Lattice Calculation of the Penguin Diagram Decay $B \to K^*\gamma$

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We calculate the leading-order matrix element for the decay $B \to K^*\gamma$ in the quenched approximation of lattice QCD on a $24^3 \times 48$ lattice at $\beta = 6.2$, using an $O(a)$-improved fermion action. Extrapolating to physical quark masses gives an on-shell form factor of $T_1(q^2{=}0) = 0.15^{+12}_{-14}$ (stat). We find T_1 is approximately independent of the spectator quark mass and extract $T_1(q^2{=}0) = 0.15^{+5}_{-4}$ if this independence is assumed. We find the results to be consistent (in the standard model) with the CLEO experimental branching ratio of $B(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$.

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Theoretical interest in the rare decay $B \to K^*\gamma$ as a test of the standard model has recently been renewed by the experimental results of the CLEO collaboration [1]. For the first time, this mode has been positively identified and a preliminary determination of its branching ratio given.

The significance of $B \to K^*\gamma$ arises from the underlying flavor-changing quark-level process $b \to s\gamma$, which first occurs through penguin-type diagrams at one-loop level in the standard model. The process is also sensitive to new physics appearing as virtual particles in the internal loops. Existing bounds on the $b \to s\gamma$ branching ratio have been used to place constraints on supersymmetry [2–4] and other extensions of the standard model [5,6].

In order to compare the experimental branching ratio with a theoretical prediction it is necessary to know the relevant hadronic matrix elements. These have been estimated using a wide range of methods, including relativistic and nonrelativistic quark models [7–9], two-point and three-point QCD sum rules [10–13], and heavy quark symmetry [14], but there remains some disagreement between the different results. It is therefore of interest to perform a direct calculation of the matrix elements using lattice QCD. The viability of the lattice approach was first demonstrated by the work of Bernard, Hsieh, and Soni [15] in 1991.

In the leading-log approximation the $B \to K^*\gamma$ transition is caused by a single chiral magnetic moment operator from the effective weak Hamiltonian. In the notation of Grinstein, Springer, and Wise [16] this is

$$O_7 = \frac{e}{16\pi^2} m_b \,\bar{s}\sigma_{\mu\nu}\frac{1}{2}(1+\gamma_5)b\,F^{\mu\nu}, \tag{1}$$

with an on-shell matrix element given by

$$\mathcal{M} = \frac{eG_F m_b}{2\sqrt{2}\pi^2} C_7(m_b) V_{tb}V_{ts}^* \eta^{\mu *}\langle K^*|\bar{s}\sigma_{\mu\nu}q^\nu b_R|B\rangle, \tag{2}$$

where q and η are the momentum and polarization of the emitted photon. The coefficient $C_7(m_b)$ arises from the mixing of O_7 with other effective operators in running the scale down from M_W to m_b. The anomalous dimension matrix of all the effective operators at the one-loop level has been calculated by several groups and is now well understood [17].

Following Bernard *et al.* the general matrix element can be parametrized in terms of the momentum, k, and polarization, ϵ, of the K^*, and the momentum, p, of the B meson, using three form factors, T_1, T_2, and T_3, where T_1 is chosen to be real, so that T_2 and T_3 are purely imaginary,

$$\langle K^*(k,\epsilon)|J_\mu|B(p)\rangle = C^1_\mu T_1(q^2) + C^2_\mu T_2(q^2) + C^3_\mu T_3(q^2), \tag{3}$$

$$J_\mu = \bar{s}\sigma_{\mu\nu}q^\nu b_R, \quad q = p - k, \tag{4}$$

$$C^1_\mu = 2\varepsilon_{\mu\nu\lambda\rho}\epsilon^\nu p^\lambda k^\rho, \tag{5}$$

$$C^2_\mu = \epsilon_\mu(m_B^2 - m_{K^*}^2) - \epsilon\cdot q(p+k)_\mu, \tag{6}$$

$$C^3_\mu = \epsilon\cdot q\left(q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2}(p+k)_\mu\right). \tag{7}$$

The on-shell ($q^2 = 0$) matrix element depends on T_1 only, since $T_2(q^2{=}0) = -iT_1(q^2{=}0)$ and the coefficient of T_3 is zero. Performing the necessary phase space integral and sums over polarization vectors gives the decay width for $B \to K^*\gamma$,

$$\Gamma(B \to K^*\gamma) = \frac{\alpha}{8\pi^4} m_b^2 G_F^2 m_B^3 \left(1 - \frac{m_{K^*}^2}{m_B^2}\right)^3 |V_{tb}V_{ts}^*|^2 |C_7(m_b)|^2 |T_1(q^2{=}0)|^2. \tag{8}$$

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By computing the matrix elements on the lattice for various q^2, the on-shell value of the form factor $T_1(0)$ can be obtained by interpolation.

