Search, Matching and Training

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Abstract

We estimate a partial and general equilibrium search model in which firms and workers choose how much time to invest in both general and match-specific human capital. To help identify the model parameters, we use NLSY data on worker training and we match moments that relate the incidence and timing of observed training episodes to outcomes such as wage growth and job-to-job transitions. We use our model to offer a novel interpretation of standard Mincer wage regressions in terms of search frictions and returns to training. Finally, we show how a minimum wage can reduce training opportunities and decrease the amount of human capital in the economy.

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1 Introduction

There is a long history of interest in human capital investment, both before and after entry into the labor market. In the latter case, it is common to speak of general and specific human capital, which are differentiated in terms of their productivity-enhancing effects across jobs (which may be defined by occupations, industries, or firms). The classic analysis of Becker (1964) considered these types of investments in competitive markets and concluded that workers should pay the full costs of general training, with the costs of specific training (that increases productivity only at the current employer) being shared in some way. Analysis of these investments in a noncompetitive setting is more recent. Acemoglu and Pischke (1999) consider how the predictions of the amount and type of human capital investment in a competitive labor market are altered when there exist market imperfections in the form of search frictions. Frictions create an imperfect "lock in" between a worker and the firm, so that increases in general or specific human capital are generally borne by both the worker and the firm.

We introduce training decisions into what is otherwise a reasonably standard search model with general and specific human capital. The training data we use to estimate the model, described briefly below, indicate that formal training is reported by a not insignificant share of workers, and that the likelihood of receiving training is a function of worker characteristics, in particular, education. Workers do not receive training only at the beginning of job spells, although the likelihood of receiving training is typically a declining function of tenure. Since training influences the likelihood of termination of the job and wages, it is important to examine training decisions in a relatively complete model of worker-firm employment relationships.

One motivation for this research is related to recent observations regarding shifts in the Beveridge curve, which is the relationship between job vacancies and job searchers. While the unemployment rate in the U.S. has been markedly higher from 2008 and beyond,[1] reported vacancies remain high. This mismatch phenomenon has been investigated through a variety of modeling frameworks (see, e.g., Cairo (2013) and Lindenlaub (2013)), typically by allowing some shift in the demand for workers' skills. In our modeling framework, such a shift could be viewed as a downward movement in the distribution of initial match productivities. Given the absence of individuals with the desired skill sets, the obvious question is why workers and firms do not engage in on-the-job investment so as to mitigate the mismatch in endowments. Using our model, we can theoretically and empirically investigate the degree to which a decentralized labor market with search frictions is able to offset deterioration in the initial match productivity distribution.

Another motivation for our research is to provide a richer model of the path of wages on the job and a more complete view of the relationship between workers and firms.

[1] Although the unemployment rate has declined recently, the employment rate in the population is at a historic low. Many of those counted as out of the labor force are in fact willing to take a "reasonable" job offer, and hence should be considered to be "unemployed" in the true sense of the term.

In this model, firms offer workers the opportunity to make mutually advantageous investments in the worker's skills, both of the general and specific (to the job) type. While investing, the worker devotes less time to productive activities, which is the only cost of investment that we include in the model.[2] Wage changes over the course of the employment spell are produced by changes in general skill levels, changes in specific skill levels, and changes in investment time. In models that include on-the-job search possibilities, which is the case for ours, wages during an employment spell may also increase due to the presence of another firm bidding for the employee's services, as in Postel-Vinay and Robin (2002), Dey and Flinn (2005), and Cahuc et al. (2006). When there exists the potential for other firms to "poach" the worker from her current firm, investments in match-specific skills will be particularly attractive to the current employer, since high levels of match-specific skills will make it less likely that the worker will exit the firm for one in which her initial match-specific skill level is higher. Other things equal, these differences in the retention value component of specific-skill investment implies that firms will reduce the employee's wage less for a given level of specific-skill investment than for general skill investment.

In our modelling framework, the complementarity between general and specific human capital gives firms an incentive to partially finance improvements in general ability. Specifically, the flow productivity of a match is given by $y(a, \theta) = a\theta$, where a is the general ability of the worker, and θ is match productivity. The gain in flow productivity from a small change in a is simply θ and the gain in flow productivity from a small change in θ is simply a. Jobs for which a is relatively high in comparison to θ will experience bigger productivity gains from investment in θ and conversely for job matches in which a is relatively low in comparison with θ. Thus, strictly from the productivity standpoint, there will be an incentive to "balance" a and θ in the investment process. This, coupled with the fact that there exist search frictions, will lead firms to be willing to finance part of the investment in general human capital, even if this does not change the expected duration of the match.

We believe that our modeling framework may be useful in understanding the link between initial labor market endowments and earnings inequality over the labor market career. Flinn and Mullins (2015) estimated a model with an identical specification for flow productivity as the one employed here and examined the pre-market entry schooling decision. In their model, initial ability endowments were altered by schooling decisions, and these decisions were a function of all of the primitive parameters characterizing the labor market. In their setting, a was fixed over the labor market career and a match draw at a firm was also fixed over the duration of the job spell. In

[2]That is, the only costs of investing in either of the skills is the lost output associated with the investment time. Moreover, we will assume that these costs are the same for either type of investment. There are no direct costs of investment as in Wasmer (2006), for example. In his case, all investment in skills occurs instantaneously at the beginning of a job spell. Lentz and Roys (2015), instead, assume that general and match-specific skills are binary, and that for a low-type worker on either dimension the cost of training is a flow cost that is increasing in the rate at which the transition to the high skill type occurs. Depreciation in skills is not considered in either paper.

the case of our model, both a and θ are subject to (endogenous and exogenous) change, although it may well be that a is more difficult and costly to change after labor market entry. This is due to the fact that employers are not equipped to offer general learning experiences as efficiently as are schools that specialize in increasing the cognitive and noncognitive abilities of their students. To the extent that a is essentially fixed over the labor market career, individuals with large a endowments will be more attractive candidates for investment in match-specific skills than will individuals with low values of a. Even if initial values of θ are drawn from the same distribution for all a types, which is the assumption that we make below, higher values of a at the time of labor market entry could lead to more investments and a more rapidly increasing wage profile over the course of an employment spell. This offers a mechanism to amplify the differences in earnings generated by the initial variability in a.

Using estimates from the model, it is possible to examine the impact of various types of labor market policies on investment in the two types of human capital. For example, Flinn and Mullins (2015) investigated the impact of minimum wage laws on pre-market investment. They found that for relatively low (yet binding for some low a workers) minimum wage levels, the minimum wage could be a disincentive for pre-market investment, since similar wage rates could be obtained without costly education. At higher levels of the minimum wage, however, most individuals invested in a to increase their probability of finding a job. For firms to earn nonnegative profit flows in that model, productivity has to be at least $a\theta \geq m$, where m is the minimum wage. For high values of m, workers will invest in general ability to increase their level of a so as to increase the likelihood of generating a flow productivity level that satisfies the firm's nonnegative flow profit condition. In our framework of post-entry investment, the impact of minimum wages is also ambiguous. As in the standard Becker story for competitive labor markets, a high minimum wage will discourage investment activity if the firm is to achieve nonnegative flow profits. On the other hand, through investment activity that raises the individual's productivity (through a and/or θ), the firm and worker can act to make the constraint nonbinding by pushing the individual's productivity into a region for which $w > m$. These possibilities may mitigate the need to increase pre-market entry investment in a.

In terms of related research, the closest paper to ours is probably Wasmer (2006). He presents a formal analysis of the human capital investment problem after market entry in a framework with search frictions and firing costs. His model is stylized, as is the one we develop below, and is not taken to data. He assumes that human capital investments, be it of the general or specific kind, are made as soon as the employment relationship between a worker and a firm begins. Investment does not explicitly involve time or learning by doing, which we believe to be an important part of learning on the job. However, due to the simplicity of the investment technology, Wasmer is able to characterize worker and firm behavior in a general equilibrium setting, and he provides elegant characterizations of the states of the economy in which workers and firms will choose only general, only specific, or both kinds of human capital investment. One of the goals of our paper is to estimate both partial and general equilibrium versions of this type of model with what we think may be a slightly more realistic form of the human capital production technology, one in which time plays the central role.

Another related paper is Bagger et al. (2014). This paper examines wage and employment dynamics in a discrete-time model with deterministic growth in general human capital in the number of years of labor market experience. There is no match-specific heterogeneity in productivity, but the authors do allow for the existence of firm and worker time-invariant heterogeneity. There is complementarity between the worker's skill level and the productivity level of the firm, so that it would be optimal to reallocate more experienced workers to better firms. The authors allow for renegotiation of wage contracts between workers and firms when an employed worker meets an alternative employer, and, due to the generality of human capital, the more productive firm always wins this competition. The model is estimated using Danish employer-employee matched data. Key distinctions between our approach and the one taken in that paper are the lack of firm heterogeneity but the presence of worker-firm match heterogeneity, the value of which can be changed by the investment decisions of the worker-firm pair. This paper also allows for worker heterogeneity that is an endogenous stochastic process partially determined through the investment decisions of workers and firms.

Lentz and Roys (2015) also examine general and specific human capital accumulation in a model that features worker-firm renegotation and the ability of firms to make lifetime welfare promises to workers in the bargaining stage. There is firm heterogeneity in productivity, and the authors find that better firms provide more training. The nature of the contracts offered to workers is more sophisticated than the ones considered here, and the authors explicitly address the issue of inefficiencies in the training and mobility process. They assume that there are only four training states in the economy (an individual can be high or low skill in general and specific productivity), which greatly aids in the theoretical analysis at the cost of not being able to generate wage and employment sample paths that can fit patterns observed at the individual level. They also assume that there is no skill depreciation, which serves to simplify the theoretical analysis of the model.

The model that we develop and estimate below has several notable features. As mentioned above, it endogenizes the general-productivity level of the individual and the match-productivity level of the worker-firm pair using a cooperative model of worker-firm interactions. In most of the literature on worker-firm sorting (e.g., Abowd et al. (1999), Postel-Vinay and Robin (2002), Cahuc et al. (2006)), match productivity is ignored and worker and firm types are assumed to be time-invariant. The focus of much of this literature is on worker-firm sorting patterns. Within our framework, it is clear that the total productivity of an employee at a particular firm is a fluid object, with "mismatches" being potentially rectified through cooperative investment choices by the worker and firm. Viewing the productivity of a worker at a given firm as an endogenous stochastic process is an important point of differentiation of our model from most of the literature.

The fact that productivity can be altered forces us to reconsider the usual constraints on firm hiring that are implied by models without endogenous productivity. When productivity is fixed, then for firms to at least break even on the employment contract, the flow profit of a firm should be non-negative. However, when productivity can be increased through investment, it is possible (and occurs given our model estimates) that firms could earn negative flow profits for some part of the employment contract. This is an especially important consideration when evaluating the impact of policies such as a mandated minimum wage. In Flinn (2006), for instance, establishing a binding minimum wage of m in a market (in which there was not one previously) immediately caused the loss of all jobs for which the worker's flow productivity was less than m. With endogenous productivity, this may no longer be the case, since, through investment, the worker's productivity could be raised sufficiently so that the firm earns positive flow profits. The extent to which this phenomenon occurs is an empirical matter, which we can investigate using our model estimates.

Another literature to which this paper contributes is that concerned with the decomposition of the sources of wage growth over the life cycle, a literature the genesis of which traces back at least to Mincer, whose work on wage determination is summarized in Mincer (1974). While much of the work in this literature focuses on the estimation of the return to schooling,[3] our contribution is to the interpretation of the part of the earnings process associated with the worker's total labor market experience and tenure on their current job. The model generates positive dependencies between the duration of time in the market and at the current job with wages since both duration measures are positively related to the values of general and match-specific human capital. As individuals age, there is a tendency for general human capital, a, to increase. Also, because the turnover decision is determined through the comparison between the match productivity value θ at the current firm, with the initial match productivity draw θ' at the competing firm, longer employment durations at the firm indicate higher current values of match productivity. Even though the Mincer earnings function is largely an atheoretical construct, it is of interest to determine the extent to which the data generating process associated with our model produces relationships between wages, schooling, general experience, and job tenure broadly consistent with what would be found when estimating a Mincerian wage function using data from our sample. We find that, by and large, the wage functions estimated using the sample and those using simulated data from the model are roughly in agreement, even though the sample regression was not used in estimating the model parameters. This gives us some confidence that the model estimated does not generate empirical implications at variance with the results in literature focused on the estimation and interpretation of Mincerian earnings functions.

The model estimates we present are obtained using data from the 1997 cohort of the National Longitudinal Survey of Youth (NLSY97), which consists of individuals between the ages of 12-16 at the end of 1997. The advantage of using these data are

[3]Heckman et al. (2006) present a comprehensive consideration of the estimation of Mincer-type earnings functions, although their main focus is on the consistent estimation of rates of return to schooling.

that we can observe individuals from the beginning of their labor market careers, which minimizes initial conditions problems. Moreover, it is well-known that job changing behavior and wage growth are more pronounced at the onset of the labor market career. One negative aspect of estimating the model using only relatively recent labor market entrants is that we are likely to get predictions for later career events that are at odds with the data. This also implies that the steady state distributions implied by the model should be interpreted with this caveat in mind. Our future research plan is to extend the model to include endogenous pre-market entry schooling decisions and to estimate the model using samples with a larger range of ages and participation histories.

The plan of the paper is as follows. In Section 2, we analyze a partial equilibrium search model with general and specific human capital and subsequently extend it to a general equilibrium framework. Section 3 discusses the data used in the estimation of the model, and presents descriptive statistics. Section 4 discusses econometric issues such as the model specification used in our estimation, the estimator we use, and identification. In Section 5, we present the estimation results and discuss the details of the estimated model, such as parameter estimates, model fit, and policy rules. Section 5 also presents a discussion regarding the implications of our estimated model for sources of wage growth and provides a novel perspective on the interpretation of the standard Mincer wage regression. In Section 6, we conduct a minimum wage experiment to determine the impact of minimum wages on general and specific human capital investment decisions, in a partial as well as general equilibrium framework. Section 7 concludes the paper.

2 Modeling Framework

Individuals are characterized in terms of a (general) ability level a, with which they enter the labor market.[4] There are M values of ability, given by

$$0 < a_1 < ... < a_M < \infty.$$

When an individual of type a_i encounters a firm, he draws a value of θ from the discrete distribution G over the K values of match productivity θ, which are given by

$$0 < \theta_1 < ... < \theta_K < \infty.$$

We denote the c.d.f. of θ by G, and we define $p_j = \Pr(\theta = \theta_j)$, $j = 1, ..., K$. The flow

[4]Flinn and Mullins (2015) examine pre-market entry education decisions in a search environment in which a hold-up problem exists. We will not explicitly model the pre-market entry schooling decision, but will merely assume that the distribution of an individual's initial value of a at the time of market entry is a stochastic function of their completed schooling level. In estimation, we will distinguish three schooling levels.

productivity value of the match is assumed to be given by the following production technology[5]

$$y(i, j) = a_i \theta_j.$$

We consider the case in which both general ability and match productivity can be changed through investment on the job. The investment level, along with the wage, are determined cooperatively in the model using a surplus division rule. At every moment of time, the individual and firm can devote an amount of time τ_a to training in general ability, in the hope of increasing a from its current level. Similarly, they can invest an amount of time τ_θ in match-specific training, in the hope of increasing θ from its current level. We normalize the allocatable amount of time at each moment to unity, so that the flow amount of time actually working and producing output is $1 - \tau_a - \tau_\theta$.

Through time investment decisions made cooperatively by the worker and firm, both a and θ can be improved whenever neither is currently at its maximum possible value. If the current state of a is a_i, with $i < M$, then the rate of improvement to the next level, a_{i+1}, is given by

$$\varphi_a(i, \tau_a)$$

with $\varphi_a(i, \tau_a) \geq 0$ for $i < M$, and $\varphi_a(i, 0) = 0$. In terms of match-specific ability, given the current state θ_j, $j < K$, the rate of improvement to the next level, θ_{j+1}, is given by

$$\varphi_\theta(j, \tau_\theta),$$

with $\varphi_\theta(j, \tau_\theta) \geq 0$ for $j < K$, and $\varphi_\theta(j, 0) = 0$. We have assumed that these growth processes are independent, in the sense that the rate of improvement in a depends only on its current level a_i and the amount of time spent investing in a, τ_a. The same is true for the growth process in θ, where the rate of improvement depends only on the current level of match-specific productivity, θ_j, and the amount of time spent investing in it, τ_θ. These restrictions are made in acknowledgment of the severe difficulties we face in credibly identifying model parameters, given that the levels of both types of human capital and the time spent investing are essentially unobservable to us. Strong functional form assumptions are required to identify our parsimoniously-parameterized model.

For purposes of estimation, we further restrict the function φ_a to have the form

$$\varphi_a(i, \tau_a) = \varphi_a^0(i)\varphi_a^1(\tau_a),$$

where φ_a^1 is strictly concave in τ_a, with $\varphi_a^1(0) = 0$. The term $\varphi_a^0(i)$ can be thought of as

[5]The assumption of strong complementary between two types of skills is often made in the matching literature, but this application is somewhat different. In the search and matching literature, it is often assumed that the productivity of an individual-firm match is the product of the two productivity values (e.g., Postel-Vinay and Robin, 2002), primarily to simplify the estimation problem. While we could, in principle, allow the flow productivity to be some function f(a,), this would create an even more challenging identification problem than we already face, since its two arguments are both unobserved by us.

total factor productivity (TFP) in a sense, and we place no restriction on whether $\varphi_a^0(i)$ is increasing or decreasing in i, although the functional form we utilize in estimation will restrict this function to be monotone.[6] Since there is no hope of increasing the level of a beyond its maximum value of α_M, there will be no investment in a when $a = a_M$.

There is an exactly analogous production technology for increasing match-specific productivity, with the rate of increase from match value j to match value $j+1$ given by

$$\varphi_\theta(j, \tau_\theta) = \varphi_\theta^0(j)\varphi_\theta^1(\tau_\theta),$$

with φ_θ^1 strictly concave in τ_θ, and $\varphi_\theta^1(0) = 0$. There is no necessary restriction on the TFP terms, as above. As was true in the case of a, if match productivity is at its highest level, θ_K, then $\tau_\theta = 0$.

We allow for depreciation of skills in the following way. Whenever an individual has a general ability level greater than the minimum value of a_1, he is subject to Poisson shocks that arrive at exogenous rate δ_a. When such a shock arrives, an individual with a general ability level of a_i has their skill level reduced to a_{i-1}. For convenience in writing the Bellman equations below, we define $\tilde{\delta}_a(i) = \delta_a$ for $i > 1$, and $\tilde{\delta}_a(1) = 0$. The same assumptions are applied to the match-specific skill process, with the rate of depreciation from a value θ_j to θ_{j-1} equal to δ_θ for all $j > 1$. We define $\tilde{\delta}_\theta(j) = \delta_\theta$ for $j > 1$, and $\tilde{\delta}_\theta(1) = 0$.

The only costs of either type of training are foregone productivity, with total productivity given by $(1 - \tau_a - \tau_\theta)y(i, j)$. The gain from an improvement in either accrues to both the worker and firm, although obviously, gains in general human capital increase the future value of labor market participation (outside of the current job spell) to the individual only. As noted by Wasmer (2006), this means that the individual's bargaining position in the current match is impacted by a change in a to a greater extent than it is due to a change in θ. Motives for investment in the two different types of human capital depend importantly on the worker's surplus share parameter α, but also on all other primitive parameters characterizing the labor market environment.

2.1 No On-the-Job Search

We first consider the case of no on-the-job search in order to fix ideas. In defining surplus, we use as the outside option of the worker the value of continued search in the unemployment state, given by $V_U(i)$, and for the firm, we will assume that the value of an unfilled vacancy is 0, produced through the standard free entry condition (FEC).

[6] By this we mean that either

$$\varphi_a^0(1) \le \varphi_a^0(2) \le ... \le \varphi_a^0(M)$$

or

$$\varphi_a^0(1) \ge \varphi_a^0(2) \ge ... \ge \varphi_a^0(M).$$

We can write the problem as

$$\max_{w,\tau_a,\tau_\theta} \left(\tilde{V}_E(i,j;w,\tau_a,\tau_\theta) - V_U(i)\right)^\alpha \tilde{V}_F(i,j;w,\tau_a,\tau_\theta)^{1-\alpha},$$

where the $\tilde{V}_E$ and $\tilde{V}_F$ functions are the value of employment to the worker and to the firm, respectively, given the wage and investment times.

