital α, the less likely that the principal implements mixed strategy effort. When $\alpha = 1$, human capital is fully firm specific and the agent earns nothing in the second period. Consequently, there are no reputational incentives because the agent's second-period utility does not depend on first-period actions. Therefore, the optimal contract implements pure strategy effort. The other polar case is when all the human capital is general, i.e., $\alpha = 0$. Then the impact of first-period actions on the agent's second-period utility is maximal since he earns the entire return to the human capital acquired in the first period. The condition for the optimality of a mixed strategy contract becomes: $\Delta\, y - g\left(\frac{\psi}{\pi}\right) < g'\left(\frac{\psi}{\pi}\right)\, u'\left(\bar{H}\right)\, \frac{\Delta H}{(1-\pi)}$. Given that $u(\cdot)$ and $g(\cdot)$ are increasing, clearly the set of values of Δy that satisfy both Assumption 1 and condition (12) is non-empty. In contrast, the effect of an increase in the human capital return to effort, ΔH, is ambiguous. An increase in ΔH unambiguously raises the expected loss in firm-specific human capital that the principal incurs by not implementing pure strategy effort. However, the magnitude of the cost saving is larger the lower the value of α and the less risk averse the agent is. Therefore, for low firm-specific human capital α and low risk aversion, mixed strategy implementation is more likely. Indeed, for $\alpha = 0$ and $u''(\cdot) = 0$, an increase of ΔH has no effect on the marginal loss (left-hand side of (12))) but has a large positive effect on the marginal gain in cost reduction (right-hand side of (12)). In contrast, for $\alpha = 1$ the cost saving effect is dominated and mixed strategy implementation becomes less likely.

In our framework, the agent earns a limited liability rent because the principal cannot make him pay for the increase in future earnings due to accumulated human capital. This rent is $u\left((1-\alpha)\,\bar{H}\right)$ in the case of pure strategy implementation and $u\left((1-\alpha)\,\left[\underline{H} + \frac{p\,(1-\pi)}{1-p\,\pi}\,\Delta\, H\right]\right)$ in the case of mixed strategy implementation. Following our analysis above, optimal contracts only involve mixed strategy effort if the limited liability rent is strictly positive, i.e., human capital is not fully specific ($\alpha < 1$). In such a setting, due to competition in the second-period labor market, the agent captures part of the productivity gains from learning by doing because limited liability prevents the principal from fully extracting rents in the first period. Remarkably, Proposition 1 shows that, for appropriate values of the parameters, the principal exposes a risk averse agent to more risk than in a static explicit incentive model: the lottery over transfers faced by an agent under a contract that implements pure strategy effort (which corresponds to the static moral hazard contract) first-order stochastically dominates the one faced under a contract with mixed strategy effort. Such exposure to risk creates uncertainty about the agent's second-period productivity and generates reputational incentives that reduce the limited liability rent. The uncertainty about the acquired human capital induced by mixed strategy effort provision is a means to transform a homogeneous group of unexperienced agents into a heterogeneous group of experienced agents for whom reputational incentives exist.[7]

An interesting feature of Proposition 1 is that the principal faces a trade-off between the riskiness of output and effort incentives, which to our knowledge is a new result in a model with explicit incentives and risk averse agents.

While mixed strategy implementation may enhance the principal's profit, it always reduces welfare compared to pure strategy implementation. Overall, mixed strategy effort decreases welfare by lowering the probability of high first-period output[8] and reducing second-period output because less human capital is accumulated. The principal can profitably create such a distortion since, when implementing mixed strategies, she does not internalize the second-period utility loss $u\left((1-\alpha)\,\bar{H}\right) - u\left((1-\alpha)\,\left[\underline{H} + \frac{p\,(1-\pi)}{1-p\,\pi}\,\Delta\,H\right]\right)$ that the agent is subject to following a low state realization in the first period.

