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A non-perturbative calculation of the mass of the B_c

UKQCD Collaboration

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Abstract

We present a calculation of the mass of the ${}^1S_0$ pseudoscalar $\bar{b}c$ (B_c) state using a non-perturbative measurement from quenched lattice QCD. We find $M_{Bc} =$ 6.386(9)(98)(15) GeV where the first error is statistical, the second systematic due to the quark mass ambiguities and quenching and the third the systematic error due to the estimation of mass of the η_b.

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1 Introduction

Little is known experimentally of the properties of the $\bar{b}c$ spectrum of states, apart from the detection of the pseudoscalar ${}^1S_0$, $\bar{b}c$ (B_c) state by the CDF collaboration [1]. At the very least these states provide an excellent "blind" test for some of the techniques used in measuring non-perturbative properties such as the mass spectrum and decay widths. The study of non-degenerate heavy quarks provides us with a unique probe of QCD. The weak decays of $\bar{b}c$ states provide a plethora of new methods for calculating the less well determined CKM matrix elements, in particular its semi-leptonic decay mode

$$B_c \to J/\psi\, l\overline{\nu} \quad , \tag{1}$$

will provide an accurate measurement of $|V_{cb}|$ [2, 3].

The spectrum of states with the ground state mass subtracted, when calculated via potential models [4, 5], for example, is in broad agreement with quenched lattice QCD

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[6, 7, 8]. Despite concerns that potential models would have large relativistic corrections (as a simple centre-of-mass argument implies $\langle v^2/c^2 \rangle$ for the charm quark is approximately 0.5) it appears that it is still a reasonable method for calculating the spectrum. Nonetheless, a non-perturbative measurement of the spectrum is important and is being actively pursued. The potential model approach unfortunately does not provide, *a priori*, a method for determining the ground state mass and hence a number of different phenomenological methods have to be used. Lattice QCD, on the other hand, can provide an *ab initio* method for determining its mass.

The use of a non-relativistic hamiltonian to describe the behaviour of a bottom quark in quenched lattice QCD [9] has been employed very effectively for the spin-independent sector of the $\bar{b}b$ spectrum [10]. For relatively coarse lattice spacings, this technique is also applicable for $\bar{c}c$ states and indeed two calculations [6, 11] of the $\bar{b}c$ spectrum and m_{B_c} using non-relativistic b and c quarks have been carried out. However, as the $\bar{c}c$ spectrum is not reproduced very accurately using this approach [12] there remains some question as to its accuracy for the $\bar{b}c$ sector. Another approach which will work for lattice spacings where $am_c < 1$ is to use a discretised relativistic Lagrangian, which, for the lattice spacing used in this letter, reproduces the charmonium spectrum reasonably well [13, 14]. Here we take the approach of combining these separate methods for treating the b and c quarks. In essence this involves computing Green's functions from a discretised NRQCD hamiltonian (NRGF) with propagators calculated using a improved discretised QCD Lagrangian devised originally by Sheikholeslami and Wohlert [17] (SWP) from the same set of quenched (zero flavour) QCD gauge configurations and then combining them into two point functions. The damped exponential behaviour of this correlation function will provide information about the mass of the states we are interested in.

The rest of this letter is organised as follows : in the following section we describe a method for computing m_{B_c} directly from zero momentum correlation functions, and demonstrate why this has the smallest systematic error; after this the computational details of the calculation are outlined, explaining briefly the details of NRQCD Hamiltonian used for the bottom quark and the Lagrangian used for the charm quark; we then present the numerical results and in the conclusions we compare the results with other calculations of m_{B_c}.

