Breakdown of the Chiral Luttinger Liquid in One Dimension

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We have developed a fermionic boot-strap method to solve a class of chiral one-dimensional fermion models. Using this scheme, we show that Luttinger liquid behavior in a gas of four interacting chiral Majorana fermions is highly sensitive to the velocity degeneracy. Upon changing the velocity of one chiral fermion, a sharp bound (or antibound) state splits off from the original Luttinger liquid continuum, cutting off the x-ray singularity to form a broad incoherent excitation with a lifetime that grows linearly with frequency.

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The anomalous normal state behavior discovered in the cuprate superconductors has stimulated enormous interest in the possibility of new kinds of electronic fluid that might provide an alternative to Fermi liquid behavior. The classic model for non-Fermi liquid behavior is provided by the one-dimensional electron gas, where the generic fixed point behavior is a Luttinger liquid [1]. This is due to two special 1D features: a Fermi surface which consists of just two points $\pm k_f$ where the electrons interact very strongly and, second, the special kinematics of one dimension, whereby energy and momentum conservation impose a *single* constraint on scattering processes near the Fermi surface, giving rise to a qualitative enhancement in scattering phase space. These two factors cause the electron to lose its eigenstate status to the collective spin and charge density bosonic modes.

In this Letter, we introduce a generalization of the Tomonaga Luttinger model, written

$$H = \int dx \bigg\{ -i \sum_{a=0}^{3} v_a \Psi^{(a)}(x)\partial_x \Psi^{(a)}(x) + g\Psi^{(0)}(x)\Psi^{(1)}(x)\Psi^{(2)}(x)\Psi^{(3)}(x)\bigg\}, \quad (1)$$

where the $\Psi^{(a)}$ $[a = (0,1,2,3)]$ represent four real (Majorana) fermions such that $\Psi^{(a)}(x) = \Psi^{(a)\dagger}(x)$. The fermions are chiral (right movers, say): this is a crucial property that ensures the system stays gapless, and allows for exact solutions in a number of cases. In the special case where all velocities are the same, this is the chiral Luttinger model with a SO(4) symmetry, where the four Majorana modes can be associated with the spin up and down, electron and hole excitations of the Fermi surface.

Here, we focus on the particular case of the model (1) where three of the velocities are equal: $v_{1,2,3} = v \neq v_0$, giving rise to a Hamiltonian with a reduced SO(3) symmetry. We shall show that once we break the degeneracy of the velocities, a qualitatively new type of behavior develops. This model is physically motivated in two ways:

(i) The transport phenomenology of the cuprates [2] suggests that electrons near the Fermi surface might divide up into two Majorana modes with different scattering rates and dispersion. To date, this kind of behavior has been realized only in impurity models [3] and their infinite dimensional generalization [4]. We shall show that by breaking the velocity degeneracy of the original chiral Luttinger model, we obtain a one-dimensional realization of this behavior: a sharp Majorana mode intimately coexisting with an incoherent continuum of excitations.

(ii) Frahm *et al.* [5] have recently proposed that the low energy Hamiltonian of an integrable spin-1 Heisenberg chain doped with mobile spin-1/2 holes is given by (1), with one Majorana fermion $\Psi^{(0)}$ describing a slow moving excitation coming from the dopant, interacting with three rapidly moving Majorana fermions that describe the spin-1 excitations of the spin chain. Such doped spin-chain models may be relevant to certain experimental systems such as $Y_{2-x}Ca_xBaNiO_5$ [6].

Whereas the SO(4) model can be treated by bosonization [1,7], by changing the velocity of a *single* Majorana fermion we introduce a nonquadratic term into the bosonized Hamiltonian that precludes a simple separation in terms of Gaussian spin and charge bosons. We shall show, using a new fermionic approach to the problem, that when we break the degeneracy of the velocities, the x-ray catastrophe associated with the Luttinger liquid behavior is cut off by the velocity shift. The "hornlike" feature in the spectral weight of the Luttinger liquid is then split into a sharp bound (or antibound) state that coexists with an incoherent spin-charge decoupled continuum, with a scattering rate linear in frequency. We summarize these results in Fig. 1.