These results contrast with those obtained when one adds explicit contracts to the Holmström (1982/99) career concerns setting. There, career concerns arise independent of the effort level that the principal implements and therefore increase the equilibrium effort relative to the pure explicit incentives case by lowering the monetary cost of providing incentives. In our setting, there is an unambiguous drop in effort when the principal introduces career concerns.

4 Unlimited liability

In the absence of limited liability, the principal can implement pure strategy effort and extract the agent's gain in future earnings due to accumulated human capital through transfers, by setting $\bar{u}_1^P = \frac{\psi}{\pi} - u\left((1-\alpha)\,\bar{H}\right)$ and $\underline{u}_1^P = -u\left((1-\alpha)\,\bar{H}\right)$. This yields expected profit:

$$\hat{\Pi}^P = \underline{y} + \pi\,\left[\Delta\,y - g\left(\bar{u}_1^P\right)\right] - (1-\pi)\,g\left(-u\left((1-\alpha)\,\bar{H}\right)\right) + \alpha\,\bar{H}. \tag{14}$$

In contrast, under a contract that implements a mixed strategy p the principal sets utility levels $\bar{u}_1^M = \bar{u}_1^P = \frac{\psi}{\pi} - u\left((1-\alpha)\,\bar{H}\right)$ and $\underline{u}_1^M(p) = -\underline{u}_2^M(p) = -u\left((1-\alpha)\,\left[\underline{H} + \frac{p\,(1-\pi)}{1-p\,\pi}\,\Delta\,H\right]\right)$, yielding expected profit

$$\hat{\Pi}^M(p) = \underline{y} + p\,\pi\,\left[\Delta\,y - g\left(\bar{u}_1^P\right)\right] - (1-p\,\pi)\,g\left(\underline{u}_1^M(p)\right) + \alpha\,\left[p\,\bar{H} + (1-p)\,\underline{H}\right]. \tag{15}$$

Implementing a mixed strategy instead of a pure strategy has two effects. First, since $0 > g\left(\underline{u}_1^M(p)\right) > g\left(\underline{u}_1^P\right)$, the payment received by the principal in the low-output state decreases

[7] Creating ambiguity about agents' types might be optimal for other reasons as well if there are heterogeneous unexperienced agents (e.g., Koch and Peyrache ((2005a),(2005b))).

[8] Recall that it is efficient for the agent to exert effort (by Assumption 1).

(note that the payoff in the high-output state does not change). Second, the probability of a high- (low-)output state decreases (increases). The overall effect unambiguously decreases the principal's profit if the payoff from pure strategy implementation received by the principal in the high-output state exceeds that received in the low-output state, i.e., if

$$\Delta y - \underbrace{g\left(\frac{\psi}{\pi} - u\left((1-\alpha)\,\bar{H}\right)\right)}_{\equiv g\left(\bar{u}_1^P\right)} > -\underbrace{g\left(-u\left((1-\alpha)\,\bar{H}\right)\right)}_{\equiv g\left(\underline{u}_1^P\right)}. \tag{16}$$

Under this condition the equilibrium contract always implements effort in pure strategies. This allows us to prove the following result:

Proposition 2

Under unlimited liability of the agent, the principal always induces effort in pure strategies.

Proof. The proof is by contradiction. Suppose that condition (16) is violated (which is a necessary condition for mixed strategy implementation to be optimal):

$$-g\left(-u\left((1-\alpha)\,\bar{H}\right)\right) \geq \Delta y - g\left(\frac{\psi}{\pi} - u\left((1-\alpha)\,\bar{H}\right)\right) \tag{17}$$

$$> g\left(\frac{\psi}{\pi}\right) - g\left(\frac{\psi}{\pi} - u\left((1-\alpha)\,\bar{H}\right)\right), \tag{18}$$

where the last relation follows from Assumption 1. Rewriting the last expression we get

