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The Chern-Simons number as an order parameter : classical sphaleron transitions for $SU(2)$-Higgs field theories for $m_H \approx 120\,\mathrm{GeV}$

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Abstract

The classical transitions between topologically distinct vacua in a $SU(2)$–Higgs model, using a Higgs field of mass approximately 120 GeV, is examined to probe the crossover region between the symmetric and broken phase. Assuming the Higgs mass is constant, we find the width of this crossover region is approximately 20% of the average temperature. We suggest that this observable is a better parameter to explore this region of phase space than equal-time correlation functions.

1 Introduction

There is evidence from a number of numerical simulations in 3 and 4 dimensions [1, 2, 3] that the first order phase transition from the symmetric to broken phase of the electroweak sector of the Standard Model ends for a Higgs mass of approximately 80 GeV. For larger Higgs masses, there is analytic crossover or very possibly a transition where higher order derivatives of the free energy experience a discontinuity (these are sometimes referred to as "third or higher order" phase transitions, whether numerical studies could ever distinguish between these two scenarios remains to be seen). The preliminary lower bound of 77 GeV for the Higgs mass [4] indicates that this latter region of phase space, where the behaviour of equal time correlation functions is much smoother, is physically far more likely.

This endpoint, and the fact that the first order phase transition is very weak for smaller Higgs masses [5] indicates that baryogenesis cannot occur via this mechanism in the Standard Model (SM). As a result, a number of other suggestions have been made to get around this. Many of these proposals make non-trivial statements about the early Universe. While the qualitative picture of supersymmetric models is no different from the SM, it is hoped that the extra fields will adjust the position and strength of the phase transition sufficiently that larger Higgs masses are feasible [6]. Other models introduce extensions to the SM, including the formation of topological defects to provide the out of equilibrium condition [7], or change the expected rate of expansion of the Universe at the EW epoch [8] so that a phase transition is unnecessary and only sufficiently rapid change is sufficient.

It is therefore of interest to explore the baryon number violating properties in this region of parameter space. A study to see if defect formation can occur in this region would also be

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of interest. In this paper we shall focus on the first point. Here we attempt to calculate the rate of diffusion for the Chern-Simons number defined as

$$N_{CS}(t) - N_{CS}(0) = \frac{1}{32\pi^2}\int_0^t dx^0 \int d^3x \epsilon^{\mu\nu\rho\lambda}\mathbf{F}_{\mu\nu}.\mathbf{F}_{\rho\lambda} \ , \tag{1}$$

which labels the change in the baryon number as a function of time (for a recent review see [9]). Such an unequal-time correlation function is extremely difficult to calculate in a full quantum field theory, discretised on a lattice, as the introduction of a density matrix operator ensures that the exponential weight in a Monte-Carlo simulation will be complex and will fluctuate enormously. However, noting that on dimensionful grounds the typical size of the sphaleron is $\mathcal{O}(1/gT) >> T^{-1}$, where T is the temperature, a classical interpretation of the problem seems to be a reasonable approximation. The accuracy of such an approach is of some debate but has been successfully utilised by an number of different groups for pure $SU(2)$ gauge theories and $SU(2)$–Higgs systems with $m_H \leq 80\,\mathrm{GeV}$ [10, 11, 12]. Here, we choose a Higgs mass of approximately $120\,\mathrm{GeV}$ and have determined the diffusion rate for a range of aT, where a is the lattice spacing, and two volumes. This paper proceeds as follows; in section 2 we outline briefly the method of calculation, describing the discretised Hamiltonian and the choice of bare lattice parameters. In section 3 we present an analysis of the results and determine their behaviour in terms of physical observables. Finally we draw some conclusions on the applications of this approach.

2 Computational Details

We employ the methods used in [10, 11, 12, 13] to evaluate the *classical* diffusion rate of the Chern Simons number. The original 4-dimensional $SU(2)$-Higgs lagrangian is dimensionally reduced by the high-temperature Matsubara formalism to a surprisingly simple 3-dimensional lagrangian, whose couplings are related to the original lagrangian's coupling by perturbation theory [14]. To measure a quasi-equilibrium quantity such as the above diffusion rate, one introduces conjugate momenta fields which allow the fields to evolve in a Hamiltonian formalism, while staying in thermal equilibrium. The $SU(2)$ scalar and gauge fields are represented in a discrete form by the fields $\Phi_{\mathbf{x}}$ and $U_{i,\mathbf{x}}$ respectively, where $\mathbf{x}$ is the site index. The conjugate momenta fields are $\pi_{\mathbf{x}}$ and $E_{i,\mathbf{x}}$. The following discretised Hamiltonian was employed