Our approach is based on the observation that for these models, the renormalized skeleton self-energy, containing full propagators, but no vertex corrections (Fig. 2), is *exact*, so that

$$\Sigma_a(x,\tau) = g^2 G_b(x,\tau) G_c(x,\tau) G_d(x,\tau), \quad (2)$$

where the G_a are the exact, interacting Greens functions and $\{a,b,c,d\}$ is a cyclic permutation of $\{0,1,2,3\}$. These equations close with the usual relations,

$$\Sigma_a(k,\omega) = (i\omega - v_a k) - G_a(k,\omega)^{-1} \quad (a = 0,1,2,3). \quad (3)$$

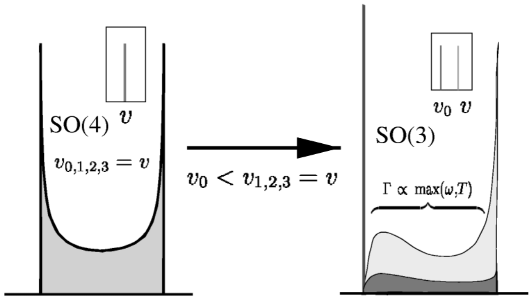


FIG. 1. Schematic diagram showing the evolution of the spectral weight as we introduce velocity difference to the fermions. Inset indicates the bare spectral function, without interactions.

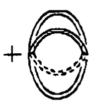
Equations (2) and (3) together define a boot-strap method to solve the problem.

Provided that we have a minimal SO(3) symmetry, then the three current densities $j^a(x) = -i\epsilon_{abc}\Psi^{(b)}(x)\Psi^{(c)}(x)$ $[a,b,c \in (1,2,3)]$ are conserved classically. Following Dzyaloshinskii and Larkin (1974) [8–10], the continuity equation guarantees that the N point connected current-current correlation functions vanish for $N > 2$ $[\mathbf{x}_i = (x_i, \tau_i)]$,

$$\langle j^a(\mathbf{x}_1) j^a(\mathbf{x}_2) \cdots j^a(\mathbf{x}_N)\rangle_C = 0, \qquad (N > 2). \tag{4}$$

For the noninteracting system, this result leads to the "loop cancellation theorem": for the amplitude associated with a closed fermion loop with $N > 2$ conserved current insertions, the sum over all possible permutations of $\{\mathbf{x}_i\}$ of the current operators must give zero [8–10]. In $d = 1$, this result depends on having a linear or quadratic dispersion. In $d > 1$, this is true only in the asymptotic limit of small momentum and energy transfer. Dzyaloshinskii and Larkin used this theorem to eliminate all diagrams that contain such closed loops, considerably simplifying the vertex function and polarization bubbles.

We use the loop cancellation theorem in a new way, to show that the vertex corrections to the skeleton self-energy (SSE) identically vanish. To illustrate the idea, consider the self-energy of the singlet Majorana mode in the SO(3) model. The Feynman diagrams contributing to the skeleton self-energy are constructed by combining loops with two current insertions. We illustrate this using the fourth order diagram in Fig. 3(iv), but it holds to all orders in perturbation theory. Nonskeleton contributions to the self-energy involve diagrams with loops containing more than two current insertions. In these diagrams, the sum over all permutations of the current insertions into the loops is automatically zero, as illustrated to order g^4 in Figs. 3(i)–3(iii). By generalizing these results to all higher order graphs, we can show that the vertex corrections to the self-energy Σ_0 of the singlet Majorana fermion cancel, leaving the fully renormalized SSE as the only remaining contribution. To complete our proof, we now use the *full* Kadanoff-Baym free energy functional $F[G] = -T\{\mathrm{Tr}\ln[G^{-1}] + \mathrm{Tr}[\Sigma G]\} + Y[G]$, where $Y[G]$ is the sum of all skeleton diagrams [11]. Now $\delta F[G]/\delta G_a = 0$ generates the equations for the self-energies, and, in particular, $\delta F[G]/\delta G_0$ must generate the skeleton self-energy Σ_0. This requires that the Kadanoff-Baym free energy functional *truncates* at the leading skeleton diagram,

$$F = -T\{\mathrm{Tr}\ln[G^{-1}] + \mathrm{Tr}[\Sigma G]\} + \tag{5}$$


Finally, by differentiating the free energy functional with respect to the Greens functions $G_{1,2,3}$ of the triplet Majorana fermions, we can show that each triplet self-energy is also given by the corresponding skeleton self-energy.