$$g\left(\frac{\psi}{\pi}\right) - g\left(\frac{\psi}{\pi} - u\left((1-\alpha)\,\bar{H}\right)\right) = \int_{\frac{\psi}{\pi}-u\left((1-\alpha)\,\bar{H}\right)}^{\frac{\psi}{\pi}} g'(q)\,dq \tag{19}$$

$$\geq \int_{-u\left((1-\alpha)\,\bar{H}\right)}^{0} g'(q)dq = -g\left(-u\left((1-\alpha)\,\bar{H}\right)\right),$$

since $g''(x) \geq 0 \;\forall x \in I\!R$. This leads to a contradiction. ■

The intuition for the result is clear when the agent is risk neutral. In the absence of limited liability, the principal can extract the entire rents accruing to the agent in the second period, e.g., by selling the project to the agent and making him the residual claimant. Pure strategy effort is optimal because it maximizes the surplus accruing from first-period production and human capital acquisition.

In the case of a risk averse agent, the principal faces a trade-off between incentives and insurance when offering the agent a contract. However, the nature of this trade-off is different from the standard one in static models since the contract affects incentives indirectly through market beliefs about the level of effort that the agent exerted and, thus, determines the agent's wage in the second period. Our result confirms the intuition that the principal does not expose a risk averse agent to the additional risk associated with mixed strategy effort when she can make him pay for acquired human capital.[9]

5 Extensions and alternative interpretations

In the previous sections we derived conditions under which a principal benefits from implementing mixed strategy effort and thereby creating ambiguity about the actions that an agent has taken. We now consider the robustness of this result in two extensions of the base model: a pure career concerns setting without explicit contracts, and a setting where contracts are not observable by outside parties. Then, to illustrate how the base model can be used as a building block in applications, we present two simple extensions that analyze the issue of optimal mission focus and optimal screening of job seekers. The latter extension demonstrates that our main insights do not necessarily require mixing over actions by the agent.

5.1 Pure career concerns

Consider the *pure* career concerns setting à la Holmström (1982/99) where the parties cannot write output contingent contracts. Since non-contingent wages have no impact on incentives and the agent is protected by limited liability, first-period wages are optimally set equal to zero. Clearly, pure strategy effort $p = 1$ cannot be an equilibrium. Given market beliefs that $p = 1$, second-period wages do not respond to first-period output so that the agent's best reply is $p = 0$.

If market beliefs are that the agent exerts no effort or mixed strategy effort, the resulting second period-utilities are given by equations (7) and (8) from Section 3.2. A pure strategy no-effort equilibrium ($p = 0$) exists if and only if the agent's incentive constraint for $p = 0$ is indeed satisfied when the market holds beliefs that $p = 0$:

$$u\left((1-\alpha)\,\bar{H}\right) - \underline{u}_2(p=0) \leq \frac{\psi}{\pi}. \tag{20}$$

Since $\underline{u}_2(p)$ is increasing in p the equilibrium then is unique. Note that

[9] However, mixed strategy implementation can be optimal if the agent's utility function has a strictly convex part. Examples can easily be constructed for the case where there exists a minimum utility level so that $u(x) \rightarrow u_{min}$ for $x \rightarrow -\infty$, or when the utility function is convex in losses and concave in gains (e.g., as in prospect theory (Kahneman and Tversky 1979)).

$\left[u\left(\left(1-\alpha\right)\bar{H}\right)-\underline{u}_2(p)\right]\rightarrow 0$ for $p\rightarrow 1$. Thus, if condition (20) does not hold, there exists a unique mixed strategy $p\in(0,1)$ sustained by corresponding market beliefs such that

$$u\left(\left(1-\alpha\right)\bar{H}\right)-\underline{u}_2(p)=\frac{\psi}{\pi}. \tag{21}$$

This leads to the following result.

Proposition 3

If parties cannot write output-contingent contracts (pure career concerns setting), there exists no pure strategy effort equilibrium ($p=1$). A pure strategy no-effort equilibrium ($p=0$) arises for parameter values satisfying condition (20). Otherwise, the equilibrium involves a mixed strategy $p\in(0,1)$.