We work in the quenched approximation on a $24^3 \times 48$ lattice at $\beta = 6.2$, which corresponds to an inverse lattice spacing $a^{-1} = 2.73(5)$ GeV, evaluated by measuring the string tension [18]. Our calculation is performed on sixty SU(3) gauge field configurations (for details see Refs. [18] and [19]). The quark propagators are calculated using an $O(a)$-improved Wilson fermion action [20]. We use gauge-invariant smeared sources for the heavy quark propagators with an rms smearing radius of 5.2 [21]. Local sources are used for the light quark propagators.

As the mass of the b quark is almost twice the inverse lattice spacing, direct computation of a b-quark propagator is not feasible. We therefore compute heavy-quark propagators with masses in the region of the charm-quark mass, and extrapolate.

Our statistical errors are calculated according to the bootstrap procedure described in Ref. [18], using 250 bootstrap samples.

To obtain the matrix element $\langle V(k)|\bar{s}\sigma_{\mu\nu}b|P(p)\rangle$, we calculate a ratio of three-point and two-point correlators,

$$C_{\rho\mu\nu}(t,t_f,\mathbf{p},\mathbf{q}) = \frac{C^{3pt}_{\rho\mu\nu}(t,t_f,\mathbf{p},\mathbf{q})}{C^{2pt}_P(t_f-t,\mathbf{p})C^{2pt}_V(t,\mathbf{p}-\mathbf{q})}, \quad (9)$$

where

$$C^{3pt}_{\rho\mu\nu}(t,t_f,\mathbf{p},\mathbf{q}) = \sum_{\mathbf{x},\mathbf{y}} e^{i\mathbf{p}\cdot\mathbf{x}}e^{-i\mathbf{q}\cdot\mathbf{y}}\langle J^\dagger_P(t_f,\mathbf{x})T_{\mu\nu}(t,\mathbf{y})J_{V\rho}(0)\rangle$$
$$\underset{t,t_f-t\to\infty}{\longrightarrow} \sum_\epsilon \frac{Z_P}{2E_P}\frac{Z_V}{2E_V}e^{-E_P(t_f-t)}e^{-E_V t}\epsilon_\rho\langle P(p)|\bar{b}\sigma_{\mu\nu}s|V(k,\epsilon)\rangle, \quad (10)$$

and

$$C^{2pt}_P(t,\mathbf{p}) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}}\langle J^\dagger_P(t,\mathbf{x})J_P(0)\rangle \underset{t\to\infty}{\longrightarrow} \frac{Z_P^2}{2E_P}e^{-E_P t},$$
$$C^{2pt}_V(t,\mathbf{k}) = -\tfrac{1}{3}\sum_{\mathbf{x}} e^{i\mathbf{k}\cdot\mathbf{x}}\langle J^\dagger_{V\sigma}(t,\mathbf{x})J^\sigma_V(0)\rangle$$
$$\underset{t\to\infty}{\longrightarrow} \frac{Z_V^2}{2E_V}e^{-E_V t}, \quad (11)$$

with J_P and J_V interpolating fields for the pseudoscalar and vector mesons, respectively. $T_{\mu\nu}$ is the $O(a)$-improved version of the operator $\bar{b}\sigma_{\mu\nu}s$ [22]. The full matrix elements can then be derived by using the relation $\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\epsilon_{\mu\nu\lambda\rho}\sigma^{\lambda\rho}$. We employ time reversal symmetry to obtain the correctly ordered matrix element, $\langle V(k)|\bar{s}\sigma_{\mu\nu}b|P(p)\rangle$. To evaluate these correlators, we use the standard source method [23]. We choose $t_f = 24$ and symmetrize the correlators about that point using Euclidean time reversal [24]. We evaluate $C_{\rho\mu\nu}$ for three values of the light quark mass ($\kappa_l = 0.141\,44, 0.142\,26, 0.142\,62$), two values of the strange quark mass ($\kappa_s = 0.141\,44, 0.142\,26$) which straddle the physical value (given by $\kappa_s^{\text{phys}} = 0.1419(1)$ [19]), and two values of the heavy quark mass ($\kappa_h = 0.121, 0.129$). We employ two values of the B meson momentum $[(12a/\pi)\mathbf{p} = (0,0,0), (1,0,0)]$, and seventeen values of the momentum, $\mathbf{q}$, injected at the operator, with magnitudes between 0 and $2\pi/12a$. To improve statistics we average over all equivalent momenta, and utilize the discrete symmetries C and P, where possible.

