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INFINITE FAMILIES OF QUASIPERFECT AND DOUBLY QUASIPERFECT BINARY ARRAYS

Indexing terms: Image processing, Matrix algebra, Binary sequences

A quasiperfect (doubly quasiperfect) binary array B is an $s \times t$ array with all entries plus or minus one, such that all periodic autocorrelation coefficients of $[-_B^B]$ (of $[-_B^B \, _{-B}^{-B}]$) are zero, except for shifts (u, v) where $u \equiv 0 \bmod s$ and $v \equiv 0 \bmod t$. We construct new infinite families of quasiperfect and of doubly quasiperfect binary arrays.

Introduction: Let $A = (a_{ij})$, $0 \le i \le s - 1$, $0 \le j \le t - 1$, be an $s \times t$ array such that $a_{ij} = 1$ or -1 for all i and j. A is called a *binary array of size s by t*. We define the *periodic autocorrelation function R of A* at displacement (u, v) as

$$R(u, v) = \sum_{i=0}^{s-1} \sum_{j=0}^{t-1} a_{ij} a_{i+u, j+v}$$

where $0 \le u \le s - 1$, $0 \le v \le t - 1$, and we consider the sums $i + u$ and $j + v$ to be addition modulo s and t, respectively. A two-dimensional binary array A is called *perfect* if $R(u, v) = 0$ for all $(u, v) \ne (0, 0)$. We write PBA(s, t) to denote a perfect binary array of size s by t. Two-dimensional perfect binary arrays were first considered by Calabro and Wolf[1] and correspond to difference sets in abelian groups.[2] They have applications in coded aperture imaging[3] and optical image alignment.[4] For two-dimensional alignment, the pattern to be aligned is projected onto a periodically extended copy of the stored image and the value of the autocorrelation function used to determine when alignment has occurred.

Quasiperfect and doubly quasiperfect binary arrays were introduced by Jedwab and Mitchell.[5] Wild[6] has shown that these arrays correspond to relative difference sets in abelian groups and has given the following definitions as alternatives to those in Reference 5:

Definition: Let B be a binary array of size $s \times t$. B is called *quasiperfect* if the $2s \times t$ array

$$B' = \begin{bmatrix} B \\ -B \end{bmatrix}$$

has $R_{B'}(u, v) = 0$ for all $(u, v) \ne (0, 0)$ or $(s, 0)$. We write QPBA(s, t) to denote a quasiperfect binary array of size $s \times t$.

Definition: Let C be a binary array of size $s \times t$. C is called *doubly quasiperfect* if the $2s \times 2t$ array

$$C' = \begin{bmatrix} C & -C \\ -C & C \end{bmatrix}$$

has $R_{C'}(u, v) = 0$ for all $(u, v) \ne (0, 0)$, $(s, 0)$, $(0, t)$ or (s, t). We write DQPBA(s, t) to denote a doubly quasiperfect binary array of size $s \times t$.

In this paper we shall construct seven infinite families of quasiperfect binary arrays and three infinite families of double

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quasiperfect binary arrays, only two each of which were previously known. Optical alignment based on these arrays is possible if the method of periodic extension of the stored image is modified. For example, using a DQPBA(s, t), the stored image consists of a chequerboard pattern of $s \times t$ arrays with the black and white shading of the board corresponding to positive and negative copies of the array. This application is particularly useful when $st \ne 4k^2$ for any integer k since in this case a PBA(s, t) cannot exist.[1] Furthermore, when $st = 4k^2$, quasiperfect and doubly quasiperfect binary arrays may be used in the construction of perfect binary arrays of larger size than $s \times t$.[5,6]

Construction theorems: We shall require the following three theorems:

Theorem 1 (Jedwab and Mitchell):[5] If there exists a QPBA(s, t) and a DQPBA(s, t) then there exists a QPBA($2s$, $2t$) and a QPBA(s, $4t$).

Proof (outline): Suppose B is a QPBA(s, t) and C is a DQPBA(s, t). Then the $2s \times 2t$ array formed by interleaving the rows of $[B \ B]$ and $[C \ -C]$, and the $s \times 4t$ array formed by interleaving the columns of $[B \ B]$ and $[C \ -C]$, are both quasiperfect.

Theorem 2: If s is odd then there exists a PBA(s, t) if and only if there exists a QPBA(s, t).