$$\begin{aligned} H = \beta \Bigg[& -\sum_{\mathbf{x}}\left(1-\frac{1}{2}TrU_{\Box,\mathbf{x}}\right) - \frac{1}{2}\sum_{\mathbf{x},i}(\Delta_i\Phi)^\dagger_{\mathbf{x}}(\Delta_i\Phi)_{\mathbf{x}} \\ & -\sum_{\mathbf{x}}\left(\frac{M_{H0}^2}{2}\Phi^\dagger_{\mathbf{x}}\Phi_{\mathbf{x}} + \frac{\lambda_L}{4}(\Phi^\dagger_{\mathbf{x}}\Phi_{\mathbf{x}})^2\right) + \frac{1}{(\Delta t)^2}\left(z_E\sum_{i,\mathbf{x}}E_{i,\mathbf{x}}E_{i,\mathbf{x}} + \frac{z_\pi}{2}\sum_{\mathbf{x}}\pi^\dagger_{\mathbf{x}}\pi_{\mathbf{x}}\right)\Bigg] \ , \end{aligned} \tag{2}$$

where

$$(\Delta_i\Phi)_{\mathbf{x}} = \Phi_{\mathbf{x}+\hat{\mathbf{i}}}U_{i,\mathbf{x}} - \Phi_{\mathbf{x}} \ , \tag{3}$$

and $U_{\Box,\mathbf{x}}$ is the elementary plaquette constructed from the gauge fields surrounding that point. The parameter Δt controls the time step size and was set to 0.05. The parameters z_E and z_π represent the effect of the renormalisation of the momentum operators by the dimensional reduction and is assumed to take the form $1 + \mathcal{O}(g^2)$. The validity of this assumption (or indeed whether higher dimensional operators should be included) is still a topic of discussion.

No estimate of the one-loop corrections to z_E and z_π have been made and we have assumed them to be 1. The remaining parameters β, M_{H0}^2 and λ_L control the physical parameters of the system; the lattice spacing a, the Higgs mass m_H and the temperature T. At this level of approximation, it is assumed that the couplings g^2 and λ (for the original 4-dimensional lagrangian) do not run and that many of the standard tree-level results can be applied. Noting that

$$\beta = \frac{4}{g^2 aT} \quad , \tag{4}$$

and that at tree-level

$$\lambda_L \approx 8\frac{\lambda}{g^2} \approx \frac{m_H^2}{m_W^2} \quad , \tag{5}$$

it is clear that one can fix m_H and aT simply from the β and λ_L. The relationship that the parameter M_{H0}^2 has to m_H, a and T is more complicated. One could vary M_{H0}^2 and β so that a is fixed. However, for this paper we shall take a more simple route by fixing λ_L and M_{H0}^2 and varying β. The ratio T/m_H as a function of β can be determined from a perturbative relationship [14].

For this paper, λ_L was set to 2.25 and M_{H0}^2 to -0.596 using two lattice sizes of 24^3 and 18^3. The coupling β was varied from 6.8 to 7.8 in steps of 0.1. For each β, 8 to 16 configurations were thermalised and then evolved using a leapfrog algorithm for time lengths greater than 6000. The Chern-Simons number was measured using the slave-field technique devised by Moore and Turok [13] at intervals of 5 time units. The simulation parameters are listed in table (1). The code used for thermalisation and evolution was developed by Guy Moore and Neil Turok and was run on a Silicon Graphics Origin 2000 made available by the U.K. Computational Cosmology Consortium (UKCCC) and an Hitachi SR2201 at the High Performance Computing Facility (HPCF) at the University of Cambridge.