Provided the three points in the correlators of Eq. (9) are sufficiently separated in time, the ground state contribution to the ratio dominates:

$$C_{\rho\mu\nu} \underset{t,t_f-t\to\infty}{\longrightarrow} \frac{1}{Z_P Z_V}\sum_\epsilon \epsilon_\rho\langle V(k,\epsilon)|\bar{s}\sigma_{\mu\nu}b|P(p)\rangle + \cdots \quad (12)$$

and $C_{\rho\mu\nu}$ approaches a plateau. The factors Z_P, Z_V and the energies of the pseudoscalar and vector particles are obtained from fits to two-point Euclidean correlators.

The form factor T_1 can be conveniently extracted from the matrix elements by considering different components of the relation

$$4(k^\alpha p^\beta - p^\alpha k^\beta)T_1(q^2) = \varepsilon^{\alpha\beta\rho\mu}\sum_\epsilon \epsilon_\rho\langle V(k,\epsilon)|\bar{s}\sigma_{\mu\nu}b|P(p)\rangle q^\nu. \quad (13)$$

We see a plateau in T_1 about $t = 12$, and fit $T_1(t;\mathbf{p},\mathbf{q})$ to a constant for $t = 11, 12, 13$, where correlations are maintained between all of the time slices. The use of smeared operators for the heavy quarks provides a very clean signal, with stable plateaus forming before time slice 11. Data with initial or final momenta greater than $(\pi/12a)\sqrt{2}$ are excluded, because they have larger statistical and systematic uncertainties.

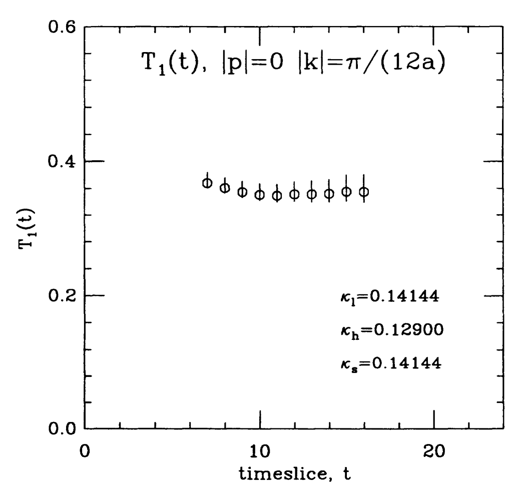

FIG. 1. T_1 vs time slice, t. (For computational reasons, only time slices 7–16 were stored.)

The data for the heaviest of our light quarks, $\kappa_l = \kappa_s = 0.141\,44$, with the smallest statistical errors, are shown in Fig. 1.

We fit $T_1(q^2)$ to a linear model in order to obtain the on-shell form factor $T_1(q^2{=}0)$,

$$T_1(q^2) = a + bq^2, \tag{14}$$

allowing for correlations between the energies of the vector and pseudoscalar particles and T_1 at each q^2. For our range of masses and momenta the differences between linear and pole model fits are small. The data and fit for $\kappa_l = \kappa_s = 0.141\,44$ are shown in Fig. 2.

The light quark mass is set to zero by a correlated chiral extrapolation to $\kappa_l = \kappa_{\text{crit}}$ [$= 0.143\,15(2)$]. We assume that the on-shell T_1 varies with the light kappa values, κ_l, according to a linear model,

$$T_1(\kappa_s, \kappa_h, \kappa_l) = T_1^{\text{crit}}(\kappa_s, \kappa_h) + \Delta_l(\kappa_s, \kappa_h)\left(\frac{1}{\kappa_l} - \frac{1}{\kappa_{\text{crit}}}\right). \tag{15}$$

The strange quark mass is set to its physical value by interpolation [$\kappa_s = \kappa_s^{\text{phys}} = 0.1419(1)$].

The momentum carried by the vector particle is of the order of the mass of the pseudoscalar and therefore heavy quark effective theory does not provide a scaling relationship [25]. As the form factor has only been evaluated at two different pseudoscalar masses, an investigation of the behavior of $T_1(q^2{=}0)$ as a function of m_P cannot be carried out. In this analysis, we therefore perform a naive extrapolation from the two pseudoscalar masses up to m_B by using the form

$$T_1(q^2{=}0; m_P) = A + \frac{B}{m_P}. \tag{16}$$

After performing this extrapolation, we obtain $T_1(q^2{=}0; m_B) = 0.15^{+12}_{-14}$, where the quoted error is purely statistical, and does not include systematic errors.

The finite renormalization needed for the lattice-continuum matching of the $\sigma_{\mu\nu}$ operator has been calculated [26] but has a negligible effect here ($\sim 2\%$) and is not included.