The result demonstrates that mixed strategy effort is not an artefact of the explicit contracts setting. Career concerns can arise endogenously and sustain mixed strategy effort in equilibrium because then different levels of output are not equally informative about the agent's effort level and thereby about the human capital acquired by the agent. In contrast, pure strategy effort is only possible with direct monetary incentives since it eliminates uncertainty about human capital and thereby decouples future wages from the current level of output produced. The difference from the career concerns mechanism in Holmström (1982/99) is that output is not influenced by ability or any other direct measure of the agent's future productivity. As a consequence differences in future wages do not arise because output is a signal about the agent's ability *level* (from which the market needs to filter out the effort exerted by the agent to "jam" this signal) but rather the market tries to learn about the *change* in the agent's human capital due to learning by doing (by inferring from output what effort level was exerted). Therefore, the two models have different implications about when it is likely that career concerns induce effort if there are no explicit incentives. In a signal-jamming setting à la Holmström (1982/99) incentives are stronger the more ex ante uncertainty there is about the level of the agent's human capital. In our setting, incentives are stronger the bigger the impact of effort on human capital and the less informative low levels of output are about the agent's effort level for any $\alpha<1$ (Holmström (1982/99) considers the case of fully general human capital, $\alpha=0$).

5.2 Unobservable contracts

Sections 3 and 5.1 demonstrated that mixed strategy equilibria arise in the two cases where output-contingent contracts are impossible and when output-contingent contracts are observable. We now briefly discuss[10] the intermediate case where output-contingent contracts are feasible but not observable by other outside parties. Since the principal can only credibly commit to output-contingent wages when contracts are verifiable and enforceable by a court, either the contract is hard evidence per se that can be directly revealed to outside parties, or it is possible to send messages to outsiders about the contract that can be verified by the court which serves as contract enforcer. Thus, without loss of generality, we will assume that contracts are hard evidence. We modify the timing of the base model by introducing a stage after (observable) output realizes where each party can unilaterally decide whether or not to disclose the contractual arrangement to the market before the second-period labor market begins. In such a game, the market holds a set of beliefs about the agent's effort conditional on a contract being disclosed as well as a belief if no contract is disclosed. Since high first-period output is evidence that the agent has accumulated high human capital the second-period wage is again pinned down by equation (7). Following low-output, one of the contracting parties will always disclose the true contract. The agent discloses if this leads the market to assign a higher probability on effort having been exerted than if no contract is disclosed, because this increases his second-period wage. The principal discloses if this leads the market to assign a lower probability on effort having been exerted than if no contract is disclosed, because this decreases the competing wage offers in the second-period labor market. It is therefore easy to show that pure and mixed strategy equilibria can be constructed as in the base model (now with the need to specify a suitable set of off-the-equilibrium path beliefs).

5.3 Multiple tasks

In this section we allow for N different tasks, which yield output $\tilde{y}_i \in \{\underline{y}, \bar{y}\},\ i = 1, ..., N$ according to the same production technology as in the base model. Through learning by doing in task i the agent accumulates task-specific human capital H_i that is fully transferable to other firms interested in this type of human capital. The marketability of task-specific human capital depends on market forces that are beyond control of the firm and is uncertain. To capture this, we assume that the value of human capital in task i in the second-period

[10] A formal derivation of the results is available from the authors.

labor market is $H_i = \bar{H}\, x_i$, where x_i is independently distributed according to the uniform distribution: $x_i \sim U[0,1]$, $i = 1, ..., N$. For simplicity, assume that the principal cares only about the sum of outputs from these tasks in the first period:

$$\tilde{Y} = \sum_{i=1}^{N} \tilde{y}_i. \tag{22}$$

For illustrative purposes, we will restrict attention to the simple case where the agent can exert effort only in a single task. If the principal wants to induce effort in a specific task i, she offers a contract that defines a *clear mission* by implementing pure strategy effort in this *specific* task. That is, transfers would be $t_1(\underline{y}_j) = t_1(\bar{y}_j) = 0$ for $j \in \{1, ..., N\}$, $j \neq i$, $t_1(\underline{y}_i) = 0$ and $t_1(\bar{y}_i) = g\left(\frac{\psi}{\pi}\right)$. In contrast, if the principal wants to create ambiguity about the agent's human capital she offers an n-task *fuzzy mission* contract that rewards the agent based on the sum of outputs over $n \leq N$ tasks. For example, if the principal wants to implement effort in one of the first n tasks in the limited liability case, transfers would be $t_1 = \begin{cases} g\left(\frac{\psi}{\pi}\right) & \text{if } \sum_{i=1}^{n} y_i = \bar{y} \\ 0 & \text{if } \sum_{i=1}^{n} y_i = 0 \end{cases}$. Under such a contract the market only imperfectly learns whether effort was exerted on a specific task or not. If it observes $y_i = \bar{y}$ for some task i, it is clear that the agent exerted effort on this task and acquired human capital $\tilde{H}_i$. Thus, ex ante expected second-period human capital in the case of high output is $\bar{H}/2$. In contrast, if the market observes $y_i = 0$, for all $i = 1, ..., N$, the market does not know which task was actually pursued and attributes probability $1/n$ to each task $i = 1, ..., n$ and probability zero to each task $i = n+1, ..., N$. The agent's best wage offer thus will be

$$\max_{i \in \{1,...,n\}} \frac{\tilde{H}_i}{n}. \tag{23}$$

Taking expectations, ex ante expected second-period human capital in the case of low output is $\underline{t}_2(n) = \frac{n}{n+1}\,\frac{\bar{H}}{n}$.[11] Thus, under an n-task fuzzy mission contract the wedge between high and low output states in terms of expected reputation is given by $\frac{(n-1)}{2(n+1)}\,\bar{H}$. Since the expected output for the principal under both types of contracts is the same, and the reputational wedge is increasing in n, it is optimal for her to set $n = N$. Thus, it is optimal for the principal to give the agent complete autonomy of decision over the production process.

Note that in this simple example focus does not matter for production and therefore it is optimal to maximize reputational incentives. The framework can easily be extended to allow

[11] This uses the fact that the k-th order statistic in a sample of n observations of the uniform distribution on $[0,1]$ follows a Beta distribution with parameters k and $n-k+1$.

for firm-specific human capital and productive gains from focus. Nevertheless, the finding that fuzzy mission contracts can dominate those that implement a clear mission is interesting, since it contrasts with the result that obtains in the pure career concerns setting of Dewatripont, Jewitt, and Tirole (1999b), in which there is no learning by doing. In their model, under some regularity conditions, a principal always prefers clear missions.

5.4 Screening of job seekers

In this section we address the issue of optimal screening of job seekers by extending the base model to allow for ex ante uncertainty about agents' types. In the first period, the principal has a vacant position to be filled and can hire from a pool of unexperienced job seekers. These start off with an initial level of human capital normalized to zero and are protected by limited liability. Once hired by the firm, an agent who does not exert effort produces low output $\underline{y}$ and acquires human capital $\underline{H}$ in the first period. A proportion λ of job seekers is *high-skilled*, and can additionally exert unobservable effort at a private cost ψ to increase their unobservable human capital from $\underline{H}$ to $\bar{H}$ through learning by doing. In that case, they produce high output $\bar{y}$ with probability π. The firm has access to a screening technology which provides an informative signal regarding the types of job applicants with probability q. To set the most favorable conditions for perfect screening, assume that the principal can choose any probability $q \in [0, 1]$ at no cost. The choice of screening precision is observable by the market.