3 Results

Assuming the behaviour of N_{CS} is diffusive, then

$$\lim_{t\to\infty} < (N_{CS}(t) - N_{CS}(0))^2 >= \Gamma t \quad . \tag{6}$$

Examples of this expectation value as a function of time are shown in figure (1). To isolate Γ the cosine transform technique, outlined in [13], was used. This involves integrating the data with a cosine weighting over the trajectory length. For a finite time spacing Δt and a trajectory length of $t_f + \Delta t$ this corresponds to evaluating

$$z_n^I = \frac{1}{t_f + \Delta_t}\left[\left(\sum_{t=\Delta_t}^{t_f} N_{CS}^I(t)\cos\frac{n\pi t}{t_f}\right) - \frac{1}{2}\left(N_{CS}^I(0) + N_{CS}^I(t_f)\cos n\pi\right)\right] \quad , \tag{7}$$

where the index I corresponds to an individual configuration. Averages over configurations are denoted with $< >$. An example of the resulting coefficients are shown in figure (2). It is possible to demonstrate that

$$n^2 < z_n^2 >= \frac{\Gamma t_f}{2\pi^2} + \zeta n^2 \quad , \tag{8}$$

where ζ is a white noise term, although a functional fit to this form was not used. The coefficients $n^2 < z_n^2 >$ were averaged in bins of 50 which considerably reduced the resulting error. An example of this is shown in in figure (3). The final central value was determined from the central value of the binned data of order 50 to 99, as higher orders will be affected by ultra-violet contributions while smaller coefficients may be affected by the finite length of the trajectory. If the other binned coefficients of 1-49 or 100-149 varied by more than a standard deviation from this, the difference was included as a systematic error.

In order to express the results in terms of a dimensionless intensive quantity $\overline{\kappa}$, the diffusion rate in the continuum Γ^{cont} is usually expressed as

$$\frac{\Gamma^{cont}}{V} = \overline{\kappa}(\alpha_W T)^4 \quad , \tag{9}$$

(although it remains unclear if $\overline{\kappa}$ has some residual dependence on α_W [15]). Equivalently, for the lattice result

$$\Gamma = V(\pi\beta)^{-4}\kappa \quad . \tag{10}$$

The results for κ are listed in table (1).

4 Conclusions

In this letter, we have calculated the diffusion rate of the Chern Simons number for an $SU(2)$–Higgs field theory at approximately fixed Higgs mass for a range of temperatures. We see that the rate is independent of volume and varies from a constant non-zero rate, in the symmetric phase to a zero rate in the broken phase (the use of the word "phase" here is perhaps slightly vague and should be redefined in this case by the region of parameter space with properties similar to the other region of parameter space where a phase transition occurs). The ratio m_H/T can be determined from perturbative matching of the 3 and 4 dimensional coefficients as described in [14] and [11]. The change from a zero to non-zero rate occurs for a variation in m_H/T of approximately 20%. This is broader than the case when $m_H \approx 80\,\mathrm{GeV}$, which was approximately 10% [11]. Assuming the Higgs mass is fixed at 120 GeV we see that the mean crossover temperature ($\sim 380\,\mathrm{GeV}$) is larger than is expected from the study of equal-time correlation functions ($\sim 210\,\mathrm{GeV}$) [2]. A more precise study in parameter space is certainly possible, where the lattice spacing is kept constant and only the temperature varied. This would involve calculating the Higgs and W mass for each possible configuration of parameters to determine the lattice spacing. However, as it unclear how accurate the classical approximation is, such an exhaustive approach may be fruitless.

Nonetheless, the Chern-Simons diffusion rate is an excellent observable for studying the complicated crossover region of such field theories. Equal-time correlation functions provide very clear signals in the region where a first-order phase transition exists. However in the region of large Higgs mass, their behaviour is non-singular and the distinction between analytic crossover and possible discontinuities becomes quite difficult. The Chern-Simons diffusion rate has behaviour that is clearer and independent of volume which defines two separate regions.

One area this study may well be of use is in the area of defect formation for large-structure formation in the early Universe. It is not clear whether defects can form in this crossover region. In the future we plan to study this.