2 Ground state mass definitions

For large Euclidean times, the two point function of a meson operator constructed from either NRGF's or SWP's will take the following form

$$\int d^3x \langle 0|M(\mathbf{x},t)M^\dagger(0)|0\rangle \overrightarrow{_{t>>1/m_1}} A\exp(-am_1 t/a) \quad , \tag{2}$$

where the integral sign can also indicate a sum over discrete lattice points. (We have also discarded the possible effect of the finite size of the lattice in the time direction for relativistic correlation functions.) As demonstrated in [18] and [19] the variable am_1

(where a is the lattice spacing), also referred to as "the pole mass", has quite different interpretations depending on whether the operator $M(\mathbf{x}, t)$ is constructed from NRGF's or SWP's. The former is the binding energy of the state with the heavy quark mass subtracted; the latter satisfies the relationship

$$am_1^{SWP} = aM + ac(am_Q, a, V) + ad(a, V) \ , \tag{3}$$

where M is the mass of the state, ad are the systematic errors from the finite lattice spacing and volume and ac is a lattice artefact due to the breaking of Lorentz invariance on the lattice This parameter vanishes for $am_Q << 1$ and grows to be a large correction as am_Q approaches 1 and beyond. Nonetheless, the difference of the pole masses, for either NRGF's or SWP's will reproduce the physical mass difference (assuming other discretisation effects are under control). Since this process of changing the zero of the quark mass is additive, one therefore expects

$$m_{1B_c} - 1/2(m_{1\eta_b}^{NRGF} + m_{1\eta_c}^{SWP}) = M_{B_c} - 1/2(M_{\eta_b} + M_{\eta_c}) \ , \tag{4}$$

where the RHS is the physical mass difference and m_{1B_c} is computed using correlation functions with the composition used in this letter. As one can easily construct a B along the same lines one can also state that

$$m_{1B_c} - (m_{1B} + m_{1D}^{SWP}) = M_{B_c} - (M_B + M_D) \ . \tag{5}$$

It is therefore possible derive an expression for M_{B_c} simply from the pole masses without resorting to using other possible mass definitions.

The mass of the η_b has yet to be determined, and has to be estimated from the Υ and a potential model calculation of the hyperfine splitting. A larger error is that resulting from the quenched approximation. A first estimate of the effect of quenching is from the variation of the effective lattice spacing between observables having different "typical" momentum scales. In this light we estimate this from the difference between the masses determined from equations 4 and 5.

A more traditional approach to calculate the mass of the B_c is to determine the "kinetic" mass, (often labelled am_2) which is derived from the dispersion relationship of pole masses at different finite momenta. This however has the drawback that it has a much larger statistical error than the pole mass. In particular, while the kinetic mass is proportional to the inverse of a difference (i.e. the zero and smallest non-zero momentum pole masses), the mass derived from equations (4) and (5) are directly proportional to it.

3 Computational Details

The calculation was carried out in quenched lattice QCD using the Wilson gluon action at $\beta = 6.2$ on a $24^3 \times 48$ lattice. Technical details on how these configurations were calculated and the exact form of the gauge action can be found in [20]. In terms of the string tension resulting from the static quark potential, the lattice spacing is approximately 0.07 fm, resulting in a spatial side length of 1.68 fm.

The non-relativistic Hamiltonian used was corrected to $\mathcal{O}(mv^4)$. The coefficients in the Hamiltonian were tree-level mean-field estimates. The mean field improvement coefficient is calculated from the plaquette. Further details on the Hamiltonian used are described in [6]. The bare mass parameter aM_b^0 for the $\overline{b}c$ states was chosen to be 1.22. From [10] this choice of the bare quark mass parameter is consistent, within quenching errors, with the b-quark mass when defined by the kinetic mass of the Υ, and the scale determined from the $\Upsilon' - \Upsilon$ splitting. As the splitting of equation (4) should be most sensitive to a variation in the b-quark mass, we use this scale to also fix the charm quark mass. In the case of the B-like states we chose a range of values for aM_b^0 from 1.1 to 1.3. A linear interpolation is carried out to determine the pole mass for the above bare parameter.