If the principal adopts the perfect screening technology $q = 1$, she can distinguish job seekers' abilities and hires a high-skilled agent. In equilibrium, the market correctly anticipates this hiring decision and the incentive problem corresponds exactly to the one in Section 3.1. Since the agent is protected by limited liability, he receives a transfer $g(\frac{\psi}{\pi})$ if he produces high output and 0 otherwise. In contrast, if the principal adopts an imperfect screening technology with $q < 1$, she ends up hiring a high-skilled agent with probability $q + \lambda(1 - q)$ and a low-skilled agent with probability $(1 - \lambda)(1 - q)$. Then, high output provides an agent with a marketable signal that reveals him to be of high ability and increases his second-period earnings compared to the situation in which he produces low output and his human capital is uncertain. The derivation of the optimal screening precision q is similar to the analysis in Section 3.2, replacing p by $q + \lambda(1 - q)$. Thus, we obtain the following corollary to Proposition 1:

Corollary 1

If agents are protected by limited liability and the principal has access to a costless screening technology, then a sufficient condition for imperfect screening (screening precision $q < 1$) is given by condition (12), replacing p by $q + \lambda(1 - q)$.

An interesting implication of our result is that the contracts offered by the firm in the first period are not type contingent. When screening is imperfect, reputational incentives arise only because contracts do not resolve the ambiguity about the agent's type. This provides a rationale for the phenomenon that wages often are less sensitive to differences in individual characteristics than predicted by incentive theory (e.g., Baker, Gibbs, and Holmström (1994)).

Our result that a firm might choose not to perfectly screen job seekers even if perfect screening is costless is reminiscent of Crémer (1995). He also shows that a principal might optimally choose not to acquire information about an agent. The rationale is however different from ours. In Crémer (1995), if the principal were to acquire information about the agent she would choose to renegotiate and retain an agent who turns out to be of high ability, despite low performance. Anticipating this, high-ability agents would exert no effort. Thus, choosing to remain ignorant about an agent's type allows the principal to credibly dissociate incentives for effort from the desire to retain high-skilled agents. Stated differently, by committing not to acquire information on the agent's true ability, the principal can cut the wage bill. This is also the effect that we highlight in our paper. Nevertheless, our rationale for limiting the amount of information that is acquired is different. Committing to an imperfect screening technology allows the principal to make the beliefs of the market regarding the agent's human capital responsive to the output produced. This provides a high-skilled agent with reputational incentives and permits the principal to cut back on monetary incentives for effort. In sum, although the benefit side of not acquiring information is the same in both models – a lower wage bill – the cost side is different: in Crémer (1995) the principal takes the risk of firing (unlucky) high-ability agents; in our setting, there is a decrease in expected output.

Also related is Fingleton and Raith's (2005) model where the principal voluntarily limits the amount of information she acquires to affect agents' mixed strategy choices. A buyer and a seller (the principals) each delegate the bargaining to agents who are solely motivated by career concerns. Two different informational settings are compared: an "open-door" process where principals observe both the actions of agents and the bargaining outcome, and a "closed-door" process where they only observe the outcome. High-skilled agents can assess the other party's reservation price and always use this information. In contrast, low-skilled agents do not have this information and engage in inefficient randomization between offers in the open-door process to mimic high-skilled types. To reduce such inefficiencies, it can be optimal for a principal to implement the less informative closed-door bargaining process.

6 Conclusion

We show that in a sequential contracting model with moral hazard and learning by doing, a principal can benefit from implementing mixed strategy effort rather than pure strategy effort when agents are protected by limited liability. Mixed strategy effort provision gives rise to reputational incentives that lower the implementation cost for the principal. If these savings exceed the expected loss in output due to lower effort, the principal implements a contract that induces mixed strategy effort provision. Moreover, we explore the robustness of this finding and demonstrate how our base model can be used as a building block to analyze issues such as mission focus or optimal screening of job applicants. In these extensions we show that it may be optimal for a principal to be vague on the type of task that an agent should pursue and delegate completely this decision to the worker. Furthermore, we derive conditions under which a principal refrains from screening heterogeneous job applicants ex ante, even if perfect screening is costless.