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β	$\frac{T}{m_H}$	N_s^3	#configurations	#time steps	κ
6.8	3.85	24^3	16	7000	$0.973 \pm 0.057 \pm 0.161$
6.9	3.67	24^3	16	7000	$0.984 \pm 0.061 \pm 0.107$
7.0	3.50	24^3	16	10000	$1.013 \pm 0.083 \pm 0.187$
7.1	3.35	24^3	8	20000	0.770 ± 0.059
7.2	3.21	24^3	8	10000	$0.388 \pm 0.034 \pm 0.038$
7.3	3.08	24^3	8	10000	0.092 ± 0.008
7.4	2.96	24^3	8	6000	$0.018 \pm 0.002 \pm 0.003$
7.5	2.85	24^3	8	9000	0.00787 ± 0.00068
7.6	2.75	24^3	16	6000	0.00088 ± 0.00006
7.7	2.66	24^3	8	6000	0.00130 ± 0.00016
7.8	2.57	24^3	8	9000	0.00055 ± 0.00005
6.8	3.85	16^3	16	10000	$1.021 \pm 0.057 \pm 0.145$
6.9	3.67	16^3	8	11500	$1.058 \pm 0.061 \pm 0.163$
7.0	3.50	16^3	16	11000	0.934 ± 0.058
7.1	3.35	16^3	16	7500	$0.754 \pm 0.044 \pm 0.078$
7.2	3.21	16^3	16	11500	0.276 ± 0.017
7.3	3.08	16^3	15	6000	$0.0700 \pm 0.0061 \pm 0.0095$
7.4	2.96	16^3	16	15500	0.0179 ± 0.0011
7.5	2.85	16^3	16	12500	0.00454 ± 0.00031

Table 1: Numbers of configurations and the time steps iterated forward for each β . The second error quoted for κ is the difference between the first and second bins of 50 coefficients when the difference was greater than 1 standard deviation.

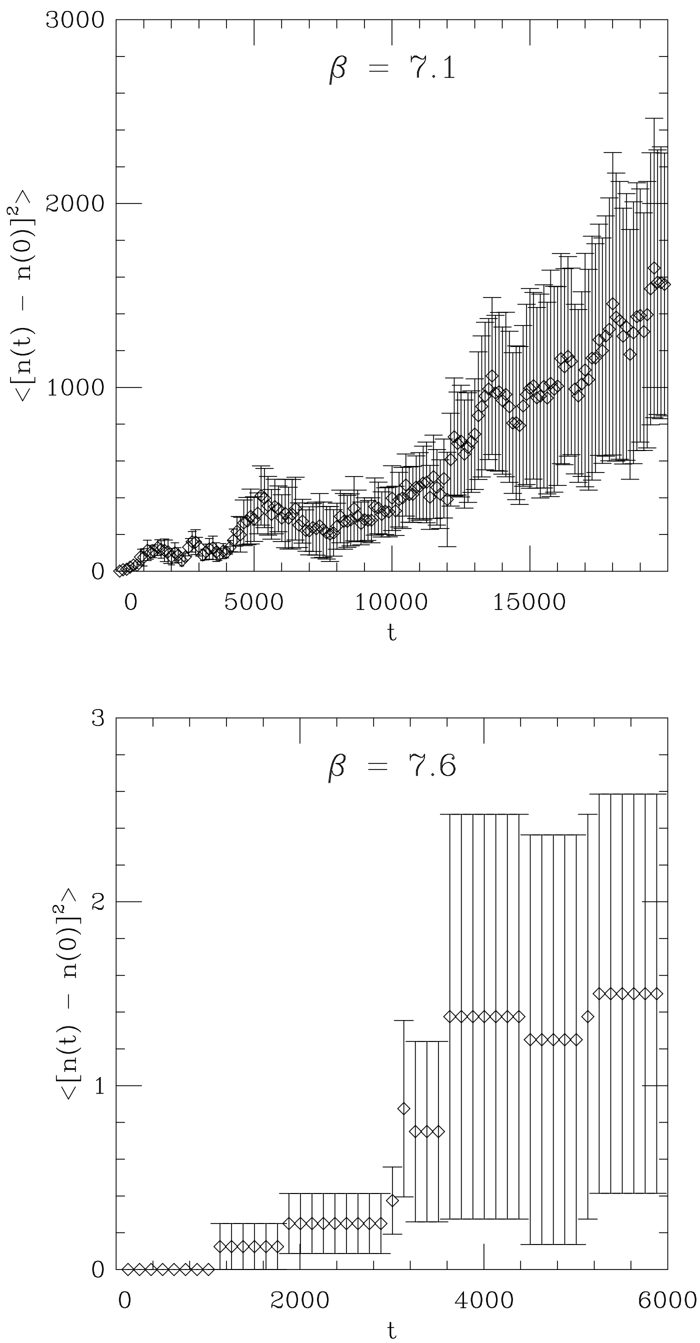

Figure 1: Diffusion rate of winding number at $\beta = 7.1$ and $\beta = 7.6$. Note the difference in the vertical scales.

