

# EDUCATION, BIRTH ORDER, AND FAMILY SIZE\*

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## Abstract

This paper introduces a framework to study parental investments in the presence of birth order preferences and/or human capital cost differentials across children. The framework yields canonical models as special cases and delivers sharp testable predictions concerning how parental investments respond to an exogenous change in family size in the presence of birth order effects. These predictions characterize a generalized quantity-quality trade-off. Danish administrative data confirms our theory's predictions. We find that for any given parity, the human capital profile of children in smaller families dominates that of large families, and that the average child's education decreases as family size increases, even after taking birth order effects into consideration.

Keywords: Quantity-quality trade-off, fertility models, education.

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# 1 Introduction

We study the effect of family size on a child’s education when parents have birth order preferences and/or face human capital cost differentials across children. The so-called “quantity-quality trade-off” (Becker and Lewis, 1973) has been widely studied in the literature. However, despite the long-standing interest, theories of this trade-off have not typically integrated birth order effects in their analyses. The earliest empirical tests of the quantity-quality trade-off assume equal human capital investments for all children and leave no room for birth order, either as a mediating factor, or as an independent determinant of parental investments; see, e.g., Rosenzweig and Wolpin (1980).<sup>1</sup>

Black *et al.* (2005) used large administrative data to document sizeable (negative) birth order effects in educational attainment. They also showed that controlling for birth order weakens an otherwise strong empirical relationship between family size and educational attainment.<sup>2</sup> Thus, they argue that birth order effects, unless appropriately controlled for, confound the effect of family size on parental investments. Estimating family size and birth order effects is challenging because family size reflects parents’ fertility choices, and the size of a family and its birth order configuration are deterministically related. To circumvent the dual endogeneity problem, Black *et al.* (2005) develop a twin-based instrumental variable strategy seeking to identify the (average) family size effect from children born prior to the exogenous change to family size induced by a twin birth.

Rosenzweig and Zhang (2009) pointed out that resource reallocation across children within a household, in response to a twin birth, may bias towards zero the estimated average family size effect that is obtained from Black *et al.* (2005). Such endogenous resource reallocations may arise from differences in endowments between twins and non-twins if parents favor higher endowment offspring. Indeed, using rich data from China, Rosenzweig and Zhang (2009) document that twins and non-twins differ markedly in birth weight, and show that endowment differences lead parents to shift resources from twins to (older) non-twin siblings. Failure to control for endowment differences results in statistically and economically significant downward bias in the estimated average family size effect.<sup>3</sup>

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<sup>1</sup>The literature on family size and education is too large to provide a detailed account here. Becker and Lewis (1973) paved the way for economic studies of parental investments in human capital. Their theoretical framework, under the assumption that parents treat all children equally, predicts that an increase in family size reduces a child’s education. This prediction has been applied and/or tested by numerous studies. Some of the empirical studies that examine the relationship between education and family size include Barro and Becker (1989), Behrman *et al.* (1989), Hanushek (1992), Parish and Willis (1993), Schultz (1997), Ahn *et al.* (1998), Glick *et al.* (2007), Li *et al.* (2008), Qian (2009), Angrist *et al.* (2010), Millet and Wang (2011), Juhn *et al.* (2013) and Oliveira (2016). From a theoretical perspective, , Becker *et al.* (1990), Becker and Tomes (1976), Doepke (2004), and Galor and Weil (2000) study family size and education in models of economic growth, also under the assumption of no birth order predispositions.

<sup>2</sup>Many previous studies of birth order effects lack large representative data and this has undermined their results; see, e.g. Booth and Kee (2009), Behrman and Taubman (1986), Hanushek (1992), Hauser and Sewell (1985), and Iacovou (2008). These previous studies provide a wide range of estimates for the effect of birth order, mostly imprecisely estimated. They are unable to control for family size indicators, indicators for children’s cohorts or for parental cohorts. An exception is Iacovou (2008) but the sample is small and the estimates are subject to considerable attrition bias. Oliveira (2019) documents gender-differences in the effect of birth order on educational attainment. There is also a large birth order literature in psychology (for a review see Eckstein *et al.* (2010)).

<sup>3</sup>Rosenzweig and Zhang (2009) examine a dataset from China with birth weight information not commonly available, measures of parental expectations about schooling attainment and health, and child-specific parental time and expenditures.

With few exceptions, recent empirical work on the influence of family size on investments in children’s human capital have advanced largely without guidance from economic theory. To address this theory-to-evidence gap, we propose and implement new tests of the quantity-quality trade-off when parents have birth order predispositions. Our analytical framework embeds the birth order predispositions considered by [Behrman \*et al.\* \(1982\)](#) and [Behrman and Taubman \(1986\)](#) in models with exogenous family size, into a model of parental choices for family size and human capital investments, as formulated by [Becker and Lewis \(1973\)](#) and [Barro and Becker \(1989\)](#). General parental preferences and investment costs drive the association between human capital and birth order, the *birth order effect*. Differences in investment costs may stem from multiple sources, including endowment heterogeneity. Our framework yields canonical models as special cases, and delivers sharp testable predictions concerning how parental investments respond to an exogenous change in family size in the presence of birth order effects. Moreover, we generalize the insight of [Rosenzweig and Zhang \(2009\)](#) regarding within-family resource allocation and provide an alternative estimation strategy that does not require detailed data on child endowments.

The model’s first prediction assesses how the distribution of education within the household changes with family size. In our theory, an exogenous increase in family size shifts downwards the children’s educational profile in the household so that children of the same birth order, but in larger families, have lower education. In other words, the educational profile of smaller families should lie above the educational profile of larger families. We refer to the difference in human capital profiles between families of different sizes as the *family size effect*. Our model predicts a negative family size effect.

The second prediction examines the difference in average educational attainment between otherwise identical families of different sizes. Changes in this dimension represent changes in the education of the average child in the family and are consistent with unrestricted birth order effects. In our theory, as family size increases, the education of the average child changes due to lower parental resources per child but also due to within household substitution of resources across birth orders, as pointed out in [Rosenzweig and Zhang \(2009\)](#). We provide general conditions under which an exogenous increase in family size reduces parental human capital investments in the average child. We refer to differences in human capital between average children in families of different sizes as the *composite family size effect* because it depends on both the family size effect and on (average) birth order effects. Our model provides conditions for a negative composite family size effect.

The predictions of negative family size and composite family size effects enable tests of a generalized quantity-quality trade-off which account for birth order effects arising through parental birth order predispositions, endowments, or human capital cost differentials across children. Existing empirical work on the relationship between family size and a child’s human capital has focused on testing for negative

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In their data, it is also possible to exploit family size restrictions to expand the set of comparisons across families, i.e., comparisons between first-born twins in areas with strict enforcement of the “one child policy” and two-children families in areas without strict enforcement.

family size effects; see, e.g., [Black \*et al.\* \(2005\)](#), [Angrist \*et al.\* \(2010\)](#), and [Mogstad and Wiswall \(2012b\)](#). Absent birth order effects, our predictions collapse to those of [Rosenzweig and Wolpin \(1980\)](#), and a negative family size effect indeed implies a [Becker and Lewis \(1973\)](#) quantity-quality trade-off. In the presence of birth order effects, however, a negative family size effect is necessary but not sufficient for the existence of a generalized quantity-quality trade-off. The composite family size effect must also be taken into account.

We develop a two-step empirical strategy that identifies birth order, family size and composite family size effects. In the first step we estimate birth order effects using within-family variation in educational attainment which controls for potential omitted variable biases emerging from family-specific heterogeneity. In the second step we estimate the family size effect by regressing average educational attainment within a family (net of average birth order effects) onto family size, using twin births as an instrumental variable for family size. The second step utilizes only between-family variation in educational attainment and is therefore unaffected by any endogenous within-family resource reallocations in response to, say, the birth of a low-endowment sibling. Having estimated birth order and family size effects, we can easily recover the composite family size effect.

While our empirical strategy identifies the effects of birth order, we are unable to separately identify the role of child endowments, parental preferences, and human capital costs in determining birth order effects.<sup>4</sup> Note, however, that we do not need to identify the underlying mechanisms leading to birth order effects to recover family size effects or to test for the presence of a generalized quantity-quality trade-off.

We carry out our empirical analyses using a population-wide comprehensive administrative panel dataset from Denmark. Danish data strongly agrees with the theoretical implications of the model. In terms of birth order, we find that birth order has a strong negative effect on a child’s education, consistent with existing empirical studies ([Black \*et al.\*, 2005](#)). Controlling for family fixed effects and with a linear birth order effect, an additional birth order reduces years of schooling by little less than 0.18 of a year. Findings are larger in magnitude when we allow for nonlinear birth order effects.

We further find that children with the same birth order, but in larger families, have lower education; that is, we estimate negative family size effects. We also find that family size has a strong negative effect on the average education in the household. That is, we find negative composite family size effects. Quantitatively, an additional child reduces the average number of years of schooling in the household by about a tenth of a year. In more flexible specifications, we find evidence that both the family size effect and the composite family size effect increase (in absolute value) with the size of the family. Taken together, our empirical analysis supports the existence of a generalized trade-off between the quantity

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<sup>4</sup>Empirical evidence on the channels through which birth order effects arise is relatively scarce. We have already noted that [Rosenzweig and Zhang \(2009\)](#) use an unusually rich dataset from China to highlight the role of endowments in parental resource allocation. Results in [Pavan \(2015\)](#) suggests that differential parental investments across siblings can account for more than 50 percent of the gap in cognitive skills among siblings. [Price \(2008\)](#) provides evidence of birth order differences in the amount of quality time that children spend with their parents, and [Hotz and Patano \(2015\)](#) show strategic parenting might explain birth order differences in school performance.

and the quality of children, even when children in the family are not treated equally. Formal statistical tests of the model predictions confirm this.

## 2 Theory

The purpose of this section is to derive testable predictions about how parental investments in human capital respond to changes in family size under birth order predispositions. We consider a model of fertility choice in which human capital investments vary with the order in which children are born. For analytical convenience, we treat family size as a continuous variable and assume that parents make all decisions in a single stage of choice. Children are born sequentially but within this single stage. The analysis is carried out at the family level.

**Preferences.** Parents derive utility from their own consumption, the household's family size, and the human capital of their children. Let  $i$  represent the order in which children are born, their birth order, and let  $N$  denote completed family size. Birth order and family size are jointly realized as  $i \in [0, N]$ . A child's human capital is a function of her birth order  $h(i)$ . Parental preferences are gender-neutral and there is no child mortality.

Let  $u(h(i), i)$  represent the parental sub-utility from having a child of birth order  $i$  with human capital  $h(i)$ . We assume that  $u(h, i)$  is increasing and concave in  $h$  but leave the dependence on  $i$  unrestricted. We assume that parents aggregate their children's sub-utilities to obtain a utility index  $U$ . We focus on a CES aggregator

$$U(\{h(i)\}, N) \equiv \left( \int_0^N u(h(i), i)^\rho di \right)^{\alpha/\rho}, \quad (1)$$

where  $\rho$  measures the degree of substitutability between the sub-utilities at different birth orders. The parameter  $\alpha$  is used to ensure concavity in  $U(\{h(i)\}, N)$ . Appendix A lists the technical assumptions we impose on  $\alpha$  and  $\rho$ . The utility index  $U(\{h(i)\}, N)$  represents the total utility associated with  $N$  children whose human capital, as a function of their birth order, is assigned according to the function  $h(i)$ . The utility index  $U(\{h(i)\}, N)$  is increasing in a child's human capital.  $U(\{h(i)\}, N)$  is also increasing in family size, i.e.,  $U_N(\{h(i)\}, N) > 0$ . While total utility increases with parental investments  $U_h(\{h(i)\}, N) > 0$ , the marginal utility of human capital investments is decreasing in family size, i.e.,  $U_{hN}(\{h(i)\}, N) < 0$ ; see (A5). The partial derivatives relevant for comparative statics are available in Appendix A.

We represent the parental utility associated with a parental consumption of  $X$ , a family size of  $N$ , and a utility index  $U$  by

$$\mathcal{U}(X, N, U(\{h(i)\}, N)),$$

where  $U(\{h(i)\}, N)$  is given by (1). Family size  $N$  has a direct effect on the parental utility as well as an

indirect effect in the total utility  $U(\{h(i)\}, N)$ .

Our setting is general and can nest previous models in the literature as special cases. On one hand, [Behrman and Taubman \(1986\)](#) is an early study of the influence of birth order on intrahousehold investments; see also [Behrman \*et al.\* \(1982\)](#). Family size, however, is not a choice variable in these papers. They define earnings  $y(i)$  as a function of a child's endowment  $e(i)$ , their human capital  $h(i)$ , and their birth order  $i$ , i.e.,  $y(i) = f(h(i), e(i), i)$ . In the parental sub-utility function  $u(h(i), i) \equiv u(f(h(i), e(i), i), i)$ , child endowments, intellectual ability, and pure parental preferences (i.e., primogeniture) would produce birth order effects in schooling. [Behrman and Taubman \(1986\)](#) and [Behrman \*et al.\* \(1982\)](#) also proposed a CES aggregator. The CES specification allows for substitutability between the children's utilities. [Behrman \*et al.\* \(1982\)](#) and [Behrman and Taubman \(1986\)](#) associate the parameter  $\rho$  in expression (1) with parental aversion to inequality. When  $\rho = 1$ , parents have no aversion to inequality and simply value the total sum of their children's utilities in  $U(\{h(i)\}, N)$ . When  $\rho \rightarrow -\infty$ , parents are unwilling to accept unequal utilities across children as the aggregator (1) becomes  $U(\{h(i)\}, N) = [\min_{i \in [0, N]} \{u(h(i), i)\}]^\alpha$ .

On the other hand, [Becker and Lewis \(1973\)](#) and [Rosenzweig and Wolpin \(1980\)](#) characterized the effect of family size on human capital investment for parents with preferences of the form  $\mathcal{U}(X, N, H)$ , which assume that all children receive the same human capital, i.e.,  $h(i) = H$  for all  $i \in [0, N]$ . If all children are treated equally, the aggregator (1) becomes  $U(\{h(i)\}, N) = N^{\alpha/\rho} u(H)^\alpha$  so one has parental preferences of the form  $\mathcal{U}(X, N, H)$ . [Barro and Becker \(1989\)](#) propose a dynastic and altruistic model of fertility choice in which human capital is also equal among all children and preferences are of the form  $\mathcal{U}(X, N, H) = v(X) + N^a V(H)$ . In [Becker and Lewis \(1973\)](#), [Rosenzweig and Wolpin \(1980\)](#), and [Barro and Becker \(1989\)](#), there are no birth order effects since all children are treated symmetrically. To nest the previous frameworks, simply restrict human capital to be equal among all children, i.e.,  $h(i) = H$  for all  $i$ , assume that  $\rho = \alpha = 1$ , and let  $\mathcal{U}(X, N, U(\{h(i)\}, N)) = v(X) + U(\{h(i)\}, N)$  with  $u(H, i) = aV(H)i^{a-1}$  and  $a > 0$  in (1). These assumptions yield  $\mathcal{U}(X, N, H) = v(X) + N^a V(H)$ , which is the functional form used by [Barro and Becker \(1989\)](#). For further remarks about the connection of our model to the literature see [Appendix A](#).

