Fully Absorbing Dynamic Compromise☆

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Abstract

I consider a repeated divide-the-dollar voting model with rejections leading to the implementation of the previous period's allocation (see Kalandrakis (2004)). I show that if proposals can be non-exhaustive, then equal division can be achieved as an absorbing steady state from any initial allocation given a large enough discount factor as a part of a Markov Perfect equilibrium. This result is robust to changes in voting thresholds and persistence in proposal power *outside* of unanimity or total persistence.

Keywords: Dynamic Legislative Bargaining, Markov Perfect Equilibria
JEL Codes: C73, D72

1. Introduction

This note considers a discrete-time infinite repeated legislative bargaining game where previous allocations serve as a status-quo for current proposals. There are N players, henceforth referred to as *legislators*. In each period, a legislator is randomly selected with equal probability to make a proposal on how to divide a budget of size one among himself and the other legislators. All legislators then vote simultaneously to accept or reject the proposal. If a majority of legislators vote accept, then the proposed division is implemented for that period. If not, then the division from the previous round is implemented instead. Thus, the division implemented in the previous round acts as an *evolving status quo*. Per-period utility is a strictly concave function of an individual legislator's budget allocation and legislators discount future payoffs in a standard exponential fashion.

Under these conditions, I show the existence of a fully-absorbing equal division Markov Perfect equilibrium when discount factors are large enough. This continues a strand of literature originally started by Kalandrakis (2004) and with more recent contributions, among others, by Kalandrakis (2010), Bowen and Zahran (2012), Anesi and Seidmann (2012) investigating the properties of different Markov perfect equilibria in the aforementioned setting. I refer to these equilibria as "fully-absorbing" because convergence to the steady state occurs

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from any initial status quo. Additionally, I show that this result is robust to changes to i). the voting threshold necessary for budget passage and ii). persistence of proposal power.

One reason why the current model may derive some interest is because it lives between the repeated game and bargaining literatures. Specifically, it differs from the standard non-cooperative bargaining literature (see Rubinstein (1982)) as i). agreements takes place via majority rule (instead of unanimity), ii). there is a new budget to be shared in every period, and iii). the default option is the previous period's allocation (as opposed to an assumed fixed default). Additionally, the current model differs from a standard repeated game setting because of he evolving status quo and thus in its analysis. Another line of interest derives from the applicability of the model (see Bowen et al. (Forthcoming) who analyze mandatory and discretionary spending in a two party system).

A basic motivation for interest in equal division outcomes is the observation that in many real-world settings, budget allocations go far beyond minimal winning coalitions. For example, see the distribution of US Federal Highway funds[2] where significant spending is allocated to each state.

Throughout the note, I focus on Markov perfect equilibrium. In the current setting, the restriction to Markovian strategies is typically justified on the grounds of simplicity and more importantly, the fact that frequent legislative turnover may lead to a lack of institutional memory.[3] In the setting considered here, for a strategy to be Markovian, the proposal strategy of legislators may only rely upon the status quo proposal, and the accept/reject decision may only rely upon the status quo proposal and the current proposal.

However, the focus on Markov equilibrium introduces difficulties that are not present when strategies can be fully history dependent. First, existence is often not a trivial matter in Markovian strategies and is typically shown through construction. Duggan and Kalandrakis (2012) show existence in a related model with shocks, which are necessary for the existence result. Second, without the Markovian restriction, the goal of supporting an equal division fully-absorbing equilibrium would be trivial as agents could monitor which agent was the last deviator (if any) and thereby keep track of who to punish and for how long. But, in a Markov Perfect environment, there is a difficulty of encoding or grouping the much larger space of all possible histories into the space of allocations. Additionally known folk theorems fail to apply in the given setting. One trivial reason is that the folk theorems for stochastic games rely on finite state spaces, but they also fail to apply to the studied model for more fundamental reasons. In particular, the irreducibility condition of Hörner et al. (2011) fails to hold, Dutta (1995) uses non-Markovian strategies, and the full-rank condition of Fudenberg and Yamamoto (2011) is not satisfied.

From a theoretical viewpoint, there are two papers that are very closely related to the present one. They both differ from the current one in that they feature multiple absorbing states. This creates a dependence on the initial status quo and thus there are no fully

[2]See http://www.fhwa.dot.gov/safetealu/fy11comptables.pdf for 2011 figures and more recent years at http://www.fhwa.dot.gov/map21/funding.cfm.