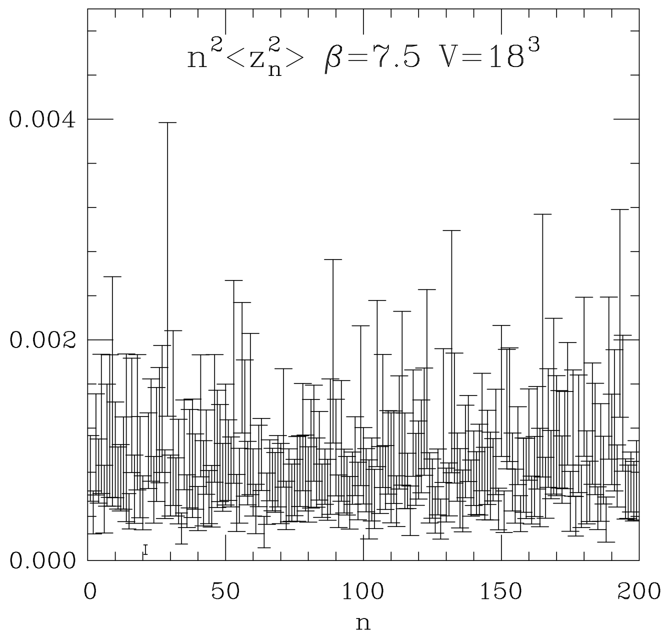

Figure 2: Cosine transform coefficients for a typical parameter set.

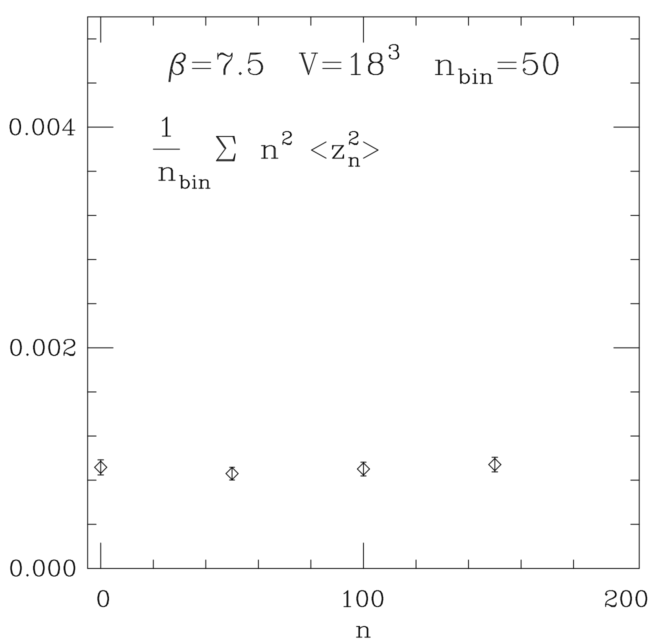

Figure 3: Cosine transform coefficients binned into groups of 50, using the same parameter set as in figure (2)

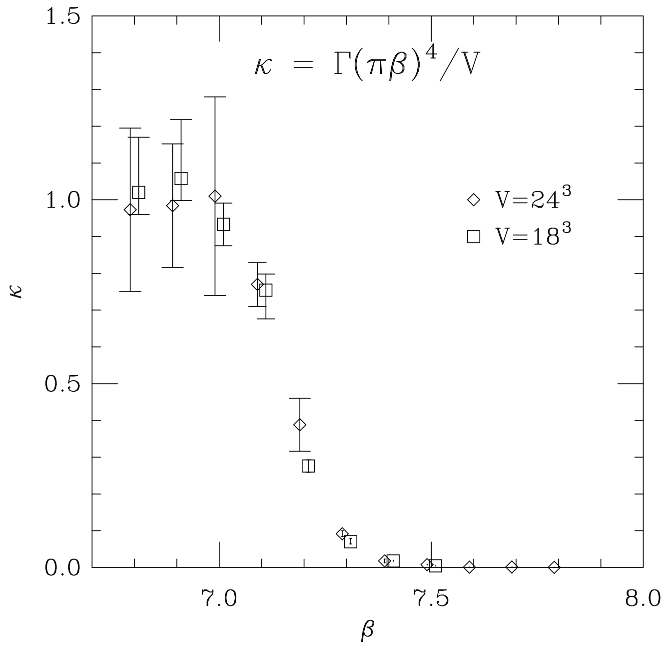

Figure 4: Resultant κ's for both volumes as a function of β. The results for different volumes are displaced slightly for clarity.