**Costs.** Let  $\mathcal{C}(X, C(\{h(i)\}, N))$  denote the total cost of attaining a parental consumption  $X$ , a family of size  $N$ , and human capital investments  $h(i)$  for  $i \in [0, N]$ . We consider a CES aggregator for human capital costs, as in

$$C(\{h(i)\}, N) \equiv \left( \int_0^N c(h(i), i)^\phi di \right)^{1/\phi}, \quad (2)$$

where  $c(h(i), i)$  is a cost function for human capital investments  $h(i)$  at order  $i$ , and  $\phi$  is a measure of substitutability between the cost of providing human capital to children of different birth orders. We assume that  $c(h, i)$  is increasing and sufficiently convex in  $h$  but leave the dependence on  $i$  unrestricted. The marginal cost of changes in family size is given by  $C_N(\{h(i)\}, N) > 0$ . As shown in [Appendix A](#),

the marginal cost of human capital investments is increasing in family size, i.e.,  $C_{hN}(\{h(i)\}, N) > 0$ ; see (A5).

Under a linear cost for parental consumption at a price  $P_X$ , the total cost function becomes

$$\mathcal{C}(X, C(\{h(i)\}, N)) = P_X X + C(\{h(i)\}, N).$$

This expression also nests existing formulations in the literature. For example, if marginal and average human capital costs are equal, i.e.,  $c(h, i)/h = c_h(h, i)$ , costs are linearly aggregated,

$$C(\{h(i)\}, N) = \int_0^N \pi(i) h(i) di, \quad (3)$$

with  $\pi(i) \equiv (c(h, i)/C(\{h(i)\}, N))^{\phi-1} c_h(h, i)$  representing birth-order-specific prices of human capital, as in [Behrman and Taubman \(1986\)](#).<sup>5</sup> As another example, assume that  $\phi = 1$ , so that costs are perfect substitutes. Let  $c(h(i), i)$  have a fixed component  $P_N$ , and a linear and constant price for  $h(i)$  given by  $\Pi + P_H/N$ , as in  $c(h(i), i) = P_N + [P_H/N + \Pi]h(i)$ . Then, the cost aggregator (2) becomes

$$C(H, N) = P_N N + P_H H + \Pi H N, \quad (4)$$

where  $H \equiv N^{-1} \int_0^N h(i) di$  represents the human capital of the average child in the household or average human capital. [Becker and Lewis \(1973\)](#) and [Rosenzweig and Wolpin \(1980\)](#) assumed the functional form (4) to study quantity-quality interactions between family size  $N$  and human capital investments  $H$  when investments are equal across birth orders, i.e.,  $h(i) = H$ . Canonical formulations of parental investments costs in models of the intrahousehold allocation of resources and the quantity-quality trade-off are also nested in (2).

**Parental choices.** Parents maximize  $\mathcal{U}(X, N, U(\{h(i)\}, N))$  subject to  $\mathcal{C}(X, C(\{h(i)\}, N)) = Y$ , where  $Y$  is the parents' total income or wealth, which we maintain as fixed throughout.

Appendix A lists sufficient conditions to ensure that  $\mathcal{U}(X, N, U)$  and  $\mathcal{C}(X, C)$  satisfy standard concavity-convexity properties. We also maintain conventional assumptions for the cross-partial terms of the parental utility. We assume that  $X$  and  $N$ , and  $X$  and  $U$ , are complements and that  $N$  and  $U$  are substitutes in the sense that  $\mathcal{U}_{XN}(X, N, U) > 0$ ,  $\mathcal{U}_{XU}(X, N, U) > 0$ , and  $\mathcal{U}_{NU}(X, N, U) < 0$  (see [Milgrom and Shannon, 1994](#)). The complementarity assumptions are fairly standard. They are sufficient to ensure, for instance, that  $N$  and  $\{h(i)\}$  are normal goods, i.e., that their maximizing values are increasing in parental wealth. The substitution assumption between  $N$  and  $U$  is sufficient to ensure that the parental problem is concave in  $N$ . Assuming  $\mathcal{U}_{NU}(X, N, U) < 0$  is also important for the qualitative predictions

<sup>5</sup>In contrast to [Behrman and Taubman \(1986\)](#), however, prices  $\pi(i)$  are endogenous since, through  $C(\{h(i)\}, N)$ , they depend on the human capital of all children as well as on family size.

of the model. For instance, [Jones and Schoonbroodt \(2010\)](#) clarified the role of  $\mathcal{U}_{NU}(X, N, U)$  in the Barro-Becker framework and defended  $\mathcal{U}_{NU}(X, N, U) < 0$  in a formulation that assumes no heterogeneity among children, uses a linear aggregator in (1), and a parental utility that values the average utility of children, i.e.,  $U/N$ .<sup>6</sup> Although not necessary,  $\mathcal{U}_{NU}(X, N, U) < 0$  is sufficient in our model to ensure a trade-off between the quantity and quality of children.

Let  $X^*$  denote optimal parental consumption. Parental choices for  $(N, \{h(i)\})$ , our focus in this paper, are represented by a family size  $N^* \in [0, N^+]$ , where  $N^+$  is a biological upper bound of fertility, and by a bounded human capital profile  $h^*(i) \in \mathcal{C}^1$  with support  $[0, N^*]$ . Family size and human capital satisfy

$$\mathcal{U}_N(X^*, N^*, U^*) + \mathcal{U}_U(X^*, N^*, U^*)\mathcal{U}_N(\{h^*(i)\}, N^*) = \frac{\mathcal{U}_X(X^*, N^*, U^*)}{P_X}C_N(\{h^*(i)\}, N^*), \quad (5)$$

$$\mathcal{U}_U(X^*, N^*, U^*)\mathcal{U}_h(\{h^*(i)\}, N^*) = \frac{\mathcal{U}_X(X^*, N^*, U^*)}{P_X}C_h(\{h^*(i)\}, N^*), \text{ for all } i \in [0, N^*]. \quad (6)$$

The first-order conditions (5) and (6) are intuitive. In (5), an additional child increases parental utility directly by  $\mathcal{U}_N(X^*, N^*, U^*)$  but also indirectly by adding  $\mathcal{U}_N(\{h^*(i)\}, N^*)$  to the aggregated utilities of the existing children. Increasing  $N$ , however, requires additional spending and this increases total costs by  $C_N(\{h^*(i)\}, N^*)$ . In (6), the utility gains from human capital investments at birth order  $i$  must, at the margin, equal their cost. As we noted before, the explicit derivatives of the aggregator functions  $U(\{h(i)\}, N)$  and  $C(\{h(i)\}, N)$  with respect to their arguments are presented in [Appendix A](#).

Parental choices can be broken into two separate parts. First, taking as given the optimal human capital investment profile, family size  $N^*$  determines the endpoint of the function  $h^*(i)$ . Second, for a family of size  $N^*$ , the profile  $h^*(i)$  determines how human capital varies across birth orders within the household. An advantage of this separation is that we can fix family size as a terminal condition for a differential equation that describes how human capital varies within the household from the first-order condition (6) alone. In particular, let  $h^*(i|N^*)$  be the optimal human capital investment of a child of birth order  $i \in [0, N^*]$  in a family of size  $N^*$ .

**Proposition 1 (Birth order effects)** *A child's optimal human capital is a function of birth order, and it varies according to*

$$\frac{\partial h^*(i|N^*)}{\partial i} \leq 0 \iff (\rho - 1)\frac{u_i(h, i)}{u(h, i)} + \frac{u_{hi}(h, i)}{u_h(h, i)} - (\phi - 1)\frac{c_i(h, i)}{c(h, i)} - \frac{c_{hi}(h, i)}{c_h(h, i)} \leq 0, \quad (7)$$

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<sup>6</sup>We use the total utility index  $U$  as an argument in the parental preferences  $\mathcal{U}(X, N, U)$  instead of the average utility  $U/N$ . Under the equal treatment assumption in [Becker and Lewis \(1973\)](#), [Rosenzweig and Wolpin \(1980\)](#), and [Barro and Becker \(1989\)](#) there is no major advantage of using one or another formulation. The use of a total utility index under birth order predispositions simplifies the comparative statics. Under  $U/N$ , we would need to bound the second-order derivative of  $\mathcal{U}(X, N, U)$  with respect to its third argument. As [Jones and Schoonbroodt \(2010, p. 677\)](#) note, in response to an exogenous decline in the price of education, “when family size and utility are complements in utility, it follows that both education and family size increase, whereas when they are substitutes, education increases substantially, but family size falls.” They also state that “there is currently no good evidence for assuming that number and well-being of children are complements.”



for all  $i \in [0, N^*]$ .

A child's human capital varies with birth order. The derivative in (7) represents the slope of the human capital profile within the household. Given the CES aggregators in (1) and (2), the slope of  $h^*(i|N^*)$  depends on how the marginal utilities and costs  $U_h(\{h^*(i)\}, N^*)$  and  $C_h(\{h^*(i)\}, N^*)$  in (6) differ by birth order. Expression (7) depends on the complementarity or substitutability between human capital and birth order in the parental sub-utilities and costs, i.e., on  $u_{hi}(h, i)$  and  $c_{hi}(h, i)$ , but also on how sub-utilities and total costs change directly with birth order, i.e., on  $u_i(h, i)$  and  $c_i(h, i)$ . Since we have not restricted any of the signs of these cross-partial and partial derivatives, Proposition 1 shows that the model delivers human capital profiles of any general shape. As a special case, if  $\partial h^*(i|N^*)/\partial i = \delta$ , the human capital profile would be linear with a constant slope  $\delta$ . In another special and testable case, if  $\partial h^*(i|N^*)/\partial i = 0$ , the human capital profile is flat. This case assumes that parents treat all their children equally and that human capital accumulation costs are independent of birth order. This special case is the [Becker and Lewis \(1973\)](#) and [Rosenzweig and Wolpin \(1980\)](#) formulation.

Proposition 1 also shows that preference and cost differentials are confounded in (7), and that without information on parental predispositions for birth orders and costs differentials it would not be possible, in general, to separately identify the motivation behind birth order effects. Negative birth order effects, for example, could result from reinforcing strategies due to a decline in the marginal value of human capital investments as birth order increases (i.e.,  $u_{hi}(h, i) < 0$ ) or to increasing differences in total valuation of human capital (i.e.,  $u_i(h, i) > 0$ ). In the extreme case of  $\rho \rightarrow -\infty$ , parents equalize utilities across children, i.e.,  $u(h, i) = \bar{u}$  for all  $i$ . If  $u_i(h, i) > 0$ , parents must lower their human capital investments as birth order increases as a compensating strategy. These principles apply unchanged to less extreme cases and to a context where birth order effects arise due to cost differences.

While our testable predictions for the effect of family size on the human capital recognize factors commonly identified by the literature as important reasons for expecting birth order effects, we will not require a separate identification of the contribution of preferences, endowments, ability, and costs to birth order.

**Quantity-quality.** Proposition 1 does not describe the effects of exogenous changes in family size on a child's human capital. The entire empirical literature analyzing the quantity-quality trade-off revolves around identifying the effects of exogenous changes in family size. Since family size is endogenous, twin births are commonly used as sources of exogenous variation for family size; see, e.g., [Rosenzweig and Wolpin \(1980\)](#). The conceptual experiment behind this logic is based on an exogenous perturbation argument described as follows. Choosing family size involves the possibility of multiple births. Suppose that twins are unanticipated and undiversifiable, and denote by  $Z \in \{0, z\}$  the occurrence of a twin birth: if there are no twin births,  $N^*(Z) = N^*$ ; if there are twins,  $N^*(Z) = N^* + z$  with  $z > 0$  representing the

exogenous additional children. Parents plan for their children's human capital according to (7); parents plan for family size according to (5) but the optimal choice is only available for families without twins. Families with twins, need to accommodate the additional children.

The following proposition characterizes how the human capital of existing children changes with an exogenous increase in family size. We refer to this as the *family size effect*.

**Proposition 2 (The family size effect)** *Regardless of the birth order effects, the human capital profile of smaller families dominates (i.e., it lays above) the human capital profile of larger families. That is,  $h^*(i|N^*) > h^*(i|N^* + z)$ , for all  $i \in [0, N^*]$ .*

Proposition 2 is a consequence of interactions between human capital investments and family size in the parental utility  $\mathcal{U}(X, N, U)$ , as well as income changes associated with the decline in available resources due to the increase in family size. To describe some of the interactions in the parental utility, recall that the marginal utility gains from human capital investments depend, among other things, on  $\mathcal{U}_U(X, N, U)$ ; see the left-hand-side of (6). Substitution between  $N$  and  $U$  in the form of  $\mathcal{U}_{NU}(X, N, U) < 0$  implies that the presence of additional children reduces the marginal value of parental investments in human capital across all birth orders. Since the aggregator  $U$  changes with the additional children, there is an additional decline in parental marginal utilities (i.e.,  $\mathcal{U}_{UU}(X, N, U) < 0$ ) that also lowers the marginal value of human capital investments at all birth orders.

To understand the income changes associated with exogenous changes in family size suppose that parental consumption is equal for parents with and without twins. Since parental spending in children  $Y - P_X X$  is constant,  $C(\{h^*(i|N^*)\}) = C(\{h^*(i|N^* + z)\})$  in the budget constraint (2). Hence,

$$\int_0^{N^*} [c(h^*(i|N^*), i)^\phi - c(h^*(i|N^* + z), i)^\phi] di = \int_{N^*}^{N^* + z} c(h^*(i|N^* + z), i)^\phi di. \quad (8)$$

The left-hand side represents the difference in the cost of human capital for the existing children (i.e.,  $i \in [0, N^*]$ ) between smaller and larger families. The right-hand side represents the cost of the human capital of the additional children in larger families. Since the human capital investments of the additional children are expensive, parents in larger families must reduce investments in existing children. That is, for the budget constraint to hold in (8), it must be the case that  $h^*(i|N^*) > h^*(i|N^* + z)$  for all  $i \in [0, N^*]$ .<sup>7</sup> This result is maintained even though parental consumption changes in response to an exogenous increase in family size. Indeed, Proposition 2 remains valid even if parents can partially (but not fully) adjust family size in response to the occurrence of a twin birth. (For further details see Appendix A.)