The relativistic Lagrangian used was the Sheikholeslami-Wohlert action [17]. For the D, $\overline{c}c$ and $\overline{b}c$ states, the value of the improvement coefficient, c_{SW}, in this action was determined by the tadpole improvement procedure [21]. For the B states, a slightly different choice of c_{SW} was used, determined by imposing PCAC for light quark masses [15]. For this very fine lattice spacing, the masses derived from these different prescriptions for c_{SW} are indistinguishable [23]. For the B and D, a range of light quark masses were used, and then extrapolated to the chiral limit.

In order to determine the charm quark mass we use the kinetic mass of the D_s [13, 14] (requoted in table 4 for clarity). This is obtained from correlation functions with one SWP with either $\kappa = 0.126$ or 0.132 and one SWP with a κ corresponding to the strange quark mass. It can be demonstrated that the kinetic masses of these correlation functions differ from the pole masses by only a few percent, so ambiguities in the definition of the charm quark mass are minimised. In particular we find that the charm quark mass is close to a bare quark mass equivalent to $\kappa = 0.126$, so the interpolation is relatively mild.

In the case of the $\overline{b}c$-like states, Coulomb gauge fixed correlation functions were computed using sources optimised to select the ground ${}^1S_0$ state and its first radial excitation. The typical radius for these functions lay in the range of 3 to 5. For $\kappa = 0.126$ the correlation functions were computed with 201 configurations and for $\kappa = 0.132$ 140 configurations were used. As a Green's function calculated using the NRQCD Hamiltonian only contains upper spin components, at $\kappa = 0.126$ the gauge field was also time reversed so as to obtain correlation functions using the lower spin components of the relativistic propagator and improve the statistics.

Correlation functions for other states in the spectrum, that is the ${}^3S_1$, ${}^1P_1$ and ${}^3P_{(2,1,0)}$ were also calculated. The absolute binding energies of the ${}^1P_1$ and ${}^3P_{1,0}$ were calculated while the ${}^3S_1$ and ${}^3P_2$ energies were determined from single exponential ratio fits relative to the ${}^1S_0$ and ${}^3P_0$ respectively.

For the B-like states, gauge invariant correlation functions were computed with 68 configurations. Smeared sources were computed using Jacobi smearing [16].

The fit criteria for obtaining the pole masses of $\overline{b}c$ states were the same as those as used in [22], using multi-exponential fits. The B-like states were fitted to a single exponential. The gauge configurations and SWP's were calculated on a Cray T3D at the Edinburgh Parallel Computing Centre (EPCC) while the Coulomb gauge fixing, NRGF's and correlation functions were computed on a Cray J90 at the same site.

4 Results

4.1 The ground state mass

The pole masses calculated for the various heavy-heavy states are listed in in table 1, as well the pole masses for charmonia listed in [13, 14]. The "pole mass" for the Υ obtained from degenerate NRGF's is listed in table 1 [10]. The chirally extrapolated results for D and B are quoted in tables 2 and 3.

From [10] the choice of the bare quark mass parameter of 1.22 is consistent, within quenching errors, with the b-quark mass. On the other hand, the choice of $\kappa = 0.126$ for the relativistic quark is not quite consistent with the charm quark mass. By performing a slight linear extrapolation on the mass difference defined in (4) using the kinetic mass, we find

$$am_{1B_c} - \frac{1}{2}(am_{1\eta_c} + am_{1\eta_b}) = 0.0557(13) \ , \quad (6)$$

and

$$am_{1B_c} - (am_{1B} + am_{1D}) = -0.2480(74) \ . \quad (7)$$

As we can see, these two differences vary by a large degree, having a different sign from each other. When converted into a dimensionful scale, they vary by approximately 1 GeV. In the case of equation (4), the choice of scale is reasonably clear; in order to be consistent in the determination of the quark masses, one should employ the scale defined via the $\Upsilon' - \Upsilon$ splitting, which for this choice of action and parameters is $a^{-1} = 3.52(14)$ GeV [10]. In the case of equation (5), the choice is less clear. For the heavy-light masses one would normally use a different definition of the lattice spacing, for example that defined from the string tension (ideally one would use the $2S - 1S$ or $1P - 1S$ split for the appropriate heavy-light combination, but the statistical error is too large in this case). Nonetheless the above argument for the heavy-heavy difference indicates that the lattice spacing from the $\Upsilon' - \Upsilon$ splitting should be used here as well. The resulting estimate for M_{B_c} from equation (7) varies by approximately 200 MeV solely by the choice of this ambiguity in the lattice spacing. The result for the mass from equation (6) lies at the centre of this interval. We therefore take the mass determined from equation (6) as our best estimate of the central value and take the variation with respect to the scale for equation (7) as a conservative estimate of the systematic error due to quenching and quark mass ambiguities.