We now apply this result, using the limiting case of the SO(4) model to check the validity of our results. Our equations are dramatically simplified by seeking solutions to (2) which satisfy a scaling form

$$G_a(x,\tau) = \frac{1}{2\pi i x}\,\mathcal{G}_a(\tau/ix). \tag{6}$$

This form is motivated by the observation that chirality prevents space from acquiring an anomalous dimension. Under a Fourier transform, this scaling form is self-dual,

$$\frac{1}{2\pi i x}\,\mathcal{G}_a(\tau/ix) \overset{F.T.}{\longleftrightarrow} \frac{1}{i\omega}\,\mathcal{G}_a(k/i\omega), \tag{7}$$

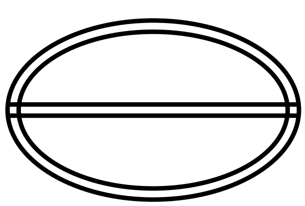
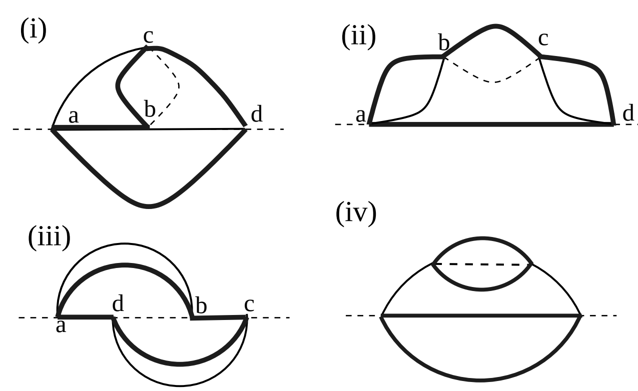


FIG. 2. Skeleton self-energy, where double lines represent full propagators.



(i)+(ii) +(iii) = 0 (Loop Cancellation Theorem).

FIG. 3. Order g^4 diagrams for the SO(3) model, demonstrating how the closed-loop theorem works. Dotted lines indicate bare propagator for the singlet Majorana fermion $\Psi^{(0)}$. Full lines indicate bare propagator for the triplet Majorana fermions. Bold line highlights the loop with 2 or 4 current insertions.

where the *same* function $\mathcal{G}_a$ appears on both sides. Inserting Eq. (7) into (3) and Fourier transforming,

$$\Sigma_a(x,\tau) = -\frac{1}{2\pi i x}\frac{d^2}{du^2}[1 - v_a u - 1/\mathcal{G}_a(u)]_{u=\tau/ix}\,. \tag{8}$$

Since the bare Green function scaling form is $1/\mathcal{G}_a^0(u) = 1 - v_a u$, it does not contribute to the self-energy. Combining Eqs. (2) and (8),

$$\frac{d^2}{du^2}[\mathcal{G}_a(u)]^{-1} = -(g/2\pi)^2\mathcal{G}_b(u)\mathcal{G}_c(u)\mathcal{G}_d(u)\,, \tag{9}$$

where $\{a,b,c,d\}$ are cyclic permutations of $\{0,1,2,3\}$. The boundary conditions are $\mathcal{G}_a(0) = 1$ and $\mathcal{G}_a'(0) = v_a$, derived from the physical requirement that at high frequencies, the fermions are free particles, moving with the *bare* velocity v_a. For the SO(4) model, where $\mathcal{G}_a(u) \equiv \mathcal{G}(u)$ $(a = 0,\dots,3)$, Eq. (9) reduces to a single differential equation, for which the solution is

$$G(x,\tau) = \frac{1}{2\pi i x}[1 - v_+\tau/ix]^{-1/2}[1 - v_-\tau/ix]^{-1/2}, \tag{10}$$

where $v_\pm = v \pm (g/2\pi)$ and v is the bare velocity. Identical results are obtained by bosonization, confirming that the skeleton self-energy is exact for the Luttinger model.

In the SO(4) model, the electron spectral weight displays two classic x-ray singularities associated with the decay of the electron into a spinon and holon continuum (Fig. 1) [7]. We now show that if $\Delta v = v - v_0$ is finite, one of these x-ray edge singularities is completely eliminated. If $v_0 < v$, we find that low velocity "Horn," originally with velocity v_-, develops a sharp bound-state pole in the singlet channel, and a broad incoherent excitation in the triplet channel with a lifetime growing linearly in energy. If $v_0 > v$, the high velocity "horn" splits off a singlet antibound state and the triplet channel develops a high-velocity incoherent excitation (Fig. 5). The sharp bound state in the singlet channel develops once a velocity difference is introduced, because energy and momentum conservation now provide distinct constraints to scattering [unlike in the SO(4) model], leading to much less phase space for $\Psi^{(0)}$ to decay into.