<sup>7</sup>The human capital profile is a differential equation; see (7). Because of the existence and uniqueness theorem for differential equations, one and only one integral curve passes through each terminal point (Hestenes, 1966, Appendix, Theorem 3.1). Accordingly,  $h^*(i|N^*)$  and  $h^*(i|N^* + z)$  should not cross (Hestenes, 1966, Appendix, Theorem 4.1). These arguments are similar in spirit to Brock (1971)'s analysis of changes in the planning horizon in the neoclassical growth model. Our proof uses standard comparative statics. This proposition might be proved by *monotone comparative statics* methods which have generalized Brock (1971); see, e.g., Amir (1996) and Milgrom and Shannon (1994). These methods may yield more general results than those presented here.

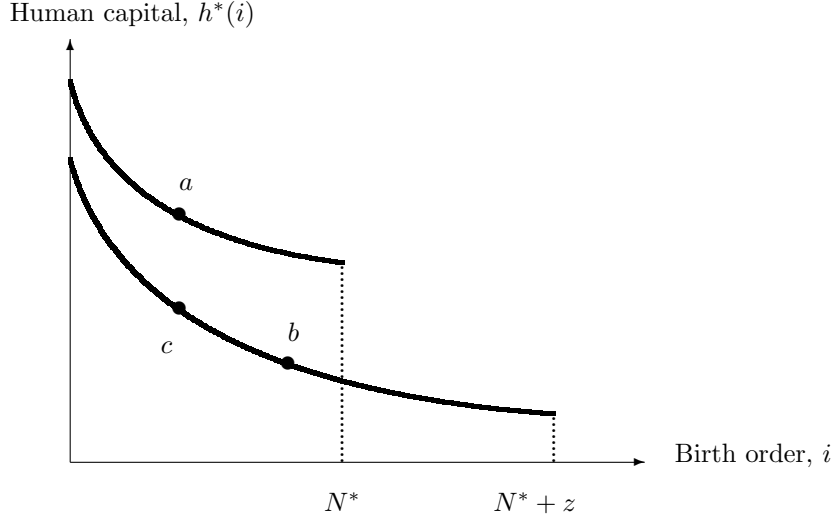


Figure 1: Human capital profiles under negative birth order effects.

Figures 1 and 2 show the effect of an increase in family size. Figure 1 represents graphically Proposition 2 in the case of negative birth order effects. Figure 2 represents the case of positive birth order effects. Both figures show that the children's human capital profile in larger families is below the human capital profile of smaller families, regardless of how birth order influences parental investments. We will later on test this prediction based on empirical human capital profiles and exogenous changes in family size.

In Proposition 2, twins shift family size but leave the shape of the human capital profile unaffected because the preference and cost differentials subsumed in (7) are independent of the presence of twins. That is, the additional children  $z$  influence the human profile only through changes in the terminal value  $N^* + z$ , something that would not be true if any of the terms in (7) were a function of family size. We can then find a solution for (7) while treating  $N^*$  and  $N^* + z$  as different terminal points.

Proposition 2 ranks individual-level human capital for the existing birth orders in families of size  $N^*$  and families of size  $N^* + z$ . Proposition 2, however, does not offer a single summary measure to test the quantity-quality trade-off. For example, Proposition 2 does not incorporate information about the human capital of the additional children in families of size  $N^* + z$ . In effect, Proposition 2 is testable but silent about how the human capital investments of the average child respond to changes in family size.

We next consider the effect of an exogenous increase in family size on the household's average human capital. Let  $H(N^* + z)$  denote the (arithmetic) average human capital of families of size  $N^* + z$ ,

$$H(N^* + z) \equiv \frac{1}{N^* + z} \int_0^{N^* + z} h^*(i|N^* + z) di. \quad (9)$$

The average human capital of families of size  $N^*$  is similarly defined. Examining average human capital

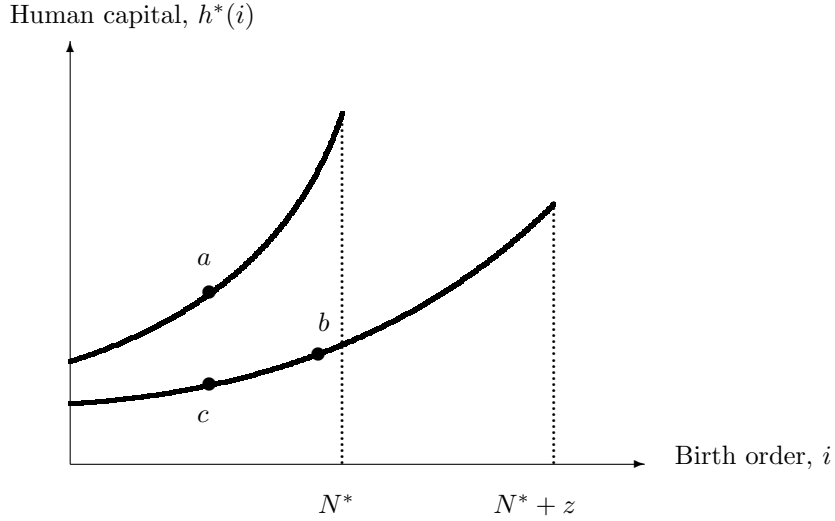


Figure 2: Human capital profiles under positive birth order effects.

is relevant because, by construction,  $H(N^*)$  and  $H(N^* + z)$  are the human capitals of the average or representative children in families of size  $N^*$  and  $N^* + z$ , respectively. Differences in the average human capital across families of different sizes reflect the family size effect (Proposition 2) and the birth order effects, and is referred to as the *composite family size effect*.

**Proposition 3 (The composite family size effect)** *If either human capital is a weakly decreasing function of birth order, or an increasing but bounded function of birth order (with bounds specified in Appendix A); then, the human capital of the average child in smaller families is larger than the human capital of the average child in larger families. That is,  $H(N^*) > H(N^* + z)$ .*

An exogenous increase in family size influences average human capital through changes in the human capital of existing children, i.e., those with birth orders  $i \in [0, N^*]$  (see Proposition 2). In addition to these children, the average human capital  $H(N^* + z)$  counts the additional children, i.e., those with birth orders  $i \in (N^*, N^* + z]$ . Thus, the sign and magnitude of birth order effects influence average human capital. If birth order effects are negative, the human capital of the additional children would be lower than the human capital of the existing children. The additional children, in other words, contribute negatively to the family's average, resulting in a negative composite family size effect. If birth order effects are positive, the additional children receive more human capital than the existing children and this might raise the family's average human capital. If the positive birth order effects are bounded, the contribution of the additional children will be unable to counter the decline associated with Proposition 2, and the composite family size effect will be negative.

Figures 1 and 2 illustrate Proposition 3. In both figures, points (a) and (b) represent  $H(N^*)$  and  $H(N^* + z)$  respectively, so their difference is associated with how the average child fares as family size

increases, the composite family size effect. In both figures, the human capital of the average child is lower in larger families, although the difference is smaller under positive birth order effects than under negative birth order effects.

Figures 1 and 2 also illustrate alternative comparisons that could be implemented in the data. Point (c) represents the human capital of a child in larger families but whose birth order coincides with the birth order of the average child in smaller families. In other words, the difference between points (a) and (c) measures human capital differences when family size increases but birth order is held constant. This is the parameter typically estimated in the recent empirical literature on the quantity-quality trade-off in the presence of birth order effects; see, e.g., Black *et al.* (2005), Angrist *et al.* (2010), and Mogstad and Wiswall (2012b). Under negative birth order effects, as in Figure 1, this is a downward biased estimate of the composite family size effect in Proposition 3. With positive birth order effects, as in Figure 2, the bias is positive.

Similar versions of Proposition 3 can be obtained for additional notions of average children. For example, if the human capital profile  $h^*(i|N^*)$  is monotone, the median child in smaller families is the one whose birth order is at the midpoint between 0 and  $N^*$ , i.e.,  $i^N = N^*/2$ . Median human capital is the value of the human capital function for the median child. Therefore, the median human capital in smaller families is  $H^{\text{med}}(N^*) = h(i^N|N^*)$ . The median child in larger families has a birth order  $i^{N+z} = (N^* + z)/2$  and the median human capital of larger families is  $H^{\text{med}}(N^* + z) = h(i^{N+z}|N^* + z)$ . Their difference,  $h(i^N|N^*) - h(i^{N+z}|N^* + z)$ , can be written as

$$H^{\text{med}}(N^*) - H^{\text{med}}(N^* + z) = [h(i^N|N^*) - h(i^N|N^* + z)] + [h(i^N|N^* + z) - h(i^{N+z}|N^* + z)]. \quad (10)$$

Quantifying the composite family size effect compares  $H^{\text{med}}(N^*)$  and  $H^{\text{med}}(N^* + z)$ , represented graphically by points (a) and (b) in Figures 1 and 2, respectively. The first term in (10) holds birth order constant but changes family size, as in Proposition 2. This term compares points (a) and (c). The second term in (10) holds family size constant but changes the birth order of the median child from  $N^*/2$  to  $(N^* + z)/2$ , as in a comparison between points (c) and (b). This term depends on the sign of the birth order effects. If birth order effects are negative,  $h(i^N|N^* + z) > h(i^{N+z}|N^* + z)$  reinforcing Proposition 2. If birth order effects are positive,  $h(i^N|N^* + z) < h(i^{N+z}|N^* + z)$  countering Proposition 2. If positive birth order effects are bounded, as in Proposition 3, the human capital of the median child in smaller families would also be larger than the human capital of the median child in larger families. We discuss more general ways of averaging human capital than (9) in Appendix A.

The human capital profiles in Figures 1 and 2 arise due to differential investments across birth orders, consistent with the discussion in Rosenzweig and Zhang (2009). They note, for example, that “the difference between the quality of first-birth children in families with and without twins at the second birth does not represent the trade-off between average child quality and the number of children” (Rosenzweig

and Zhang, 2009, p. 1155). In our setting, this comparison corresponds to that between points (a) and (c) in the figures, and as noted above, this is not an exhaustive test of the generalized quantity-quality trade-off. Such a test should also consider the composite family size effect. Of course, Rosenzweig and Zhang (2009) focused on differences in birth endowments, which are particularly salient for twinning; see their Figures 1 and 2. Our framework shows that biased estimates of the quantity-quality trade-off are not exclusively associated with differences in birth endowments. Biased estimates are generally the result of birth order effects, regardless of their source.

Our model is deliberately parsimonious. We focused on limited choices for parents whose only constraint is given by their available wealth. We used a continuous fertility choice and particular utility and cost aggregators. In Appendix A, we demonstrate the robustness of the basic theoretical predictions. To stress the advantages of our approach, note that our analytical findings do not assume that children are treated equally and that we do not need to identify the motivation behind birth order effects. Therefore, the quantity-quality trade-off studied here allows for a general analysis of birth order effects, which, as we shall see in the next section, are empirically relevant. Note also, on an empirical level, that our testable predictions are based on quantities that are easily obtainable in the data.

### 3 Empirical Analysis

Our empirical analysis focuses on quantifying the impact of birth order and family size on human capital investments, as well as testing the predictions of Propositions 2 and 3. To facilitate the exposition of the econometric model, the remainder of the paper treats birth order  $i$  as a discrete variable. We also use subindices differently from our previous notation. We index birth order specific objects by subscript  $i$ , and family specific objects by the subscript  $j$ . For example,  $h_{ij}$  is the human capital of a child with birth order  $i$  in family  $j$ , measured as years of education in the empirical analysis.

Recall that parental choices of  $N_j$  and  $h_{ij}$  satisfy (5) and (6), respectively. The following regression equation matches our theoretical human capital profile across birth orders:

$$h_{ij} = \alpha + \boldsymbol{\nu}'_i \boldsymbol{\delta} + \boldsymbol{\nu}'_{N_j} \boldsymbol{\beta} + \xi_j + \varepsilon_{ij}, \quad (11)$$

for  $i = 1, \dots, N_j$ . In (11),  $h_{ij}$  is an unrestricted function of birth order  $i$  and family size  $N_j$ :  $\boldsymbol{\nu}_i$  is a vector whose  $i$ -th entry equals 1 and all other entries equal 0. The dimension of  $\boldsymbol{\nu}_i$  is  $N^+$ ,  $\boldsymbol{\delta} = (\delta_1, \dots, \delta_{N^+})$  is a vector of birth order coefficients, and  $\boldsymbol{\beta} = (\beta_1, \dots, \beta_{N^+})$  is a vector of family size coefficients. Individual human capital  $h_{ij}$  is also a function of household-level characteristics such as parental spending, parental preferences, and human capital costs. This dependence is implicit in the vector of family fixed effects  $\xi_j$ .

Finally, (11) contains an idiosyncratic error term  $\varepsilon_{ij}$  that satisfies<sup>8</sup>

$$\mathbb{E}[\varepsilon_{ij}|i, N_j, \xi_j] = 0. \quad (12)$$

Existing empirical studies of birth order and family size effects are based on reduced-form regressions of the form of (11). If  $\iota'_i \boldsymbol{\delta} = \delta i$  and  $\iota'_{N_j} \boldsymbol{\beta} = \beta N_j$ , we obtain an empirical model with linear birth order and family size profiles:  $h_{ij} = \alpha + \delta i + \beta N_j + \xi_j + \varepsilon_{ij}$ .

Equation (11) implies the following expression for the average human capital in a family of size  $N_j$ , denoted  $H_j$ ,

$$H_j = \alpha + \iota'_{N_j} (\boldsymbol{\beta} + \bar{\boldsymbol{\delta}}) + \xi_j + \bar{\varepsilon}_j, \quad (13)$$

where  $\bar{\boldsymbol{\delta}} \equiv (\bar{\delta}_1, \dots, \bar{\delta}_{N^+})$  is the vector of the average birth order effects in families of size  $1, \dots, N^+$ , and  $\bar{\varepsilon}_j$  is the within-family average of the idiosyncratic error term in (12). If  $\iota'_i \boldsymbol{\delta} = \delta i$  and  $\iota'_{N_j} \boldsymbol{\beta} = \beta N$ , we obtain:  $H_j = \alpha + (\beta + \delta/2)N_j + \xi_j + \bar{\varepsilon}_j$ .