We first consider the unemployment state. We will assume that the flow value of unemployment to an individual of type a_i is proportional to a_i, or ba_i, $i = 1, ..., M$, where b is a scalar parameter. The value of unemployed search can be written as

$$V_U(i) = \frac{ba_i + \lambda_U \sum_{j=r^*(i)}^{K} p_j V_E(i,j)}{\rho + \lambda_U \tilde{G}(\theta_{r^*(i)})}, \tag{1}$$

where λ_U is the rate of arrival of potential employment opportunities to the individual. The discount rate ρ is the sum of the subjective discount rate of the individual, ρ_0, and a constant death rate of ℓ, so that $\rho = \rho_0 + \ell$.[7] It is assumed that the value associated with the state of death is 0. The critical (index) value $r^*(i)$ is defined by

$$\begin{aligned} V_U(i) &\geq V_E(i, \theta_{r^*(i)}) \\ V_U(i) &< V_E(i, \theta_{r^*(i)+1}). \end{aligned}$$

An agent of general ability a_i will reject any match values of $\theta_{r^*(i)}$ or less, and accept any match values greater than this.[8]

Given a wage of w and a training level of τ_a and τ_θ, the value of employment of type a_i at a match of θ_j is

$$\begin{aligned} \tilde{V}_E(i,j;w,\tau_a,\tau_\theta) &= (\rho + \varphi_a(i,\tau_a) + \varphi_\theta(j,\tau_\theta) + \tilde{\delta}_a(i) + \tilde{\delta}_\theta(j) + \eta)^{-1} \\ &\times[w + \varphi_a(i,\tau_a)Q(i+1,j) + \varphi_\theta(j,\tau_\theta)V_E(i,j+1) + \tilde{\delta}_a(i)Q(i-1,j) \\ &+\tilde{\delta}_\theta(j)Q(i,j-1) + \eta V_U(a_i)], \end{aligned}$$

where $\tilde{\delta}_k(i) = 0$ if $i = 1$ and $\tilde{\delta}_k(i) = \delta_k$ if $i > 1$, for $k = a, \theta$. The term

$$Q(i,j) \equiv \max[V_E(i,j), V_U(i)],$$

allows for the possibility that a reduction in the value of a or θ could lead to an endogenous termination of the employment contract, with the employee returning to the unemployment state. It also allows for the possibility that an increase in a from a_i

[7] The death rate is introduced so as to produce more reasonable steady state distributions than the ones we generated using model estimates and an assumption that $\ell = 0$. Less dramatically, we can think of this state as corresponding to retirement, but it should be borne in mind that we assign the value of this absorbing state to be 0.

[8] Note that we assume that there are no shocks to the individuals' ability level during unemployment.

to a_{i+1} could lead to an endogenous separation. This could occur if the reservation θ, $r^*(i)$, is increasing in i. In this case, an individual employed at the minimally acceptable match $r^*(i)+1$, may quit if a improves and $r^*(i+1) \geq r^*(i)+1$.

The corresponding value to the firm is

$$\begin{gathered}\tilde{V}_F(i,j;w,\tau_a,\tau_\theta) = (\rho + \varphi_a(i,\tau_a) + \varphi_\theta(j,\tau_\theta) + \tilde{\delta}_a(i) + \tilde{\delta}_\theta(j) + \eta)^{-1} \\ \times [(1-\tau_a-\tau_\theta)y(i,j) - w + \varphi_a(i,\tau_a)Q_F(i+1,j) + \varphi_\theta(j,\tau_\theta)V_F(i,j+1) \\ + \tilde{\delta}_a Q_F(i-1,j) + \tilde{\delta}_\theta Q_F(i,j-1)\Big]\end{gathered}$$

where $Q_F(i,j)=0$ if $Q(i,j)=V_U(i)$ and $Q_F(i,j)=V_F(i,j)$ if $Q(i,j)=V_E(i,j)$.[9] Then the solution to the surplus division problem is given by

$$\begin{aligned}\{w^*(i,j),\tau_a^*(i,j),\tau_\theta^*(i,j)\} &= \arg\max_{w,\tau_a,\tau_\theta}\left(\tilde{V}_E(i,j;w,\tau_a,\tau_\theta) - V_U(i)\right)^\alpha \\ &\quad \times \tilde{V}_F(i,j;w,\tau_a,\tau_\theta)^{1-\alpha}; \\ V_E(i,j) &= \tilde{V}_E(i,j;w^*(i,j),\tau_a^*(i,j),\tau_\theta^*(i,j)), \\ V_F(i,j) &= \tilde{V}_F(i,j;w^*(i,j),\tau^*(i,j),\tau_\theta^*(i,j)).\end{aligned}$$

More specifically, the surplus division problem is given by

$$\begin{gathered}\max_{w,\tau_a,\tau_\theta}(\rho + \varphi_a(i,\tau_a) + \varphi_\theta(j,\tau_\theta) + \tilde{\delta}_a(i) + \tilde{\delta}_\theta(j) + \eta)^{-1} \\ \times \left[\begin{array}{c} w + \varphi_a(i,\tau_a)[Q_E(i+1,j) - V_U(i)] + \varphi_\theta(j,\tau_\theta)[V_E(i,j+1) - V_U(i)] \\ + \tilde{\delta}_a(i)[Q(i-1,j) - V_U(i)] + \tilde{\delta}_\theta(j)[Q(i,j-1) - V_U(i)] - \rho V_U(a_i) \end{array}\right]^\alpha \\ \times \left[\begin{array}{c} (1-\tau)y(i,j) - w + \varphi_a(i,\tau_\alpha)Q_F(i+1,j) + \varphi_\theta(j,\tau_\theta)V_F(i,j+1) \\ + \tilde{\delta}_a(i)Q_F(i-1,j) + \tilde{\delta}_\theta(j)Q_F(i,j-1) \end{array}\right]^{1-\alpha}.\end{gathered}$$

The first order conditions for this problem can be manipulated to get the reasonably standard wage-setting equation,

$$\begin{gathered}w^*(i,j) = \alpha\{(1-\tau_a^*-\tau_\theta^*)y(i,j) + \varphi_a(i,\tau_a^*)Q_F(i+1,j) + \varphi_\theta(j,\tau_\theta^*)V_F(i,j+1) \\ + \tilde{\delta}_a(i)Q_F(i-1,j) + \tilde{\delta}_\theta(j)Q_F(i,j-1)\} \\ +(1-\alpha)\{\rho V_U(i) - \varphi_a(i,\tau_a^*)(V_E(i+1,j) - V_U(i)) \\ -\varphi_\theta(j,\tau_\theta^*)(V_E(i,j+1) - V_U(i))] - \tilde{\delta}_a(i)Q(i-1,j) - \tilde{\delta}_\theta(j)Q(i,j-1)\}.\end{gathered}$$

The first order conditions for the investment times τ_a and τ_θ are also easily derived, but

[9] Note that the discount rate ρ is the same for both workers and firms, even if we do not think of firms as being subject to a death shock. In this framework, there is essentially one worker per firm, and the worker's "death" terminates the match just as does a shock dissolving that particular job, η, assumed to arise due to changes in demand conditions or other exogenous events. In both cases, the firm is left without an employee, and we use the FEC associated with vacancies to apply the value of 0 in either case.

are slightly more complex than is the first order condition associated with wage setting. The assumptions regarding the investment technologies φ_a and φ_θ will obviously have important implications for the investment rules. The time flow constraint is

$$\begin{aligned} 1 &\geq \tau_a + \tau_\theta, \\ \tau_a &\geq 0 \\ \tau_\theta &\geq 0. \end{aligned}$$

Depending on the parameterization of the production technology, it is possible that optimal flow investment of either type is 0, that one type of investment is 0 while the other is strictly positive, and even that all time is spent in investment activity, whether it be in one kind of training or both. In such a case, it is possible to produce the implication of negative flow wages, and we will not explicitly assume these away by imposing a minimum wage requirement in estimation. In the case of internships, for example, which are supposed to be mainly investment activities, wage payments are low or zero. What is true is that no worker-firm pair will be willing to engage in such activity without the future expected payoffs being positive, which means that the worker would be expected to generate positive flow profits to the firm at some point during the job match.

2.2 On-the-Job Search

In the case of on-the-job search, individuals who are employed are assumed to receive offers from alternative employers at a rate λ_E, and it is usually the case that $\lambda_E < \lambda_U$. If the employee meets a new employer, the match value at the alternative employer, $\theta_{j'}$, is immediately revealed to the searcher. Whether or not the employee leaves for the new job and what the new wage of the employee is after the encounter depends on assumptions made regarding whether, and if so, how, the two employers compete for the individual's labor services. In Flinn and Mabli (2009) and Flinn and Mullins (2015), two cases were considered. In the first, in which employers are not able to commit to wage offers, or unable to verify the employee's claims regarding the existence of such outside options, the outside option in the wage determination problem remains the value of unemployed search, since this is the action available to the employee at any moment in time and is the minimum welfare level of the individual. This model produces an implication of efficient mobility, in that individuals will only leave a current employer if the match productivity at the new employer is at least as great as current match productivity (general productivity has the same value at all potential employers). An alternative assumption, utilized in Postel-Vinay and Robin (2002), Dey and Flinn (2005), and Cahuc et al. (2006), for example, is to allow competing employers to engage in Bertrand competition for the employee's services. In this case, efficient mobility will also result, but the wage distribution will differ in the two cases, with employees able to capture more of the surplus (at the same values of the primitive parameters) in the case of Bertrand competition.

We begin by discussing the reasoning behind our decision to assume no renegotiation of employment contracts due to the arrival of alternative employment opportunities to the worker. We do assume that contracts are renegotiated when the productivity of the individual on the job changes, whether due to an increase or decrease in a or θ. After providing some motivation for the renegotiation protocal we assume, we turn to describing the model with OTJ search.

2.2.1 The Negotiating Environment

We assume that the firm has no credible information concerning the worker's outside options, other than knowledge of her type, a_i. In particular, the employer is not able to verify claims that the individual has received an offer from another firm. Given that both sides are risk neutral, long term employment contracts that guarantee the worker fixed welfare levels across future states of the world, as in Harris and Holmstrom (1982) and Lentz and Roys (2015), are not necessary, although they would be of value in the case when the firm is risk neutral and the worker is risk averse, which is the case considered in those two papers. The environment analyzed by Harris and Holmstrom is one in which the productivity of the worker is only learned gradually over time, and the wage contracts offered by the firm provide insurance against low productivity outcomes. In our model, in which there is no learning and information regarding the productivity of the worker at any moment in time is symmetric, productivity decreases are possible over the course of a job spell, so that in the case of risk-averse workers, downwardly-rigid wage contracts would dominate the contracts we consider. In the case of Lentz and Roys, the contracts they analyze allow risk-averse agents to smooth consumption, but under their assumptions, the productivity of the individual at a job or over time is non-decreasing. Allowing for decreases in productivity would further increase the value of the types of contracts considered by Lentz and Roys.

Even under the assumption of risk neutrality, the kind of short-term surplus redivision that we consider faces diffiulties in the case of Bertand competition between firms. This is due to the fact that a surplus division rule that utilizes the last rejected employment opportunity as an outside option may be greater than the current productivity value when one or more setbacks in terms of a or θ has occurred since the last time the contract was renegotiated. Such a nonzero-probability event would require additional assumptions to deal with surplus division in such cases.

In the end, since the goal of this exercise is to estimate a model of on-the-job investment and wages using individual-level data, our assumptions regarding the surplus division problem were made so as to be relatively consistent with the sample paths of wages within and across job spells that are observed in the data. Although the within-spell wage data from the NLSY97 are not all that one might wish for, both in terms of quality and the frequency of observation, there is substantial indication in these data, as well in the Survey of Income and Program Participation panels, that wage changes within job spells are not infrequent, and include decreases as well as increases. Our surplus division rule that is utilized everytime the value of a or θ changes within a spell is capable of generating these patterns. Models of Bertrand competition with no investment on the job are not capable of generating wage decreases within a job spell, generally speaking. A final consideration in our decision to exclude interfirm competition in wage-setting regards identification of the primitive parameters of the model, and is primarily a pragmatic one. We face many challenges in identifying movements in the underlying levels of the unobservable a and θ given the paucity of high-quality data on individual wage rates within and between job spells. Wage increases within job spells generally will indicate an increase in the value of a or θ at a particular point in time. Were we also to allow wage increases due to the arrival of outside offers, this would further complicate the estimation of the hazard rates (human capital production technologies) associated with increases in a and θ.

2.2.2 Bargaining Problem with On-the-Job Search

Under our bargaining protocol, the employment contract is only a function of the individual's type and the current match value, (i, j). The property of efficient turnover decisions holds, with the employee accepting all jobs with a match value $j' > j$, and refusing all others. The formal structure of the problem as follows.

$$\tilde{V}_E(i, j; w, \tau_a, \tau_\theta) = \frac{N_E(w, \tau_a, \tau_\theta; i, j)}{D(\tau_a, \tau_\theta; i, j)},$$

where

$$\begin{aligned} N_E(w, \tau_a, \tau_\theta; i, j) = w + \lambda_E \sum_{s=j+1} p_s V_E(i, s) \\ +\varphi_a(i, \tau_a) Q(i+1, j) + \varphi_\theta(j, \tau_\theta) V_E(i, j+1) \\ +\tilde{\delta}_a(i) Q(i-1, j) + \tilde{\delta}_\theta(j) Q(i, j-1) + \eta V_U(i); \end{aligned}$$

$$\begin{aligned} D(\tau_a, \tau_\theta; i, j) = \rho + \lambda_E \tilde{G}(\theta_j) + \varphi_a(i, \tau_a) + \varphi_\theta(j, \tau_\theta) \\ +\tilde{\delta}_a(i) + \tilde{\delta}_\theta(j) + \eta. \end{aligned}$$

The RHS of the first line of N_E contains the flow wage rate and the rate of meeting other firms multiplied by the weighted sum of the value of meeting a firm with a higher match productivity value. The second line is the sum of the rate of improvements in a, as a function of the current state of a, i, and the flow rate of time invested in a, τ_a. The term $Q(i+1, j) = \max\{V_E(i+1, j), V_U(i+1)\}$, and reflects the fact that at a higher level of a, a_{i+1}, the match value θ_j may no longer be in the acceptance set of employment contracts, in which case the worker becomes an unemployed searcher. The second term on the line is the rate of improvement in θ given the current state of θ and investment time (j, τ_θ). An improvement in match-specific capital can never lead to a voluntary separation, of course. The final line on the RHS of N_E is the flow rate of decreases in

a and θ. Either of these events can lead to a voluntary separation, so that $Q(i-1,j) = \max\{V(i-1,j), V_U(i-1)\}$ and $Q(i,j-1) = \max\{V_E(i,j-1), V_U(i)\}$. Finally, the match can be exogenously terminated at rate η, in which case the individual enters the unemployment state with ability level a_i. One feature of our model is that transitions from employment to unemployment can be exogenous, at rate η, or endogenous, when a change in the value of a or a decrease in the value of θ leads to a separation. Our estimates indicate that approximately 15 percent of transitions from employment to unemployment are associated with changes in a or θ. The denominator D is simply the effective discount rate, which is ρ plus the sum of all of the rates associated with the various events described in the numerator, N_E.

The value to the firm conditional on the wage and investment decisions is given by

$$\tilde{V}_F(i,j;w,\tau_a,\tau_\theta) = \frac{N_F(w,\tau_a,\tau_\theta;i,j)}{D(\tau_a,\tau_\theta;i,j)},$$

where

$$\begin{aligned} N_F(w,\tau_a,\tau_\theta;i,j) = {} & y(i,j)(1-\tau_a-\tau_\theta) - w + \varphi_a(i,\tau_a)Q_F(i+1,j) \\ & + \varphi_\theta(j,\tau_\theta)V_F(i,j+1) + \tilde{\delta}_a(i)Q_F(i-1,j) \\ & + \tilde{\delta}_\theta(j)Q_F(i,j-1), \end{aligned}$$

and where $Q_F(i+1,j) = V_F(i+1,j)$ if $V(i+1,j) > V_U(i+1)$ and equals 0 otherwise, $Q_F(i-1,j) = V_F(i-1,j)$ if $V(i-1,j) > V_U(i-1)$ and equals 0 otherwise, and $Q_F(i,j-1) = V_F(i,j-1)$ if $j-1 > r^*(i)$. Now the surplus division problem becomes:

$$\begin{aligned} \max_{w,\tau_a,\tau_\theta} D(\tau_a,\tau_\theta;i,j)^{-1}[N_E(w,\tau_a,\tau_\theta;i,j) - V_U(i)]^\alpha \\ \times N_F(w,\tau_a,\tau_\theta;i,j)^{1-\alpha}. \end{aligned}$$

The value of unemployed search in this case is simply

$$V_U(i) = \frac{ba_i + \lambda_U \sum_{j=r^*(i)+1} p_j V_E(i,j)}{\rho+\lambda_U \tilde{G}(\theta_{r^*(i)})}.$$

2.3 Equilibrium Model

The model described to this point is one set in partial equilibrium, with contact rates between unemployed and employed searchers and firms viewed as exogenous. The model can be closed most simply by employing the matching function framework of Mortensen and Pissaridies (1994). We let the measure of searchers be given by $S = U + \xi E$, where U is the steady state measure of unemployed and E is the measure of the employed ($E = 1-U$, since we assume that all individuals are participants in the labor market). The parameter ξ reflects the relative efficiency of search in the employed state, and it is expected that $0 < \xi < 1$. We denote the measure of vacancies posted by firms by v. The flow contact rate between workers and firms is given by

$$M = S^{\phi} v^{1-\phi},$$

with $\phi \in (0,1)$.[10] Letting $k \equiv v/S$ be a measure of labor market tightness, we can write the rate at which searchers contact firms holding vacancies by

$$\lambda_F = \frac{M}{v} = k^{\phi}.$$

The proportion of searchers who are employed is given by $\xi E/S$, so that the mass of matches that involve an employed worker is simply $\xi E/S \times M$, which means that the flow rate of contacts for the employed is

$$\begin{aligned} \lambda_E &= \frac{\xi E}{S} \frac{S^{\phi} v^{1-\phi}}{E} \\ &= \xi k^{\phi-1}. \end{aligned}$$

By a similar argument, the mass of matches involving an unemployed worker is $U/S \times M$, and the contact rate for unemployed searchers is

$$\lambda_U = k^{\phi-1}.$$

A fact that will be utilized in the estimation of demand side parameters below is that $\xi = \lambda_E/\lambda_U$.

Turning to the firm's problem, let the flow cost of holding a vacancy be given by $\psi > 0$. The distribution of potential hires is determined by the steady state distributions of a among the unemployed and (a, θ) among the employed, which are complex objects that have no closed form solution, due to the (endogenous) dynamics of the a and θ processes in the population. However, these distributions are well-defined objects, the values of which can be obtained through simulation. The way in which we obtain the steady state distributions through simulation is described in Appendix B.

Let the steady state distribution of a among the unemployed be given by $\{\pi_i^U\}$, $i = 1, ..., M$, and the steady state distribution of (a, θ) among the employed by $\{\pi_{i,j}^E\}$, $i = 1, ..., M, j = 1, ..., K$. Then the expected flow value of a vacancy in the steady state is given by

[10] We have fixed $TFP = 1$ in the Cobb Douglas matching function due to the impossibility of identifying this parameter given the data available. The number of matches is unobserved, so that this essentially amounts to a normalization.

$$-\psi + \frac{\lambda_F}{S} \times \{U \sum_i \sum_{j \geq r^*(i)+1} p_j V_F(i,j) \pi_i^U$$
$$+\xi E \sum_i \sum_{j'>j} \sum_j p_{j'} V_F(i,j') \pi_{i,j}^E\}.$$

By imposing a free entry condition on firms that equates this value to zero, the equation can be solved for equilibrium values of λ_U and λ_E given knowledge of the parameters ψ, ϕ, and ξ.