We also require an estimate for the hyperfine splitting for $\Upsilon - \eta_b$ as experimental data for M_{η_b} is not yet available. Quenched lattice studies of the charmonium spectrum have always underestimated the hyperfine splitting (as the central charge from the Coulombic term is smaller than usual for QCD where $n_f = 0$). In order to estimate the size of this splitting we use the value of the hyperfine splitting determined in [10] in conjunction with the the potential model results quoted in [4]. From this we estimate the $\Upsilon - \eta_b$ splitting to be 60(30)MeV. Hence we take $M_{\eta_b} = 9.400(30)$ GeV. Using these parameters, one finds :

$$M_{B_c} = 6.386(9)(98)(15) \;\; \text{GeV} \;\; , \tag{8}$$

where the first error is statistical, the second is the systematic error due to quenching and the third is the systematic error in the estimation of M_{η_b}

4.2 The spectrum

The absolute or relative binding energies for the higher states of the spectrum are listed in table 5. In figure 1 we plot these results along with the final lattice results at $\beta = 5.7$ quoted in [6] and the potential model predictions in [4], with the scale determined from the spin averaged $1P - 1S$ splitting, which has the smallest statistical errors. In general these results are in broad agreement with each other, although there does appear to be a significant difference in the hyperfine splitting between the lattice results and the potential model calculation.

5 Conclusions

In this letter we have presented a calculation of the ground state mass of the B_c and the immediate spectrum of states. Although the calculation has been carried in the quenched approximation, by using two different possible derivations of the ground state mass which use a wide range of different possible quark masses, we believe we have conservatively estimated its effect.

The results for the spectrum are, for the most part, in broad agreement with the results of [4] and [6]. The ground state mass quoted here is consistent with previous lattice calculations [6, 11, 25] in the quenched approximation calculated using the kinetic mass definition, using NRGF's for both the charm and bottom quarks on coarser lattices. QCD sum rule calculations which are listed in [26] predict that M_{B_c} lies in the interval of 6.3-6.5 GeV. Kwong and Rosner [27] surveyed the phenomenological techniques used in determining M_{B_c} and estimated it to lie in the range of 6.194 GeV to 6.292 GeV. Most phenomenological estimates for the mass have been taken to lie in the centre of this interval [4] although more recently other work by Fulcher [5] has suggested it to be slightly larger at $6.286\,^{+15}_{-6}$ GeV. Nonetheless, the mass we have determined is certainly consistent with the present estimate of the mass from the CDF collaboration [1] of $M_{B_c} = 6.40 \pm 0.39(stat.) \pm 0.13(sys.)$GeV.

Despite the systematic error due to the hyperfine splitting in $\bar{b}b$ sector, the largest error is the effect of quenching and quark mass ambiguities. Eliminating this approximation by the use of *dynamical* configurations, where sea quark effects are included appears to be the most important step in reducing the error on this mass. Likewise, a more complete understanding of how the differences of equations (4) and (5) behave as a function of the heavy quark masses also needs to be carried out. Such configurations (albeit on coarser lattices than used here) are now available and a measurement of the spin-averaged B_c would be the next course of action.