The parameterization of the empirical model implies that  $\delta_i - \delta_{i-1}$  represent the birth order effect comparing children of parity  $i - 1$  and  $i$ , and  $\beta_N - \beta_{N-1}$  and  $\beta_N + \bar{\delta}_N - (\beta_{N-1} + \bar{\delta}_{N-1})$  represent the family size and composite family size effects comparing families of size  $N - 1$  and  $N$ , respectively. If  $\iota'_i \boldsymbol{\delta} = \delta i$  and  $\iota'_{N_j} \boldsymbol{\beta} = \beta N_j$ , the birth order effect is  $\delta$ , the family size effect is  $\beta$ , and the composite family size effect is  $\beta + \delta/2$ . Our model predicts that the family size effect and the composite family size effect are negative at all family sizes. Testing the theory thus requires estimates of  $\boldsymbol{\delta}$  and  $\boldsymbol{\beta}$ .

### 3.1 A Two-Step Empirical Strategy

Estimation of the regression coefficients  $\boldsymbol{\delta}$  and  $\boldsymbol{\beta}$  from (11) is complicated by a dual endogeneity problem. As noted above, human capital  $h_{ij}$  and family size  $N_j$  reflect optimal behavior on the part of parents, i.e., both depend on expenditures, preferences and costs. Hence,  $N_j$  is endogenous in relation to  $\xi_j$ . Since family size determines a family's birth order configuration, and vice versa, the endogeneity of family size  $N_j$  spills over to birth order  $i$ .

We propose a simple two-step estimation procedure to overcome this complication. In the first step, we estimate  $\boldsymbol{\delta}$  from (11) using the within-family estimator. The within-family transformation sweeps out all time-invariant family characteristics that impact parental choices. Hence, while the first step delivers a consistent estimate of  $\boldsymbol{\delta}$  using within-family variation in educational outcomes, it cannot be used to estimate the impact of family size on education because family size does not vary within a household.

In the second step, we estimate  $\boldsymbol{\beta}$  from between-family variation in average years of education net of the variation that stems from differences in average birth order across families,  $\tilde{H}_j = H_j - \iota'_N \bar{\boldsymbol{\delta}}$ . This

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<sup>8</sup>In the empirical analysis we augment (11) with a set of exogenous individual and family specific observable controls.

object can be obtained as the fixed effects from the first step regressions. By (13),

$$\tilde{H}_j = \boldsymbol{\iota}'_{N_j} \boldsymbol{\beta} + \xi_j + \varepsilon_j. \quad (14)$$

Birth orders do not appear directly in this equation and family size  $N_j$  is endogenous in relation to the family fixed effect  $\xi_j$ . We follow previous literature and overcome the endogeneity problem by using twin births as an instrumental variable for family size when estimating  $\boldsymbol{\beta}$  by regressing  $\tilde{H}_j$  onto family size according to (14). This yields a consistent estimate of the coefficient vector  $\boldsymbol{\beta}$ . Since  $\boldsymbol{\delta}$  is known from the within-family regression analysis, it is straightforward to recover the composite parameter  $\boldsymbol{\beta} + \bar{\boldsymbol{\delta}}$ . We have thus identified and estimated the parameters of interest and we can proceed to test the predictions in Propositions 2 and 3.<sup>9</sup>

**Instrumental variables.** Our empirical analysis includes various specifications of the family size profile. In the simple case where the family size profile is linear, i.e.,  $\boldsymbol{\iota}'_{N_j} \boldsymbol{\beta} = \beta N_j$ , only a single instrumental variable is required for identification. In the general case of a nonlinear family size profile, (13) contains a vector of endogenous variables  $\boldsymbol{\iota}_N$ . We have excluded families with  $N = 1$  and treated  $N = 2$  as the reference case. Hence,  $\boldsymbol{\iota}_N$  contains  $N^+ - 2$  endogenous variables.

Let  $\tilde{z}_j^k$  take the value of 1 if family  $j$  experienced a twin birth in the  $k$ -th birth order and  $\tilde{z}_j^k = 0$  otherwise. For the empirical specifications with linear family size profiles, we present estimates of  $\beta$  using both twin birth at the last birth, i.e. using  $z_j = \tilde{z}_j^{N_j-1}$  to instrument  $N_j$ , and twin birth at any birth parity, i.e. using  $z_j = \mathbb{I}(\tilde{z}_j^k = 1)$  for some  $k = 1, 2, \dots, N_j - 1$ , where  $\mathbb{I}(\cdot)$  is the indicator function.

Using twin birth as an instrumental variable when the family size profile is nonlinear is slightly more involved. Since  $\tilde{z}_j^k = 0$  when information on twin birth at parity  $k$  is missing due to truncation at  $N_j^*$ ,  $\tilde{z}_j^k$  is correlated with the family fixed effect  $\xi_j$ . This invalidates its use as an instrumental variable. Angrist *et al.* (2010), Mogstad and Wiswall (2012b), and Mogstad and Wiswall (2012a), however, have shown that one can construct a set of valid instruments in the following way:

$$z_j^k = (\tilde{z}_j^k - \mathbb{E}[\tilde{z}_j^k | N_j \geq k]) \mathbb{I}(N_j \geq k), \quad (15)$$

for  $k = 3, 4, \dots, N^+ - 1$ . The vector  $z_j^k$  constitutes our instrumental variables for family size in (13) when the family size profile is nonlinear.<sup>10</sup>

<sup>9</sup>We obtain the variance-covariance matrix of the two-step estimator by block-bootstrapping at the family-level using 100 repetitions.

<sup>10</sup>In practice, we estimate  $\mathbb{E}[\tilde{z}_j^k | \mathbf{w}_j, N_j \geq k]$ , where  $\mathbf{w}_j$  denotes a vector of family-level controls. The empirical specification also includes interactions of the variables in  $\mathbf{w}_j$  within the relevant subsamples defined by  $N_j \geq k$ .



## 4 Data

Our analysis data is extracted from IDA (*Integreret Database for Arbejdsmarkedsforskning*), a comprehensive Danish administrative panel dataset for the period 1980-2006 with annual observations on all individuals aged 15-74 and residing in Denmark with a social security number. IDA contains detailed individual-level information on socioeconomic characteristics, including date of birth, gender and educational attainment. The data is constructed and collected for administrative purposes and contains very few measurement errors. Moreover, the data is population-wide with a long period of observation. We can link children and parents, and thus identify siblings; families are defined as sets of children born to the same mother.

We select all individuals in IDA in 2006 with non-missing mother ID and father ID; that is, all outcome measurements are taken in 2006. This is the children-data. We then locate the parents in IDA and merge parental characteristics onto the children-data. We retain only the children where we are able to locate both parents in the IDA. From this data we can compute family size and assign birth orders to children. Finally, we impose a standard set of selection criteria, the most important of which are as follows: First, we only retain children aged 25 or above in 2006 to ensure that our outcome measurements represent completed education. Second, we exclude families with children aged 0-14 in 2006. This ensures that our family size measure represents completed fertility. Third, we exclude families in which at least one member (a child or one of the parents) has missing education data, families where the mother was below 17 or above 49 when giving birth, and families containing siblings with different fathers. Fourth, as birth order is not defined for multiple births (i.e. for twins, triplets etc.) we remove individuals that are part of multiple births instances, retaining only an indicator for a multiple birth occurrence in the family and the birth parity at which the multiple birth occurred. We note that family size and birth order are recorded before the individual level selection criteria are imposed, to ensure our family size measure represents completed fertility. These restrictions reduce our sample size from 2,361,083 individuals in 1,220,477 families to 1,438,994 individuals in 756,776 families.

Effectively, our analysis data contains all individuals in Denmark aged 25-74 in 2006, with non-missing mother and father IDs, whose parents were both alive, aged 15-74 and present in the IDA-files at some point during 1980-2006, and who satisfy the additional restrictions described just above. Further details on the construction and selection of our analysis data can be found in online supplementary material.

**Descriptive statistics.** Table 1 presents descriptive statistics for the analysis data, excluding single child families. The average individual is 38.3 years old and has 12.8 years of education. Forty eight percent are females and, on average, their mothers and fathers have completed 10.2 and 10.9 years of schooling, respectively. Conditional on having at least one sibling, an individual has on average 1.7 siblings.

Table 1: Descriptive statistics. Analysis data

	Mean	Standard deviation	Minimum	Maximum
Age in 2006	38.3	7.6	25	67
Female	0.48	0.5	0	1
Education	12.8	2.8	7	20
Mother's education	10.2	3.5	7	20
Father's education	10.9	3.5	7	20
Mother's age in 2006	64.6	8.2		
Father's age in 2006	67.5	8.7		
Number of siblings	1.7	0.9	1	10
Twins in family	0.01	0.11	0	1

*Note:* Descriptive statistics are from our analysis data consisting of 1,438,994 individuals in 756,776 families. Single children are excluded from the data, and the reported statistics are based on 1,278,510 individuals in 596,292 families. To ensure anonymity, we cannot report minimum and maximum values for mother's and father's age in 2006.

Table 2: Number of children in the family

Number of children	Frequency	Percentage
1	160,484	21.2
2	380,259	50.3
3	163,426	21.6
4	40,785	5.4
5	8,705	1.2
6+	3,157	0.3

*Note:* Descriptive statistics are obtained using 756,776 families including single child families.

Table 2 presents the distribution of family size, including single child families: 50.3 percent of families have two children and less than one-third of the families have more than two children. The average number of children in the family is 2.5. Table 3 presents the average education by family size and birth order. This table shows a clear negative association between an individual's education and family size, as well as between an individual's education and her birth order. Similar patterns are documented for the mother's and father's education.

## 5 Main Findings

**OLS findings.** We start by estimating the impact of family size on a child's education from (11) by OLS. This naive procedure ignores any endogeneity issues. The first column of Table 4 reports findings from a linear specification of family size on child's education, controlling for the age and sex of the child.

Table 3: Average education by family size and birth order

Education	Mother's education	Father's education	Family size		Observations
			Fraction with < 12 years	Fraction with 12 years	
1	10.1	10.8	0.20	0.47	160,484
2	10.6	11.3	0.15	0.48	676,112
3	10.1	10.9	0.18	0.47	413,602
4	9.3	10.1	0.24	0.46	135,869
5	8.5	9.2	0.32	0.45	36,101
6+	8.0	8.7	0.39	0.42	16,826
			Birth order		
1	10.4	11.1	0.17	0.46	715,957
2	10.3	11.0	0.18	0.49	503,678
3	9.6	10.5	0.21	0.48	166,996
4	8.8	9.7	0.26	0.47	39,954
5	8.1	8.9	0.32	0.47	8,988
6+	7.8	8.6	0.37	0.45	3,421

Note: Descriptive statistics are from our analysis data consisting of 1,438,994 individuals in 756,776 families. Single children are included.

As expected, the coefficient on family size is negative and implies that an additional child decreases schooling by a little more than a quarter of a year. Because family-specific characteristics might impact the choice of completed family size and educational choices, column (4) adds unrestricted indicators for the mother’s and father’s education, and a 5-year interval set of indicator variables for mother’s and father’s age. Adding demographic controls reduces the magnitude of the relationship by about 36 percent but the coefficient remains statistically significant.

In order to account for birth order effects, column (5) adds a linear control for birth order. Consistent with previous findings reported in the literature, the coefficient on family size is considerably reduced to  $-0.038$ , but remains significant. Allowing for a more flexible estimation by including indicator variables for birth order in column (6) does not change the findings markedly. The impact of family size is  $-0.063$ , smaller but comparable to the coefficient ( $-0.013$ ) reported in Black *et al.* (2005) using Norwegian data.<sup>11</sup>

Birth order coefficients, whether included linearly or nonlinearly, are negative, large and highly significant. They suggest, for example, that a third child in a family has, on average, 0.657 fewer years of education than the first child (column (6) in Table 4). The negative impact of birth order could reflect family-specific unobservable factors. In columns (7) and (8) of Table 4, we report findings from estimating birth order coefficients while controlling for family fixed effects. The family indicators capture any time-invariant characteristics, including completed family size. Once we control for family fixed effects, the linear birth order coefficient is reduced by about a half to  $-0.179$ . Similar changes in magnitude occur when we estimate the regression including a nonlinear birth order profile (column (8)). The birth order effects are similar in magnitude to the results reported elsewhere in the literature.<sup>12</sup>

**Two-step estimation findings.** We now present the estimates from our two-step strategy described earlier. The first step uses within-family variation to estimate birth order coefficients  $\delta$ , as reported in columns (7) and (8) of Table 4. In the second step, we use between-family variation to estimate the impact of family size on the average education in the family netting out the effect of average birth order, as in (14). The second step identifies family size coefficients  $\beta$ . With estimates of  $\delta$  and  $\beta$ , we can compute birth order effects, family size effects, and composite family size effects. The latter two objects allow us to test Propositions 2 and 3, which restrict family size and composite family size effects to be negative.

The results from our two main specifications are reported in Tables 5 and 6. Table 5 contains results for the case of linear birth order and family size profiles. Table 6 contains results for the case of nonlinear birth order and linear family size profiles. We further present a specification with nonlinear birth order and nonlinear family size profiles in Table B.1 in Appendix B.

<sup>11</sup>Estimating the effect of family size using the instrumental variable strategy in Black *et al.* (2005) produces comparable estimates.

<sup>12</sup>Our theory based empirical analysis abstracts from the potential impact of spacing/timing of children. Extending our analysis to a dynamic framework is beyond the scope of this papers and we leave it as future research.

Table 4: Family size, birth orders and children's education—OLS regressions

Dependent variable: Child's education	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Family size	-0.283*** (0.004)	-0.196*** (0.004)	-0.215*** (0.004)	-0.181*** (0.004)	-0.038*** (0.004)	-0.063*** (0.004)		
Birth order		-0.173*** (0.003)			-0.328*** (0.004)		-0.179*** (0.006)	
Birth order indicators								
Second			-0.247*** (0.005)			-0.412*** (0.005)		-0.274*** (0.007)
Third			-0.314*** (0.008)			-0.657*** (0.009)		-0.423*** (0.014)
Fourth			-0.473*** (0.014)			-0.895*** (0.016)		-0.488*** (0.021)
Fifth			-0.635*** (0.027)			-1.111*** (0.028)		-0.437*** (0.034)
Sixth or later			-0.652*** (0.047)			-1.135*** (0.046)		-0.437*** (0.052)
Demographic controls	No	No	No	Yes	Yes	Yes	Yes	Yes
Family fixed effects	No	No	No	No	No	No	Yes	Yes
Observations	1,278,510	1,278,510	1,278,510	1,278,510	1,278,510	1,278,510	1,278,510	1,278,510

*Note:* \*\*\* indicates statistical significance at the 1 percent level. Standard errors (in parentheses) are clustered at the family level. All regressions include indicators for age and sex. Demographic controls include indicators for mother's education, mother's age, father's education, and father's age. Single child families are excluded from the estimation.