Proof: Let $A = (a_{ij})$ and $B = (b_{ij})$ where $b_{ij} = (-1)^i a_{ij}$. Then A is a PBA(s, t) if and only if B is a QPBA(s, t).

Theorem 3 (Wild):[7] If $t/\gcd(s, t)$ is odd then there exists a QPBA(s, t) if and only if there exists a DQPBA(s, t).

Constructing infinite families of arrays:

Corollary 1: If there exists a QPBA(s, s) then there exist the following infinite families of arrays:

$$\text{QPBA}(2^n s, 2^n s) \quad \text{DQPBA}(2^n s, 2^n s) \quad \text{QPBA}(2^n s, 2^{n+2} s)$$

$$(n \ge 0)$$

Proof: We use induction on n. Given a QPBA($2^n s$, $2^n s$), by Theorem 3 there exists a DQPBA($2^n s$, $2^n s$). By Theorem 1 we may construct from these two arrays a QPBA($2^{n+1} s$, $2^{n+1} s$) and a QPBA($2^n s$, $2^{n+2} s$), establishing the induction.

Corollary 2: There exist the following infinite families of arrays:

$$\text{QPBA}(2^n, 2^n) \quad \text{DQPBA}(2^n, 2^n) \quad \text{QPBA}(2^n, 2^{n+2})$$

$$\text{QPBA}(3 \cdot 2^{n+1}, 3 \cdot 2^{n+1}) \quad \text{DQPBA}(3 \cdot 2^{n+1}, 3 \cdot 2^{n+1}),$$

$$\text{QPBA}(3 \cdot 2^n, 3 \cdot 2^{n+2}) \qquad\qquad (n \ge 0)$$

Proof: There exists a QPBA(1, 1) and a QPBA(6, 6).[5] Apply Corollary 1. Also the existence of a PBA(3, 12)[2] implies the existence of a QPBA(3, 12) by Theorem 2 (represented by case $n = 0$ of the sixth family).

Whereas the third and sixth families above are new, the other families were implicitly constructed in Reference 6.

Corollary 3: If there exists a QPBA($2s$, s) then there exist the following infinite families of arrays:

$$\text{QPBA}(2^{n+1} s, 2^n s) \quad \text{DQPBA}(2^{n+1} s, 2^n s)$$

$$\text{QPBA}(2^{n+1} s, 2^{n+2} s) \quad \text{QPBA}(2^{n+1} s, 2^{n+4} s) \qquad (n \ge 0)$$

Proof: We use induction on n. Given a QPBA($2^{n+1} s$, $2^n s$), by Theorem 3 there exists a DQPBA($2^{n+1} s$, $2^n s$). By Theorem 1 we may construct from these two arrays a QPBA($2^{n+2} s$, $2^{n+1} s$) and a QPBA($2^{n+1} s$, $2^{n+2} s$), establishing the induction for the first three families. Transposing a DQPBA($2^{n+2} s$, $2^{n+1} s$) we obtain a DQPBA($2^{n+1} s$, $2^{n+2} s$) which, together with a QPBA($2^{n+1} s$, $2^{n+2} s$), gives a QPBA($2^{n+1} s$, $2^{n+4} s$) by

Theorem 1. This establishes the induction for the fourth family.

Corollary 4: There exist the following infinite families of arrays:

$$QPBA(2^{n+1}, 2^n) \quad DQPBA(2^{n+1}, 2^n)$$

$$QPBA(2^{n+1}, 2^{n+2}) \quad QPBA(2^{n+1}, 2^{n+4}) \qquad (n \geq 0)$$

Proof: The array $\begin{bmatrix} * \\ + \end{bmatrix}$ is a QPBA(2, 1). The result follows by Corollary 3.

None of the above families has been previously reported.

Further construction methods: A binary sequence (a_i) of length s may be identified with an $s \times 1$ binary array (a_{ij}) by dropping the second subscript of the array. We may then define a *perfect binary sequence of length s* to be a PBA(s, 1) and a *quasiperfect binary sequence of length s* to be a QPBA(s, 1), calling the sequence *nontrivial* if $s > 1$.

Suppose that $A = (a_i)$ and $B = (b_j)$ are binary sequences of length s and t, respectively, and that $C = (c_{ij})$ is the $s \times t$ binary array formed by $c_{ij} = a_i b_j$. It is straightforward to establish the following:

(i) If A is a quasiperfect and B is perfect then C is quasiperfect.
(ii) If A and B are quasiperfect then C is doubly quasiperfect.