[3]For example, if the discount factor comes not only from time discounting, but is also dependent upon the legislator possibly being not reelected, then a fully history dependent equilibrium may require a legislator to look at his predecessors' history in determining his history dependent action and possibly to continue punishment schemes based upon those historical actions.

absorbing states. Bowen and Zahran (2012) (hereafter BZ), serves as a primary inspiration for this paper, and considers a setting where proposals must be exhaustive. They find that with sufficient additional conditions on the period utility function and the discount factors, a form of compromise is indeed possible. Specifically, the compromise state features all but one legislator equally dividing the dollar and the remaining legislator is frozen out (i.e. this legislator receives 0). Which legislator is frozen out changes over time, because, when the frozen legislator obtains proposal power, he proposes that a different legislator becomes the frozen legislator, and this is accepted. Some initial status quos will lead to this near-equal sharing allocation and other initial status quos will lead to a permanent punishment regime.[4]

The other quite closely related theory paper is Anesi and Seidmann (2012) (hereafter AS). They show how almost any allocation can be supported as an absorbing state (out of multiple absorbing states) using *no-delay equilibria*. Each legislator has a coalition containing himself which he proposes to be a winning coalition. In every state, a legislator chooses a coalition that he is a member of and all legislators in the chosen coalition receive a good allocation while all other legislators receive a bad allocation. If the status quo is not an absorbing state, then all legislators in a chosen coalition accept the proposal. If the status quo is an absorbing state, then all legislators reject the proposal. Importantly, because some legislators are receiving bad allocations in some absorbing states and good allocations in others, there are always multiple absorbing states in any equilibrium. Thus, equal division is not supportable as a fully-absorbing state according to this type of "no-delay" equilibrium. A key difference between this paper and BZ is that in AS, some legislators are in a reward state and some are in a punishment state, whereas in BZ, some initial status quos lead to a reward state for (almost all) legislators and other initial status quos lead to a punishment state for (almost all) legislators.

In my construction, the fact that I can find the existence of fully-absorbing equal division Markov perfect equilibria relies fundamentally on the fact that budget allocations need not be fully exhaustive. As mentioned previously, this agrees with the setup of AS and not BZ. These stronger results are achieved by allowing budget proposals to be non-exhaustive. Intuitively, the construction is enabled as non-exhaustive budget allocations makes it possible for legislators to separate "punishing" from "stealing". Imagine that a proposer has to divide a dollar and wishes to punish legislators who receive a larger than equal share of the budget. The proposer punishes them by allocating them a smaller than equal share of that dollar. However, if the proposer must make exhaustive proposals, then somebody else will end up with a greater than equal share of the budget. This is undesirable for that legislator because then that legislator will merit future punishment. However, if allocations need not be exhaustive, then it becomes possible to punish some legislators without making it appear as though other legislators are stealing, and thereby meriting future punishment for them. This solves the problem that legislators don't want to be caught stealing and subsequently punished, and therefore would not want to agree to a punishment of another legislator which would force them to be perceived as stealing. One could phrase the above finding as

[4]In this punishment regime, legislators follow the same strategy as in Kalandrakis (2004), namely it is a proposer-takes-all equilibrium.

"wastefulness may enable cooperation".[5]

Additionally, Baron and Ferejohn (1989) permits non-exhaustive budget allocations, but in equilibrium no proposer will make an accepted proposal where all of the budget is not allocated. This is because, in their model, agreement is once and for all. Therefore, if a legislator were to make an acceptable proposal that fails to allocate the entire dollar, then the proposer could profitably give ϵ more to every other legislator and the excess to himself. All legislators are now better off (including the proposer); thus legislators who agreed to the smaller allocation will also agree to this one. In the model that I present here, an agreed-upon allocation additionally impacts future allocations in its role as a status quo. So, there may be (and, in the constructed equilibrium, are) strategic reasons why a legislator would wish to propose a non-exhaustive budget.

Non-exhaustive budgets allocations can be though of as a kind of "money burning" although in the constructed equilibrium there will be used as punishment by the proposer towards agents who are perceived as deviating. In this fashion, it differs from, say Ben-Porath and Dekel (1992) where an agent's ability to burn money goes unused, but enables the selection of a favorable equilibrium. In a more related Rubinstein bargaining setting, Manzini (1999) and Avery and Zemsky (1994), among others, consider a one-side ability to burn part of the pie being bargained over. There, money burning may be used in equilibrium and can serve to enhance a player's negotiating ability. Incorporating Avery and Zemsky (1994) into a repeated game setting, Houba et al. (2000) find that MPE are inefficient relative to the setting where strategies can be history-dependent. On the other hand, Goldlücke and Kranz (2012) provide a stationary strategy equilibrium that supports all public perfect equilibria and find that money burning may or may not increase the set of possible stationary equilibrium payoffs in a repeated game setting depending upon the monitoring structure and the available stage game payoffs. This is in line with Sannikov and Skrzypacz (2007) who consider a repeated duopoly model with decreasing time periods and find collusion to be impossible, even with the ability to destroy money or make monetary transfers.