Table 5: Two-step estimation with linear family size and birth order profiles

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Step 1: OLS w/ family FE		Step 2: OLS	Step 2: OLS	Step 2: 2SLS	Step 2: 2SLS	Step 2: 2SLS	Step 2: 2SLS
Dependent variable:	Child's years of education	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>
Instrumental variable:		Twin birth at last birth	Twin birth at last birth	Twin birth at last birth	Twin birth at any parity	Twin birth at any parity	Twin birth at any parity
Birth order	-0.179*** (0.006)						
Family size		-0.163*** (0.007)	-0.081*** (0.005)	-0.012 (0.030)	-0.022 (0.027)	-0.038* (0.021)	-0.024 (0.019)
Composite family size		-0.253*** (0.005)	-0.170*** (0.004)	-0.102*** (0.031)	-0.112*** (0.028)	-0.127*** (0.021)	-0.114*** (0.019)
First stage:							
Min. eigenvalue stat. <sup>2</sup>		No	Yes	No	Yes	No	Yes
Demographic controls		No	Yes	No	Yes	No	Yes
Observations	1,278,510	596,292	596,292	596,292	596,292	596,292	596,292

Note: \*\*\* indicates statistical significance at the 1 percent level. Standard errors for all regressions are computed by block-bootstrapping at the family level (100 repetitions) and are given in brackets. The family fixed effect regression in column (1) includes controls for age and sex. Demographic controls include indicators for mother's education, mother's age, father's education, and father's age. Single child families are excluded from the analysis data.

<sup>1</sup>The family-level average education in columns (2)-(7) nets out the effects of age, sex as well as the average birth order effect, see (14).

<sup>2</sup>See Stock and Yogo (2002) for critical values. We clearly reject the null hypothesis of weak instruments.

Table 6: Two-step estimation with linear family size and nonlinear birth order profiles

	(1)	(2)	(3)	(4)
	Step 1: OLS w/ family FE	Step 2: OLS	Step 2: 2SLS	Step 2: 2SLS
Dependent variable:	Child's years of education	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>
Instrumental variable:			Twin birth at last birth	Twin birth at any parity
Second child	-0.274*** (0.007)			
Third child	-0.423*** (0.014)			
Fourth child	-0.488*** (0.021)			
Fifth child	-0.437*** (0.034)			
Sixth child or later	-0.437*** (0.052)			
Family size		-0.095*** (0.005)	-0.040 (0.027)	-0.049*** (0.019)
Composite family size 2		-0.137*** (0.004)	-0.137*** (0.004)	-0.137*** (0.004)
Composite family size 3		-0.327*** (0.006)	-0.273*** (0.029)	-0.281*** (0.020)
Composite family size 4		-0.485*** (0.009)	-0.376*** (0.057)	-0.394*** (0.040)
Composite family size 5		-0.608*** (0.013)	-0.445*** (0.084)	-0.471*** (0.058)
Composite family size 6		-0.721*** (0.017)	-0.503*** (0.113)	-0.538*** (0.078)
First stage:				
Min. eigenvalue stat. <sup>2</sup>			15, 825.5	28, 156.6
Observations	1,278,510	596,292	596,292	596,292

*Note:* \*\*\* indicates statistical significance at the 1 percent level. Standard errors for all regressions are computed by block-bootstrapping at the family level (100 repetitions) and are given in brackets. The family fixed effect regression in column (1) includes controls for age and sex. Demographic controls in the regression in columns (2), (3), and (4) include indicators for mother's education, mother's age, father's education, and father's age. Single child families are excluded from the analysis data.

<sup>1</sup>The family-level average education in columns (2)-(4) nets out the effects of age, sex as well as the average birth order effect, see (14).

<sup>2</sup>See [Stock and Yogo \(2002\)](#) for critical values. We clearly reject the null hypothesis of weak instruments.

To ease the exposition, column (1) of Table 5 reports the effect of birth order on educational attainment from the first step in the estimation procedure, the same estimates that can be found in column (7) of Table 4. Column (2) of Table 5 reports OLS estimates of the family size effect,  $\beta$ , and the composite family size effect implied by the estimated birth order and family size effects;  $\beta + \delta/2$  in the linear case (the second step). The results indicate that, holding birth order constant, an additional child in the family reduces years of education by about 0.163 years. Adding family-level demographic controls in the second-step regression in column (3) reduces the magnitude of the effect to  $-0.081$ , but it remains highly significant. The estimates of the composite family size effect are large, negative, and highly significant. For instance, the estimate in column (3) indicates that an increase of one child reduces the average years of schooling in the family by about 0.170 of a year.

Columns (4) and (5) of Table 5 report the coefficients on family size, with and without demographic controls, using twins at last birth as the instrument. In columns (6) and (7) we report the results of analogous specifications using twins at any parity as the instrument.<sup>13</sup> We note that the Minimum Eigenvalue Statistic indicates that our instruments are strong. The results indicate that, regardless of the instrument we use and the inclusion of demographic controls, increasing the family size by one child decreases the family’s average education by about one tenth of a year. Although the estimates of  $\beta$  in these specifications are negative, they are smaller in magnitude, and are with one exception, statistically insignificant.

Table 6 reports estimates of a specification commonly used in the literature where the birth order profile in the first step is unrestricted and therefore possibly nonlinear. Family size impacts educational attainment linearly through a single slope coefficient  $\beta$  in the second step regression. Again, to ease the exposition, column (1) reports the estimated birth order effects. The IV results in columns (3) and (4) indicate that an increase of one child decreases years of schooling by 0.040 and 0.049 years, respectively, although only the latter estimate is significant at the 1 percent level. The estimates of the composite family size effects are also negative and monotonically increasing (in absolute value) with family size. The estimates reported in Table 6 provide strong support for Propositions 2 and 3.

Table B.1 in Appendix B reports estimates of a fully flexible specification with unrestricted nonlinear birth order and family size effects. Specifically, the empirical model includes separate indicators for families with 3, 4, 5 and 6 or more children, using families with 2 children as the omitted category. Because the second stage includes four endogenous family size effects, we follow the methodology proposed by Angrist *et al.* (2010) and Mogstad and Wiswall (2012a) in constructing appropriate instrumental variables.<sup>14</sup> The IV results are reported in column (3) of Table B.1. We note again that the Minimum

<sup>13</sup>Using twins at last birth ensures that desired family size is, on average, the same for families with singletons and for families with a twin birth. It also ensures that the family size changes without also changing the birth order of subsequent children. However, we also report results from using twins at any parity because the incidence of twins at last birth could endogenously impact subsequent fertility.

<sup>14</sup>We estimate the family size profile nonparametrically, although we restrict the estimated family size effects to be constant for families with  $N \geq 6$ . Given that the effect of  $N = 1$  and  $N = 2$  are normalized to zero, we need four



Eigenvalue Statistic indicates that our instruments are strong. Qualitatively, the results reported in Table B.1 are in line with those obtained in Table 6. First, the elements in the vector of family size coefficients,  $\beta$ , are negative, although not always statistically significant. For instance, a family with four children has about 0.134 fewer years of schooling compared to a family with two children, with the effect significant at the 10 percent level. Importantly, the composite family size effects are all negative, large, and statistically significant at the 1 percent level. Overall, the results support the predictions of Propositions 2 and 3 and the presence of a generalized quality-quantity trade-off as hypothesized by our model.

Finally, we investigate the impact of allowing birth order effects to vary by family size in the first step on the family size coefficients estimated in the second step. While these specifications do not meet the sufficient conditions for Proposition 2, they are at the very least of descriptive interest. Table 7 presents the estimates from a parsimonious specification with a linear birth order effect, a linear interaction with family size, and a linear family size effect. As can be seen in column (1), the birth order profile becomes flatter as family size increases. Specifically, the birth order effect is smaller by 0.061 years of education for each additional child. As a result, the estimated coefficients on family size,  $\beta$ , in columns (3) and (4) increase in magnitude and imply that an increase of one child lowers years of schooling by about one tenth of a year. These effects are significantly larger than the ones reported in Table 5. The composite family size effects remain negative, large, and statistically significant. We report the results of family-specific non-linear birth order effects and a linear family size effect in Table B.4 in Appendix B, while in Table B.5 we include unrestricted family-size specific birth order profiles and unrestricted family size profiles. Both sets of the results support the presence of a generalized quality-quantity trade-off.

**Empirical Content of Theoretical Predictions.** Proposition 2 states that, *ceteris paribus*, the human capital profiles in smaller families is always above those of larger families. In our terminology: the family size effect is negative. Proposition 3 states that, *ceteris paribus*, the average human capital is larger in smaller families. In our terminology: the composite family size effect is negative. We formally test the validity of these restrictions using the estimated empirical models, reiterating that the existing literature studying the roles of family size and birth order in human capital formation has focused primarily on the prediction from Proposition 2, the negative family size effect. This prediction carries over from the canonical framework of Becker and Lewis (1973), where birth order effects are absent. The additional restrictions imposed by Proposition 3, the negative composite family size effect, arise from our generalized theoretical framework where parents may have birth order predispositions that influence human capital investments in their children, and are not identified, nor tested, elsewhere in the literature.<sup>15</sup> Testing the

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instrumental variables. See the description in Section 3, or consult Angrist *et al.* (2010), Mogstad and Wiswall (2012a), or Mogstad and Wiswall (2012b) for details.

<sup>15</sup>An exception is Guo *et al.* (2017) who show that estimating the family size effect controlling for birth order indicators provides an estimate of the family size effect on the first child, and not on the average child in the family. They also show that using twinning at low parities as instrumental variables identifies the effect of family size on low-parity children and not on the average child in the family.

Table 7: Two-step estimation with linear family size and family size-specific birth order profiles

	(1)	(2)	(3)	(4)
	Step 1: OLS w/ family FE	Step 2: OLS	Step 2: 2SLS	Step 2: 2SLS
Dependent variable:	Child's years of education	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>
Instrumental variable:			Twin birth in last birth	Twin birth at any parity
Birth order	-0.435*** (0.011)			
Birth order × family size	0.061*** (0.002)			
Family size		-0.180*** (0.007)	-0.123*** (0.028)	-0.124*** (0.019)
Composite family size 2		-0.157*** (0.004)	-0.157*** (0.004)	-0.157*** (0.004)
Composite family size 3		-0.372*** (0.007)	-0.314*** (0.029)	-0.315*** (0.020)
Composite family size 4		-0.526*** (0.009)	-0.411*** (0.057)	-0.413*** (0.040)
Composite family size 5		-0.619*** (0.012)	-0.447*** (0.085)	-0.450*** (0.059)
Composite family size 6		-0.651*** (0.016)	-0.422*** (0.112)	-0.427*** (0.079)
First stage:				
Min. eigenvalue stat. <sup>2</sup>			15, 825.5	28, 156.6
Observations	1,278,510	596,292	596,292	596,292

*Note:* \*\*\* indicates statistical significance at the 1 percent level. Standard errors for all regressions are computed by block-bootstrapping at the family level (100 repetitions) and are given in brackets. The family fixed effect regression in column (1) includes controls for age and sex. Demographic controls in the regression in columns (2), (3), and (4) include indicators for mother's education, mother's age, father's education, and father's age. Single child families are excluded from the analysis data.

<sup>1</sup>The family-level average education in columns (2)-(4) nets out the effects of age, sex as well as the average birth order effect, see (14).

<sup>2</sup>See Stock and Yogo (2002) for critical values. We clearly reject the null hypothesis of weak instruments.

generalized quantity-quality trade-off thus requires a joint test of the restrictions in Propositions 2 and 3.

For ease of exposition we focus on the specification with linear birth order profiles and linear family size profiles. In this specification,  $\beta$  is the family size effect and  $\delta$  is the birth order effect. The composite family size effect is  $\beta + \delta/2$ . Let  $\boldsymbol{\theta} = (\beta, \delta)'$ . Propositions 2 and 3 impose two restrictions on  $\boldsymbol{\theta}$ , represented by the following hypothesis structure:

$$H_0 : \mathbf{R}\boldsymbol{\theta} \leq \mathbf{0} \text{ against } H_1 : \mathbf{R}\boldsymbol{\theta} \not\leq \mathbf{0}, \quad (16)$$

where

$$\mathbf{R} = \begin{pmatrix} 1 & 0 \\ 1 & 1/2 \end{pmatrix}.$$

Our empirical models provide estimates of  $\boldsymbol{\theta}$ , and the hypothesis structure (16) lends itself naturally to a Wald-type test procedure. Note that  $H_0$  in (16) is a composite hypothesis that is consistent with many parameter configurations. Univariate composite hypotheses are appropriately handled using conventional one-sided  $z$ -tests. However, our theory explicitly delivers a multivariate composite hypothesis, rendering inference difficult because the asymptotic distribution of the Wald test statistic is a complicated mixture of  $\chi^2$ -distributions. We overcome this difficulty by applying a relatively simple test procedure developed in Kodde and Palm (1986).<sup>16</sup>

Let  $\hat{\boldsymbol{\theta}}$  be the unrestricted estimate of  $\boldsymbol{\theta}$ , define  $\boldsymbol{\vartheta} = \mathbf{R}\boldsymbol{\theta}$ , and consider the unrestricted estimate of this transformed parameter vector  $\hat{\boldsymbol{\vartheta}} = \mathbf{R}\hat{\boldsymbol{\theta}}$  along with a consistent estimate of its variance-covariance matrix  $\hat{\boldsymbol{\Sigma}}$ . Let  $\Theta_0$  be the admissible parameter space under the null.<sup>17</sup> The Kodde and Palm test statistic  $D$  is the  $\hat{\boldsymbol{\Sigma}}$ -metric distance between  $\hat{\boldsymbol{\vartheta}}$  and the closest parameter vector admissible under the null, denoted  $\tilde{\boldsymbol{\vartheta}}$ . That is,  $\tilde{\boldsymbol{\vartheta}} = \arg \min_{\boldsymbol{\vartheta} \in \Theta_0} (\hat{\boldsymbol{\vartheta}} - \boldsymbol{\vartheta})' \hat{\boldsymbol{\Sigma}}^{-1} (\hat{\boldsymbol{\vartheta}} - \boldsymbol{\vartheta})$  and  $D = (\hat{\boldsymbol{\vartheta}} - \tilde{\boldsymbol{\vartheta}})' \hat{\boldsymbol{\Sigma}}^{-1} (\hat{\boldsymbol{\vartheta}} - \tilde{\boldsymbol{\vartheta}})$ . Clearly, if  $\hat{\boldsymbol{\vartheta}} \in \Theta_0$ , i.e., if the unrestricted parameter estimates  $\hat{\boldsymbol{\theta}}$  satisfy the restrictions from Propositions 2 and 3 encoded in  $\mathbf{R}$ , then  $D = 0$ . If  $\hat{\boldsymbol{\vartheta}} \notin \Theta_0$ , then  $D > 0$ . The null is rejected for “large” values of  $D$ . In our case, failure to reject the null constitutes evidence in favor of our generalized quantity-quality trade-off. Kodde and Palm (1986) show that, asymptotically,  $D$  is a mixture of  $\chi^2$ -variables and tabulate critical values of  $D$  for tests of different sizes.