3 Data

We utilize data from the National Longitudinal Survey of Youth 1997 (NLSY97) to construct our estimation sample. The NLSY97 consists of a cross-sectional sample of $6,748$ respondents designed to be representative of people living in the United States during the initial survey round and born between January $1,1980$ and December $31,1984$, and a supplemental sample of $2,236$ respondents designed to oversample Hispanic, Latino and African-American individuals. At the time of first interview, respondents' ages range from 12 to 18, and at the time of the interview from the latest survey round, their ages range from 26 to 32.

For our analysis, we use a subsample of 1,994 respondents from the NLSY97. We obtain this sample through three main selection criteria: (1) the oversample of Hispanic, Latino, and African-American respondents is excluded so that the final sample comprises only the nationally representative cross-sectional sample, (2) the military sample is excluded, and (3) all females and high-school dropouts are excluded. A respondent who satisfies these criteria enters our sample after having completed all schooling.

The estimation sample is constructed this way since our model is not designed to explain behavior while in school; staying in school or continuing education are not endogenous choices. These sample selection criteria give us an unbalanced sample of $1,994$ individuals and $661,452$ person-week observations. The proportion of high school graduates is 37 percent and the proportion of those with some college and those with a college degree are 30 and 33 percent, respectively.

The NLSY97 provides detailed retrospective data on the labor market histories and the wage profiles of each respondent. This retrospective data is included in the employment roster, which gives the start and end dates of each employment spell experienced by the respondent since the last interview, wage profiles and other characteristics of each employment (or unemployment) episode. We use the employment roster to construct weekly data on individual labor market histories. This information provides us with some of the key moments that identify the parameters of the search environment faced by the agents in our model, including transitions between jobs.

While we make extensive use of the weekly data constructed retrospectively from the NLSY97 employment rosters for obtaining moments related to employment transition, our empirical analysis of wages uses information collected from respondents about current wages as of each interview date. This information is likely to have fewer measurement problems than wage information in the employment roster, which is collected as part of a set of retrospective questions about all current and previously held jobs since the previous interview.

In the context of the model, the duration of a job spell is indicative of the value of the match between worker and firm. Therefore, looking at job spells of different lengths provides information about how wages and training differ at different match values. Our decision to only use wage observations from interview dates suggests defining the length of a job spell as the number of annual wage observations rather than using length from the employment roster. Table 1 shows the percentage of job spells by the number of interview dates they span for each schooling level. This distribution closely mirrors the actual duration distribution of jobs obtained from the employment rosters, suggesting that the two approaches should yield similar conclusions. For high school graduates, the table shows that about 61 percent of all observed job spells cover no interview dates at all, 23 percent of spells span one interview date, 7 percent last long enough to span two interview dates, and 9 percent span three or more. The proportion of job spells with longer durations and therefore spanning more interview dates, increases by education level. For example, for individuals with a college degree, 15 percent of job spells span more than two interview dates.

Given the importance of schooling on the labor market environment faced by the agents in our model, we distinguish between three groups of individuals in our empirical analysis: (1) individuals who have a high-school degree, (2) individuals who have attended college but who do not have a college degree, and (3) individuals with a college degree or more. In what follows, we refer to these three levels as low, medium, and high education groups, respectively.

In addition to key labor market variables, NLSY97 contains a wealth of information about training, which in our model is the way workers and firms invest to build human capital. For this aspect of our analysis, we use NLSY97's training roster, where respondents are asked about what types of training they receive over the survey year and about the start and end dates of training periods by source of training.[11] Combining the information from the employment and training rosters, we construct a weekly event history of employment and training for each respondent. We do not make assumptions regarding the specificity of human capital acquired during a training episode. Instead, we use the empirical relationship between the patterns of training and previous/future employment and wage transitions in order to make inferences about the degree of specificity in the human capital accumulation process.

[11]Examples of sources of training are business colleges, nursing programs, apprenticeships, vocational and technical institutes, barber and beauty schools, correspondence courses and company training. Training received in formal regular schooling programs is included in the schooling variables.

Tables 2-3 present some descriptive statistics on the training patterns observed in our sample. Specifically, these tables display the incidence of training by schooling and the timing of training spells by job tenure. The proportion of respondents with at least one training spell is 18, 13, and 13 percent for workers with low, medium, and high education, respectively. This suggests a negative relationship between schooling and training.

We next discuss training in relation to employment and wage transitions in our sample. Table 4 provides detailed information about employment and wage transitions between interview dates. We distinguish between three types of employment-to-employment transitions that may occur between interview dates $t-1$ and t: (1) transitions that do not involve a change in employer, (2) transitions that involve a change in employer, with no intervening spell of non-employment between the two jobs, and (3) transitions that involve a change in employer, with an intervening spell of non-employment.[12] The transitions that involve a change in employer are usually referred to as job-to-job transitions in the literature and we follow the same definitions in our discussion. Among workers who are employed in two successive interviews, the fraction of workers who change jobs decreases with education. For example, among high school graduates who remain employed at consecutive interview dates $t-1$ and t, 19 percent had a different employer, compared to only 12 percent of college graduates. Among workers who do change jobs, those with more education are more likely to do so without an intervening spell of non-employment.

As discussed previously, wage growth within and across job spells is an important indicator of which type of human capital investment behavior workers engage in. Panel B of Table 4 shows the difference between log wages for employment-to-employment transitions between interview dates. Again, we distinguish between the three types of transitions described above. We observe that log wage difference ($logw_t - logw_{t-1}$) for job-to-job transitions increases by education level: the average log wage difference is 0.11, 0.15 and 0.20 for low, medium and high education groups, respectively. The differences by education are particularly large for job-to-job transitions with an intervening non-employment spell.

Finally, Table 5 shows average log wage difference between consecutive interview dates $t-1$ and t, broken down by whether the worker receives training at the job he held at $t-1$. An individual is considered to have received training if the training roster reports him as enrolled in a training program during a week that was (1) before interview date $t-1$ and (2) while he was also employed at the job he held at time $t-1$. We see that for employment transitions that do not involve a change in employer, the average log wage difference does not change by whether the worker obtained any training in the past. On the other hand, for employment transitions that do entail an

[12] Using the employment rosters, we determine that there was an intervening non-employment spell between two consecutive jobs, if individuals are observed to be not working for a period of at least 4 weeks between the first employment episode that covers their interview date $t-1$ and second employment episode that covers their interview date t.

employer change (i.e. job-to-job transitions), the average log wage difference between t and $t-1$ for those individuals who obtained some form of training in the first job spell is smaller. It is also instructive to examine the wage differences for job-to-job transitions that involve an intervening non-employment spell. These are displayed in the last two rows of Table 5. We see that for individuals who moved to their next job with no intervening spell of non-employment, the average log wage difference is 0.09 if they received training in the previous job and 0.15 if they did not.

These moments will serve as a basis for our estimation of the parameters in the model described above.

4 Econometric Issues

4.1 Empirical Implementation of the Model

We make several assumptions in order to solve the model, which (clearly) does not produce closed-form solutions. We restrict workers and firms to choose training times from a discrete choice set consisting of multiples of two percent of the worker's total time in the range of times actually chosen and a coarser grid at higher off-equilibrium values.[13] The production functions are assumed to have the following functional forms. Recall that there are M values of a, $0 < a_1 < ... < a_M$. There is no possibility of increasing ability if an individual is already at the highest level, so the hazard rate for improvements from the state a_M is equal to 0. For $i < M$, we specify the hazard rate to level $i+1$ as

$$\varphi_a(i, \tau_a) = \delta_a^0 \times a_i^{\delta_a^1} \times (\tau_a)^{\delta_a^2},$$

where δ_a^0, δ_a^1, and δ_a^2 are scalar constants. Similarly, there are K values of θ, $0 < \theta_1 < ... < \theta_K$, and no possibility to increase match productivity when $\theta = \theta_K$. For a worker with $j < K$ who spends a fraction τ_θ of her time in firm-specific training, the value of the match increases at rate

$$\varphi_\theta(j, \tau_\theta) = \delta_\theta^0 \times \theta_j^{\delta_\theta^1} \times (\tau_\theta)^{\delta_\theta^2},$$

where, once again, δ_θ^0, δ_θ^1, and δ_θ^2 are scalar constants.

Because it is difficult to separately identify the level of general ability and match quality, we attempted to make the support of the distributions of a and θ as symmetric as possible. Therefore, we choose identical grids for for a_i and θ_j. We chose grid points to cover the range of likely values of θ including the possibility that workers with high values of θ will receive match-specific training that will produce match values above the set of values that they would naturally receive from searching. In the end, we use a grid containing 24 points which are spaced logarithmically from 2.5 standard deviations below the mean of the theta distribution to 3.5 standard deviations above

[13] Specifically, $\tau_a, \tau_\theta \in \{.00, .005, .02, .04, .06, ..., .32, .35, .4, .45, .5, .6, .7, .8, 1.00\}$

it. At the estimated parameters of our baseline model, moving up by one grid point in either a or θ corresponds to a roughly 7 percent increase in productivity.

Several model parameters are fixed outside of the estimation. We choose $\alpha = 0.5$, giving the worker and firm equal bargaining weight, and we also explore the sensitivity of our results to variation in this parameter value. All rate parameters are expressed at a weekly frequency and we set the time discount rate $\rho_0 = 0.0016$, corresponding to to a four percent annual discount rate. Finally, we set the death shock to produce an average career length of 45 years, $\ell = 1/(45 \cdot 52) = 0.00043$. This produces a "total" discount rate of $\rho = 0.00203$.

Training observed in the data is likely a very rough proxy for the amount of time spent developing workers' human capital. To relate our observed measures of training in the data to the training time chosen in the model simulations, we assume that a worker who spends a fraction of time τ engaged in training is observed to receive training is that period with probability

$$\text{Prob}(\text{Training observed} \mid \tau) = \Phi(\beta_0 + \beta_1 \tau)$$

where Φ is the c.d.f. for the normal distribution. In calculating τ from the simulations, we compute the average fraction of time spent training over each six month period, or, for job spells lasting less than six months, over the entire job spell. In theory, we would estimate the parameters β_0 and β_1. However, in practice, we find that β_1 is poorly identified and fix its value at $\beta_1 = 1$.[14] We estimate β_0 jointly with the other parameters of the model, giving us a total of 19 parameters to estimate.

4.2 Estimator

4.2.1 Estimation of Supply-Side Parameters

We utilize a method of simulated moments estimator (MSM) in order to estimate all of the parameters of the model with the exception of those characterizing firms' vacancy decisions. Under the data generating process (DGP) of the model, there are a number of sharp restrictions on the wage and mobility process that are generally not consistent with the empirical distributions observed. In such a case, measurement error in wage observations is often added to the model, with the variance of this measurement error estimated together with the other model parameters. This is not really a feasible alternative here given that we are already trying to estimate what is essentially a convolution, to be explained more precisely in our discussion of identification that follows. This implies that the addition of another random variable to the wage and mobility processes can only exacerbate the difficulty of separately identifying the distributions

[14] Robustness checks with different values of β_1 confirm that the parameter is not precisely estimated and that the effect of its value on the other parameter estimates is small.

of a and θ, particularly given their endogeneity with respect to investment decisions.[15] We chose to use a moment-based estimator which employs a large amount of information characterizing wage distributions within and across jobs, often by schooling class, as well as some training information, as was described in the previous section.

The information from the sample that is used to define the estimator is given by M_N, where there are N sample observations. Under the DGP of the model, the analogous characteristics are given by $\tilde{M}(\omega)$, where ω is the vector of all identified parameters. Then the estimator is given by

$$\hat{\omega}_{N,W_N} = \arg\min_{\omega\in\Omega}(M_N - \tilde{M}(\omega))' W_N (M_N - \tilde{M}(\omega)),$$

where W_N is a symmetric, positive-definite weighting matrix and Ω is the parameter space. The weighting matrix, W_N, is a diagonal matrix with elements proportional to the inverse of the variance of the corresponding element of M_N. Under our random sampling assumption, $\text{plim}_{N\to\infty} M_N = M$, the population value of the sample characteristics used in estimation. Since W_N is a positive-definite matrix by construction, our moment-based estimator is consistent since $\text{plim}_{N\to\infty}\hat{\omega}_{N,Q} = \omega$ for any positive-definite matrix Q. We compute the standard errors using an approximation to the asymptotic distribution of the parameter estimates and correcting for simulation error based on the number of draws in the simulation.[16] Our estimate for the asymptotic variance-covariance matrix of the MSM estimator is given by,

$$\hat{V} = (\hat{D}'\hat{W}\hat{D})\hat{D}\hat{W}\hat{S}\hat{W}\hat{D}(\hat{D}'\hat{W}\hat{D})^{-1}$$

where $D = \frac{\partial m}{\partial \omega'}$, $S = \left[1 + \frac{1}{R}\right] E(m'm)$, $m = \left(M_N - \tilde{M}(\omega)\right)$ and R is the number of simulation draws we use.

4.2.2 Demand-Side Parameter Estimation

Recall that the matching function was defined as

$$M = \nu^{\phi} S^{1-\phi},$$

with $\nu \in (0, 1)$, and the measure of searchers was given by $S = U + \xi E$, where U is the measure of unemployed and E is its complement. The parameter ξ is the search

[15]We do introduce measurement error into the wage observations, but the variance of this error is fixed rather than being estimated with the other model parameters. Including some realistic measurement error allows us to consider higher moments of the wage distribution, while imposing a fixed variance prevents the estimator from having to separately identify yet another source of variation in observed wages. Following Bound *et. al*) (1994), we set the standard deviation of the measurement error on observed log wages at $\sigma_e = 0.15$

[16]When we compute bootstrapped standard errors using different draws of the random shocks in our simulations, we find that this approach yields comparable estimates for the effect of simulation error on the precision of our estimates.

efficiency of employed agents relative to that of the unemployed, and it is expected that $\xi \in (0,1)$. The rate at which employers with vacancies contact applicants is

$$\begin{aligned}\lambda_F &= \frac{M}{\nu} \\ &= \nu^{\phi-1} S^{1-\phi} \\ &= k^{\phi-1},\end{aligned}$$

where $k \equiv \nu/S$ is our measure of labor market tightness.

The proportion of matches that involve an unemployed worker is given by

$$\frac{U}{U+\xi E} M,$$

so that the contact rate per unemployed searcher is

$$\begin{aligned}\lambda_U &= \frac{U}{S}\frac{M}{U} \\ &= k^{\phi}.\end{aligned}$$

The contact rate for employed searchers is

$$\begin{aligned}\lambda_E &= \frac{\xi E}{S}\frac{M}{E} \\ &= \xi k^{\phi}.\end{aligned}$$

It follows that $\xi = \lambda_E/\lambda_U$. Since we can consistently estimate the parameters λ_E and λ_U, it follows that a consistent estimator of ξ is given by $\hat{\xi} = \hat{\lambda}_E/\hat{\lambda}_U$. Then a consistent estimator for the measure of search effort on the supply side of the market is $\hat{S} = \hat{U} + \hat{\xi}\hat{E}$.

Without a consistent estimate of the vacancy cost parameter, ψ, the parameters of the matching function are not identified. We follow the usual approach for recovering an estimate of ψ. Under the assumption of a given value of the Cobb-Douglas parameter, ϕ, and that the TFP parameter in the matching function is equal to unity, we first find an estimator for unobserved vacancies, ν. We have

$$\begin{aligned}\lambda_U &= k^{\phi} \\ &= (\nu/S)^{\phi} \\ \Rightarrow \nu &= S(\lambda_U)^{\frac{1}{\phi}}.\end{aligned}$$

Using consistent estimates of the relevant parameters, a consistent estimate of ν is given by

$$\hat{\nu} = \hat{S}(\hat{\lambda}_U)^{\frac{1}{\phi}}.$$

Of course, consistency of $\hat{\nu}$ is based on the assumption that we have used the true matching function parameter, ϕ. In practice, we utilize the value of 0.5, which is common in the literature (see Petrongolo and Pissarides (2001)).

Using this estimator of ν, we then find a consistent estimator of λ_F, which is simply

$$\hat{\lambda}_F = (\hat{v}/\hat{S})^{\phi-1}.$$

We can then find a consistent estimator of ψ, which is given by

$$\hat{\psi} = \hat{\lambda}_F \hat{p}(A) \widehat{E(V_F|A)}.$$

The estimate of ψ is used in our counterfactual experiments involving the minimum wage. It should be borne in mind that this estimator is based on the assumption that the Cobb-Douglas matching function parameter is equal to a given value, which in our application is assumed to be 0.5.[17]

4.3 Identification of Primitive Parameters

The model is relatively complex, and many of the variables that play a major role in it are difficult, if not impossible, to measure. Since the estimator we employ is not based on a likelihood function, it is not possible to give a rigorous proof of identification or lack thereof, as in the simple partial-partial equilbrium model analyzed by Flinn and Heckman (1982). Instead, we will try to indicate the identification issues that we face, and what features of the data may be particularly useful in estimating certain subsets of the vector of primitive parameters. We also conduct an empirical exercise, the results of which are described in Section 5.3, that gives us some further guidance regarding which features of the data are most useful in separately identifying the a and θ processes, which is the main identification problem we face.

For the moment, imagine that we have access to productivity data associated with a worker-firm match at every moment in time. At a given point in time, potential output is given by

$$y(i,j) = a_i \theta_j,$$

so that

$$\ln y(i,j) = \ln a_i + \ln \theta_j.$$

The actual productivity, after accounting for training time, is

$$\begin{aligned} \tilde{y}(i,j) &= a_i \theta_j (1 - \tau_a - \tau_j) \\ \Rightarrow \ln \tilde{y}(i,j) &= \ln a_i + \ln \theta_j + \ln(1 - \tau_a - \tau_\theta), \end{aligned}$$

[17] In Flinn and Mullins (2015), a similar assumption was made. They also conducted some counterfactual policy experiments involving the minimum wage and found that varying ϕ did not greatly change the policy implications from the experiments.

where the τ_a and τ_θ training times are functions of the state (i, j), as is the wage. We see that the log of potential output is a convolution of the random variables $\ln a$ and $\ln \theta$. In Flinn and Mullins (2015), with no investment in a or θ considered after entry into the labor market, the distribution of $\ln a$ was fixed and $\ln \theta$ was a time-invariant characteristic of the match. There, as in this paper, turnover decisions are not a function of a, so that an individual moving from a match value θ to a new match value θ' had a change in log productivity of

$$\ln y(a, \theta') - \ln y(a, \theta) = \ln \theta' - \ln \theta.$$

In their version of the model in which the outside option of the worker was always the value of unemployed search, the bargained wage was $w(a, \theta) = \alpha(a\theta) + (1 - \alpha)r(a)$, where $r(a)$ is the reservation value of a type a individual, so that the log of the difference in wages across the two jobs is

$$\ln\{w(a, \theta') - w(a, \theta)\} = \ln \alpha + \ln a + \ln(\theta' - \theta), \tag{2}$$

and the variance of this quantity will only be a function of the distribution of θ. Using moments based on these types of statistics greatly facilitates the separate identification of the parameters characterizing the population distributions of a and θ. Flinn and Mullins had success in producing reasonable estimates of these parametric distributions, as well as the standard transition rate parameters studied in Flinn and Heckman (1982).

The estimation problem we face in this paper is considerably more complex than that of Flinn and Mullins (2015). One problem is the fact that there are three endogenous variables chosen by the worker-firm pair, the wage, training time in general human capital, and training time in match-specific human capital. We do not observe the two levels of human capital at any point in time (a, θ), which is a problem shared with Flinn and Mullins. However, by utilizing some training data, albeit of questionable quality, we can get some additional information on the values of (a, θ) not available in most search models that do not allow for training. That is, our model produces a mapping from the state variables $(a, \theta) \to (\tau_a, \tau_\theta)$. While this mapping is not, in general, invertible, it still conveys information on the set of values (a, θ) consist with the reported training time of the individual.