Finally, it should be mentioned that there are a few other papers related to the current one. The first is Kalandrakis (2004) who originated the study of this type of model and considers a setting with 3 legislators and linear per-period utility in budget allocations. He finds the existence of an equilibrium which eventually degenerates into a proposer-takes-all equilibrium. Kalandrakis (2010) extends this result to the setting where the number of legislators is an odd number equal to 5 or greater. Additionally, in a case where legislators have concave per-period utility, which also satisfies some additional restrictions, he finds the existence of Markov Perfect equilibria that feature minimum winning coalitions. Finally, Battaglini and Palfrey (2012) conducts numerical simulations and laboratory experiments to test behavior. In their numerical simulations, they find convergence to a proposer-takes-all equilibrium when agents have a linear period utility function and to a compromise state when the period utility function is a specific CRRA utility function. In their laboratory experiment they find "significant evidence of concave utility functions".

The rest of the paper proceeds as follows. In the next section I set up the framework and model. The main result on the achievability of a fully-absorbing equal division equilibrium is

[5]Alternatively, "bridges to nowhere" can serve efficiency purposes.

presented in the third section along with extensions regarding changes to voting thresholds and proposer persistence. The final section concludes. Proofs and calculations in general are relegated to the appendix.

2. The Model

2.1. Framework

There are N agents, who I refer to as "legislators." These legislators bargain in a discrete-time infinitely-repeated game, where in each period, there is a new budget of size 1 to be divided. At the beginning of each period, nature selects a legislator at random to make a proposal. The proposer makes a proposal on how to share the budget, x^t so that $\sum_{i=1}^{N} x_i^t \leq 1$, where x_i^t represents the proposed allocation for legislator i in period t. Denote the space of possible proposals as $\Delta^N = \{(x_1, \ldots, x_N) : \sum_{i=1}^{N} x_i \leq 1\}$. As mentioned earlier, this inequality constraint is also found in Anesi and Seidmann (2012) and provides the main difference to Bowen and Zahran (2012) who require equality.

In addition, in every period t, there is a status quo proposal s^{t-1}. Each legislator votes on whether to accept or reject the proposal. If more than a voting threshold q, vote accept, then the proposal x^t is enacted and becomes the new status quo, $s^t := x^t$. If less than q vote accept, then the proposal is rejected and the status quo proposal s^{t-1} is implemented and remains the status quo, $s^t := s^{t-1}$. Notice that s^t is both the status quo for period $t+1$ and the realized outcome for period t. Let h^t be composed of all proposals, proposer selections, and voting histories in rounds $1, \ldots, t-1$.

There are two decisions to be made in each round, proposing and voting. Proposal strategies may theoretically depend upon play in an all previous rounds whereas acceptance strategies may also condition on the identity of the proposer and the new proposal. Thus, legislators' strategies consist of two functions:

- $\sigma_i^t(h^t)$ - the proposal legislator i would make in period t if he happens to be the proposer conditional on the entire history of h^t of proposals and voting histories, and
- $\alpha_i^t(h^t, p^t, x^t)$ - the accept/reject strategy of legislator i in period t where x^t is the proposal made and p^t is the identity of the proposer.

Concavity: Legislators have a strictly concave, strictly increasing, period utility function u which is normalized so that $u(0) = 0,\ u(1) = 1$. Concavity of the legislators' utility function creates an incentive for compromise as the allocation $(1/N, \ldots, 1/N)$ maximizes the sum of the legislators' utility in any given period.

Utility for a sequence of allocations is geometrically discounted with discount factor δ, so in period t, a legislator's evaluation of a deterministic sequence of outcomes $y^t,\ y^{t+1},\ \ldots$ is

$$U_i^t(y^t,\ y^{t+1},\ \ldots) = \sum_{k=0}^{\infty} \delta^k u(y_i^{t+k}).$$

However, due to the stochastic nature of proposer selection, and possibly due to mixed strategies employed by other legislators, a legislator attaches continuation value V_i^t to the game after history h^t and continuation value v_i^t to the game after history h^t, p^t, x^t:

$$v_i^t(h^t, p^t, x^t, \sigma, \alpha) = \mathbb{E}\left[\sum_{k=0}^{\infty} \delta^k u(s_i^{t+k}) : h^t, p^t, x^t, \sigma, \alpha\right]$$

$$V_i^t(h^t, \sigma, \alpha) = \mathbb{E}\left[v_i^t(h^t, p^t, x^t, \sigma, \alpha) : h^t, \sigma, \alpha\right]$$

Both expectations are taken over nature choosing the proposer and the (perhaps) random plays of other legislators. I use the above notation because in every period, there are two actions to be taken.