Panel A in Table 8 reports  $D$  for tests of the joint validity of Propositions 2 and 3 for the empirical specification with linear birth order and family size profiles, including demographic controls in the second step.<sup>18</sup> With linear birth order and linear family size profiles, the point estimates of  $\boldsymbol{\vartheta} = (\beta, \beta + \delta/2)'$  are all admissible under the null. Hence, the Kodde-Palm test statistics are always 0, we always fail to

<sup>16</sup>A multiple-testing strategy where a multivariate hypothesis is treated as a sequence of univariate hypotheses results in well-known size problems.

<sup>17</sup>When testing  $H_0$  in (16),  $\Theta_0 = \mathbb{R}_- \times \mathbb{R}_-$ .

<sup>18</sup>The parameter estimates of this specification are reported in Table 5.

reject the restrictions, and the tests invariably come out in favor of our model.

As  $D = 0$  for all the tests in panel A of Table 8, it is tempting to consider “reverse hypotheses” to force a strictly positive Kodde-Palm test statistic. Note, however, that a reversal of the null and the alternative in (16), as in  $H_0 : \mathbf{R}\boldsymbol{\theta} \not\leq \mathbf{0}$  against  $H_1 : \mathbf{R}\boldsymbol{\theta} \leq \mathbf{0}$ , leaves the admissible parameter vector under the null unrestricted. If we are willing to ignore size distortions from a multiple-testing procedure, we can test each of the restrictions of the two propositions individually using standard univariate one-sided  $z$ -tests. Thus, we can consider “direct” as well as “reverse” hypotheses. We designate as direct hypotheses structures those that impose a “less than or equal to” restriction.<sup>19</sup> For the direct hypotheses, failure to reject the null is evidence in favor of our model, while for reverse hypotheses, rejection of the null is interpreted as favorable evidence. Panel B in Table 8 reports  $z$ -statistics and  $P$ -values for individual one-sided tests of Propositions 2 and 3. We report only  $P$ -values for the direct hypotheses, but if the  $P$ -value for the direct hypotheses is, say,  $p$ , the  $P$ -value for the reverse hypotheses is  $1 - p$ , and the conclusion from tests of both direct and reverse hypotheses structures can be gauged from the table.

Consider first the univariate tests of Proposition 3. We note that both direct and reverse tests provide strong evidence in favor of the validity of Proposition 3 across the three estimated empirical models reported in Table 8. We cannot reject the direct nulls, and always reject the reverse nulls. Turning attention to the tests of Proposition 2, test based on the OLS estimates in column (1) provide strong evidence in favor of Proposition 2. We fail to reject the direct null, and consequently, reject the reverse null for all conventional significance levels. For the instrumental variable regression that uses twin birth in the last birth in column (2), the univariate  $z$ -tests provide somewhat mixed evidence on the validity of Proposition 2. On the one hand, with a  $P$ -value of 0.792, we fail to reject the direct null, thus validating the proposition. On the other hand, with the  $P$ -value for the reverse null at 0.208, we also fail to reject the reverse null at the usual levels.<sup>20</sup> For the instrumental variable regression using twin birth at any parity in column (3), the individual univariate tests come out in favor of our model. We fail to reject the direct null with a  $P$ -value of 0.901, and thus (admittedly, just) reject the reverse null at a 10% significance level.

In Tables B.2 and B.3 in Appendix B, we report multivariate Kodde-Palm tests and univariate one-sided  $z$ -tests for the empirical specifications with nonlinear birth order and family size profiles. We note that, for these richer empirical specifications, there are instances where the Kodde-Palm test statistic  $D$  is strictly positive, but even in those cases, the test comes out in favor of the restrictions in Propositions 2 and 3. We also note that Propositions 2 and 3 impose a greater number of restrictions on the parameter vector in the nonlinear specifications, which exacerbates the size distortions in the multiple-testing procedure

<sup>19</sup>For example, considering only the single restriction imposed by Propositions 2, the direct hypothesis structure is  $H_0 : \beta_0 \leq 0$  against  $H_1 : \beta_0 > 0$ , and the reverse hypothesis structure is  $H_0 : \beta_0 \geq 0$  against  $H_1 : \beta_0 < 0$ .

<sup>20</sup>That the direct and reverse tests yield different conclusions regarding Proposition 2 for this set of estimates is not surprising as the point estimate of  $\beta$  in Table 5 is insignificant at conventional levels.

Table 8: Testing Propositions 2 and 3 with linear birth order and family size profiles

	(1)	(2)	(3)
	Step 2: OLS	Step 2: 2SLS	Step 2: 2SLS
Dependent variable:	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>
Instrumental variable:		Twin birth in last birth	Twin birth at any parity
Panel A: Kodde-Palm tests (joint inequality restrictions from Propositions 2 and 3)			
Kodde-Palm test statistic $D^1$	0.000	0.000	0.000
Conclusion	Fail to reject $H_0$	Fail to reject $H_0$	Fail to reject $H_0$
Panel B: One-sided $z$ -tests (individual inequality restrictions from Propositions 2 and 3)			
Proposition 2 <sup>2</sup>			
$z$ -statistic	-16.921	-0.814	-1.290
$P$ -value	0.000	0.208	0.099
Proposition 3 <sup>3</sup>			
$z$ -statistic	-44.790	-3.984	-5.858
$P$ -value	0.000	0.000	0.000

*Note:* The step 1 estimator of the birth order profile is always a family fixed effect regression that includes controls for age and sex. Demographic controls in the step 2 regression include indicators for mother's education, mother's age, father's education, and father's age. The parameter estimates of  $\beta$  and  $\delta$  are reported in Table 5.

<sup>1</sup>The hypothesis structure is  $H_0 : \beta \leq 0$  and  $\beta + \delta/2 < 0$  against  $H_1 : \beta < 0$  or  $\beta + \delta/2 > 0$ . Failure to reject the null constitutes evidence in favor of our model. For critical values for the Kodde-Palm test statistic, see [Kodde and Palm \(1986, Table 1\)](#).

<sup>2</sup>The direct hypothesis structure is  $H_0 : \beta \leq 0$  against  $H_1 : \beta > 0$ . Failure to reject the null constitutes evidence in favor of our model. The reported  $P$ -value refer to this direct null. If the reported  $P$ -value for the direct null is  $p$ , the  $P$ -value for the reverse null is  $1 - p$ .

<sup>3</sup>The direct hypothesis structure is  $H_0 : \beta + \delta/2 \leq 0$  against  $H_1 : \beta + \delta/2 > 0$ . Failure to reject the null constitutes evidence in favor of our model. The reported  $P$ -value refer to this direct null. If the reported  $P$ -value for the direct null is  $p$ , the  $P$ -value for the reverse null is  $1 - p$ .

involving direct and reverse hypotheses.<sup>21</sup> Such distortions notwithstanding, as can be seen from Tables B.2 and B.3, the tests are generally favorable to Propositions 2 and 3. This is particularly true for the empirical specification with nonlinear birth order profiles and linear family size profiles. Overall, the battery of tests conducted on various estimated model specifications provide formal empirical support for our general theory for the analysis of the “quality-quantity trade-off” in the presence of birth order predispositions.

## 6 Conclusions

This paper developed a framework to study the relationship between family size and children’s education when parents allocate resources differentially according to a child’s birth order. We derived a testable generalized quantity-quality trade-off and develop an estimation strategy that recognizes that the within-family resource allocation can vary with birth order, and that family size and birth order cannot be varied independently. We tested our model’s predictions using a population-wide comprehensive panel data set from Denmark. Danish data confirm our theory’s predictions. Particularly, the human capital profile of smaller families lies above the profile of larger families and an increase in family size reduces the average education in the family, or the education of the average child.

Understanding the causal relationship between family size and children’s education is central to many areas in economics. Theoretical studies in population economics have extensively relied on a quantity-quality trade-off but in a context in which children are treated symmetrically (see, e.g., footnote 1). We have shown that the notion of a trade-off between family size and parental investments in education survives the introduction of birth order predispositions if such trade-off is applied to the average child in the household. The relationship between family size and education also figures prominently in economic policy circles in both developing and developed countries. For example, family planning and educational policies have been a central part of development agendas across the developing world. Likewise, the significant decline in fertility rates in a number of developed countries has prompted policies designed to promote larger families. Identifying the effect of family size in the presence of birth order effects helps assessing the costs, benefits, and distributional impacts associated with such policies.

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<sup>21</sup>With nonlinear birth order profiles and linear family size profiles there are 5 testable restrictions. With nonlinear birth order and family size profiles there are 8 testable restrictions.

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## Appendix

### A Omitted derivations and remarks

This Appendix collects derivations and proofs that are not directly obtainable from the text. We also present some remarks and supplemental results. For notational convenience, we will suppress indices when such suppression does not lead to confusion. For example, we write the aggregators  $U(\{h(i)\}, N)$  and  $C(\{h(i)\}, N)$  simply as  $U(\{h\})$  and  $C(\{h\})$ .

**Preliminaries.** We first establish some properties for the aggregator functions  $U(\{h\})$  and  $C(\{h\})$ . These properties are not explicit in the first-order conditions (5) and (6) but are needed to prove Propositions 1 and 2.

Consider  $N$  first. Then,  $U_N(\{h\}) = (\alpha/\rho) (U(\{h\}))^{(\alpha-\rho)/\alpha} u(h(N), N)^\rho > 0$ , if  $\alpha$  and  $\rho$  are of the same sign, which we assume from now on. Also,  $C_N(\{h\}) = (1/\phi) (C(\{h\}))^{1-\phi} C(h(N), N)^\phi > 0$  if  $\phi > 0$ , which we also assume. Second derivatives can be written as

$$\frac{U_{NN}(\{h\})}{U_N(\{h\})} = \left( \frac{\alpha - \rho}{\alpha} \right) \frac{u(h(N), N)^\rho}{U(\{h\})} + \rho \frac{u_h(h(N), N)h_i(N) + u_i(h(N), N)}{u(h(N), N)}, \text{ and} \quad (\text{A1})$$

$$\frac{C_{NN}(\{h\})}{C_N(\{h\})} = (1 - \phi) \frac{c(h(N), N)^\phi}{C(\{h\})} + \phi \frac{c_h(h(N), N)h_i(N) + c_i(h(N), N)}{c(h(N), N)}. \quad (\text{A2})$$

The first terms in (A1) and (A2) are related to the symmetric case in which  $u_i = c_i = h_i = 0$ . In a symmetric case,  $u(h, i) = \hat{u}$  and  $U(\{h\}) = N^{\alpha/\rho} \hat{u}^\alpha$  is strictly concave in  $N$  if  $\alpha < \rho$ , which we now assume. Likewise, if  $c(h, i) = \hat{c}$  then  $C(\{h\}) = N^{1/\phi} \hat{c}$ , which is convex in  $N$  if  $\phi \leq 1$ . We assume that  $0 < \phi \leq 1$ . It is not possible to sign the second derivatives unambiguously as  $h_i(N)$ ,  $u_i(h(N), N)$ , and  $c_i(h(N), N)$  are of unrestricted signs. The ambiguity in the sign of (A1) and (A2) arises in a nonsymmetric assignment and depends on the value of the human capital at  $i = N$ . Sufficient conditions to ensure concavity-convexity can be given in terms of  $u_i(h, i)$  and  $c_i(h, i)$ .

Consider next human capital  $h$ . Notice that  $U_h(\{h\}) = \alpha U(\{h\})^{(\alpha-\rho)/\alpha} u(h, i)^{\rho-1} u_h(h, i) > 0$ , and that  $C_h(\{h\}) = (C(\{h\}))^{1-\phi} c(h, i)^{\phi-1} c_h(h, i) > 0$ . Their second derivatives can be written as

$$\frac{U_{hh}(\{h\})}{U_h(\{h\})} = \left( \frac{\alpha - \rho}{\alpha} \right) \frac{u(h, i)^\rho}{U(\{h\})} \frac{u_h(h, i)}{u(h, i)} + (\rho - 1) \frac{u_h(h, i)}{u(h, i)} + \frac{u_{hh}(h, i)}{u_h(h, i)}, \text{ and} \quad (\text{A3})$$

$$\frac{C_{hh}(\{h\})}{C_h(\{h\})} = (1 - \phi) \frac{c(h, i)^\phi}{C(\{h\})} \frac{c_h(h, i)}{c(h, i)} + (\phi - 1) \frac{c_h(h, i)}{c(h, i)} + \frac{c_{hh}(h, i)}{c_h(h, i)}. \quad (\text{A4})$$

It is possible to see from (A3) that  $U_{hh}(\{h\}) < 0$ . In (A4),  $C_{hh}(\{h\}) > 0$  for  $0 < \phi \leq 1$  if  $c(h, i)$  is sufficiently convex, i.e.,  $c_{hh}(h, i)h/c_h(h, i) \geq (1 - \phi)c_h(h, i)h/c(h, i) > 0$ , which we now assume.

In terms of cross-partial derivatives, the previous restrictions on  $\alpha$ ,  $\rho$ , and  $\phi$  yield

$$\frac{U_{hN}(\{h\})}{U_h(\{h\})} = \left( \frac{\alpha - \rho}{\alpha} \right) \frac{u(h(N), N)^\rho}{U(\{h\})} < 0, \text{ and } \frac{C_{hN}(\{h\})}{C_h(\{h\})} = (1 - \phi) \frac{c(h(N), N)^\phi}{C(\{h\})} > 0. \quad (\text{A5})$$

For birth order,

$$\frac{U_{hi}(\{h\})}{U_h(\{h\})} = (\rho - 1) \frac{u_i(h, i)}{u(h, i)} + \frac{u_{hi}(h, i)}{u_h(h, i)}, \text{ and } \frac{C_{hi}(\{h\})}{C_h(\{h\})} = (\phi - 1) \frac{c_i(h, i)}{c(h, i)} + \frac{c_{hi}(h, i)}{c_h(h, i)}. \quad (\text{A6})$$

The signs of the expressions in (A6) are unrestricted.