Given these issues, it is clear that the identification of the primitive parameters is challenging. In order to convince ourselves, and the reader, that all of the moving pieces in the model were necessary in order to fit the set of sample characteristics we utilize, we preformed the following exercise. Since without training, the model is quite similar to that used in Flinn and Mullins (although the papers used different data sources), it seemed clear that we could estimate the model successfully if we shut down the growth and depreciation processes in both a and θ. We then sought to determine whether we could adequately fit sample characteristics by allowing growth and depreciation in only one of a or θ. We first estimated a model in which the individual's draw of a was fixed from the time of labor market entry, while we continued to allow growth and depreciation of match-specific capital, θ. We then estimated a version of the model that allowed for growth and depreciation in a over during employment spells while forcing the match-specific productivity value θ to remain constant over the course of a job spell. We briefly describe the results we obtained below, but it did emerge that growth and depreciation in both a and θ was necessary to more satisfactorily fit the sample characteristics that we targetted. This gave us some faith in the estimability of the model, although more and better information on training episodes would greatly enhance faith in the estimates we report below.

5 Estimation Results

5.1 Parameter Estimates

The estimated parameter values are shown in Table 6 together with the estimated standard errors. We start with a discussion of the parameters that control employment transition rates. First, the flow value of unemployment for a worker of ability a is estimated to be $\hat{b}a = 6.47a$. The output of a worker of ability a at a firm with the median match quality is estimated to be slightly lower at $\exp(\hat{\mu}_\theta) \cdot a = 4.09a$. For unemployed workers, an offer arrives at a rate of $\hat{\lambda}_u = 0.138$ or approximately once every seven weeks. Workers with medium levels of general ability accept 31 percent of job offers, implying that the average unemployment spell lasts 23 weeks. Conversely, matches are exogenously dissolved at a rate of $\hat{\eta} = .004$, or approximately once every five years. Matches may also be dissolved endogenously if a shock to general ability or match quality makes unemployment preferable to the worker's current match. To assess the relative importance of these two shocks, we observe that the overall unemployment rate in the model is 13.23 percent, close to the data target of 14.10 percent. The steady state unemployment rate is higher at 19.54 percent. Together with the job finding rate and exogenous job separation rate, this equilibrium unemployment rate implies that approximately 15 percent of separations are endogenous. For employed workers, new offers arrive at rate $\hat{\lambda}_e = 0.069$, or approximately once every 15 weeks, about half as frequently as for unemployed workers.

The parameters $\mu_a(e)$ and σ_a control the distribution of starting values for general ability, where e denotes the education level of the worker. The estimated values imply that workers with some college education begin their labor force careers with 30 percent more human capital than high school graduates, on average, and those with at least a bachelor's degree begin with an additional 26 percent. These parameters are identified largely from wages of new workers entering the labor force. We match the starting wages of workers for the two higher education groups almost exactly. For workers with only a high school degree, starting wages are slightly higher in the model than in the data but subsequently increase at a slower rate. The variance for the initial distribution of ability $\hat{\sigma}_a^2 = 0.042$, approximately half the variance of the distribution of match qualities.

The parameters that govern the technologies for the rate of increase in general ability are δ_a^0, δ_a^1 and δ_a^2. As specified in Section 4.1, for an individual with general ability a_i, the hazard rate of improvement to ability level a_{i+1} is given by

$$\varphi_a(i, \tau_a) = \delta_a^0 \times a_i^{\delta_a^1} \times (\tau_a)^{\delta_a^2} \qquad i < M$$

with the analogous expression specified for the θ process. In the estimated model, $\hat{\delta}_a^0$ and $\hat{\delta}_\theta^0$ are very similar: $\hat{\delta}_a^0 = 0.029$, and $\hat{\delta}_\theta^0 = 0.023$. However, the remaining components of the general and match-specific skill processes look considerably different. In Table 6, we see that $\hat{\delta}_a^1$ is -0.135, whereas $\hat{\delta}_\theta^1$ is 0.443. In other words, the parameter estimates show that general training becomes less productive as a increases, whereas match-specific training becomes more productive with increases in θ. This is a reasonable finding since a is likely to be more difficult and costly to change after labor market entry due to the fact that employers are not equipped to offer general learning experiences as efficiently as are schools that specialize in increasing students' cognitive abilities. These parameter estimates also provide a bridge between this model and the Flinn and Mullins (2015) specification, where a is assumed to be fixed over the labor market career. In our model, we allow a to change over the labor market career, but the estimated model shows that it can indeed be thought of as quasi-fixed since it is difficult to change after labor market entry.

As mentioned earlier, the training observed in the data is likely a very rough proxy for the amount of time spent developing workers' human capital. Despite the predictions of our model that most workers are generally receiving some kind of training, only five percent of workers in the data report training in their current job. The parameters β_0 and β_τ control the relationship between training in the model and the probability that we observe a worker to be receiving training in the data. We normalize β_1 to 1 and estimate β_0 to be -2.73. This means that although the median worker in our model spends 20 percent of her time training, we expect that this worker will be observed to be involved in training only one percent of the time. For a worker engaged in full-time training, we would expect to observe this training in the data only 35 percent of the time.

We fix the value of the surplus share parameter α to 0.5 in the estimation. We believe that it is difficult to estimate this parameter given our data limitations, despite the fact that it should in theory be identified under our functional form assumptions. In Appendix A, we perform a sensitivity analysis by reestimating the model under different values of α. The results show that worker's surplus share plays a significant role in determining the type of investment that takes place as well as in determining the importance of initial labor market endowments. More specifically, when α increases, general training increases, leading to a more rapidly increasing wage profile during the course of a worker's labor market career. These findings suggest that with better training data, α can be estimated.

5.2 Model Fit

In this section, we compare the fit of the simulations from our estimated model to the corresponding moments in the data. We begin by comparing the model-predicted and observed wage distributions. Conditioning on tenure and education, we construct histogram plots from the simulated and observed wages, shown in Figures 1, 2 and 3 for workers with 0-2, 3-5 and 6-8 years of tenure, respectively. Overall, the model-predicted wage distributions are close to the empirical ones.

Next, we evaluate the ability of the model to replicate the training patterns in the data. In Table 7, we see that the observed and model-predicted proportion of individuals who have participated in at least one training spell during the time they are observed is 15 and 17 percent, respectively. Columns (2) and (3) of Table 7 further reveal that the model accurately captures the decreasing pattern of training with education, albeit with a small tendency to overstate the incidence of training for low-education workers: the proportion of individuals who get training at least once is 18, 13 and 13 percent for the low, medium and high education workers, respectively; whereas in the model simulations these moments are 21, 16 and 12 percent.

One of the distinguishing features of our model is our focus on within-job spell investment in human capital. This investment behavior, whether general or match-specific, impacts transition rates and wage processes within and across job spells. In the estimation, we match moments related to the joint distribution of wages between consecutive interview dates *within* a job spell as well as the joint distribution of wages between consecutive interview dates *between* different job spells. Table 8 displays how the model performs in generating some of these transition moments. Here, we limit our discussion to events that span only two consecutive interview dates, $t-1$ and t, and to workers who are employed at both dates. As described in the data section, we consider three possible events that may occur between $t-1$ and t: 1) no job change, 2) job-to-job transition with an intervening spell of non-employment, and 3) job-to-job transition with no intervening spell of non-employment.[18]

First, we consider the the proportion of job-to-job transitions with no intervening spell of non-employment, shown in Panel A of Table 8. In the data, the proportion of job-to-job transitions are 15, 12, and 10 percent for the low, medium and high-education groups, respectively. The corresponding model-predictions for these moments are 17, 17, and 16 percent, respectively. Hence, the model-predicted transition rates are reasonably close to the actual ones in the data for the lower education workers. However, for the higher education groups, the model overstates the proportion of job-to-job transitions with no intervening spell of non-employment.

Next, we examine the distribution of wage changes between consecutive interview dates, shown in Panels B and C of Table 8. We focus our discussion on the moments

[18]As described in the data section, we define a job-to-job transition to not involve a intervening non-employment spell if the time between the end of first job and beginning date of second job is a non-employment spell of 4 weeks or less. This allows us to distinguish between instantaneous turnovers from those that involve a period of search between consecutive jobs.

for high-school graduates, shown in Column (1), though the patterns for the other two education groups, shown in Columns (2) and (3), are similar. Panel B shows the average wage growth during these employment spells. In the data, for job-to-job transitions with an intervening non-employment spell the average log wage difference is 0.06. For job-to-job transitions with no intervening non-employment spell, it is 0.12. The corresponding moments in the model simulations are -0.10 and 0.18, respectively. These numbers show that the estimated model captures correctly the direction of the implications of an intervening non-employment spell, but that it performs poorly in matching the levels of these changes. Similarly, Panel C shows that in the data, the proportion of job-to-job transitions that are associated with a wage decrease is 40 percent with an intervening non-employment spell, and only 30 percent without. The corresponding moments in the model simulations are 63 and 27 percent so that the model overstates the fraction of negative changes for transitions that include a spell of non-employment. Nevertheless, the model correctly captures the fact that transitions spanning a non-employment spell are more likely to have negative wage growth than those that do not.

To understand the intuition behind these moments, we recall that in the model, values of θ and any investment made in match-specific productivity do not carry over into future employment, whereas general human capital does. For a worker who directly switches to a new job, the only acceptable jobs are those that have θ values higher than his current value. Therefore, the average wage gains from a job-to-job transition are high and the proportion of negative wage transitions is low (and would be zero without any measurement error). In contrast, for transitions that do involve an intervening non-employment spell, the worker loses the value from his accumulated match-specific human capital. Once he enters unemployment, he is willing to accept offers with a wider range of θ values, including values that are lower than the match quality at his recent job. His willingness to accept such offers decreases the average wage change across such transitions and results in a higher fraction of wage changes that are negative. The wage losses predicted by this mechanism in the model are evidently larger than those observed in the data.

5.3 Empirical Results on Identification Issues

In this section, we reestimate the model under two restrictions on human capital investment. The first exercise entails eliminating the possibility of investment in general human capital and the second entails eliminating the possibility of match-specific investment. As discussed in Section 4.3, the purpose of these empirical exercises is to illustrate which features of the data are most useful in separately identifying the a and θ processes.

First, we compare the model predictions and parameter estimates we obtain from the constrained estimation with no possibility of investment in general human capital to the ones we obtain from the baseline estimation. In the absence of general human capital accumulation, all wage growth within a job spell is attributed to growth in the worker's match-specific human capital, which must increase more quickly than in the baseline model to match the overall rate of wage growth. Because workers have more match-specific human capital, fewer potential offers would cause them to leave their current employers and we would therefore expect, holding all else constant, a decrease in the job-to-job transition rates. In the absence of fewer acceptable outside job offers for employed workers, the estimation with no general human capital accumulation generates job-to-job transitions by adjusting up the estimate of the job offer rate (λ_e) relative to the baseline estimate. This can be seen in Table 9, where our estimate for the job offer rate (λ_e) increases to 0.082 from a baseline value of 0.069. The job-to-job transitions are displayed in Table 10, which compares the moments for the constrained estimations and the baseline estimation. Panel A of Table 10 shows that the job-to-job transition rates are slightly lower for the constrained estimation with no a, relative to the baseline estimation.

A second consequence of the increase in match-specific human capital is that the the wage gains from job-to-job transitions with no intervening non-employment spell tend to be smaller.[19] As a result, the restricted model is less able to match the wage growth across job-to-job transitions with no intervening non-employment spell. We see this change in Panel C of Table 10, where the proportion of job-to-job transitions associated with a wage decrease is 26 percent in the baseline estimated model and is 44 percent in the estimated model with no a.

This reveals an important aspect of identification between general and match-specific human capital in the model estimation: transition data alone is not sufficient to distinguish between the different types of human capital and we need transition data in conjunction with moments that pertain to the joint distribution of wages within and across job spells in order to isolate one form of investment from the other.

Next, we reestimate the model with only general training. This version of the model does a better job of matching average wage growth within and across job spells. However, without match-specific training, the model is unable to capture the differences in wage growth and separation rates that we observe between short and long job spells. In all versions of the model, longer job spells are associated with higher match qualities. In the baseline model, the productivity of match-specific training rises with the quality of the match since $\hat{\delta}_2^\theta > 0$. This explains why jobs with better matches experience more wage growth. In addition, the increase in match quality over time due to match-specific training explains the decrease in the job-to-job transition rate with increasing job tenure. The alternative model with only general training is unable to match these features of the data.

The results from the constrained estimations discussed above demonstrate the empirical content of general and match-specific investments. As the results illustrate, general and match-specific investment have distinct implications for subsequent employment and wage transitions. These distinct implications can be summarized through plotting the life-cycle wage profiles obtained from each estimation. These graphs are

[19] Formally, θ is log-normally distributed so $E(\theta' - \theta|\theta' > \theta)$ is decreasing in θ.

displayed in Figure 4. Each panel in Figure 4 corresponds to a different education level. We see the same pattern for all three education groups: the initial accepted wages (wages that correspond to Year 0-2 of labor market tenure) are lower in the estimated model with no investment in θ and higher in the estimated model with no investment in a, with the baseline in between. This order gets reversed as the worker accumulates more labor market experience. In other words, by Year 6-8, the average log wages that correspond to the estimated model with no investment in θ overtakes the other two so that average log wages for the case with no investment in a is the smallest among all three estimated models. In the estimated model with only general human capital investment, the benefits of any training undertaken by the worker can be carried over to other jobs. The resulting wage growth throughout the labor market career of a worker is consequently larger relative to the estimated model with only θ investment.

The opposite is true for the estimated model with no a investment: the only training that a worker is able to engage in is training in θ, which is something that he cannot carry over to future jobs. Hence, the amount of benefits that he can accrue and transfer to future periods is smaller in the estimated model with no a investment and, consequently, overall wage growth throughout the labor market career is much lower. This is why the average log wage for the case with no a is the lowest at the end of the Year 6-8 tenure profile.

The comparison between these wage plots shows us that the difference between wage growth rates during the course of a worker's career within a firm and over the course of a worker's overall labor market career is an important indication of the type of training as well.

5.4 Policy Rules

5.4.1 Acceptable Job Offers

We next explore the choices of workers and firms implied by these parameter values. First, we consider the worker's decision to accept a match. A worker of ability a who receives an offer with match value θ will accept the offer if $\theta > \theta^*(a)$, and will otherwise remain unemployed and continue to search. For our estimated parameters, we plot $\theta^*(a)$ in Figure 5. As shown in the graph, the value of $\theta^*(a)$ rises as a increases. To understand the reason for this increase, we need to examine the choice of how much general training the firm provides at each combination of a and θ, which is plotted in Figure 7. At low values of a, for values of θ just above $\theta^*(a)$, workers spend a full 25 percent of their time engaged in general training. This suggests that these marginal matches become feasible only because of the opportunity they provide for the workers to build their general human capital. As a increases and general training becomes less productive (because $\delta_1^a < 0$), these marginal draws no longer deliver positive surplus relative to unemployment, and workers raise their reservation value of θ. In other words, low ability workers are more eager to accept lower quality matches because of the training opportunities, while higher ability workers find this training less valuable and prefer to search for more productive matches.

Next, we consider more broadly the choices of how much general and match-specific training firms and workers choose to provide for different levels of (a, θ). As background for this discussion, we need to understand how much workers value each kind of training relative to simply receiving wages. To this end, we examine Figure 6, which shows combinations of training and wages that solve the bargaining problem between the worker and the firm. Near the actual solution, the wages decrease quickly as the firm chooses to provide more general training, suggesting that workers regard general training as a good substitute for wages. In contrast, increasing match-specific training results in a much smaller decrease in wages, implying that the worker's value from additional match-specific training is small, and that most of the value from match-specific training goes to the firm. However, the worker does seem to receive some benefit from the match-specific training, which is reflected in her willingness to trade off some wages for more match-specific training.

5.4.2 Training Policies

Having identified the trade-offs between wages and training, we next examine the three outcomes of the bargaining process: the two types of training and the wage. In Figures 8 to 9, we plot the amount of general and match-specific training and wages that workers receive at different combinations of a and θ. For ease of illustration, both states are shown on a log scale and the lines on the graph show contours along which wages or the amount of training remains constant. The bottom of the figures, corresponding to low values of θ, are combinations for which workers will not accept the job offer.

Looking first at the policy for firm-specific training plotted in Figure 8, we see that the amount of firm-specific training is largely a function of the current value of θ with much less dependence on the worker's level of general ability. At values of θ just above the minimum $\theta^*(a)$ threshold, the amount of training is small. Firm-specific training increases for higher values of θ, reaching a maximum of 14 percent of the worker's time at roughly the 85th percentile of the distribution of acceptable θ draws. Two different mechanisms contribute to this pattern. First, in the estimated model, $\hat{\delta}^1_\theta > 0$ so firm-specific training is more productive at higher values of θ. Second, at higher values of θ, the expected duration of the current match increases as it becomes less likely that the worker will leave to take an outside offer. Because firm-specific training increases future output only for as long as the worker remains with her current employer, this increase in expected duration raises the value of match-specific training. Offsetting these effects is the incentive for the firm to provide match-specific training in order to raise the value of θ and thereby increase the length of the current match. This incentive is stronger at lower values of θ because the density of potential job offers is higher so that increase in θ yields a greater reduction in the fraction of outside offers that would cause the worker to leave.

Next, we look at the amount of general training provided to the workers, which we plot in Figure 7. In matches with low values of a and θ, workers spend about 25 percent of their time engaged in general training. The amount of training decreases at higher values of either a or θ. The decreasing amount of general training with increasing ability is attributable simply to the decreasing productivity of this training for higher ability worker ($\hat{\delta}^1_\theta < 0$). Meanwhile, the decrease in general training at higher values of θ reflects a decline in the bargaining position of workers as match quality increases. To understand this explanation, we note that unlike the match-specific training discussed above, a worker retains her accumulated general human capital even after the current match is dissolved, so general training does not become more valuable as the expected duration of the match increases. Rather, as in a standard Ben-Porath model, the benefits of general training flow largely to the worker and the amount of general training is determined by the worker's trade-off in allocating time between production and the accumulation of general human capital. In the context of our model, this implies that negotiations over the amount of general training should look similar to the negotiations over wages. In states where the bargaining process yields higher compensation for the worker, he will choose to receive some of this compensation as higher wages and some as general training. Indeed, in Figure 9, we show that, like general training, the fraction of worker's output that is paid in wages also decreases with match quality.

Given these policy rules, how much training do workers actually receive? Based on the simulations from the estimated model, Figure 10 plots the fraction of time that workers spend training as they move through the first years of their careers. When workers first enter the labor force, initial match qualities are relatively low and therefore most of the training takes the form of general training. In the model simulations, workers in their first year in the labor force spend about 19 percent of their time in general training and 7 percent in firm-specific training. Over time, match quality increases as workers sort into jobs of higher match quality. Because the expected duration of these jobs is higher and also because firm-specific training becomes more productive at higher values of θ, a larger fraction of the training becomes firm-specific. As both kinds of human capital increase further over time, workers spend less time training and more time engaged in production. In the long-run, workers in our model spend about 11 percent of their time training, split evenly between the general and match-specific types.

5.5 Efficiency

We next consider what would be an efficient amount of training in our model and ask how the quantity of training undertaken in our baseline model compares to this ideal. For this discussion, we think of the efficient amount of training as the choice that maximizes the present discounted value of the worker's expected future output (including unemployment benefits, which are here interpreted as home production).[20]

[20] To solve the full social planner problem, we would also want to choose a market tightness that takes into account the firms' vacancy posting costs in maximizing overall surplus. However, because of the difficulty in

Intuitively, inefficiencies arise because the firm does not internalize the full benefits of general training insofar as this training also increases the worker's productivity at future jobs. In contrast, the worker does fully internalize these benefits. Similarly, the firm has a strong incentive to provide match-specific training, which both increases productivity and decreases the likelihood that the worker will move to a different firm. In contrast, the worker prefers to spend less time on match-specific training and instead to increase his match-specific capital by searching for a better match with a different firm, a process that is costless in our framework. Because the worker's preferences incorporate the true benefits of both kinds of training, solving the Nash bargaining problem where the worker has all the bargaining weight ($\alpha = 1$) should produce training decisions that achieve this efficient outcome.

When we resolve the model with $\alpha = 1$, we find, unsurprisingly, that the amount of general training increases while the amount of match-specific training decreases. In other words, the inefficiencies in the baseline model take the form of an under-provision of general training and an over-provision of match-specific training, reflecting the effect of the firm's preferences on the bargaining solution. Quantitatively, the efficient solution includes only slightly more general training than the baseline solution. Intuitively, it appears that even with a lower bargaining weight, the worker can come close to her preferred level of general training by accepting lower wage payments instead. Meanwhile, the baseline model includes a significant amount of excess match-specific training compared to the efficient solution, suggesting that this may be a much larger source of inefficiency than the slight under-provision of general training.