Notation: I will denote the game just described as $G(u, \delta, q)$.

The equilibrium notion that I employ is Markov Perfect Equilibrium with pivotal voting and tie-breaking towards acceptance. The pivotal voting requirement avoids many unrealistic outcomes, such as all legislators accepting a proposal that none of them prefer. Each legislator votes "Accept" for all proposals which weakly improve themselves and rejects all other proposals. Note that when legislators are voting, they are taking into account future outcomes and thus consider the expected discounted value of all possible future payoffs when voting to accept or reject a budget proposal.

Definition: A Markov Perfect Equilibrium (MPE) with pivotal voting is a vector of proposal strategies $(\sigma_1, \ldots, \sigma_n)$ and acceptance strategies $(\alpha_1, \ldots, \alpha_n)$ such that there exists a vector of strategies $(\hat{\sigma_1}, \ldots, \hat{\sigma_n})$, $(\hat{\alpha}_1, \ldots, \hat{\alpha}_N)$, where $\hat{\sigma_i} : \Delta^N \to \Delta^N$, $\hat{\alpha}_i : \Delta^N \times \Delta^N \to \{A, R\}$ and

1. For any history h^t with status quo s^t, it is the case that $\sigma_i(h^t) = \hat{\sigma}_i(s^t)$.
2. For any history coupled with a selected proposer and proposal (h^t, p^t, x^t), it is the case that $\alpha_i(h^t, p^t, x^t) = \hat{\alpha}_i(s^t, x^t)$.
3. $\sigma_i(h^t) \in \text{argmax}_x v_i^t(h^t, i, x, \sigma, \alpha)$.
4. $\alpha_i(h^t, p^t, x^t) = \begin{cases} \mathrm{A} & \text{if } \mathrm{u(x_i^t) + \delta V_i(x^t, \sigma, \alpha) \geq u(s_i^{t-1}) + \delta V_i(s^{t-1}, \sigma, \alpha)} \\ \mathrm{R} & \text{if } \mathrm{u(x_i^t) + \delta V_i(x^t, \sigma, \alpha) < u(s_i^{t-1}) + \delta V_i(s^{t-1}, \sigma, \alpha)}. \end{cases}$

The first two conditions above state the Markov properties. Specifically, i) planned proposals depend only upon the status quo; and ii) accept/reject decisions depend only upon the status quo proposal s^t and the new proposal x^t. The last two conditions specify that a legislator is playing an equilibrium - specifically, he maximizes his discounted utility through his proposal choices.

Also, note that I changed the inputs for the continuation value function V above by omitting full history dependence and time superscripts due to the Markov assumptions of parts (1) and (2). While I did not, it would have been legitimate to do so for the continuation value function v as well.

At first glance, it is unclear whether allowing part of the dollar to be unallocated should make compromise easier or harder. On the one hand, legislators now have access to more proposals, and there are now more paths to the equal division allocation. On the other hand, equilibria may be harder to sustain because a proposer now has more possible proposals to deviate to. I find that by allowing for part of the budget to be unallocated, the compromise allocation $(1/N, \ldots, 1/N)$ is supportable as a fully-absorbing state of a MPE.

3. Fully-Absorbing Compromise

3.1. Construction

In this section, I will present the main equilibrium construction. Specifically a MPE that supports the equal division allocation as a fully-absorbing state. Because the equilibrium will be Markov Perfect, it will be enough to specify legislator i's proposal strategy σ_i for every possible status quo and his acceptance strategy by α_i for every possible status quo paired with the current proposal.

Definition: The budget proposal b is a *fully-absorbing* state of the equilibrium $\sigma^*,\ \alpha^*$ if for any initial status quo allocation s, it is the case that $\lim_{t\to\infty} Pr\left(s^t = b|s^0\right) = 1$, where s^t is a random variable denoting the status quo allocations in round t whose probability depends upon the possible sequences of nature's proposer selection in rounds $1, \ldots, t-1$.

Recall that in the model, the status quo allocation in round t is equal to the implemented allocation in round $t-1$. Therefore, while the above condition requires the convergence of status quos, this is equivalent to requiring convergence of the implemented allocations. In addition, note that the above definition of convergence is strong in two respects: i). it is stronger than convergence in probability, because it requires the allocation to eventually equal the absorbing allocation, rather than approach it arbitrarily closely as $t \to \infty$, and ii). convergence occurs from any initial status quo allocation. Thus, I call the limit allocation, a "fully-absorbing state."