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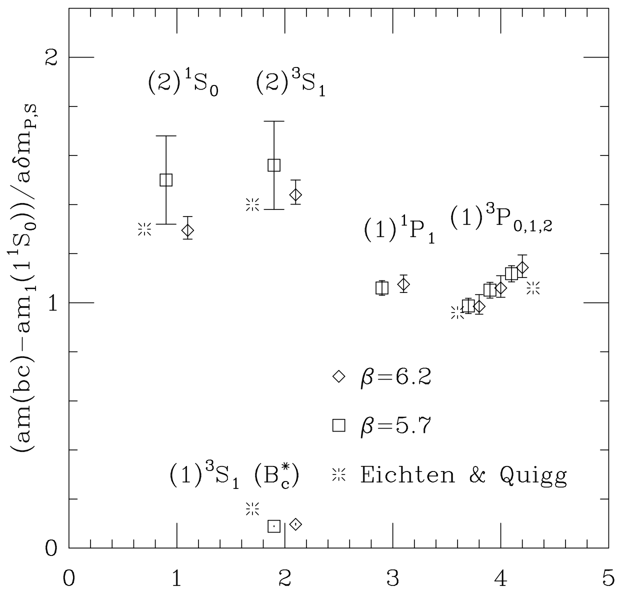

Figure 1: A comparison of the spectrum of states in this letter with other calculations. The results denoted by the square and burst were originally quoted in, respectively in [6] and [4]. The diamond points represent the results determined in this paper. To eliminate the scale for the lattice calculations, the spin averaged $1P - 1S$ splitting has been used. The $2E$ and $2T$ labels for the $^3P_2$ states represent different possible representations in the correlation functions that have overlap with that state.

κ	aM_b	$am_1(\overline{c}c)$	$am_1(\overline{b}c)$	$am_1(\overline{b}b)$
0.126	1.22	1.035(1)	0.7207(9)	0.3027(3)
0.132	1.22	0.661(1)	0.5532(18)	

Table 1: Pole masses determined from correlation functions composed of SWP's or NRGF's, for heavy-heavy states.

κ	$am_1(D)$
0.132	0.4332(15)
0.126	0.6424(19)

Table 2: Chirally extrapolated pole masses determined for D-like states.

aM_b	$am_1(B)$
1.1	0.3192(92)
1.2	0.3354(96)
1.3	0.347(10)

Table 3: Chirally extrapolated pole masses determined for B-like states.

κ	$a(\Upsilon' - \Upsilon)/am_2(\overline{c}s)$
0.126	0.255(16)
0.132	0.389(23)

Table 4: Inverse kinetic masses of heavy-strange correlation functions composed SWF's and the heavy is in the region of charm. The scale determined from the $\Upsilon' - \Upsilon$ splitting is chosen in order to maintain consistency with the b-quark mass definition. The physical ratio is 0.283(3).

	$\kappa = 0.126$	$\kappa = 0.132$	$am_{\overline{b}c}(m_2(D_s))$
$aE(^3S_1) - aE(^1S_0)$	$0.0127\pm^3_3$	$0.0145\pm^4_5$	$0.0131\pm^3_3$
$aE((2)^1S_0)$	$0.886\pm^4_3$	$0.756\pm^{11}_7$	$0.854\pm^6_4$
$aE((2)^3S_1)$	$0.906\pm^4_3$	$0.775\pm^9_8$	$0.876\pm^5_4$
$aE(^1P_1)$	$0.864\pm^5_3$	$0.701\pm^6_6$	$0.822\pm^5_4$
$aE(^3P_0)$	$0.852\pm^4_3$	$0.691\pm^4_6$	$0.814\pm^4_4$
$aE(^3P_{2T}) - aE(^3P_0)$	$0.021\pm^2_2$	$0.018\pm^5_3$	$0.020\pm^3_2$
$aE(^3P_{2E}) - aE(^3P_0)$	$0.021\pm^2_2$	$0.027\pm^3_2$	$0.022\pm^2_2$
$aE(^3P_1)$	$0.861\pm^5_4$	$0.703\pm^6_5$	$0.822\pm^5_4$

Table 5: Spectrum of states for bare quark masses and the interpolated estimate at the charm mass.