5.6 Sources of Wage Growth

We now examine the factors that drive wage growth in the model and provide a link between our structural approach and the well-known results from the literature on Mincer-type earnings functions. The model contains five possible sources of wage growth. First, workers can increase their productivity by building general human capital through on-the job training. Second, workers can increase productivity by searching for a new job with a higher match quality. Third, workers can improve the quality of the match with their current employers by engaging in firm-specific training. Fourth, as workers spend less time training and more time engaged in production, some of this increased output will flow to them in the form of higher wages.[21] Finally, the bargaining between workers and firms can result in different shares of worker output being paid as wages depend on the value of the workers' outside options. The current section aims to quantify the importance of each of these channels.

identifying these costs (see the discussion in Section 4.2.2), we leave this exercise for future research.

[21] Alternatively, a shift towards less training and higher wages could be interpreted as a shift in the workers' compensation towards higher current wages and away from expected higher future wages.

5.6.1 Growth of Wage Components

To start, we recall that output $y = a \cdot \theta \cdot (1 - \tau_a - \tau_\theta)$. Additionally, it is useful to decompose θ into two separate components, as $\theta = \theta_0 \cdot \theta_\tau$ where θ_0 the match quality at the start of the match, and θ_τ is the additional match quality accumulated through match-specific training (net of depreciation). This allows us to formally write the wage w as the product of the five pieces described above:

$$w = a \cdot \theta_0 \cdot \theta_\tau \cdot (1 - \tau_a - \tau_\theta) \cdot (w/y)$$

or in logs,

$$\log(w) = \log(a) + \log(\theta_0) + \log(\theta_\tau) + \log(1 - \tau_a - \tau_\theta) + \log(w/y)$$

In Figure 11, we plot the evolution of each of these five components, together with the total wage, as workers move through the early years of their careers. The figure shows that in the worker's first few years in the labor force, the two most important sources of wage growth are the development of match-specific human capital through search and growth of general human capital through training. In later years, increases in general ability account for a larger fraction of total wage growth as the rise in match-specific human capital slows over time. As described above, the time spent training decreases over a worker's career and this reduction contributes noticeably to the worker's overall output and therefore to his wage. The last component, the fraction of output represented by the worker's wage, declines slowly over time as both a and θ rise, producing some drag on wage growth through workers' careers.

5.6.2 Interpreting Mincer Regressions

Our structural model also allows us to interpret the coefficients of a standard wage regression in terms of the different source of wage growth discussed above. This analysis helps relate our results to the well-known results from the literature on earnings dynamics. As an example, we consider a simple Mincer wage regression of the form

$$log(wage_{it}) = \sum_j \beta_j^w X_{it}^j + \varepsilon_{it}^w$$

where $log(wage_{it})$ is the log of the wage for person i at time t. Specifically, we estimate

$$log(wage_{it}) = \beta_0^w + \sum_e \beta_e^w ed_{ie} + \beta_y^w years_{it} + \beta_t^w tenure_{it} + \varepsilon_{it}^w. \tag{3}$$

where ed_{ie} is a dummy variable indicating that person i has education level e (for each level of education except HS graduate), $years_{it}$ denotes the number of years in the labor force and $tenure_{it}$ the length of time with current employer. We first estimate this model on the actual NLSY data and then on the simulated data from the model.

Results are shown in the first two lines of Table 11. As expected, more education, more years in the labor force and greater job tenure are all associated with higher wages. Comparing the regressions on the real and simulated data, we find that additional education is associated with less of an increase in wages in the simulated data than the actual data. Also, relative to the data, slightly more of the wage growth in the model is attributed to tenure with particular employers and less to overall labor-market experience.

Focusing on the regression using the simulated data, we next aim to understand how the increases in wages associated with education, labor market experience and job tenure reflect the different determinants of wages present in the model. Similar to the decomposition described in the previous section, we can decompose log wages in the model as a sum of the logs of i) general ability, ii) match quality at the start of the match iii) additional match quality accumulated through match-specific training, iv) time spent not training and v) wage as a fraction of output. Additionally, because we are interested in the level of wages rather than just the growth rate, we express the worker's general ability as a combination of her initial endowment (a_0) and the additional human capital he accumulates through training (a_τ). This defines six components of wages, which we denote $Y_{it}^k, k = 1, ..., 6$, allowing us to write

$$\begin{aligned} \log(w_{it}) &= \textstyle\sum_{k=1}^6 \log(Y_{it}^k) \\ &= \log(a_{0,it}) + \log(a_{\tau,it}) + \log(\theta_{0,it}) + \log(\theta_{\tau,it}) + \log(1 - \tau_{a,it} - \tau_{\theta,it}) + \log(\tfrac{w_{it}}{y_{it}}) \end{aligned} \tag{4}$$

In order to measure how education, labor-market experience and job tenure affect each of these components, we repeat the regression from Equation 3 on each of these six pieces separately, i.e. we estimate

$$\log(Y_{it}^k) = \sum_j \beta_j^k X_{it}^j + \varepsilon_{it}^k, \quad k = 1, ..., 6.$$

It is straightforward to show that for each covariate (indexed by j), the sum of the regression coefficients from these six regressions must equal the coefficient for regression using the total log wage, i.e.

$$\beta_j^w = \sum_{k=1}^6 \beta_j^k.$$

This allows us to interpret each of the coefficients β_j^w from Equation 3 as reflecting different combinations of the components of wages defined in Equation 4. The results of this exercise, which we present in Table 11, are all quite reasonable. The increase in wages with more education is largely picking up differences in initial ability and, to a lesser extent, the fact that more educated workers spend less of their time training. The positive coefficient on labor market experience is mostly capturing general ability acquired through training and, to some extent, workers' ability to find better matches over time. Finally, the increase in wages for workers with more tenure mostly reflects improved match quality from training. Wages also appear to increase with tenure because workers engage in less training as they remain with a firm longer, and there is an additional selection effect whereby longer-tenured workers have received better initial matches with their firms.

In addition to studying Mincer wage regressions on the entire sample, we can run these regressions separately on workers with different amounts of education to help understand differences in wage growth between these groups. Results from this exercise are shown in Table 12. The first panel shows the difference in the constant term. The model captures the higher wages of more educated workers and, as one would expect, most of this difference is captured simply by the higher amounts of general human capital that more educated workers have upon entering the labor force. In addition, a portion of the differences is explained by the fact that more highly educated workers spend less time training and more time engaged in production.

In the middle panel of Table 12, we show the effect of increasing labor market experience on wages for the three different education groups. In the data, high school graduates receive a larger increase in wages for each year in the labor market than do more educated workers. In the model, the wages of less-educated workers do increase more quickly because they receive more general training. However, the effect of these differences in the simulations is small and is largely offset by the tendency of less-educated workers to move more slowly into matches of higher quality.

The final panel of Table 12 describes the returns to increasing tenure for each of the three education groups. In both the data and the model, workers with more education have higher returns to tenure, though the relationship is somewhat weaker in the model than in the data. In the model, most of this difference is accounted for by more educated workers building more match-specific human capital from training during their job spell. This difference emerges because more educated workers are more likely to accept jobs where the match quality is higher, which makes match-specific training more productive.

6 Policy Analysis: The Minimum Wage and Investment

In competitive markets, minimum wages are expected to impact human capital formation in relatively immediate ways (Leighton and Mincer (1981), Hashimoto (1982)). In the case of general human capital, the returns to which accrue to the worker, the theory implies that the sum of the cost of investment and the wage paid to the worker should be equal to the worker's (marginal) productivity. Restrictions on the wage that must be paid to the worker act as a constraint on the amount of general human capital investment that the individual can undertake, which typically results in lower levels of investment early in the labor market career than would be efficient. Within our modeling framework, in which there exist two types of human capital and search frictions, these types of considerations still apply, but in a much more subtle manner.

One of the more interesting implications of our model regarding the impact of minimum wage laws is that, given the existence of search frictions and the possibility of improving the productivity of the match, the introduction of a minimum wage that exceeds the current flow productivity of the match need not lead to the termination of the employment contract. Models in which the productivity of the match is fixed produce the implication that all matches with flow productivities less than the value of the minimum wage will be terminated (Flinn (2006), Flinn and Mullins (2015)). The possibility of increasing the flow productivity means that the decision to keep the match alive involves a comparison of the expected profits of the match (to the firm) with its outside option of zero, while the worker must receive an expected value of continuing on the job at the binding minimum wage that exceeds the value of unemployed search under the new minimum wage. In general, not all matches with current flow net productivity $a\theta < m$, where m is the newly imposed minimum wage, will be terminated. The likelihood of termination in such a case will be a function not only of net productivity, but also the mix of general and match-specific capital possessed by the worker. For a given shortfall in net productivity with respect to the minimum wage, firms will be more likely to continue the match when match-specific capital is greater. To eventually earn positive flow profits from the match, the employee will have to be likely to remain with the firm, and this is an increasing function of the current level of θ. Moreover, this consideration will make it more likely for the worker-firm pair to invest in match-specific human capital than general human capital. The extent of these effects will depend on the parameters characterizing the model.

As has been found in Flinn (2006) and Flinn and Mullins (2015), the impact of the minimum wage is likely to vary significantly depending on whether we use the partial or general equilibrium version of the model. In the partial equilibrium version, contact rates (λ_U and λ_E) are assumed to be fixed. In the general equilibrium version of the model, these contact rates are a function of the job vacancy creation decisions of firms. As minimum wages increase, the share of the match surplus accruing to firms shrinks, which decreases firms' incentives to create vacancies. This corresponds, roughly, to a shift downward in the demand function, and exacerbates the minimum wage's negative impact on the employment rate. We begin our analysis with the partial equilibrium version of the model.

6.1 Minimum Wage in Partial Equilibrium

In order to study the effect of a minimum wage in the context of our model, we use the estimates of the model parameters and then solve the model imposing a minimum wage of \$15 per hour (in 2014 dollars, corresponding to \$10.17 in the 1994 dollars we use in our analysis). As expected, we find that imposing a minimum wage increases the unemployment rate by rendering low quality matches unprofitable for firms.[22] Figure 12 shows the minimal acceptable match quality draw in the baseline model and under the minimum wage. In our simulations, the minimum wage has the largest effect on employment for workers who have recently entered the labor force, raising the unemployment rate of workers with just one year of experience from 12.2 percent to 14.0 percent. The impact fades over time so that the unemployment rate for workers with 10 year experience is just two-tenths lower with the minimum wage than without it (8.4 percent compared to 8.2 percent).

In addition to the effect on employment, our model shows how the minimum wage can also affect the amount of training provided to employed workers. Because employers must pay workers a higher wage, they decrease the amount of compensation that is provided in the form of general training. Figure 13 compares the amount of training that workers receive in the baseline model and under the minimum wage. With a minimum wage in place, employed workers spend 5-10 percent less time on general training than they do in the baseline model. For workers who have been in the labor force for 10 years, average general ability is about 1 percent lower as a result of this decrease in training.

Conditional on employment, workers receive higher wages with a minimum wage in place than they do in the baseline model. Average wages are 7 percent higher for workers entering the labor force, though this difference disappears within 6-7 years. Most directly, when the minimum wage is binding, employers must pay workers a higher fraction of their output then they otherwise would in order to raise their wages to the required level. In addition, part of this increase is due to selection on both general ability and match quality as the presence of a minimum wage raises the distribution of acceptable match values and also disproportionately keeps low-ability workers unemployed. Finally, the lower amount of training discussed above means that workers are spending more of their time engaged in production and some of this additional output naturally flows to the worker in the form of higher wages. Quantitatively, this total increase in wages is mostly attributable to the increase in accepted match quality, the selection of higher ability workers and the decrease in training time, with the increase in the workers' wages as a fraction of total output contributing a smaller amount.

Looking in more detail at the effect of the minimum wage on the distribution of wages, Figure 14 shows how various percentiles of the wage distribution evolve with labor market experience, with and without the minimum wage. As expected, most of the of effect of the minimum wage occurs at the bottom of the distribution. For workers entering the labor market, the minimum wage raises the first percentile of the wage distribution by 50 percent and this difference persists even as workers gain more experience. At the tenth percentile, wages start out 20 percent higher for new workers but the effect fades after workers have been working for several years. Meanwhile, the effect on workers further up the distribution is insignificant.

[22] Recall that our model does not include a labor force participation decision. An increase in unemployment translates into a decrease in employment, since these rates sum to one.

Overall, the welfare effects from the minimum wage are small as the loss to workers from a higher unemployment rate and lower amounts of general training is largely offset by the higher wages they receive. Workers at the very lowest value of general ability experience a welfare loss of just 0.1 percent as they are most impacted by the lower job-finding rates. For high school graduates on average, the welfare loss is just 0.02 percent and it is even smaller for those with more education. These relatively small effects are due to the minimum wage being set at a relatively low level taking into account the baseline marginal wage distribution. Obviously, at higher values, the minimum wage impacts extend further up the ability distribution.

6.2 Minimum Wage in General Equilibrium

We now consider the impact of minimum wage increases in a simple general equilibrium version of the search and matching model in which firms' vacancy creation decisions and the measure of unemployed and employed searchers determine the contact rates λ_U and λ_E. In order to move to the general equilibrium framework, we need to define the steady-state distribution of workers, which is used in computing the expected return to a filled vacancy when solving for firms' vacancy creation decisions. We describe the computation of this steady-state distribution in Appendix B.

Having identified the baseline steady-state distribution of workers, the next challenge is to is to identify the firm's cost for posting a vacancy, ψ, and the value of the Cobb-Douglas parameter in the aggregate matching function, ϕ. Following the discussion in Section 4.2.2, we fix the Cobb-Douglas parameter at $\phi = 0.5$ and back out the vacancy cost. This procedure produces an estimate of the vacancy cost $\hat{\psi} = 190$.

In general equilibrium, the minimum wage constraint reduces the firm's incentive to post a vacancy and therefore decreases the number of vacancies and the rate at which workers receive job offers. With a minimum wage of $15 (in 2014 dollars), the minimum wage binds on a very small fraction of matches in the steady state distribution so that the impact on transition rates is small: the change in job-finding rates is negligible and the steady-state unemployment rate only rises 0.15 percentage points relative to the baseline. The average level of general human capital in the economy falls by 0.3 percent, with the losses concentrated among high school graduates, for whom general ability decreases by an average of 0.6 percent. Combining the slightly higher unemployment rate and the slightly lower stock of human capital, total output falls 0.3 percent relative to the baseline with no minimum wage. Finally, looking at the welfare effects, the minimum wage produces a small average welfare loss for high school graduates, equal to just 0.05 percent on average, with an even smaller effect for workers with more education.

The general equilibrium effects become more significant at higher values of the minimum wages. As an illustration, we consider a minimum wage equal to $20.[23] With the minimum wage set at $20, the job offer rate falls by one percent and the steady-

[23]Twenty dollars, measured in 2014 dollars, becomes $13.56 when expressed in our units of 1994 dollars.

state unemployment rate increases an additional 1.2 percentage points. In addition, because low-ability workers are employed less and receive less training, the amount of general human capital in the economy falls 4.1 percent for high school graduates and an average of 1.8 percent across all workers. At this higher minimum wage, total output is 2 percent lower than in the baseline economy and welfare losses increase to 0.8 percent for high school graduates and 0.2 percent for those with some college.

7 Conclusion

We have developed an estimable model of investment in both (completely) general and (completely) match-specific human capital while individuals are active members of the labor force, which we assume follows the completion of formal full-time schooling. While other researchers have examined investment decisions in a search, matching, and borrowing framework, ours is perhaps the first to attempt to estimate such a model in a reasonably general framework. Perhaps the greatest challenge we face in estimation is to attain credible identification of such a model when human capital stocks and investments essentially are unobservable. In this we are aided by having access to (self-reported) data on whether a worker engaged in formal training during a job spell. We heavily exploit this information in our moment-based estimation procedure. Furthermore, our assumptions regarding the specificity of human capital imply that changes in the stock of general human capital have no impact on the future mobility decisions of an individual during the employment spell. This stands in stark contrast to changes in the stock of match-specific human capital, which strictly reduce the likelihood of accepting a job with another firm during the employment spell. Thus job-to-job mobility along with wage changes during the current job spell can be utilized to infer whether the wage change was the result of general or match-specific investment.

Our estimates of the human capital production technology exhibit decreasing returns to investment in both types of human capital, of approximately the same degree. Our production technology also includes a TFP term that captures how the current level of both types of human capital affect the returns to investment. Here we find that the payoffs from time investment in match-specific human capital are increasing in its current level, while there is no impact of the current level of general human capital on the return to investment in it. These results imply fairly complex dynamic patterns in investment and wage growth. They also serve to produce a job acceptance probability from the unemployment state that is non-monotone in the individual's level of general human capital.

We use our estimates to examine the impact of minimum wage policy on investment in human capital and equilibrium outcomes. Our experiments are conducted in both partial and general equilibrium frameworks, where the general equilibrium specification relies on the typical matching function approach. Unlike previous estimates of these minimum wage effects, as in Flinn (2006) and Flinn and Mullins (2015), the deleterious effects on high minimum wages on job finding rates can partially be alleviated by increases in the productivity of workers, which is obtained by investment in general and match-specific human capital on the job. In the general equilibrium setting, we find that a minimum wage of $15 does little to affect the labor market equilibrium, since the steady state distributions of worker and match quality used to determine the vacancy creation decisions of firms already produce a vast majority of matches that have productive quality levels greater than this amount. A minimum wage of $20 dollars an hour also has fairly minor impacts on unemployment and other features of the labor market equilibrium. On the other hand, the impact of a minimum wage of $25 dollars an hour does have notable impacts on unemployment and the steady state distribution of human capital and wages. To some extent, these results parallel those found in Flinn and Mullins (2015), which examined minimum wage impacts within a model of formal schooling decisions that did not allow post-schooling investment in human capital of either type.

In our future research, we intend to endogenize the formal schooling decision, as in Flinn and Mullins. This will provide us with a relatively complete model of human capital investment over the entire life-cycle, and will allow us to examine the relationship between the human capital acquired during the formal schooling phase and that acquired while in the labor market. Our belief is that formal school training is to some extent similar to what we are calling general human capital in this paper, but that the two are not perfect substitutes in production. We believe that much of what one acquires during formal schooling is a technology for learning, which impacts the production of both general and match-specific human capital during the individual's labor market career. In this view, skills acquired or not acquired during early periods of development and formal schooling will have long-lasting effects on labor market outcomes and lifetime welfare.

Appendix

A Sensitivity Analysis with Different Bargaining Weights

In our baseline estimation, we fix the value of the surplus division parameter α at 0.5. In this section, we re-estimate the model with different values of α in order to assess the sensitivity of our parameter estimates as well as the estimated model's equilibrium labor market properties to the value of this parameter.

The results from our sensitivity analysis confirm that investment in the two different types of human capital strongly depend on the worker's surplus share α. In Table A2, we see that total investment in θ (τ_θ) decreases from 6.9 percent to 3.4 percent (as proportion of total time) when α increases from 0.2 to 0.8. The baseline investment in θ falls in between these values at 5.3 percent. On the other hand, the total investment in worker ability a (τ_a) increases from 5.3 percent to 5.9 percent when α increases from 0.2 to 0.8. The baseline investment in worker ability also falls in between these values at 5.6 percent.

The decrease in the total steady state level of match-specific training observed when worker's surplus share is increased reflects the differences between firms and workers in their motives for general and specific training. The worker incorporates the impact of general training on her lifetime productivity and any increase in his bargaining position results in a larger weight on this benefit. On the other hand, the firm values match-specific training also due to turnover concerns, since specific training increases the expected duration of the match. With an increase in the worker's bargaining position, these turnover concerns get a much smaller weight in the bargaining problem. Consequently, when α increases, the proportion of time devoted to general training increases, while time devoted to match-specific training falls. The change in the steady state distributions of general and match-specific human capital can also be seen in Figure A1 and Figure A2.