Theorem 1. *For any $q \le N-1$, u, there exists a symmetric MPE $\sigma^*,\ \alpha^*$ and a $\delta^* < 1$ so that the proposal $(1/N, ..., 1/N)$ is a fully-absorbing state of the game $G(u, \delta, q)$.*

Proof. See Appendix. ■

While the proof is provided in the appendix, I here provide its construction. First, I define proposal strategies by partitioning all of the possible status quos (denoted by ω) into three cases depending on how many legislators merit punishment. In the following, let j denote the proposer and $(\sigma_j(\omega))_i$ denote legislator j's proposal for legislator i's allocation when the status quo is ω. Denote by $D = \{i : \omega_i > \frac{1}{N}\}$, the set of legislators who are receiving a greater-than-equal share of the budget - the set of deviators.

Case 1: $\forall i,\ \omega_i \le \frac{1}{N}$. Then, $\forall i$

$$(\sigma_j(\omega))_i = \begin{cases} \frac{1}{N} & i = j \\ \omega_i & i \neq j \end{cases} \qquad \alpha_i(\omega, \sigma_j(\omega)) = \mathrm{A}$$

Case 2: There is exactly one legislator to be punished, that is $D = \{k\}$. Then, $\forall i$

$$(\sigma_j(\omega))_i = \begin{cases} \frac{1}{N} & i = j \\ 0 & i = k, j \neq k \\ \omega_i & \text{otherwise} \end{cases} \qquad \alpha_i(\omega, \sigma_j(\omega)) = \begin{cases} \mathrm{A} & i \neq k \text{ or } i = j \\ \mathrm{R} & i = k, i \neq j \end{cases}$$

Case 3: $|D| > 1$. Let $k \neq j$ be an legislator randomly selected from D with probability $\frac{w_i - 1/N}{1-2/N}$. Notice that the sum of punishment probabilities may be strictly less than 1, so it is possible no legislator will be punished. In this case, one may consider $k = 0$ in the following definition.

$$(\sigma_j(\omega))_i = \begin{cases} \frac{1}{N} & i = j \text{ or } i \in D\backslash\{k\} \\ 0 & i = k \\ \omega_i & \text{otherwise} \end{cases} \qquad \forall i,\ \alpha_j(\omega, \sigma_j(\omega)) = \begin{cases} \text{A} & i \neq k \\ \text{R} & i = k \end{cases}$$

Note that the acceptance strategy has not yet been completely specified because it has not been defined for off-equilibrium proposals. However, in the studied setting, an acceptance strategy can be simply extended to cover off-equilibrium proposals by requiring that legislators accept a proposal if it is weakly in their best interest and not otherwise. This is simply pivotal voting with tie-breaking towards acceptance. In general, in a dynamic game, there is a concern that this type of extension is not well-defined because legislators' utility depends upon future undefined acceptances. However, this does not prove to be a concern here because on-path acceptance strategies have all been explicitly defined and if a legislator deviates, given that strategies are Markovian, all future periods are still on-path. Thus, the off-path accept/reject decisions need only take into account future proposals (which are all necessarily on-path) and on-path accept/reject strategies. Therefore, the above extension of accept/reject strategies is not circular and well-defined.

The above equilibrium construction can be intuitively understood as follows. Any legislator, when she proposes, gives herself the compromise share of the budget $1/N$, even if she is receiving a greater then $1/N$ share in the status quo allocation. If a single legislator is receiving more than a $1/N$ share of the budget (either because of a previous deviation, or because they were just born with a "bad" status quo allocation), then she punishes that legislator. Agreement is secured as all other legislators are willing to consent to this punishment.

However, there can also be situations where a number of legislators are to be punished. The proposer may not be able to punish them all simultaneously (depending upon the voting threshold and the number of deviators) because each legislator would disagree with being punished. Thus, acceptance of such a proposal could not be secured. The above construction sidesteps this problem by having the proposer choose a single legislator to punish. All other deviating legislators receive a reprieve and are brought to the compromise allocation, thereby securing their votes. Importantly, the legislator takes into account how much above the compromise allocation a deviating legislator in determining the probability with which that legislator is to be punished. This maintains incentives because the more stolen, the greater the likelihood of punishment.

In the figure above, I consider three different possible allocations. In each of them, legislator 1 is the proposer. In the first allocation, no legislator merits punishment, so the resulting proposal is $(1/3, 1/4, 1/4)$. In the second allocation, only legislator 2 merits punishment, and thus the resulting proposal is $(1/3, 0, 1/6)$. Finally, in the third scenario, two legislators warrant punishment, so the proposer punishes each with a 50% probability by randomizing between the two proposals 3a (when legislator 2 is to be punished) and 3b (when legislator 3 is to be punished). For all proposals, any unpunished legislator along with the proposer will accept the proposal.