Appendix

Proof of Lemma 1

This section proves that each contract choice induces a subgame with a unique[12] equilibrium.

Contract space

Partition the space of contracts $c = \left[t_1\left(\underline{y}\right), t_1\left(\bar{y}\right)\right] \in I\!R \times I\!R$ as follows. Define, for each $p \in [0, 1]$, by $\mathcal{C}^p$ the subset of contracts that satisfy the agent's incentive, individual rationality, and limited liability constraints for effort p if the agent's second-period continuation utilities

[12]To be precise, the equilibrium is unique in the sense that all equilibria implement the same effort and are payoff equivalent (as in a standard Bertrand competition model many profiles of bidding strategies for competitors are consistent with equilibrium payoffs).

are equal to

$$\bar{u}_2^p = u\left((1-\alpha)\,\bar{H}\right), \tag{24}$$

$$\underline{u}_2^p = u\left((1-\alpha)\,\left[\underline{H} + \frac{p\,(1-\pi)}{1-p\,\pi}\,\Delta\,H\right]\right). \tag{25}$$

Market beliefs

Denote by $\hat{p}$ the market beliefs about agent's effort choice p. Then the second-period wages in the continuation game are

$$\bar{u}_2^{\hat{p}} \equiv u(t_2(\bar{y};\hat{p})) = u\left((1-\alpha)\,\bar{H}\right), \tag{26}$$

$$\underline{u}_2^{\hat{p}} \equiv u(t_2(\underline{y};\hat{p})) = u\left((1-\alpha)\,\left[\underline{H} + \frac{\hat{p}\,(1-\pi)}{1-\hat{p}\,\pi}\,\Delta\,H\right]\right). \tag{27}$$

Subgames induced by contracts $\mathbf{c} \in \mathcal{C}^{\mathbf{1}}$

We will show that pure strategy effort $p = 1$ is the unique equilibrium outcome of such a subgame. By definition, contracts $c \in \mathcal{C}^1$ involve first-period wages that induce first-period utilities $\bar{u}_1^c \equiv u(t_1^c(\bar{y}))$ and $\underline{u}_1^c \equiv u(t_1^c(\underline{y}))$ which satisfy:

$$\pi\,\left[\bar{u}_1^c + u\left((1-\alpha)\,\bar{H}\right)\right] + (1-\pi)\,\left[\underline{u}_1^c + u\left((1-\alpha)\,\bar{H}\right)\right] - \psi \geq \underline{u}_1^c + u\left((1-\alpha)\,\bar{H}\right). \tag{28}$$

Suppose market beliefs are that the agent exerts pure strategy effort, i.e., $\hat{p} = 1$. Then, inequality (28) is exactly the agent's incentive constraint. Since c satisfies also the individual rationality and limited liability constraints, $p = 1$ is indeed a best response for the agent, confirming market beliefs.

We now show that $c \in \mathcal{C}^1$ cannot induce any effort level $p \neq 1$ in equilibrium. Suppose that market beliefs are $\hat{p} \in (0,1)$. Then, given contract $c \in \mathcal{C}^1$, effort $p = \hat{p}$ is a best response for the agent only if

$$\pi\,\left[\bar{u}_1^c + \bar{u}_2^{\hat{p}}\right] + (1-\pi)\,\left[\underline{u}_1^c + \underline{u}_2^{\hat{p}}\right] - \psi = \underline{u}_1^c + \underline{u}_2^{\hat{p}}. \tag{29}$$

By (26) and (27) we have that for all $\hat{p} \in [0,1)$ second-period continuation wages satisfy $\bar{u}_2^{\hat{p}} = u\left((1-\alpha)\,\bar{H}\right)$ and $\underline{u}_2^{\hat{p}} < u\left((1-\alpha)\,\bar{H}\right)$. Therefore, inequality (28) implies that the left-hand side of (29) is strictly greater than the right-hand side. Hence, the agent's best response is $p = 1 \neq \hat{p}$, leading to a contradiction of market beliefs. Repeating the argument for $\hat{p} = 0$ also leads to a contradiction since the only thing that changes is that one needs to replace in (29) "=" by "<".