To see this, we must analyze Eq. (9) for the SO(3) case,

$$\frac{d^2}{du^2}\mathcal{G}_3^{-1} = -(g/2\pi)^2(\mathcal{G}_3)^2\mathcal{G}_0\,,$$
$$\frac{d^2}{du^2}\mathcal{G}_0^{-1} = -(g/2\pi)^2(\mathcal{G}_3)^3. \tag{11}$$

A very convenient way to discuss these equations is to map them onto a central force problem. If we write $\mathbf{r} = (\mathcal{G}_3^{-1},\mathcal{G}_0^{-1})$, $\mathbf{F} = -(g\mathcal{G}_3/2\pi)^2(\mathcal{G}_0,\mathcal{G}_3)$, then $\ddot{\mathbf{r}} = \mathbf{F}$. By inspection, $\mathbf{r}\times\mathbf{F} = 0$, so the force is radial, thus the "angular momentum," $\mathbf{r}\times\dot{\mathbf{r}} = \Delta v$, is a constant. If we use polar coordinates $(\mathcal{G}_3^{-1},\mathcal{G}_0^{-1}) = r(\cos\theta,\sin\theta)$, the

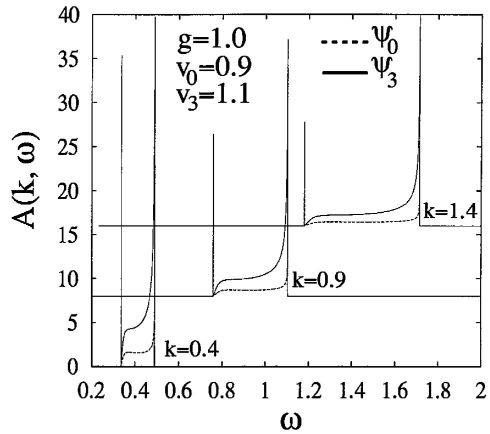


FIG. 4 Spectral weight of the SO(3) model. For clarity, we have shifted up the curves for various momenta by 8 units.

equations for the Green function resemble the motion of a fictitious particle under the influence of an anisotropic central force,

$$\ddot{r} - \frac{\Delta v^2}{r^3} = -(g/2\pi)^2\frac{1}{r^3\cos^3\theta\sin\theta}\,,$$
$$r^2\dot{\theta} = \Delta v\,. \tag{12}$$

The velocity difference $\Delta v = v - v_0$ provides a repulsive centrifugal force. The boundary conditions mean that the "particle" starts out at $r(0) = \sqrt{2}, \theta(0) = \pi/4$. When $\Delta v = 0$, the particle falls directly into the origin, and both $\mathcal{G}_3$ and $\mathcal{G}_0$ diverge with x-ray singularities. However, once Δv is finite, the orbit does not pass through the origin at $u \sim 1/v_+$ or $1/v_-$, thereby eliminating the associated x-ray singularity in the spectral function. The quantity $\zeta = (v - v_0)/g$ plays the role of a coupling constant, and approximate analytic solutions are possible in the limiting cases of small and large ζ.

Suppose first that $\Delta v < 0$. In this case, $\theta \to 0$ at some finite "time" $u = 1/v_0^*$, at which $r = C$ and $\dot{\theta} = \Delta v/C^2$. For $u \sim 1/v_0^*$, it follows that $(r,\theta) = [C, \dot{\theta}(u - 1/v_0^*)]$, from which we can read off the following asymptotics:

$$\mathcal{G}_3(u)^{-1} \sim C, \tag{13}$$

$$\mathcal{G}_0(u)^{-1} \sim (1 - uv_0^*)/Z, \qquad Z = Cv_0^*/|\Delta v|\,. \tag{14}$$

Thus when one Majorana fermion moves faster than the others, an antibound state with spectral weight Z, moving with velocity v_0^*, splits off above the continuum. For $|\Delta v| \gg \frac{|g|}{2\pi}$ interactions can be ignored, so $v_0^* \to v_0$, and $Z \to 1^-$. For $|\Delta v| \ll \frac{|g|}{2\pi}$, the "motion" of the fictitious particle emulates that of the SO(4) model until the angle θ approaches zero. We may estimate v_0^* and C by integrating Eq. (12) with the approximation $r(u) \approx \tilde{r}(u)$, where $\tilde{r} = [2(1 - v_+u)(1 - v_-u)]^{1/2}$ is the SO(4) solution:

$$-\frac{\pi}{4} = \int_0^{1/v_0^*} \frac{\Delta v}{\tilde{r}^2(u)}\, du, \qquad C \approx \tilde{r}(1/v_0^*). \tag{15}$$