Table A1 displays the parameter estimates for the three scenarios. The first column displays the parameter estimates for the baseline, and the second and third columns display the parameters for the models with $\alpha = 0.2$ and $\alpha = 0.8$. We see that the parameter estimates are mostly similar between the three cases, with a few exceptions. The largest differences are observed for the parameters that govern the technologies for the rate of increase in general and specific human capital and the parameters for the initial distributions of general ability which depend on schooling. Specifically, we see that $\hat{\delta}_a^1$ is -0.135 for the baseline, whereas it is -0.177 and 0.006 for the models with $\alpha = 0.2$ and $\alpha = 0.8$, respectively. This is the parameter that governs the state-dependence of general ability investment and the comparison of the estimates between the three scenarios shows that it increases with the values of the surplus division parameter α. In the baseline with $\alpha = 0.5$, general training becomes less productive as a increases, whereas with $\alpha = 0.8$, this negative state-dependence is reversed and general training becomes more productive with increases in a. In other words, in the baseline case, a is considerably more costly to change and consequently, initial worker ability level is important for determining the human capital level of the worker throughout his lifetime. However, in the estimated model with higher α, a is relatively less costly to change and the role of initial endowments is diminished. This seems to reflect the fact that worker's ability to devote more time to general human capital investment due to his larger surplus share also diminishes the role of schooling in the model.

The parameters $\mu_a(e)$ control the distribution of starting values for general ability, where e denotes the education level of the worker. Table A1 shows that we obtain larger values for $\mu_a(e)$ when we reestimate the model with a larger α value. The estimated model with $\alpha = 0.8$ also exhibits lower initial observed wages with higher wage growth rates, relative to the baseline. This is consistent with the above discussion regarding the higher level of general human capital investment and the diminished role of initial endowments that are observed due to the increased bargaining position of the worker.

A.1 Minimum Wage with Different Bargaining Weights

We also repeat our minimum wage policy experiment using the parameters estimated with the different values of the Nash bargaining weight. The effects of the minimum wage on unemployment, training and total output are almost identical to those using the baseline parameters. However, we find slightly different effects on welfare driven by the differences in the amount of general training provided under each set of estimates. We recall that the benefits of general training flow mostly to the worker so that workers receive more general training when their bargaining weight is higher. Also, as described in the text, one of the welfare-reducing consequences of the minimum wage is the reduction in general training. Putting these two facts together, we find that when the worker has a low bargaining weight ($\alpha = 0.2$), there is less general training to begin with so that the effect of the minimum wage on training is smaller than in the baseline model, as are the associated welfare losses. As a result, due to other the benefits of minimum wage for the worker, the policy has an overall positive effect on welfare when the worker's bargaining weight is low. Similarly, when the worker's bargaining weight is high ($\alpha = 0.8$), the effect of the minimum wage on training is larger and the welfare losses are 2-3 times larger than in the baseline model.

B Deriving the Steady State Distribution in the Labor Market

In order to simplify notation, we first expand the space of match values to include 0, which signifies that the agent is unmatched, that is, unemployed. All employed individuals at an arbitrary point in time are characterized by the labor market state (j, k), which signifies a_j and θ_k. Let the probability of (j, k) be denoted by $\pi(j, k)$. The steady state marginal distributions of a and θ are given by $\pi_a(j)$ and $\pi_\theta(k)$, respectively. There are J possible values of a, and K possible values of θ (for employed agents), with

$$\begin{aligned} 0 &< a_1 < ... < a_J \\ 0 &< \theta_1 < ... < \theta_K. \end{aligned}$$

From our estimates, we have determined the minimal value of θ that is acceptable when an agent characterized by a_j is in the unemployment state, which we denote by $\underline{k}(j)$ (that is, $\theta_{\underline{k}(j)}$ is the minimal acceptable match value to an individual with ability level a_j). We define the indicator variable

$$d(j,k) = \begin{cases} 1 & \text{if} \quad k \geq \underline{k}(j) \\ 0 & \text{if} \quad k < \underline{k}(j) \end{cases}, \; j = 1, ..., J; k = 1, ..., K.$$

Tautologically, $\pi(j,k) = 0$ for all (j,k) such that $d(j,k) = 0$, $j = 1, ..., J$, $k = 1, ..., K$.

Unemployed agents of type a_j occupy the state $(j, 0)$, with the probability of a type j individual being unemployed given by $\pi(j, 0)$. Then we have

$$\pi_a(j) = \pi(j,0) + \sum_{k>0} \pi(j,k)$$

is the marginal distribution of a in the population. The conditional probability that a type j individual is unemployed is $U(i) = \pi(j,0)/\pi_a(j)$.

We begin by considering movements in the probability of being unemployed for an agent of ability type j. We have

$$\begin{aligned} \dot{\pi}(j,0) &= \eta \sum_{k>0} \pi(j,k) \\ &\quad + \tilde{\delta}_a(j+1)[\sum_{k>0} \pi(j+1,k)(1-d(j,k))] \\ &\quad + \sum_{k>0} \tilde{\delta}_\theta(k)\pi(j,k)(1-d(j,k-1)) \\ &\quad - \lambda_U \pi(j,0) \sum_{k>0} p_\theta(k) d(j,k). \end{aligned}$$

The right hand side terms correspond to the following events. On the first line is the rate at which jobs of any acceptable type (all k for which $d(j,k) = 1$) are destroyed times the probability that type j individuals are employed. The second line represents inflows in to the unemployment state that result from depreciation in general skills that are associated with "endogenous" quits into unemployment. In this case, an individual employed with skills $(j+1,k)$ will quit into unemployment if they would not accept employment at (j,k). The third line represents inflows into unemployment from individuals with skills (j,k) when their match skill level depreciates to $k-1$ and

$(j,k-1)$ is not an acceptable match. The last line is the outflow from the unemployment state, which is the the product of the rate of receiving job offers in the unemployed state, the probability of being in the state $(j,0)$, and the probability of receiving an acceptable job offer. Then, in the steady state,

$$\begin{aligned}\pi^*(j,0) &= [\lambda_U \sum_{k>0} p_\theta(k)]^{-1} \\ &\times\{\eta \sum_{k>0} \pi^*(j,k) \\ &+\tilde{\delta}_a(j+1)[\sum_{k>0} \pi^*(j+1,k)(1-d(j,k))] \\ &+\sum_{k>0} \tilde{\delta}_\theta(k)\pi(j,k)(1-d(j,k-1))\},\end{aligned} \tag{5}$$

for $j = 1, ..., J$.

Now consider the determination of the probabilities associated with employment, those for which $k > 0$. The generic expression for the time derivative of $\pi(j,k)$ is

$$\begin{aligned}\dot{\pi}(j,k) &= \pi(j-1,k)\varphi_a(j-1,k) + \pi(j,k-1)\varphi_\theta(j,k-1) \\ &+\tilde{\delta}_a(j+1)\pi(j+1,k) + \tilde{\delta}_\theta(k+1)\pi(j,k+1) \\ &+\lambda_U \pi(j,0)p_\theta(j) + \lambda_E \sum_{l<k} \pi(j,l) \\ &-[\eta + \tilde{\delta}_a(j) + \tilde{\delta}_\theta(k) + \lambda_E \sum_{l>k} \pi(j,l)]\pi(j,k).\end{aligned}$$

In terms of the expressions on the right hand side of this equation, the first line represents improvements resulting in attaining state (j,k) from the states $(j-1,k)$ and $(j,k-1)$. The second line represents inflows from human capital depreciations from the states $(j+1,k)$ and $(j,k+1)$. The third lines represent inflows from the unemployment state and from contacts with other employed individuals of ability type j who are currently working at jobs in which there match value is less than k. The final line represents all of the ways in which individuals from (j,k) exit the state. These are the exogenous dismissals, decreases in j or k, or finding another job for which match productivity is greater than k. Then in the steady state we have

$$\begin{aligned}\pi^*(j,k) &= [\eta + \tilde{\delta}_a(j) + \tilde{\delta}_\theta(k) + \lambda_E \sum_{l>k} \pi^*(j,l)]^{-1} \\ &\times\{\varphi_a(j-1,k)\pi^*(j-1,k) + \varphi_\theta(j,k-1)\pi^*(j,k-1) \\ &+\tilde{\delta}_a(j+1)\pi^*(j+1,k) + \tilde{\delta}_\theta(k+1)\pi^*(j,k+1) \\ &+\lambda_U \pi^*(j,0)p_\theta(k) + \lambda_E \sum_{l<k} \pi^*(j,l)\},\end{aligned}$$

for $j = 1, ..., J$, $k = 1, ..., K$.

We can vectorize the π matrix, and define the column vector

$$\Pi = \begin{bmatrix} \pi(1, \cdot) \\ \pi(2, \cdot) \\ \vdots \\ \pi(J, \cdot) \end{bmatrix},$$

where $\pi(j, \cdot) = (\pi(j, 0)\ \pi(j, 1)\ ...\ \pi(j, K))'$. Some elements of this vector are identically equally to 0, those for which $d(j, k) = 0$. Let the number of nonzero elements of Π be denoted $N(\Pi)$, where $N(\Pi) \leq J \times (K + 1)$. Denote the entire system of equations by $D(\Pi)$. Then we seek

$$\Pi^* = D(\Pi^*).$$

With no on-the-job search, this mapping is monotone on a compact space, and hence the solution is unique. With on-the-job search, it is clear that an equilibrium always exists, although we have not yet proven uniqueness. Simulations of the model and computation of the fixed point have consistently agreed, however, so that we are confident in the uniqueness property.

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Table 1: **Proportion of Job Spells by the number of interview dates they span**

	High School Graduates	Some College	College Graduates
0	61%	55%	53%
1	23%	24%	21%
2	7%	9%	11%
3	3%	4%	6%
4	2%	3%	3%
5	1%	2%	3%
6	3%	3%	3%

Table 2: **Incidence of Training**

	All	HS	Some College	College or More
% who got training at least once	15%	18 %	13 %	13%
% who got training at the start of job spell	6%	10 %	5 %	3%

Table 3: **Proportion by Number of Training Spells (Conditional on Having Participated At Least Once)**

	Percentage over All Workers with at Least One Training Spell
1	72 %
2	18 %
3	6 %
4	3 %
5	1 %

Table 4: **Annual Labor Turnover Rates and Wage Growth**

	HS	Some College	College or More
Panel A: Employment-to-Employment (EE) transitions btw $t-1$ and t	% of EE transitions *with no* job change		81 %
% of EE transitions *with no* job change	81 %	85 %	88%
% of EE transitions *with* job change (job-to-job transitions)	19 %	15 %	12%
..... % of job-to-job transitions *with* non-employment btw $t-1$ and t	4 %	3%	2%
..... % of job-to-job transitions *with no* non-employment btw $t-1$ and t	15%	12%	10%
Panel B: Wage Growth btw $t-1$ and t	EE transitions *with no* job change		0.08
EE transitions *with no* job change	0.08	0.08	0.09
EE transitions *with* job change (job-to-job transitions)	0.11	0.15	0.20
..... job-to-job transitions *with* non-employment btw $t-1$ and t	0.06	0.06	0.23
..... job-to-job transitions *with no* non-employment btw $t-1$ and t	0.12	0.17	0.20
Panel C: % of Negative Wage Growth btw $t-1$ and t	EE transitions *with no* job change		17 %
EE transitions *with no* job change	17 %	19 %	23%
EE transitions *with* job change	32 %	32 %	28%
..... job-to-job transitions *with* non-employment btw $t-1$ and t	40 %	47%	27%
..... job-to-job transitions *with no* non-employment btw $t-1$ and t	30%	28%	28%

Table 5: **Log Wage Difference btw Interview Dates $t-1$ and t by Training**

	$logw_t - logw_{t-1}$ No Training	Got Training
transitions with no job change	0.08	0.08
transitions with job change (job-to-job transitions)	0.14	0.10
job-to-job transitions with non-employment spell btw $t-1$ and t	0.08	0.15
job-to-job transitions with no non-employment spell btw $t-1$ and t	0.15	0.09

Table 6: **Parameter Estimates**

PARAMETERS FOR EMPLOYMENT TRANSITIONS			
flow value of unemployment	b	6.471	(0.857)
job offer rate - unemployed	λ_u	0.138	(0.019)
job offer rate - employed	λ_e	0.069	(0.023)
exogenous job separation rate	η	.004	(0.001)
PARAMETERS OF INVESTMENT FUNCTIONS			
General ability investment TFP	δ_a^0	0.029	(0.005)
Firm-specific investment TFP	δ_θ^0	0.023	(0.003)
State-dependence of general ability investment	δ_a^1	-0.135	(0.144)
State-dependence of firm-specific investment	δ_θ^1	0.577	(0.098)
Curvature of general ability investment	δ_a^2	0.283	(0.084)
Curvature of firm-specific investment	δ_θ^2	0.443	(0.062)
Rate of decrease in general ability	$\tilde{\varphi}_a^-$	0.001	(0.0003)
Rate of decrease in match quality	$\tilde{\varphi}_\theta^-$	0.011	(.003)
PARAMETERS OF INITIAL ABILITY DISTRIBUTIONS			
Mean of initial general ability - High School	$\mu_a(e_i = 1)$	0.884	(0.159)
Mean of initial general ability - Some College	$\mu_a(e_i = 2)$	1.152	(0.112)
Mean of initial general ability - BA or higher	$\mu_a(e_i = 3)$	1.449	(0.083)
Variance of initial general ability	σ_a	.204	(.086)
PARAMETERS OF JOB OFFERS			
Mean of match quality distribution	μ_θ	1.408	(0.040)
Variance of match quality distribution	σ_θ	0.289	(0.034)
PARAMETERS GOVERNING TRAINING OBSERVATION			
Intercept for training observation	β_0	-2.633	(0.039)
Coefficient on τ for training observation	β_1	1.0000	-

Table 7: **Model Fit: Incidence of Training**

	All	HS	Some College	College or More
% who got training at least once				
.... Data	15%	18 %	13 %	13%
.... Model	17%	21 %	16 %	12%
% who got training at the start of job spell				
.... Data	6%	10 %	5 %	3%
.... Model	5%	6 %	4 %	3%