Another issue that arises is that it may seem curious that proposers bring only themselves to the compromise allocation and do not do so for other legislators who are receiving smaller shares of the budget. Doing so would cause the legislators to reach the equal division

Legislator	1	2	3
Status Quo # 1	1/4	1/4	1/4
Proposal # 1	1/3	1/4	1/4
Status Quo # 2	1/4	1/2	1/6
Proposal # 2	1/3	0	1/6
Status Quo # 3	0	1/2	1/2
Proposal # 3a	1/3	0	1/3
Proposal # 3b	1/3	1/3	0

Figure 1: Examples of Status Quos and Proposals by Legislator #1

allocation faster and should make it easier to garner votes. Moreover, such a proposal is feasible and would waste less of the budget along the way. However, the problem is that this would short circuit others legislators' punishments. Punishments need to be severe enough to deter deviation, but the worst punishment in a single period is the allocation of 0. Thus, punishments must achieve their strength due to the length of time that they are imposed. This length is stochastic, but in a strategy where each individual j is the only one to "fix" her own allocation, the resultant punishments are of sufficient duration (in expectation) to deter deviations.

As mentioned earlier, one advantage of non-exhaustive allocations is that it becomes possible to separate "punishing" and "stealing". While these non-exhaustive budgets are not used in the steady state, there naturally raises a question of how costly is any waste that may occur from following a punishment regime. As it turns out, the constructed equilibrium uses a punishment scheme that is independent of the discount factor. An upper bound for the amount of punishment incurred can then be calculated as $(N-1)u(1/N)$. Thus, expressed as a fraction of agents' total discounted utility, the amount of waste actually incurred in a particular punishment is bounded above by $(N-1)(1-\delta)$ which is vanishingly small (as $\delta \to 1$).

Finally, in the above theorem, the equilibrium is constructed not just for majoritarian voting systems, but for any voting threshold short of unanimity. This is a natural extension to consider as, for example, in the Senate, 60 votes out of 100 are required to pass a bill over a filibuster. A typical motivation given for this type of restriction is to protect a minority from the "tyranny of the majority" Tocqueville (1839). Additionally, the analysis performed here extends to the case where voting thresholds are less than 50%. One prominent example of such a setting is the case where the proposer is a dictator. In this case, the voting threshold is just one vote, and the proposer can implement any outcome by proposing it and then accepting it. Another example where there may be an implicit voting threshold of less than 50% is when votes are taken by roll calls and herding may occur. This could enable a minority of legislators to pass legislation.

Comment: If $q = N$, then equal division is not sustainable as a fully-absorbing state. To see this, consider a case where the initial status quo is fully allocative and unequal. In any equilibrium, each legislator must receive at least the value of them continuing to have their initial allocation forever. The only possible stream of allocations that offers this is the one where initial status quo allocation is implemented in every period. Thus, every fully allocative status quo is a steady state.

3.2. Persistence in Proposer Power

Here, I change the studied setting to allow for persistence (or lack of persistence) in proposal power. Specifically, there is a γ chance that the proposer does not change. Each legislator who is not the proposer has an equal $\frac{1-\gamma}{N-1}$ chance of becoming the proposer in the next period. Note that the case where $\gamma = 0$ is also admitted – which captures the situation where proposal power never persists. Denote the augmented game as $G(u, \delta, q, \gamma)$.

Theorem 2. *For any $\gamma < 1$, $q \leq N-1$, u, there exists a symmetric MPE σ^*, α^* and a $\delta^* < 1$ so that the proposal $(1/N, ..., 1/N)$ is a fully-absorbing state of the game $G(u, \delta, q, \gamma)$.*

Comment: As full unanimity rules out equal division, so does full persistence, that is $\gamma = 1$. For an any initial status quo, s^0, if the proposer has an allocation greater than $1/N$, then he will never settle on a sequence in which he eventually receives $1/N$ because at any consumption level, he can guarantee himself that consumption level forever. Thus, the proposal $(1/N, \ldots, 1/N)$ is not sustainable as a fully-absorbing state in any MPE.

4. Conclusion

In this note, I have shown that equal division of a budget is supportable as a fully-absorbing steady state in an MPE. This result is appealing because equitable divisions are often observed in legislative settings and because of the simplicity of Markovian strategies. Other papers have shown the existence of different possible steady states, but not the case of fully-absorbing equal division.

To get the above result, I allowed budget proposals to be interior (i.e., non-exhausting). This enabled legislators to distinguish "punishment" and "stealing". In a working paper version of this note, I provide an example that shows equal division is not sustainable as a fully-absorbing state in a pure strategy MPE when $N = 2$, $q = 1$, and $\gamma = 0$ and budgets are required to be exhaustive for any concave utility function[6] As for this legislative model with dynamic status quos, there remain interesting open questions. One such question would be to characterize the full set of MPE (or pure strategy MPE) in the above setting.

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[6] See https://sites.google.com/site/richtereconomics/DynamicCompromise.pdf.