**Parental choices.** To characterize parental choices for  $N^*$  and  $h^*$ , let the objective function of the parental problem be

$$\bar{u}(N, \{h\}) \equiv \mathcal{U}((Y - C(\{h\})/P_X, N, U(\{h\})).$$

Its first-order conditions  $\bar{u}_N(N^*, \{h^*\}) = 0$  and  $\bar{u}_h(N^*, \{h^*\}) = 0$  coincide with (5) and (6). To ensure that  $\bar{u}$  is concave, and that the optimal choices are interior, we need to assume that  $\mathcal{U} \in \mathcal{C}^2$  is strictly increasing, strictly concave, and satisfy Inada conditions.<sup>22</sup>

In particular,  $\bar{u}_{NN} = \mathcal{U}_{XX}(C_N/P_X)^2 - 2\mathcal{U}_{XN}(C_N/P_X) - 2\mathcal{U}_{XU}(C_N U_N/P_X) - \mathcal{U}_X(C_{NN}/P_X) + \mathcal{U}_{NN} + 2\mathcal{U}_{NU}U_N + \mathcal{U}_{UU}(U_N)^2 + \mathcal{U}_U U_{NN}$ . The terms  $\mathcal{U}_{XX}$ ,  $\mathcal{U}_{NN}$ , and  $\mathcal{U}_{UU}$  are all negative. The terms  $\mathcal{U}_{XN}$  and  $\mathcal{U}_{XU}$  are positive by assumption. Assuming that  $\mathcal{U}_{NU} < 0$  is sufficient for  $\bar{u}_{NN} < 0$  provided that  $U_{NN}$  is negative in (A1). If  $U_{NN} > 0$ , one needs to assume that  $\mathcal{U}$  is sufficiently concave in  $U$  so that  $\mathcal{U}_{UU}U_N/\mathcal{U}_U + U_{NN}/U_N < 0$ . No extra assumptions are needed for  $\bar{u}_{hh} = \mathcal{U}_{XX}(C_h/P_X)^2 - 2\mathcal{U}_{XU}(C_h U_h/P_X) - \mathcal{U}_X(C_{hh}/P_X) + \mathcal{U}_{UU}(U_h)^2 + \mathcal{U}_U U_{hh} < 0$ , whose sign follows from previous assumptions.

**Proof of Proposition 1.** The derivation of (7) is an application of the *Implicit Function Theorem* for concave and differentiable objective functions with interior solutions, which states that the function  $h^*(i|N^*)$  is increasing (resp. decreasing) in  $i \in [0, N^*]$  iff  $\bar{u}_{hi} > (<)0$  whenever  $\bar{u}_h = 0$ . From (6),  $\bar{u}_{hi} = -\mathcal{U}_X(C_{hi}/P_X) + \mathcal{U}_U U_{hi}$ , which can be written as  $-C_{hi}/C_h + U_{hi}/U_h$ , with the cross-partial derivatives listed in (A6). ■

**Proof of Proposition 2.** For any value of  $z$ , including  $z = 0$ , optimal choices  $h^*(i|N^* + z)$  satisfy (6) or  $\bar{u}_h(N^* + z, \{h^*\}) = 0$ . Since parents cannot respond to exogenous changes in family size, (5) is irrelevant and we can treat  $N^* + z$  as a parameter in (6). As an application of the *Implicit Function Theorem*,  $h^*(i|N^* + z)$  would be a decreasing function of  $z$  at each birth order  $i$  iff  $(\bar{u}_{hz} =)\bar{u}_{hN} < 0$ . In particular,  $\bar{u}_{hN} = \mathcal{U}_{XX}C_N C_h(1/P_X)^2 - \mathcal{U}_{XN}(C_h/P_X) - \mathcal{U}_{XU}(U_N C_h/P_X) - \mathcal{U}_X(C_{hN}/P_X) - \mathcal{U}_{UX}(U_h C_N/P_X) + \mathcal{U}_{UN}U_h + \mathcal{U}_{UU}U_h U_N + \mathcal{U}_U U_{hN}$ , which is negative provided the complementarity assumptions  $\mathcal{U}_{XN} > 0$  and  $\mathcal{U}_{XU} > 0$ , the substitution assumption  $\mathcal{U}_{NU} < 0$ , and the cross-partial derivatives  $C_{hN} > 0$  and  $U_{hN} < 0$  obtained in (A5). ■

<sup>22</sup>Concavity conditions for  $\bar{u}$  are standard (i.e.,  $\bar{u}_{NN} < 0$ ,  $\bar{u}_{hh} < 0$ , and  $\bar{u}_{NN}\bar{u}_{hh} - \bar{u}_{Nh}^2 > 0$ ). From a variational point of view, the parental problem can be seen as an *isoperimetric* problem; see Hestenes (1966). Its second-order condition is standard in variational problems with optimal endpoints; see Vincent and Bruschi (1970, Theorem 3.1).

**Some remarks.** (i) The cross-partial derivative  $\bar{u}_{hN}$  is not informative about the channels by which family size lowers human capital. The term  $\bar{u}_{hN}$  can be written as  $-\bar{u}_{hY}C_N + \mathcal{U}_{UN}U_h - \mathcal{U}_{XN}(C_h/P_X) - \mathcal{U}_X(C_{hN}/P_X) - \mathcal{U}_{UX}(U_hC_N/P_X) + \mathcal{U}_{UU}U_hU_N + \mathcal{U}_U U_{hN}$ . Assume that  $\mathcal{U}_{XN} = \mathcal{U}_{XU} = 0$ , as in the Barro-Becker formulation, and take  $\alpha = \rho = \phi = 1$  for expositional purposes. Focus on  $-\bar{u}_{hY}C_N + \mathcal{U}_{UN}U_h + \mathcal{U}_{UU}U_hU_N$ . The first term is associated with the response of human capital to changes in parental income. Since human capital is a normal good,  $\bar{u}_{hY} > 0$ , this term arises because the cost of children increases  $C_N > 0$  leaving less resources for parental investments.<sup>23</sup> The second term  $\mathcal{U}_{UN}U_h$  is negative when  $U$  and  $N$  are substitutes. There, the marginal value of human capital investments declines as  $N$  increases. The last term arises due to the influence of  $N$  in the marginal utility  $\mathcal{U}_U$  due to its effects in  $U$ .

(ii) The assumptions needed for Proposition 2 apply to the special cases studied by Becker and Lewis (1973) and Rosenzweig and Wolpin (1980). Let the aggregator in (1) be  $U(N, H)$  with  $U_{NH}(N, H) < 0$  and the cost  $C(H, N)$  as in (4). The cross-partial derivative between  $H$  and  $N$  (or  $z$ ) for  $\bar{u}(N, H) \equiv \mathcal{U}((Y - C(N, H))/P_X, N, U(N, H))$  is  $\bar{u}_{HN} = \mathcal{U}_{XX}(P_N + \Pi H)(P_H + \Pi N)/P_X^2 - \mathcal{U}_{XN}(P_H + \Pi N)/P_X - \mathcal{U}_{XU}U_N(P_H + \Pi N)/P_X - \mathcal{U}_X(\Pi/P_X) - \mathcal{U}_{UX}U_H(P_N + \Pi H)/P_X + \mathcal{U}_{UN}U_H + \mathcal{U}_{UU}U_NU_H + \mathcal{U}_U U_{NH}$ , which depends on the same preference terms as in Proposition 2. To invalidate Proposition 2, human capital should be inferior,  $\bar{u}_{hY} < 0$ , family size and the aggregate utility of children should be complements  $\mathcal{U}_{UN} > 0$ , and parental consumption should be a substitute with family size and the children's utilities, as in  $\mathcal{U}_{XN} < 0$  and  $\mathcal{U}_{XU} < 0$ .

(iii) Parents cannot adjust their fertility choices in response to additional children. If parents could perfectly insure against twinning, then  $N^*(Z) = N^*$  if  $Z = 0$  and  $N^*(Z) = N^* - z$  if  $Z = z$ . Family size for parents with twins would be the same as for parents without twins. Consider an intermediate case in which parents can *partially* adjust to the presence of twins. As before, the first-order condition for  $h^*(i|N^* + z)$  is  $\bar{u}_h(N^* + z, \{h^*\}) = 0$ . Totally differentiating this expression yields  $\bar{u}_{hN}(dN^* + dz) + \bar{u}_{hh}dh^* = 0$ . In Proposition 2,  $dN^* = 0$  and  $h_z^*(i|N^* + z) = -(\bar{u}_{hN}/\bar{u}_{hh}) < 0$ , as there is no margin of adjustment. Assume instead that parents with twins adjust their fertility choice as if they had no additional children. That is, suppose that  $N^*$  changes while maintaining  $\bar{u}_N(N^*, \{h^*\}) = 0$ , which the first-order condition for parents without twins. This means that parents can partly 'undo' the increase in family size. Since  $\bar{u}_{NN}dN^* + \bar{u}_{Nh}dh^* = 0$ , simple substitutions yield  $h_z^*(i|N^* + z) = -(\bar{u}_{hN}/\bar{u}_{hh})(\bar{u}_{hh}\bar{u}_{NN}/(\bar{u}_{Nh}^2 - \bar{u}_{hh}\bar{u}_{NN}))$ , which is of the same sign as  $\bar{u}_{hN}$  although quantitatively smaller than in the case of no adjustment.

(iv) Proposition 2 examines the response to an exogenous change in family size. By symmetry, one could obtain the same qualitative trade-off by studying the fertility response to an exogenous decline in human capital. Consider the first-order condition (5) for parents who experience an exogenous (and uniform, for simplicity) decrease in human capital by  $z'$ . Since  $\bar{u}_N(N^*, \{h^*\} - z') = 0$ ,  $\bar{u}_{NN}dN^* = \bar{u}_{Nh}dz'$ ,

<sup>23</sup>Income effects in  $N$  and  $\{h\}$  depend on  $\bar{u}_{NY} = -\mathcal{U}_{XX}C_N(1/P_X)^2 + \mathcal{U}_{NX}/P_X + \mathcal{U}_{UX}U_N/P_X$ , and  $\bar{u}_{hY} = -\mathcal{U}_{XX}C_h(1/P_X)^2 + \mathcal{U}_{UX}U_h/P_X$ . Complementarity in  $(X, N)$  and  $(X, U)$  is sufficient to ensure that both income effects are positive.

such that these parents would increase family size.

**Proof of Proposition 3.** Differentiating  $H(N^* + z)$  in (9) yields

$$H_z(N^* + z) = \frac{1}{N^* + z} \left[ h^*(N^* + z|N^* + z) + \int_0^{N^* + z} h_z^*(i|N^* + z) di - H(N^* + z) \right]. \quad (\text{A7})$$

Since  $h_z^*(i|N^* + z) < 0$ , the only ambiguity in (A7) is  $h^*(N^* + z|N^* + z)$ .

If  $h^*(i|N^*)$  is decreasing in  $i$ , then  $H(N^* + z) > h^*(N^* + z|N^* + z)$  and  $H_z(N^* + z) < 0$ . This case corresponds to the case of negative birth orders. Consider next positive birth order effects. We can write

$$h^*(N^* + z|N^* + z) = h^*(0|N^* + z) + \int_0^{N^* + z} h_i^*(i|N^* + z) di, \quad (\text{A8})$$

with  $h^*(0|N^* + z) \leq H(N^* + z)$ . A sufficient condition for a negative sign in (A7) is  $h_z^*(i|N^* + z) + h_i^*(i|N^* + z) \leq 0$ , or  $\bar{u}_{hN} + \bar{u}_{hi} \leq 0$ . Reorganizing terms from  $\bar{u}_{hN}$  and  $\bar{u}_{hi}$  shows that strengthening the degree of complementarity  $\mathcal{U}_{XN} > 0$  and substitutability  $\mathcal{U}_{UN} < 0$  such that

$$\frac{\mathcal{U}_{UN}}{\mathcal{U}_U} - \frac{\mathcal{U}_{XN}}{\mathcal{U}_X} - \frac{C_{hi}}{C_h} + \frac{U_{hi}}{U_h} \leq 0, \quad (\text{A9})$$

ensures that  $\bar{u}_{hN} + \bar{u}_{hi} \leq 0$ . It is also possible to bound the positive birth order effects by strengthening the concavity of the parental utility function, as in

$$\frac{\mathcal{U}_{UU}}{\mathcal{U}_U} U_N + \frac{\mathcal{U}_{XX}}{\mathcal{U}_X} \frac{C_N}{P_X} - \frac{C_{hi}}{C_h} + \frac{U_{hi}}{U_h} \leq 0. \quad (\text{A10})$$

Under either of the previous bounds,  $H_z(N^* + z) < 0$  for positive birth order effects. ■

**Some remarks.** (i) The relevant term to sign  $H_z(N^* + z)$  under positive birth order effects in (A7) and (A8) is  $h_z^*(i|N^* + z) + h_i^*(i|N^* + z)$ . These expressions arise because family size and birth order cannot vary independently. They also serve to sign the differences between median children under positive birth order effects in the text; see (10). If the bounds in (A9) or (A10) hold, median human capital will be ranked as average human capital.

(ii) A *generalized* average human capital of order  $p$  is given by

$$H(N^* + z; p) = \left( \frac{1}{N^* + z} \int_0^{N^* + z} h(i|N^* + z)^p \right)^{1/p},$$

with the harmonic ( $p = -1$ ) and geometric ( $p = 0$ ) average as special cases. The arithmetic average used in (9) and Proposition 3 assumes  $p = 1$ . In the absence of birth order effects, all averages are equal. The function  $H(N^* + z; p)$  is positive and decreasing in  $p$ . For instance,  $H(N^* + z; p = +\infty) \equiv \max_i h^*(i|N^* + z) = h^*(N^* + z|N^* + z)$ . Therefore,  $H(N^*; +\infty) - H(N^* + z; +\infty) = h^*(N^*|N^*) - h^*(N^* + z|N^* + z)$ , which can be written as  $[h^*(N^*|N^*) - h^*(N^*|N^* + z)] + [h^*(N^*|N^* + z) - h^*(N^* + z|N^* + z)]$ . The first

term is positive (Proposition 2) but the second term is negative. Negative values of  $p$  give more weight to the lowest human capital in the household. For instance,  $H(N^* + z; p = -\infty) \equiv \min_i h^*(i|N^* + z) = h^*(0|N^* + z)$  under positive birth order effects. By Proposition 2,  $H(N^*; -\infty) - H(N^* + z; -\infty) = h^*(0|N^*) - h^*(0|N^* + z) > 0$ , even under positive birth order effects.

(ii) We considered continuous fertility choices. Under discrete family size, the relevant derivative for  $N$  is  $\bar{u}_N(N, \{h\}) \equiv \bar{u}(N + 1, \{h\}) - \bar{u}(N, \{h\})$ , and the first-order condition for  $N^*$  is  $\bar{u}_N(N^* - 1, \{h\}) \geq 0 > \bar{u}_N(N^*, \{h\})$ . If  $\bar{u}(N, \{h\})$  is strictly concave, either the optimal family size is unique or there are two neighboring numbers that are optimal and leave parents indifferent between them. If twins change family size, the response would induce a trade-off between quantity and quality.