Figure 1: **Model Fit: Log Wage Distribution - Year 0-2**

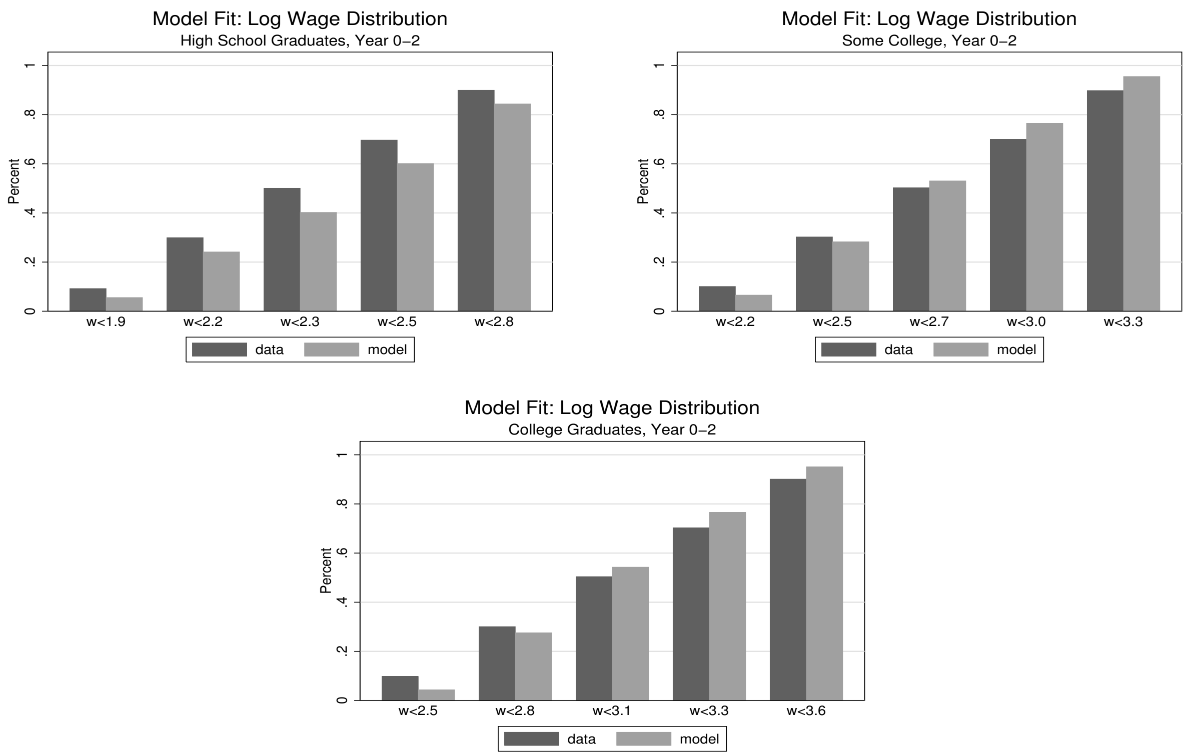


Figure 2: **Model Fit: Log Wage Distribution - Year 3-5**

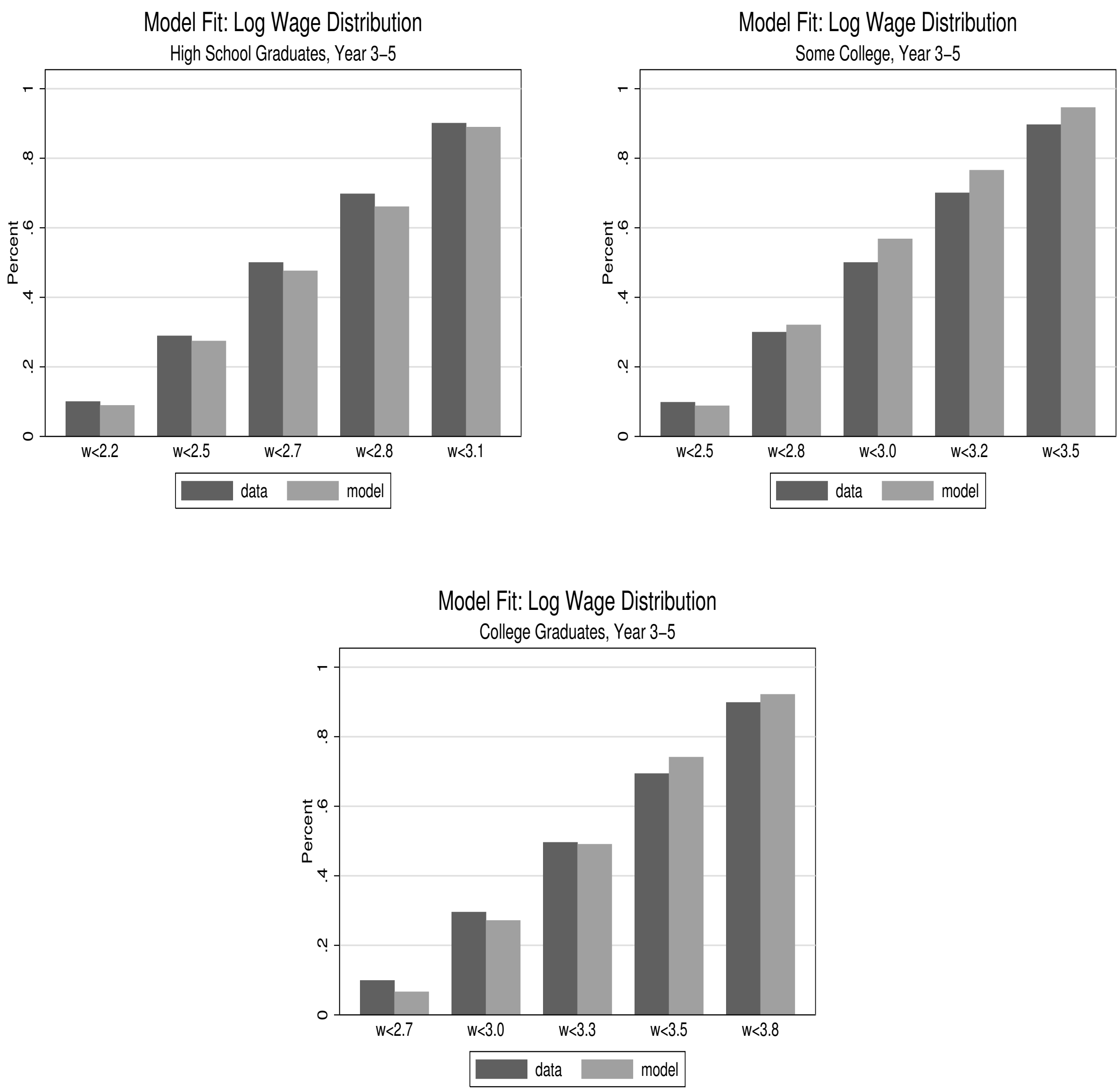


Figure 3: **Model Fit: Log Wage Distribution - Year 6-8**

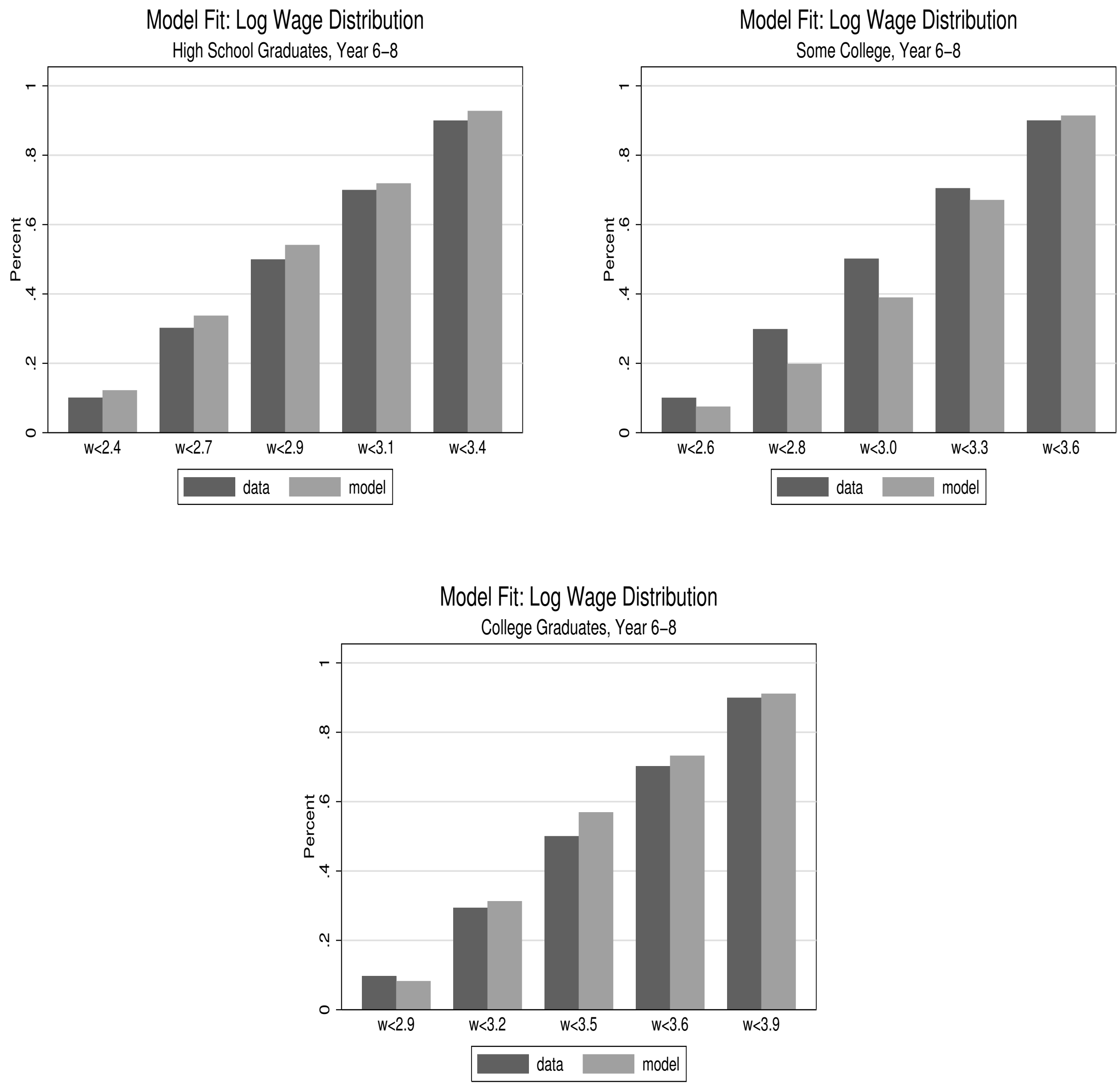


Table 8: **Model Fit: Annual Labor Turnover Rates and Wage Growth**

	HS	Some College	College or More
Panel A: Employment-to-Employment (EE) transitions btw $t-1$ and t			
% of EE transitions *with no* job change			
.... Data	81 %	85 %	88%
.... Model	74%	74 %	76%
% of EE transitions *with* job change (job-to-job transitions)			
.... Data	19 %	15 %	12%
.... Model	25 %	26 %	24%
% of job-to-job transitions *with* non-employment btw $t-1$ and t			
.... Data	4 %	3%	2%
.... Model	8 %	8%	8%
% of job-to-job transitions *with no* non-employment btw $t-1$ and t			
.... Data	15%	12%	10%
.... Model	17%	17%	16%
Panel B: Wage Growth btw $t-1$ and t			
EE transitions *with no* job change			
.... Data	0.08	0.08	0.09
.... Model	0.08	0.08	0.08
EE transitions *with* job change			
.... Data	0.11	0.15	0.20
.... Model	0.09	0.11	0.11
job-to-job transitions *with* non-employment btw $t-1$ and t			
.... Data	0.06	0.06	0.23
.... Model	-0.10	-0.06	-0.02
job-to-job transitions *with no* non-employment btw $t-1$ and t			
.... Data	0.12	0.17	0.20
.... Model	0.18	0.19	0.17
Panel C: % of Negative Wage Growth btw $t-1$ and t			
EE transitions *with no* job change			
.... Data	17 %	19 %	23%
.... Model	38 %	37 %	37%
EE transitions *with* job change			
.... Data	32%	32%	28%
.... Model	37%	35%	34%
job-to-job transitions *with* non-employment btw $t-1$ and t			
.... Data	40 %	47%	27%
.... Model	61 %	57%	50%
job-to-job transitions *with no* non-employment btw $t-1$ and t			
.... Data	30%	28%	28%
.... Model	26%	24%	26%

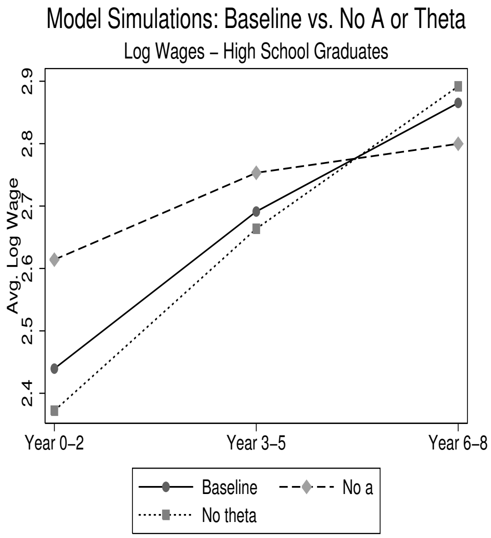
Table 9: **Parameter Estimates: Baseline vs. Constrained Estimations with No a/No θ**

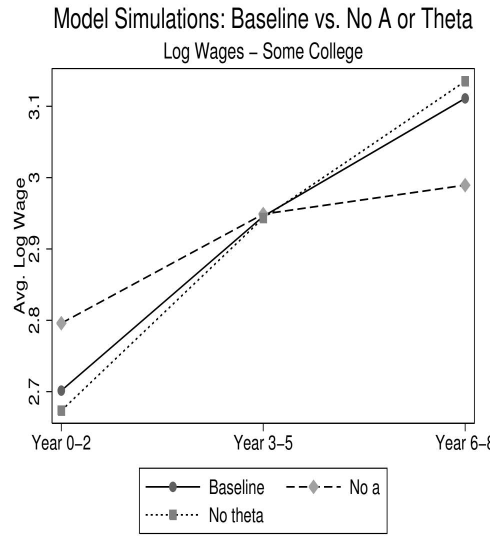
		Baseline	No a	No θ
PARAMETERS FOR EMPLOYMENT TRANSITIONS				
flow value of unemployment	b	6.4705	4.2083	7.1923
job offer rate - unemployed	λ_u	0.1375	0.1416	0.2040
job offer rate - employed	λ_e	0.0685	0.0823	0.0823
exogenous job separation rate	η	0.0036	0.0026	0.0026
PARAMETERS OF INVESTMENT FUNCTIONS				
General ability investment TFP	δ_a^0	0.0285	0.0	0.0372
Firm-specific investment TFP	δ_θ^0	0.0229	0.0225	0.0
State-dependence of general ability investment	δ_a^1	-0.1353	-0.1353	-0.0304
State-dependence of firm-specific investment	δ_θ^1	0.4426	0.6586	0.5767
Curvature of general ability investment	δ_a^2	0.2826	0.2826	0.1459
Curvature of firm-specific investment	δ_θ^2	0.4426	0.6561	0.4426
Rate of decrease in general ability	$\tilde{\varphi}_a^-$	0.0014	0.0014	0.0036
Rate of decrease in match quality	$\tilde{\varphi}_\theta^-$	0.0113	0.0195	0.0016
PARAMETERS OF INITIAL ABILITY DISTRIBUTIONS				
Mean of initial general ability - High School	$\mu_a(e_i = 1)$	0.8844	0.8621	0.9117
Mean of initial general ability - Some College	$\mu_a(e_i = 2)$	1.1515	1.0546	1.2009
Mean of initial general ability - BA or higher	$\mu_a(e_i = 3)$	1.4489	1.3429	1.4695
Variance of initial general ability	σ_a	0.2041	0.1355	0.1992
PARAMETERS OF JOB OFFERS				
Mean of match quality distribution	μ_θ	1.4078	1.3900	1.2990
Variance of match quality distribution	σ_θ	0.2895	0.3757	0.2400
PARAMETERS GOVERNING TRAINING OBSERVATION				
Intercept for training observation	β_0	-2.7331	-2.8469	-2.6084
Coefficient on τ for training observation	β_a	1.0000	1.0000	1.0000

Table 10: **Simulations: Baseline vs. Estimations with No a/No θ**

Annual Labor Turnover Rates and Wage Growth	HS	Some College	College or More
Panel A: Employment-to-Employment (EE) transitions btw $t-1$ and t			
% of EE transitions *with no* job change			
.... Baseline	74%	74 %	76%
.... No a	84 %	82 %	81%
.... No θ	70 %	70 %	72%
% of EE transitions *with* job change (job-to-job transitions)			
.... Baseline	25 %	26 %	24%
.... No a	16 %	18 %	19%
.... No θ	30 %	30 %	27%
% of job-to-job transitions *with* non-employment btw $t-1$ and t			
.... Baseline	8 %	8%	8%
.... No a	7 %	7 %	7%
.... No θ	6%	7 %	7%
% of job-to-job transitions *with no* non-employment btw $t-1$ and t			
.... Baseline	17%	17%	16%
.... No a	9%	11%	12%
.... No θ	23%	23%	21%
Panel B: Wage Growth btw $t-1$ and t			
EE transitions *with no* job change			
.... Baseline	0.08	0.08	0.08
.... No a	0.06	0.07	0.08
.... No θ	0.07	0.07	0.06
EE transitions *with* job change			
.... Baseline	0.09	0.11	0.11
.... No a	-0.03	-0.002	0.02
.... No θ	0.15	0.15	0.15
job-to-job transitions *with* non-employment btw $t-1$ and t			
.... Baseline	-0.10	-0.06	-0.02
.... No a	-0.17	-0.13	-0.09
.... No θ	-0.02	0.02	0.08
job-to-job transitions *with no* non-employment btw $t-1$ and t			
.... Baseline	0.18	0.19	0.17
.... No a	0.07	0.07	0.09
.... No θ	0.19	0.19	0.18
Panel C: % of Negative Wage Growth btw $t-1$ and t			
EE transitions *with no* job change			
.... Baseline	38 %	37 %	37%
.... No a	43%	41%	40%
.... No θ	39%	39%	41%
EE transitions *with* job change			
.... Baseline	37%	35%	34%
.... No a	54%	51%	49%
.... No θ	29%	28%	27%
job-to-job transitions *with* non-employment btw $t-1$ and t			
.... Baseline	61 %	57%	50%
.... No a	69%	63%	61%
.... No θ	53%	48%	41%
job-to-job transitions *with no* non-employment btw $t-1$ and t			
.... Baseline	26%	24%	26%
.... No a	44%	44%	42%
.... No θ	23%	22%	23%

Figure 4: **Avg. Log Wages by Tenure**





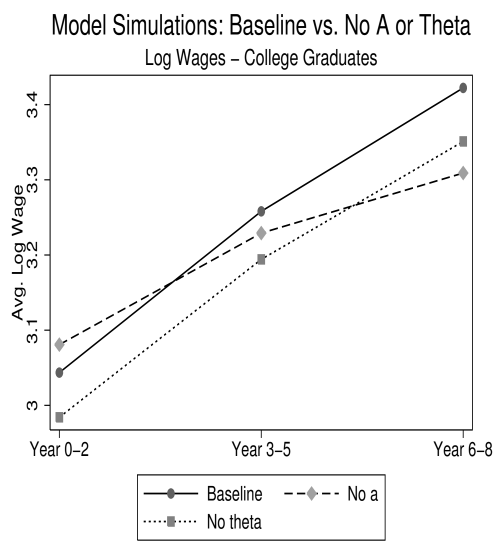


Figure 5: Minimum Acceptable Match-Quality

This figure shows the $\theta^*(a)$, the lowest match quality that workers with each level of general ability will accept.

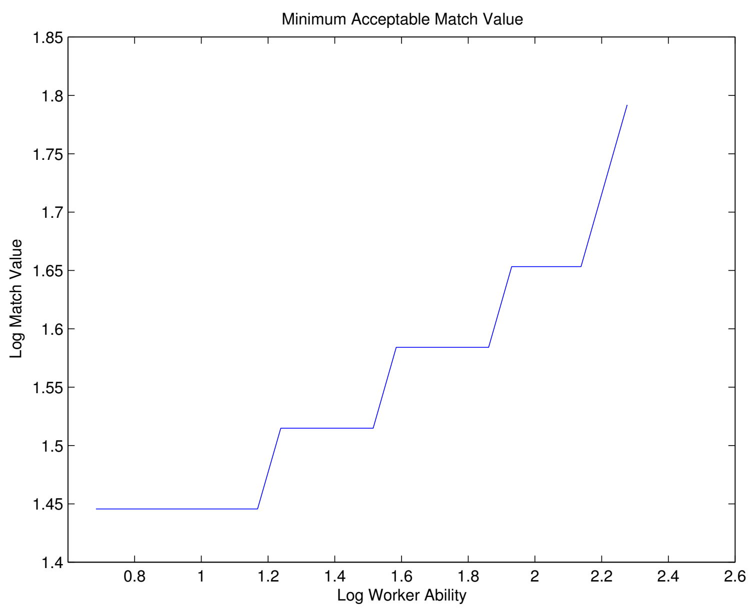


Figure 6: Trade-Off between Wages and Training in Bargaining Problem

This figure shows combinations of general training (τ_a), match specific training (τ_θ) and wages that solve the bargaining problem between the worker and the firm at the median values of a and θ. Lines on the graph show combinations of τ_a and τ_θ along which the negotiated wage remains constant. The blue dot shows the surplus maximizing combination, which is the model solution for τ_a and τ_θ.

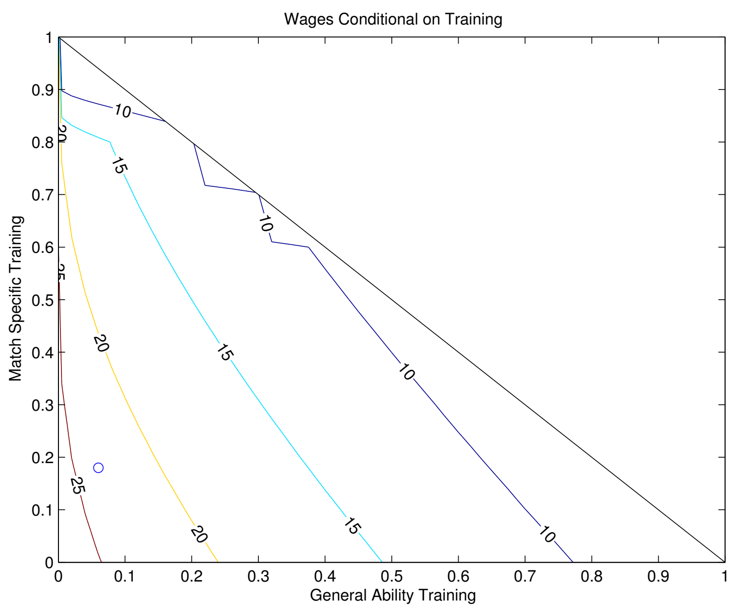


Figure 7: General Training

This figure shows the amount of general training that workers receive at different combinations on a and θ. Lines on the graph show contours along which the amount of training remains constant. The blank area below the black line, shows states for which workers will not accept the job offer.

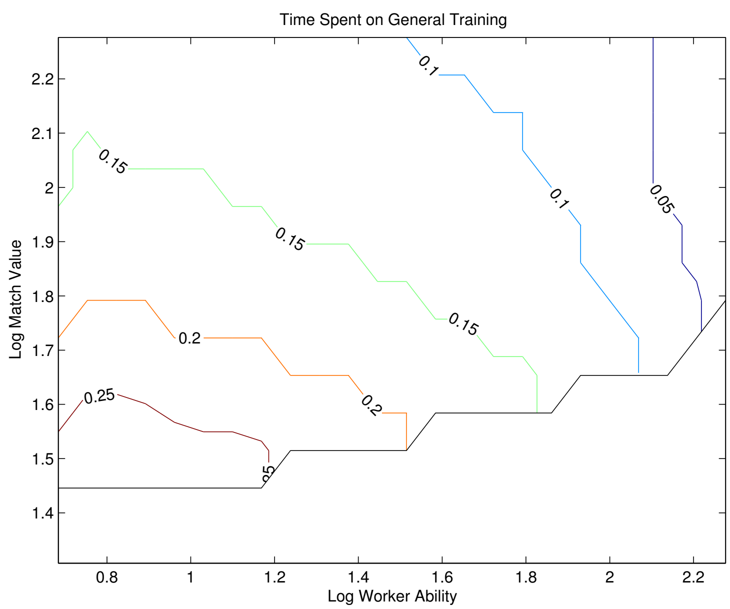


Figure 8: Match-Specific Training

This figure shows the amount of match-specific training that wokers receive at different combinations on a and θ. Lines on the graph show contours along which the amount of training remains constant. The blank area below the black line, shows states for which workers will not accept the job offer.

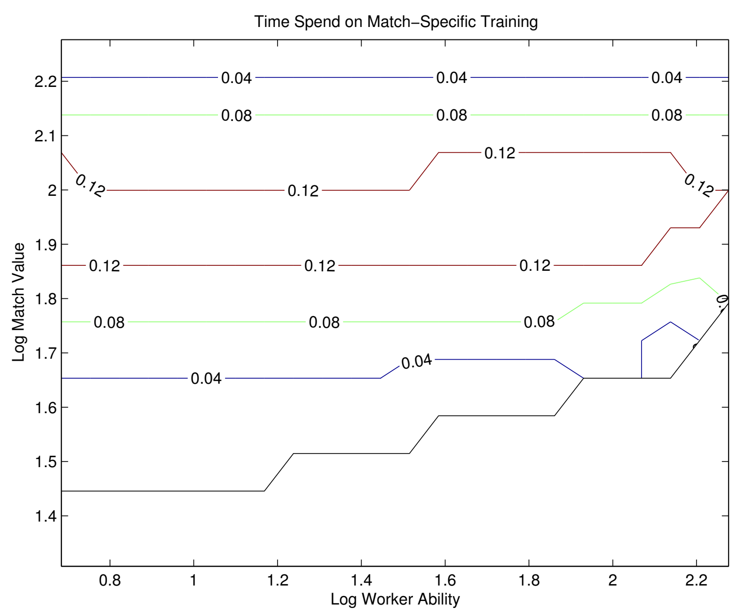


Figure 9: Wage as Fraction of Output

This figure shows the worker's wage as a fraction of his total output at different combinations on a and θ. Lines on the graph show contours along which the fraction remains constant. The blank area below the black line, shows states for which workers will not accept the job offer.