This estimate gives (for $|\Delta v| \ll |g|/2\pi$)

$$v_0^* = v_+ + \frac{g}{\pi} \exp - \left|\frac{g}{2\Delta v}\right|, \tag{16}$$

$$Z = \left|\frac{\sqrt{2}\, g}{\pi \Delta v}\right| \exp - \left|\frac{g}{4\Delta v}\right|, \tag{17}$$

indicating that the formation of the sharp antibound state is nonperturbative in the velocity difference. These results can be generalized to the case where $\Delta v > 0$ by replacing $v_+ \to v_-$, $g \to -g$. To illustrate these results further, we have carried out numerical solutions of the differential equations (11) for intermediate values of the coupling constant ζ, using a standard adaptive integration routine. Results are summarized in Figs. 5 and 6.

In summary, we have demonstrated that by breaking the velocity degeneracy of a system of interacting chiral fermions we restrict the scattering phase space in a way which causes a sharp bound or antibound state to split off from the spin-charge continuum, leading to a system with two qualitatively distinct spectral peaks and scattering rates. This is a significant departure from the Luttinger liquid scenario and demonstrating a new class of one-dimensional fixed point behavior.

This new fixed point exhibits properties in common with both Luttinger and Fermi liquids, and is perhaps closest in character to the marginal Fermi liquid phenomenology introduced in the context of cuprate metals [12]. Like the Fermi liquid, there is a sharp quasiparticle bound state, but this coexists with a Luttinger liquidlike continuum which is bounded by two extremal velocities.

Had we chosen to change two Majorana velocities at the same time, so that $v_0 = v_1$ and $v_2 = v_3$, we would have reduced the symmetry still further, to an SO(2) $\times$ SO(2) symmetry. In this case, the Hamiltonian is readily bosonized, giving the results

$$G_3(x,\tau) = \frac{1}{2\pi i x}\left[1 - \frac{v_+\tau}{ix}\right]^{-(1/2)+\gamma}\left[1 - \frac{v_-\tau}{ix}\right]^{-(1/2)-\gamma},$$

$$v_\pm = \frac{1}{2}\{v_0 + v_3 \pm [(v_3 - v_0)^2 + (g/\pi)^2]^{1/2}\}, \tag{18}$$

and $\gamma = \frac{1}{2}(v_3 - v_0)\{(v_3 - v_0)^2 + (g/\pi)^2\}^{-1/2}$. Curiously, this result may also be obtained as a solution of Eq. (9), even though, as far as we can see, the closed-loop cancellation is not sufficient in the case of the SO(2) $\times$ SO(2) model to cancel all vertex corrections. This suggests that a more general cancellation principle is at work, and that the range of validity of our solution may extend to models with a still smaller SO(2) symmetry. To date, we have not been able to prove this result.

Our work raises the question whether this kind of non-Fermi liquid behavior might survive in dimensions higher than one. In higher dimensions energy conservation and momentum conservation are distinct constraints on scattering phase space, and the Luttinger liquid reverts to a Fermi liquid, at least for short-range interactions [9,13]. By contrast, the unusual properties of the SO(3) model have reduced reliance on the coincidence between momentum and energy conservation in scattering processes. In very large dimensions [4], the two lifetime behavior persists in the SO(3) model, but undergoes a crossover to a Fermi liquid asymptotically. The case of small, but finite dimensions is however, still open.

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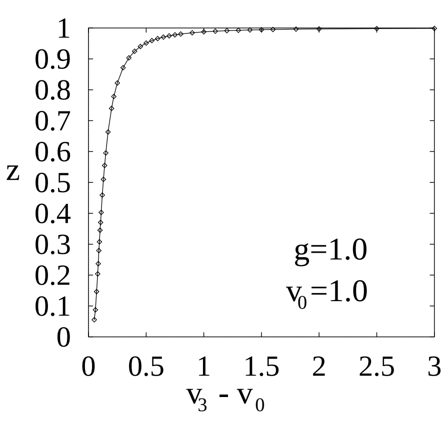

FIG. 5 Quasiparticle weight Z of $\Psi^{(0)}$ in the SO(3) model.

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