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Appendix

Proof of Theorem 1

Consider a legislator j who is making a proposal. For the moment, assume that the proposal is accepted. She can propose according to σ or she can make an alternate proposal $\{b_i\}_{i=1}^N$. For her to strictly prefer the proposal $\{b_i\}_{i=1}^N$ she must be getting above $\frac{1}{N}$ in at least one period, as equilibrium strategies deliver $\frac{1}{N}$ in every period for the proposer. But any proposal that has $b_j \leq \frac{1}{N}$ delivers less than or equal to $\frac{1}{N}$ in every period in equilibrium. Therefore for there to be any possible strict improvement, it must be that $b_j > \frac{1}{N}$.

Now, suppose that she makes such a proposal that will result in Case 2 in the next period. By the one-shot deviation principle, for the proposal to be strictly improving, it must be that[7]

$$u(b_j) + \sum_{t=1}^{\infty} \delta^t \left(\left(\frac{N-1}{N} \right)^t u(0) + \left(1 - \left(\frac{N-1}{N} \right)^t \right) u\left(\frac{1}{N} \right) \right) > u\left(\frac{1}{N} \right) + \sum_{t=1}^{\infty} \delta^t u\left(\frac{1}{N} \right), \tag{1}$$

$$\text{and thus } \left(\mathrm{u}(\mathrm{b_j}) - \mathrm{u}\left(\frac{1}{\mathrm{N}} \right) \right) > \sum_{\mathrm{t}=1}^{\infty} \left(\frac{\mathrm{N}-1}{\mathrm{N}} \right)^{\mathrm{t}} \delta^{\mathrm{t}} \left(\mathrm{u}\left(\frac{1}{\mathrm{N}} \right) - \mathrm{u}(0) \right). \tag{2}$$

Notice that the left hand side of (2) represents the proposer's immediate gains from deviating and the right hand side her different continuation payoffs. The deviating legislator will be punished until he again has proposal power, and thus has a $\left(\frac{N-1}{N}\right)^t$ probability of being punished in round t. Simplifying yields

$$u(b_j) - u\left(\frac{1}{N} \right) > \frac{\delta \left(\frac{N-1}{N} \right) \left(u\left(\frac{1}{N} \right) - u(0) \right)}{1 - \delta \left(\frac{N-1}{N} \right)}. \tag{3}$$

Rearranging and bounding b_j above by 1 yields

$$\frac{u(1) - u\left(\frac{1}{N} \right)}{u\left(\frac{1}{N} \right) - u(0)} > \frac{\delta(N-1)}{N - (N-1)\delta}. \tag{4}$$

By strict concavity of u

$$N - 1 > \frac{u(1) - u\left(\frac{1}{N} \right)}{u\left(\frac{1}{N} \right) - u(0)} > \frac{\delta(N-1)}{N - (N-1)\delta}. \tag{5}$$

As $\delta \to 1$, the above equation becomes

$$N - 1 > \frac{u(1) - u\left(\frac{1}{N} \right)}{u\left(\frac{1}{N} \right) - u(0)} \geq N - 1. \tag{6}$$

[7] To check the existence of a sufficient lower bound, the overtaking criterion of Rubinstein (1979) also suffices, but the calculations are less direct.

This contradiction, along with monotonicity in δ of the RHS in (5), gives that $\exists\bar{\delta}$ such that $\forall\delta \in [\bar{\delta}, 1]$ it is the case that

$$\frac{u(1) - u\left(\frac{1}{N}\right)}{u\left(\frac{1}{N}\right) - u(0)} \leq \frac{\delta(N-1)}{N-(N-1)\delta}. \tag{7}$$

So, for high enough δ, a proposer would not like to deviate to a Case 2 proposal. As for an explicit bound, if I let $\psi_n \equiv \frac{u(1)-u\left(\frac{1}{N}\right)}{u\left(\frac{1}{N}\right)-u(0)}$, then a lower bound on δ is given by

$$\frac{\psi_n N}{(N-1)(1+\psi_n)} = \bar{\delta}, \tag{8}$$

or alternatively

$$1 - \frac{N-1-\psi_n}{(N-1)(1+\psi_n)} = \bar{\delta} < 1 \tag{9}$$

where $0 < \psi_n < N-1$. This lower bound is increasing in ψ_n, and for any fixed N, as ψ_n converges to 0 (or $N-1$ respectively), it is the case that $\bar{\delta}$ converges to 0 (or 1 respectively).