We note that the slopes of the form factor T_1 with respect to κ_l in the chiral extrapolations are consistent with zero (Fig. 3), which indicates that $T_1(\kappa_s, \kappa_h, \kappa_l)$ is almost independent of κ_l. However, this behavior occurs only for the spectator quark, and is not seen to the same extent for the interacting strange quark. We explore this by fitting T_1 to a constant for the three values of κ_l. We find that the χ^2 per degree of freedom is comparable to the original linear model, indicating that the model is statistically valid. Using this approach, the final statistical error is significantly reduced, and we obtain $T_1(q^2{=}0; m_B) = 0.15^{+5}_{-4}$. Given the unknown systematic errors in the calculation, and in particular those resulting from extrapolating $T_1(q^2{=}0; m_p \approx m_D)$ to $T_1(q^2{=}0; m_B)$, this value should only be taken as a guide at the present stage. The results for T_1, using both analysis procedures, are shown in Fig. 4.

In this Letter we have reported on an *ab initio* computation of the form factor for the decay $B \to K^*\gamma$. The large number of gauge configurations used in this calculation enables an extrapolation to the appropriate masses to be made and gives a statistically meaningful result. To compare this result with experiment we convert the preliminary branching ratio from CLEO, $B(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ based on 13 events, into its corresponding T_1 form factor, assuming the standard model. We work at the scale $\mu = m_b = 4.39$ GeV and use values from the Particle Data Group [27], combined with Eq. (8). Setting the mass of the top quark to be $m_t = 100$, 150, and 200 GeV we find T_1^{exp} to be 0.23(6), 0.21(5), and 0.19(5), respectively. The two lattice results are consistent with these experimental numbers within statistical

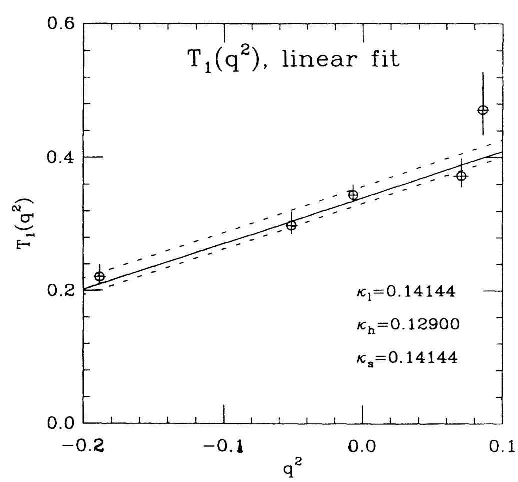

FIG. 2. $T_1(q^2)$, with a linear fit. The dotted lines represent the 68% confidence levels of the fit at $q^2 = 0$.

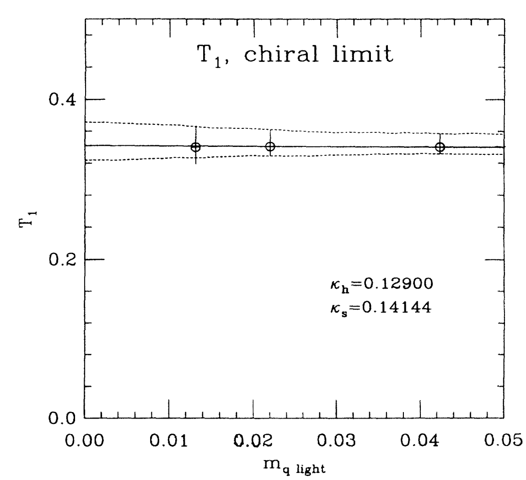

FIG. 3. Chiral extrapolation of $T_1(q^2{=}0)$. The dotted lines indicate the 68% confidence levels of the fit. $m_{q\text{ light}}$ is the lattice pole mass.

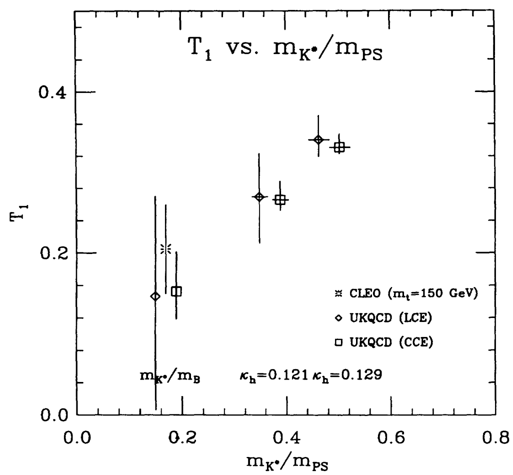

FIG. 4. Extrapolation of $T_1(q^2=0)$ to m_B. LCE, using linear chiral extrapolation; CCE, using constant chiral extrapolation for the spectator quark. (N.B. for clarity, the LCE and CCE points have been displaced horizontally by 0.02 to the left and right, respectively.)

errors. This is also shown in Fig. 4.

Although the systematic errors of this calculation resulting from the extrapolation of T_1 to the appropriate mass scales, the quenched approximation, finite volume, and other lattice artifacts remain to be explored, we believe that we have shown the phenomenological utility of the lattice for probing the limits of the standard model.

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