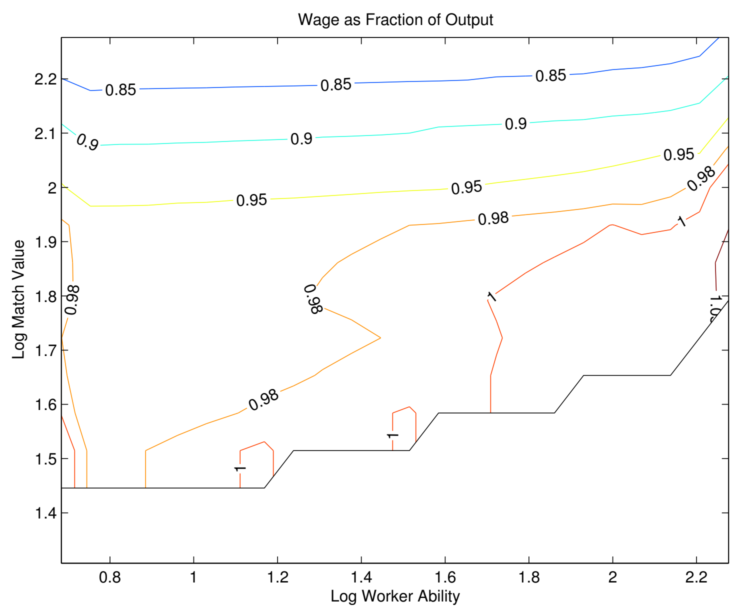


Figure 10: Training by Years in Labor Market

This figure shows the average fraction of their time that workers spend on general and match-specific training in the model simulation as a function of the number of years in the labor market.

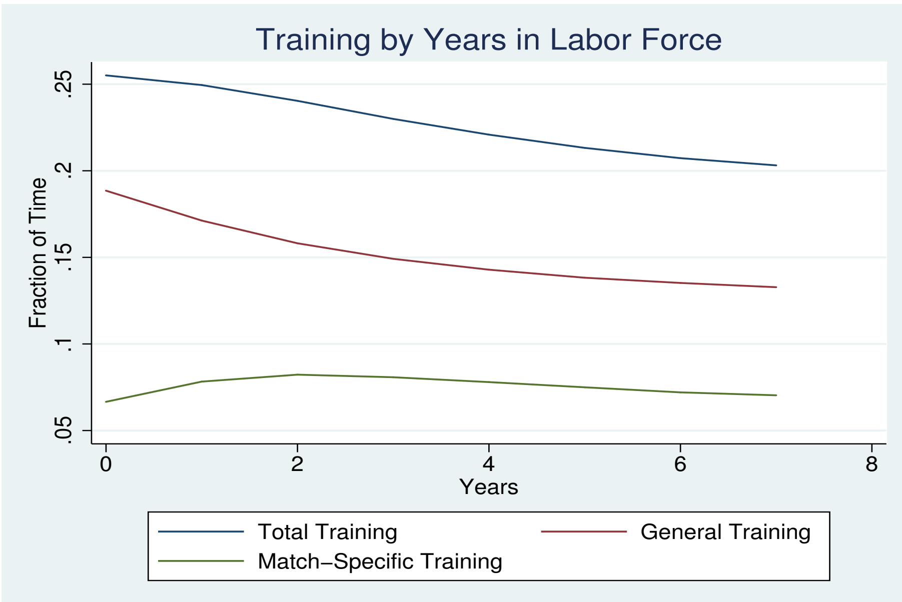


Figure 11: Sources of Wage Growth by Years in Labor Market

This figure shows the average of the log of general human capital, the amount of match-specific capital accumulated through search and through training, the log of the fraction of time they spend not training, and the log of the average wage as a fraction of worker output for simulated workers as a function of the number of years in the labor market. In the absence of employment costs, these five components would add up to the total log wage, which is also shown.

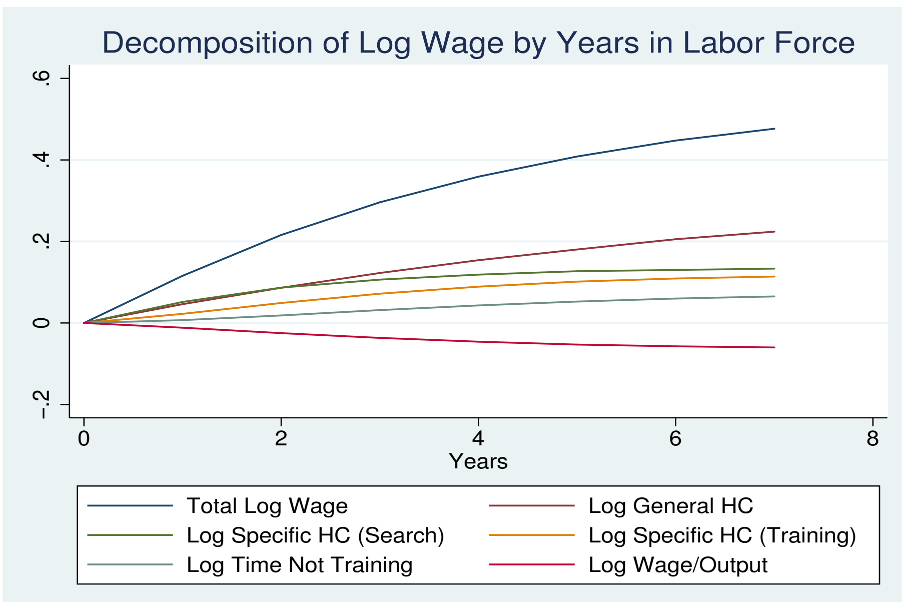


Table 11: **Mincer Regressions**

	Const	Some College	BA	Years in LF	Tenure
DATA					
log wage	2.141	0.316	0.796	0.051	0.045
	(0.098)	(0.098)	(0.098)	(0.002)	(0.003)
MODEL					
log wage	2.390	0.260	0.594	0.047	0.055
starting a	.902	0.239	0.537	-0.000	0.002
a from training	0.037	-0.015	-0.031	0.037	0.004
θ from search	1.789	0.010	0.029	0.008	0.013
θ from training	0.017	0.005	0.007	-0.000	0.038
hours worked	-0.315	0.017	0.044	0.004	0.016
wage/output	-0.040	0.003	0.008	-0.001	-0.017

Table 12: **Mincer Regressions by Education**

	Constant			Years in LF			Tenure		
	HS	College	BA	HS	College	BA	HS	College	BA
DATA									
log wage	2.305	2.667	2.951	0.069	0.039	0.044	0.033	0.052	0.053
	(0.011)	(0.015)	(0.015)	(0.002)	(0.004)	(0.006)	(0.003)	(0.006)	(0.007)
MODEL									
log wage	2.390	2.645	2.990	0.048	0.047	0.042	0.053	0.058	0.060
starting a	0.902	1.139	1.440	-0.000	-0.001	-0.001	0.001	0.004	0.002
a from training	0.029	0.027	0.021	0.038	0.035	0.031	0.003	0.004	0.006
θ from search	1.794	1.796	1.813	0.006	0.009	0.011	0.014	0.011	0.011
θ from training	0.023	0.018	0.014	-0.000	-0.000	-0.000	0.035	0.041	0.045
hours worked	-0.314	-0.299	-0.273	0.004	0.004	0.004	0.016	0.015	0.016
wage/output	-0.044	-0.035	-0.026	-0.000	-0.001	-0.001	-0.016	-0.017	-0.020

Figure 12: Mininimum Acceptable Wage with Minimum Wage

This figure shows the $\theta^*(a)$, the lowest match quality that workers with each level of general ability will accept. The solid line shows $\theta^*(a)$ for the baseline model, the dashed line when we impose a minimum wage.

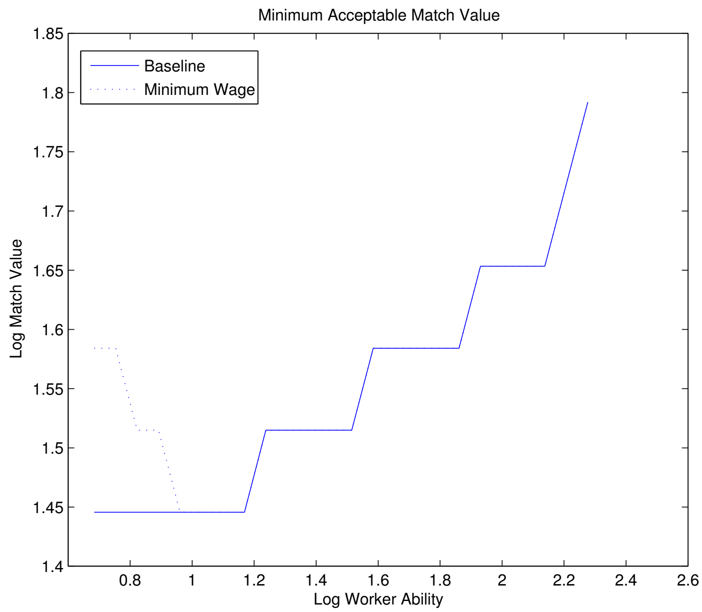


Figure 13: Training by Years in Labor Market with Minimum Wage

This figure shows the average fraction of their time that workers spend on general and match specific training in the model simulation as a function of the number of years in the labor market. The solid lines show the amount of training in the baseline model. The corresponding dashed lines show the amount of training when we impose a minimum wage.

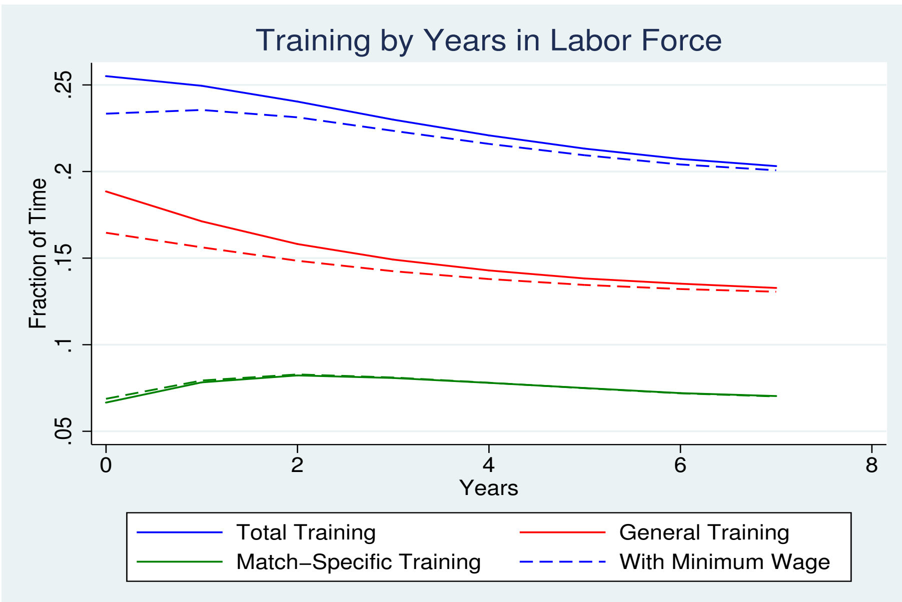


Figure 14: Log Wage Distribution by Years in Labor Market with Minimum Wage

This figure shows the distribution of log wages in the model simulation as a function of the number of years in the labor market. The solid lines show percentiles of the distribution in the baseline model. The corresponding dashed lines show the same percentiles when we impose a minimum wage.

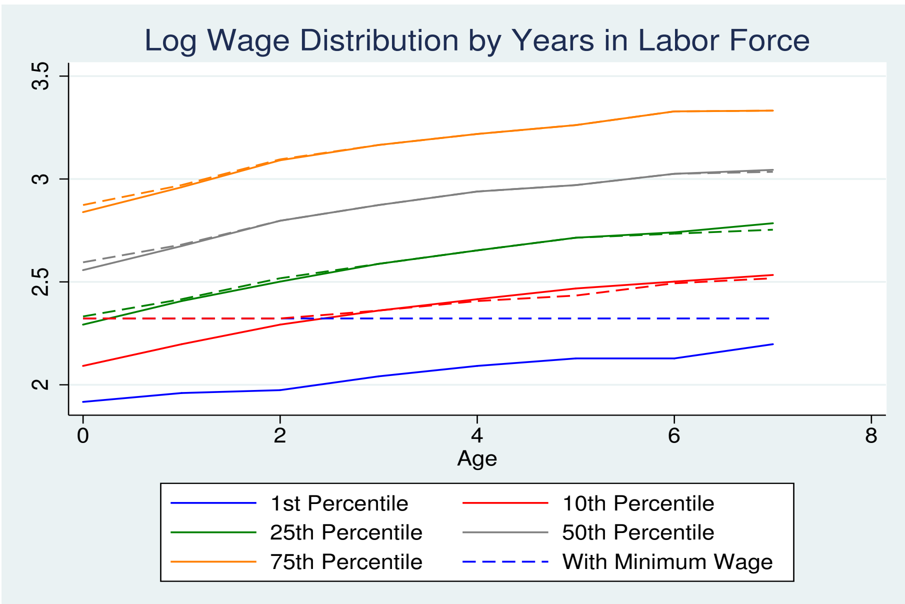


Table A1: **Sensitivity Analysis with Different α Values**

		Baseline ($\alpha = 0.5$)	$\alpha = 0.2$	$\alpha = 0.8$
PARAMETERS FOR EMPLOYMENT TRANSITIONS				
flow value of unemployment	b	6.471	7.069	6.816
job offer rate - unemployed	λ_u	0.138	0.143	0.141
job offer rate - employed	λ_e	0.069	0.059	0.061
exogenous job separation rate	η	0.004	0.004	0.004
PARAMETERS OF INVESTMENT FUNCTIONS				
General ability investment TFP	δ_a^0	0.029	0.029	0.027
Firm-specific investment TFP	δ_θ^0	0.023	0.023	0.024
State-dependence of general ability investment	δ_a^1	-0.135	-0.177	0.006
State-dependence of firm-specific investment	δ_θ^1	0.443	0.593	0.542
Curvature of general ability investment	δ_a^2	0.283	0.263	0.278
Curvature of firm-specific investment	δ_θ^2	0.443	0.463	0.392
Rate of decrease in general ability	$\tilde{\varphi}_a^-$	0.001	0.001	0.001
Rate of decrease in match quality	$\tilde{\varphi}_\theta^-$	0.011	0.010	0.011
PARAMETERS OF INITIAL ABILITY DISTRIBUTIONS				
Mean of initial general ability - High School	$\mu_a(e_i = 1)$	0.884	0.921	0.745
Mean of initial general ability - Some College	$\mu_a(e_i = 2)$	1.152	1.184	1.099
Mean of initial general ability - BA or higher	$\mu_a(e_i = 3)$	1.449	1.445	1.383
Variance of initial general ability	σ_a	0.204	0.254	0.219
PARAMETERS OF JOB OFFERS				
Mean of match quality distribution	μ_θ	1.408	1.439	1.446
Variance of match quality distribution	σ_θ	0.289	0.303	0.270
PARAMETERS GOVERNING TRAINING OBSERVATION				
Intercept for training observation	β_0	-2.733	-2.777	-2.713
Coefficient on τ for training observation	β_a	1.0000	1.0000	1.0000

Table A2: **Sensitivity Analysis: Steady State**

	Baseline ($\alpha = 0.5$)	$\alpha = 0.2$	$\alpha = 0.8$
Unemployment Rate (%)	19.54	19.47	27.67
Average Log Worker Ability	1.91	1.98	1.91
Total Investment in Worker Ability (%)	5.56	5.25	5.98
Total Theta Investment (%)	5.28	6.99	3.40
Total Output	41.71	47.94	36.87
Mean Log Wage	3.71	3.75	3.73
Mean Productivity	57.73	67.08	56.17

Figure A1: **Sensitivity Analysis: Steady State Distributions - Log Match Values**

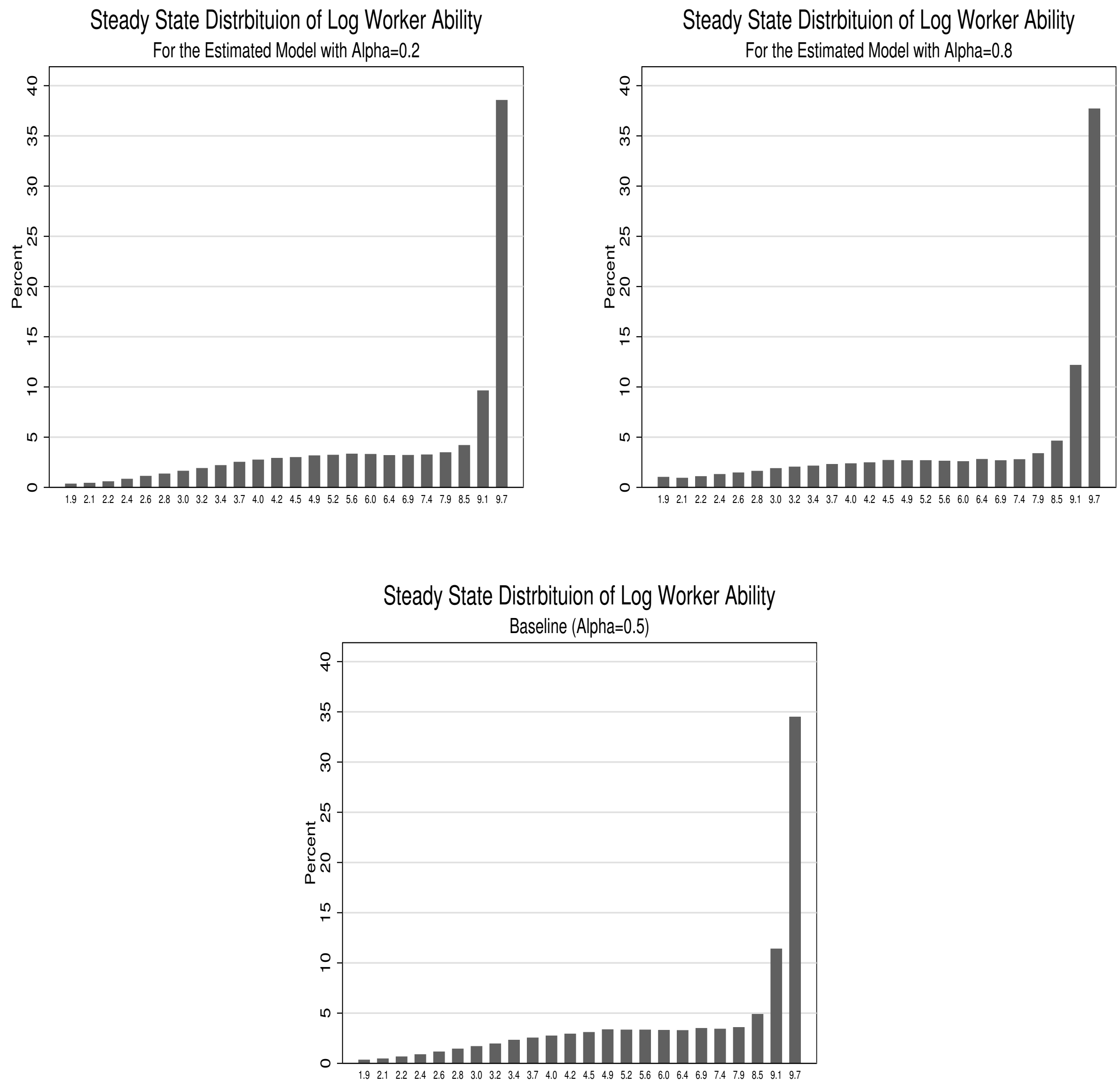


Figure A2: **Sensitivity Analysis: Steady State Distributions - Log Match Values**