Finally, it needs to be checked that a legislator j does not wish to deviate to a Case 3 proposal. In this case, she would deviate only if there is a b so that

$$\left(u(b_j) - u\left(\frac{1}{N}\right)\right) + \frac{b_j - 1/N}{1-2/N}\sum_{t=1}^{\infty}\left(\frac{N-1}{N}\right)^t \delta^t \left(u(0) - u\left(\frac{1}{N}\right)\right) > 0. \tag{10}$$

Rearrange as before to get

$$\frac{b_j - 1/N}{1/N - 0} > \frac{u(b_j) - u\left(\frac{1}{N}\right)}{u\left(\frac{1}{N}\right) - u(0)} > \frac{b_j - 1/N}{1-2/N} \cdot \frac{\delta(N-1)}{N-\delta(N-1)}. \tag{11}$$

This reduces to

$$\frac{N(N-2)}{(N-1)^2} > \delta. \tag{12}$$

Importantly, the LHS is less than 1. Thus, if

$$\delta^* = \max\left(\frac{N(N-2)}{(N-1)^2}, \frac{\psi_n N}{(N-1)(1+\psi_n)}\right), \tag{13}$$

then as long as $\delta \geq \delta^*$, it is the case that assuming that other legislators play their equilibrium proposal and acceptance strategies, σ is an optimal proposal strategy.

From the above argument, one can see that whenever a legislator is proposed to receive $1/N$, it is weakly in his best interest to accept, because that is optimal for him to propose if he could dictate the outcome.

As for acceptance strategies, in Case 1, it is the case that all legislators per-period payoffs are unaffected except for the proposer's when comparing ω and $\sigma_j(\omega)$. Therefore acceptance by every legislator is equilibrium behavior.

In Case 2, the punished legislator has a worse flow of payoffs (specifically, he receive 0 now and the same payoffs thereafter) and rejects the proposal. The proposer is weakly better off (obtaining the best possible allocation conditional on δ above) and all other legislators are indifferent, hence their acceptance is equilibrium behavior. So, for any voting threshold, proposals of this type are accepted.

In Case 3, the legislator randomly selected to be punished has worse per-period payoffs (they receive 0 now and until they propose, this is the worst possible allocation) and hence rejects the proposal. All other legislators are weakly better off receiving either the best possible allocation $1/N$ or being unaffected and thus accept. So, again, for any voting threshold, proposals of this type are accepted.

As for pairs of proposals where the proposal x^t is not an equilibrium proposal, I let legislators accept according to whichever alternative is best for them, and therefore, this portion of the acceptance strategy trivially satisfies the equilibrium definition. Note this is not important for establishing the Nash equilibrium because even a dictator has no strict incentive to deviate in his proposals.

Finally, note that in the above equilibrium, once each legislator has been the proposer, the equilibrium allocation will be $(1/N, 1/N, \ldots, 1/N)$. The probability that this has not happened by period t is bounded above by $N \cdot \left(\frac{N-1}{N}\right)^t \to 0$ as $t \to \infty$. ■

Proof of Proposition 2:

First, notice as before that there are no profitable deviations for a proposer to Case 1 proposals. In the case where the persistence parameter is γ, Eq. (5) becomes

$$N - 1 > \frac{u(b_j) - u(1/N)}{u(1/N)} \geq \frac{\delta(1-\gamma)(N-1)}{N - 1 - \delta(N - 2 + \gamma)} \tag{14}$$

and the lower bound on the RHS converges to N-1 as $\delta \to 1$. Moreover, the right hand side is monotonic in δ. Thus, there are no profitable deviations to Case 2 proposals when $\delta \geq \bar{\delta}$ where

$$\bar{\delta} = \frac{\psi_N(N-1)}{(1-\gamma)(N-1) + \psi_N(N-2+\gamma)} < 1. \tag{15}$$

Finally, to prevent Case 3 deviations, equation 11 becomes:

$$\frac{b_j - 1/N}{1/N - 0} > \frac{b_j - 1/N}{1 - 2/N} \cdot \frac{(1-\gamma)\delta(N-1)}{N - 1 - \delta(\gamma(N-1) + (1-\gamma)(N-2))}. \tag{16}$$

Algebraic manipulation yields:

$$(N-1)(N-2) > \delta((N-1)(N-2) + (1-\gamma)) \tag{17}$$

.

Thus, a sufficient lower bound for delta is now

$$\delta^* = \max\left(\frac{(N-1)(N-2)}{(N-1)(N-2) + (1-\gamma)}, \frac{\psi_N(N-1)}{(1-\gamma)(N-1) + \psi_N(N-2+\gamma)}\right). \tag{18}$$

Notice that if $\gamma = 1/N$, then proposer recognition probabilities are exactly as in the standard case and the left lower bound on δ reduces to the familiar $(N)(N-2)/(N-1)^2$.

Finally, as before, acceptance strategies are that legislators accept if they weakly prefer the proposal to the status quo (taking future payoffs into account). ■