Subgames induced by contracts $\mathbf{c} \in \mathcal{C}^{\mathbf{p}'}$, $\mathbf{p}' \in [\mathbf{0}, \mathbf{1})$

Consider some $p' \in [0,1)$ and contracts $c \in \mathcal{C}^{p'}$. We will show that mixed strategy effort $p = p'$ is the unique equilibrium outcome of these subgames. By definition, contracts $c \in \mathcal{C}^{p'}$ induce first-period utilities $\bar{u}_1^c$ and $\underline{u}_1^c$ that satisfy:

$$\pi\,\left[\bar{u}_1^c + u\left((1-\alpha)\,\bar{H}\right)\right] + (1-\pi)\,\left[\underline{u}_1^c + u\left((1-\alpha)\,\left[\underline{H} + \frac{p'\,(1-\pi)}{1-p'\,\pi}\,\Delta\,H\right]\right)\right] - \psi$$
$$= \underline{u}_1^c + u\left((1-\alpha)\,\left[\underline{H} + \frac{p'\,(1-\pi)}{1-p'\,\pi}\,\Delta\,H\right]\right). \tag{30}$$

Suppose that market beliefs are $\hat{p} = p'$. Then, (30) is exactly the agent's incentive constraint. Since c satisfies also the individual rationality and limited liability constraints, $p = p'$ is indeed a best response for the agent, confirming market beliefs.

Now we show that $c \in \mathcal{C}^{p'}$ cannot induce any effort level $p \neq p'$ in equilibrium. Suppose first that market beliefs are $\hat{p} \in (0,1) \neq p'$. Then, given contract $c \in \mathcal{C}^{p'}$, effort $p = \hat{p}$ is a best response for the agent only if (29) is satisfied. Since for all $\hat{p} \in (0,1) \neq p'$ we have that $\bar{u}_2^{\hat{p}} = \bar{u}_2^{p'}$ while $\underline{u}_2^{\hat{p}} \neq \underline{u}_2^{p'}$, the constraint is violated and the agent's best response is either $p = 1$ or $p = 0$, contradicting market beliefs. This rules out equilibria with mixed strategy $p \in (0,1) \neq p'$. Now suppose market beliefs are $\hat{p} = 0$. Then, given contract $c \in \mathcal{C}^{p'}$, effort $p = \hat{p} = 0 \neq p'$ is a best response for the agent only if

$$\pi\,\left[\bar{u}_1^c + u\left((1-\alpha)\,\bar{H}\right)\right] + (1-\pi)\,\left[\underline{u}_1^c + \underline{u}_2^0\right] - \psi \leq \underline{u}_1^c + \underline{u}_2^0,$$
$$\Leftrightarrow \qquad \bar{u}_1^c - \underline{u}_1^c + u\left((1-\alpha)\,\bar{H}\right) - \underline{u}_2^0 \leq \frac{\psi}{\pi} \tag{31}$$

However, the definition of the contract (30) implies that

$$\bar{u}_1^c - \underline{u}_1^c + u\left((1-\alpha)\,\bar{H}\right) - \underline{u}_2^{p'} = \frac{\psi}{\pi}, \tag{32}$$

so that (31) is violated since $\underline{u}_2^{p'} > \underline{u}_2^0$ for $p' \in (0,1)$. Thus, the agent's best response is $p = 1 \neq \hat{p}$, contradicting market beliefs. This rules out equilibria with pure strategy $p = 0 \neq p'$. Suppose that market beliefs are $\hat{p} = 1$. Then, second-period wages do not respond to the output and the first-period monetary incentives based on (30) are too weak so that $p = 0 \neq \hat{p}$ is the agent's best response, leading to a contradiction. This rules out equilibria with pure strategy $p = 1 \neq p'$.

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