No nontrivial perfect binary sequence is known except with length 4, and it has long been conjectured that no others exist. If a nontrivial quasiperfect binary sequence with length other than 2 could be found, then the construction methods above, as well as Theorems 1 and 3, could be used to construct new quasiperfect and doubly quasiperfect binary arrays. Unfortunately a computer search has shown that no quasiperfect binary sequences with length greater than 2 and less than 34 exist.

Summary: We have constructed seven infinite families of quasiperfect binary arrays whose sizes are

$$2^n \times 2^n \quad 3 \cdot 2^{n+1} \times 3 \cdot 2^{n+1} \quad 2^n \times 2^{n+2} \quad 3 \cdot 2^n \times 3 \cdot 2^{n+2}$$

$$2^{n+1} \times 2^n \quad 2^{n+1} \times 2^{n+2} \quad \text{and} \quad 2^{n+1} \times 2^{n+4}$$

$$(n \geq 0)$$

of which only the first two families were previously known. Every QPBA(s, t) known to the authors has (s, t) belonging to one of the above parameter sets.

We have also constructed three infinite families of doubly quasiperfect binary arrays whose sizes are

$$2^n \times 2^n \quad 3 \cdot 2^{n+1} \times 3 \cdot 2^{n+1}$$

and

$$2^{n+1} \times 2^n \qquad (n \geq 0)$$

of which only the first two families were previously known. Every DQPBA(s, t) known to the authors has (s, t) or (t, s) belonging to one of the above parameter sets.

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MICROWAVE SCATTERING BY METAL CUBES AND THE EFFECT OF PERTURBING THE CUBE GEOMETRY

Indexing terms: Microwave circuits, Microwave techniques and systems, Wave scattering, Microwaves, Microwave measurement

New measurements are made of the total scattering cross-sections of metal cubes having a range of sizes, for the frequency interval 8 to 12 GHz. The effect of small changes in the geometry of certain edges has been identified as influencing the scattering properties.

The detailed calculation and measurement of the backscattering properties of metallic cubes have been discussed recently.[1,2] Presentation of the theoretical work necessary to predict the backscatter cross-sections of perfectly conducting cubes having side sizes in the range 0·15 to 4 wavelengths, together with backscatter measurements, has demonstrated that this difficult problem can be described analytically.[1,2] Cote *et al.*[1] have also calculated and plotted the total-loss cross-section (σ_T) of metallic cubes as a function of cube circumference, and this is plotted together with calculated results for conducting spheres, but there are no directly measured results.

The aim of this letter is (a) to present measurements of σ_T for a range of sizes of aluminium cubes and (b) to investigate possible changes in the values of σ_T when the cube geometry is altered slightly by machining a small amount of metal off certain edges.

Measurements of σ_T were taken between 8 and 12 GHz when cubes were inserted as perturbing objects in a large open resonator (mirror diameter = 1 m; mirror radius of curvature, and separation = 10·5 m and 4·0 m, respectively; unloaded Q-factor $\simeq 110000$). This was operated on each of 100 fundamental modes within 8 to 12 GHz and a microcomputer recorded and processed the experimental data to give 99 measured values of σ_T. The basic operation of the system and its general application to the measurement of σ_T have been described previously.[3]

In Fig. 1, measured values of total cross-sections are presented for a range of sizes of cubes machined accurately from free-cutting aluminium. Results were taken throughout the frequency interval of 8 to 12 GHz and each cube was positioned in two orientations, I and II. For orientation I, one face of the cube was broadside to the wave, i.e. the face lay in a plane of constant phase of the wave with four edges parallel to the E-field. For orientation II, the cube was rotated through 45° to a position where, with one (leading) edge parallel to the E-field and phase fronts, it presented a symmetrical geometry to the wave. It is interesting to note (Fig. 1) that for the smaller cubes, measured values for σ_T depend little on the orientation with greater differences (between orientations I and II) being observed for the larger cube sizes.

Selected results, taken at 0·5 GHz intervals from the data of Fig. 1 are replotted in Fig. 2, in the form, $\sigma_T/s^2 = \sigma_{TN}$ against $4s/\lambda$, s being the length of the cube edge and λ the wavelength. Five differently sized cubes were used and, for each one, results cover the frequency range 8 to 12 GHz. These discrete results are for cubes in orientation I. A similar set of results taken for cubes in orientation II, are represented (for clarity)