(iii) Our testable predictions assume that parental wealth is unchanged in response to exogenous changes in family size. The budget constraint can be generalized without affecting the main conclusions of the analysis. Suppose that parents incur in a time cost  $\omega \leq 1/N^+$  proportional to each child. The generalized budget constraint is  $P_X X + C(\{h(i)\}) = Y(1 - \omega N)$ , where time worked is  $(1 - \omega N) \in (0, 1)$ . An exogenous increase in family size reduces parental resources even more compared to our baseline case. In the baseline case, the marginal cost of an additional child is  $C_N(\{h(i)\})$  whereas now the marginal cost is  $C_N(\{h(i)\}) + \omega Y$ . Time costs can vary by birth order, but they would simply act as additions to the marginal cost  $C_h(\{h(i)\})$ .

## B Additional empirical results

This appendix provide additional empirical results references in the main text. Table B.1 contains the estimated coefficients from a flexible empirical specification with nonlinear birth order profiles and nonlinear family size profiles. Table B.2 contains the tests of Propositions 2 and 3 for a specification with nonlinear birth order profiles and linear family size profiles, and Table B.3 contains the tests for the empirical specification with nonlinear birth order profiles and nonlinear family size profiles. Finally, Table B.4 presents estimated parameters for the empirical specification with nonlinear and family-size dependent birth order profiles and linear family size profiles, and Table B.5 presents the estimates for the specification with nonlinear and family-size dependent birth order profiles and nonlinear family size profiles. In addition, results for three types of families (families with only boys, families with only girls, and mixed gender families) are available upon request from the authors.

Table B.1: Two-step estimation with nonlinear family size and birth order profiles

	(1)	(2)	(3)
	Step 1: OLS w/ family fixed effects	Step 2: OLS	Step 2: 2SLS
Dependent variable:	Child's years of education	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>
Second child	-0.274*** (0.007)		
Third child	-0.423*** (0.014)		
Fourth child	-0.488*** (0.021)		
Fifth child	-0.437*** (0.034)		
Sixth child or later	-0.437*** (0.052)		
Family size 3		-0.015** (0.007)	-0.027 (0.049)
Family size 4		-0.155*** (0.013)	-0.134* (0.073)
Family size 5		-0.468*** (0.023)	-0.084 (0.121)
Family size 6+		-0.778*** (0.038)	-0.284 (0.264)
Composite family size 2		-0.137*** (0.004)	-0.137*** (0.004)
Composite family size 3		-0.248*** (0.008)	-0.259*** (0.050)
Composite family size 4		-0.452*** (0.012)	-0.430*** (0.076)
Composite family size 5		-0.793*** (0.018)	-0.409*** (0.123)
Composite family size 6		-1.121*** (0.033)	-0.627** (0.265)
First stage:			
Minimum eigenvalue statistic <sup>2</sup>			2, 657.7
Observations	1,278,510	596,292	596,292

*Note:* \*\*\* indicates statistical significance at the 1 percent level. \*\* indicates statistical significance at the 5 percent level. \* indicates statistical significance at the 10 percent level. Standard errors for all regressions are computed by block-bootstrapping at the family level (100 repetitions) and are given in brackets. The family fixed effect regression in column (1) includes controls for age and sex. Demographic controls in the regression in columns (2) and (3) include indicators for mother's education, mother's age, father's education, and father's age. Single child families are excluded from the analysis data.

Single child families are excluded from the analysis data.

<sup>1</sup>The family-level average education in columns (2)-(3) nets out the effects of age, sex as well as the average birth order effect, see (14).

<sup>2</sup>See Stock and Yogo (2002) for critical values. We clearly reject the null hypothesis of weak instruments.

Table B.2: Testing Propositions 2 and 3 with nonlinear birth order profiles and linear family size profiles

	(1)	(2)	(3)
	Step 2: OLS	Step 2: 2SLS	Step 2: 2SLS
Dependent variable:	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>
Instrumental variable:		Twin birth in last birth	Twin birth at any parity
Panel A: Kodde-Palm tests (joint inequality restrictions from Propositions 2 and 3)			
Kodde-Palm test statistic $D^1$	0.000	0.000	0.000
Conclusion	Fail to reject $H_0$	Fail to reject $H_0$	Fail to reject $H_0$
Panel B: One-sided $z$ -tests (individual inequality restrictions from Propositions 2 and 3)			
Proposition 2 <sup>2</sup>			
$t$ -statistic	-19.239	-1.464	-2.597
$P$ -value	1.000	0.928	0.995
Proposition 3 <sup>3</sup>			
$t$ -statistic, 3- versus 2-child families	-45.038	-4.806	-7.436
$P$ -value	1.000	1.000	1.000
$t$ -statistic, 4- versus 3-child families	-32.814	-3.652	-5.625
$P$ -value	1.000	0.998	1.000
$t$ -statistic, 5- versus 4-child families	-20.024	-2.492	-4.025
$P$ -value	1.000	0.994	1.000
$t$ -statistic, 6- versus 5-child families	-14.839	-2.021	-3.242
$P$ -value	1.000	0.978	1.000

*Note:* The step 1 estimator of the birth order profile is always a family fixed effect regression that includes controls for age and sex. Demographic controls in the step 2 regressions include indicators for mother's education, mother's age, father's education, and father's age. The parameter estimates of  $\beta$  and  $\delta$  are reported in Table 6.

<sup>1</sup>The hypothesis structure is  $H_0 : \beta \leq 0$  and  $\beta + \bar{\delta}_N - \bar{\delta}_{N-1} \leq 0$  for all  $N \in \{3, 4, 5, 6\}$  against  $H_1 : \beta > 0$  or  $\beta + \bar{\delta}_N - \bar{\delta}_{N-1} > 0$  for some  $N \in \{3, 4, 5, 6\}$ . Failure to reject the null constitutes evidence in favor of our model. For critical values for the Kodde-Palm test statistic, see [Kodde and Palm \(1986, Table 1\)](#).

<sup>2</sup>The direct hypothesis structure is  $H_0 : \beta \leq 0$  against  $H_1 : \beta > 0$ . Failure to reject the null constitutes evidence in favor of our model. The reported  $P$ -value refer to this direct null. If the reported  $P$ -value for the direct null is  $p$ , the  $P$ -value for the reverse null is  $1 - p$ .

<sup>3</sup>The direct hypothesis structure comparing  $N$ - and  $(N - 1)$ -child families is  $H_0 : \beta + \bar{\delta}_N - \bar{\delta}_{N-1} \leq 0$  against  $H_1 : \beta + \bar{\delta}_N - \bar{\delta}_{N-1} > 0$  where  $N \in \{3, 4, 5, 6\}$ . Failure to reject the null constitutes evidence in favor of our model. The reported  $P$ -value refer to this direct null. If the reported  $P$ -value for the direct null is  $p$ , the  $P$ -value for the reverse null is  $1 - p$ .



Table B.3: Testing Propositions 2 and 3 with nonlinear birth order profiles and family size profiles

	(1)	(2)
	Step 2: OLS	Step 2: 2SLS
Dependent variable:	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>
Panel A: Kodde-Palm tests (joint inequality restrictions from Propositions 2 and 3)		
Kodde-Palm statistic $D^1$	0.000	0.215
Conclusion	Fail to reject $H_0$	Fail to reject $H_0$
Panel B: One-sided $z$ -tests (individual inequality restrictions from Propositions 2 and 3)		
Proposition 2 <sup>2</sup>		
$t$ -statistic, 3- versus 2-child families	-2.253	-0.547
$P$ -value	0.988	0.618
$t$ -statistic, 4- versus 3-child families	-11.950	-1.473
$P$ -value	1.000	0.930
$t$ -statistic, 5- versus 4-child families	-14.749	0.464
$P$ -value	1.000	0.321
$t$ -statistic, 6- versus 5-child families	-7.857	-0.784
$P$ -value	1.000	0.783
Proposition 3 <sup>3</sup>		
$t$ -statistic, 3- versus 2-child families	-17.802	-2.454
$P$ -value	1.000	0.993
$t$ -statistic, 4- versus 3-child families	-18.183	-2.334
$P$ -value	1.000	0.990
$t$ -statistic, 5- versus 4-child families	-18.320	0.201
$P$ -value	1.000	0.420
$t$ -statistic, 6- versus 5-child families	-8.883	-0.854
$P$ -value	1.000	0.803

*Note:* The step 1 estimator of the birth order profile is always a family fixed effect regression that includes controls for age and sex. Demographic controls in the step 2 regression include indicators for mother's education, mother's age, father's education, and father's age. The parameter estimates of  $\beta$  and  $\delta$  are reported in Table B.1.

<sup>1</sup>The hypothesis structure is  $H_0 : \beta_N - \beta_{N-1} \leq 0$  and  $\beta_N + \bar{\delta}_N - (\beta_{N-1} + \bar{\delta}_{N-1}) \leq 0$  for all  $N \in \{3, 4, 5, 6\}$  against  $H_1 : \beta_N - \beta_{N-1} > 0$  or  $\beta_N + \bar{\delta}_N - (\beta_{N-1} + \bar{\delta}_{N-1}) > 0$  for some  $N \in \{3, 4, 5, 6\}$ . Failure to reject the null constitutes evidence in favor of our model. For critical values for the Kodde-Palm test statistic, see [Kodde and Palm \(1986, Table 1\)](#).

<sup>2</sup>The direct hypothesis structure comparing  $N$ - and  $(N - 1)$ -child families is  $H_0 : \beta_N - \beta_{N-1} \leq 0$  against  $H_1 : \beta_N - \beta_{N-1} > 0$  where  $N \in \{3, 4, 5, 6\}$ . Failure to reject the null constitutes evidence in favor of our model. The reported  $P$ -value refer to this direct null. If the reported  $P$ -value for the direct null is  $p$ , the  $P$ -value for the reverse null is  $1 - p$ .

<sup>3</sup>The direct hypothesis structure comparing  $N$ - and  $(N - 1)$ -child families is  $H_0 : \beta + \bar{\delta}_N - \bar{\delta}_{N-1} \leq 0$  against  $H_1 : \beta + \bar{\delta}_N - \bar{\delta}_{N-1} > 0$  where  $N \in \{3, 4, 5, 6\}$ . Failure to reject the null constitutes evidence in favor of our model. The reported  $P$ -value refer to this direct null. If the reported  $P$ -value for the direct null is  $p$ , the  $P$ -value for the reverse null is  $1 - p$ .

Table B.4: Two-step estimation with linear family size and nonlinear family size-specific birth order profiles

	(1)	(2)	(3)	(4)
	Step 1: OLS w/ family FE	Step 2: OLS	Step 2: 2SLS	Step 2: 2SLS
Dependent variable:	Child's years of education	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>	Family-level average years of education <sup>1</sup>
Instrumental variable:			Twin birth in last birth	Twin birth at any parity
Second in 2 child family	-0.346*** (0.009)			
Second in 3 child family	-0.249*** (0.010)			
Third in 3 child family	-0.491*** (0.017)			
Second in 4 child family	-0.180*** (0.020)			
Third in 4 child family	-0.330*** (0.022)			
Fourth in 4 child family	-0.509*** (0.025)			
Second in 5 child family	-0.112*** (0.039)			
Third in 5 child family	-0.245*** (0.044)			
Fourth in 5 child family	-0.318*** (0.046)			
Fifth in 5 child family	-0.394*** (0.045)			
Second in 6+ child family	-0.073 (0.068)			
Third in 6+ child family	-0.140** (0.068)			
Fourth in 6+ child family	-0.277*** (0.072)			
Fifth in 6+ child family	0.242*** (0.074)			
6th or later in 6+ child family	-0.325*** (0.066)			
Family size		-0.132*** (0.006)	-0.068** (0.028)	-0.074*** (0.019)
Composite family size 2		-0.173*** (0.004)	-0.173*** (0.004)	-0.173*** (0.004)
Composite family size 3		-0.379*** (0.008)	-0.315*** (0.030)	-0.320*** (0.021)
Composite family size 4		-0.520*** (0.013)	-0.391*** (0.058)	-0.402*** (0.040)
Composite family size 5		-0.611*** (0.027)	-0.419*** (0.090)	-0.434*** (0.067)
Composite family size 6		-0.706*** (0.045)	-0.449*** (0.113)	-0.470*** (0.083)
First stage:				
Min. eigenvalue stat. <sup>2</sup>			15,825.5	28,156.6
Observations	1,278,510	596,292	596,292	596,292

Note: For explanatory table notes, consult tables in the main text.

Table B.5: Two-step estimation with nonlinear family size and family size-specific birth order profiles

	(1)	(2)	(3)
	Step 1: OLS w/ family fixed effects	Step 2: OLS	Step 2: 2SLS
Dependent variable:	Child's years of education	Family-level average years of education	Family-level average years of education
Second in 2 child family	-0.346*** (0.009)		
Second in 3 child family	-0.249*** (0.010)		
Third in 3 child family	-0.491*** (0.017)		
Second in 4 child family	-0.180*** (0.020)		
Third in 4 child family	-0.330*** (0.022)		
Fourth in 4 child family	-0.509*** (0.025)		
Second in 5 child family	-0.112*** (0.039)		
Third in 5 child family	-0.245*** (0.044)		
Fourth in 5 child family	-0.318*** (0.046)		
Fifth in 5 child family	-0.394*** (0.045)		
Second in 6+ child family	-0.073 (0.068)		
Third in 6+ child family	-0.140** (0.068)		
Fourth in 6+ child family	-0.277*** (0.072)		
Fifth in 6+ child family	0.242*** (0.074)		
6th or later in 6+ child family	-0.325*** (0.066)		
Family size 3		-0.038*** (0.009)	-0.034 (0.049)
Family size 4		-0.233*** (0.018)	-0.175** (0.074)
Family size 5		-0.617*** (0.037)	-0.181 (0.124)
Family size 6+		-0.986*** (0.063)	-0.432 (0.278)
Composite family size 2		-0.173*** (0.004)	-0.173*** (0.004)
Composite family size 3		-0.284*** (0.008)	-0.280*** 0.050
Composite family size 4		-0.488*** (0.012)	-0.430*** 0.077
Composite family size 5		-0.830*** (0.019)	-0.395*** 0.124
Composite family size 6		-1.162*** (0.033)	-0.608** 0.264
First stage:			
Minimum eigenvalue statistic			2,657.7
Observations	1,278,510	596,292	596,292

Note: For explanatory table notes, consult tables in the main text.