Family-Specific Investments and Divorce with Dynamically Inconsistent Households: Marital Contracts and Policy☆

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Abstract

This paper bridges two distinct areas of inquiry: the economic theory of the family and behavioral research on time-inconsistent preferences. Motivated by anecdotal and survey evidence suggesting that present bias plays a role in intra-household decision making, we propose a model in which hyperbolic discounting couples engage in home production activities, thereby accumulating family-specific capital over time. At any given point in time, the gains to continued marriage depend on the accumulated stock of this capital and a temporary random shock to match quality. Couples whose match quality deteriorates may choose to divorce, and this is more likely to happen if past investments in family-specific capital have been low. We obtain three main sets of results. First, present-biased preferences induce couples to underinvest in family-specific capital and to "overdivorce". Second, sophisticated couples but not naive ones may choose to enter marriage on terms which make divorce more costly to obtain. Third, the inefficiencies in the behavior of time-inconsistent couples can be completely undone by means of earnings and divorce taxes that vary over the marital life-cycle.

Keywords: intra-household decision-making; dynamic inconsistency

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"*Of course there are cases where divorce is inevitable. I havent sat in the courts for 40 years without knowing that there are cases where it is just as well the parties separated. But it has been obvious to me that, by and large, a significant proportion of people who separate wish they had not five years down the line.*"—Sir Paul Coleridge (Former High Court Judge, 9 February 2016)

1. Introduction

With Gary Becker's path-breaking *Treatise on the Family*, scholars started to use economic theory to describe a broad variety of intrahousehold decision problems. A common thread in the literature that has developed since then is the argument that the basic dynamics of family life can be explained within a framework in which family members behave fully rationally over time. Put differently, it has been routinely assumed that decisions about marriage and divorce, household production, labor supply, childbearing and childrearing are made by perfectly foresighted and time-consistent partners.

In a parallel but unrelated development, the last decades have also witnessed the emergence of behavioral research in economics showing that instant gratification overpowers long-term considerations when individuals make decisions in a dynamic setting (e.g., Laibson, 1997; O'Donoghue and Rabin, 1999a). Economists have coined the notion of *time inconsistency* to describe this phenomenon. Individuals with time-inconsistent preferences discount the future hyperbolically rather than exponentially and therefore make their decisions with a present bias. Thus, they are unable to consistently follow a utility-maximizing plan over their lifetime. Today it is well understood how time-inconsistent preferences affect consumption decisions (e.g., Laibson, 1998; Angeletos et al., 2001), saving for retirement (Diamond and Kőszegi, 2003), health behavior (e.g., Gruber and Kőszegi, 2004) or performance on work tasks (e.g., Herweg and Mller, 2011), and the policy implications that come with it have received considerable attention.[1]

Our paper argues that family decisions can also be expected to be affected by cognitive biases, including present bias. This is particularly salient in the context of divorce. At a first casual level, evidence clearly suggest a cognitive bias in relation to expectations about marital breakups: as noted by Thaler and Sunstein (2008), there is almost universal over-optimism among newly-wed couples about their marriage survival chances.[2] More directly related to the incidence of marital breakups, there is evidence that divorce is associated with

[1] In particular, a lot of thought has gone into understanding the implications of time-inconsistent preferences for social security schemes (İmrohoroğlu *et al.*, 2003; Schwarz and Sheshinski, 2007), the optimal design of sin taxes (O'Donoghue and Rabin, 2003, 2006; Gruber and Kőszegi, 2004), or uniform saving floors (Malin, 2008).

[2] See also Baker and Emery, 1993, for evidence showing that couples are overly optimistic regarding the survival chances of their own marriage, despite correctly assessing the likelihood of divorce in the population at large.

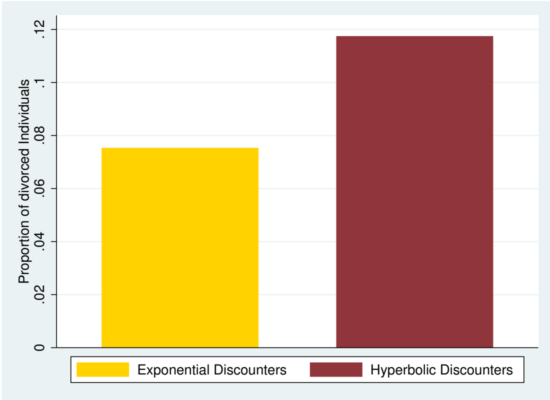


Figure 1: *Proportion of divorcees among exponential and hyperbolic discounters. The difference between the two means is statistically significant at the 5% level.*

personality traits that are predictive of short-sighted and impulsive behavior. Lundberg (2011) in particular notes that the incidence of divorce is related to low conscientiousness and neuroticism, which have also been shown to be negatively associated with investments in savings and health. Direct evidence linking divorce to measures of present-biasedness is, however, harder to come by. In Figure 1, we present some exploratory evidence based on the 2000 Bank of Italy Survey of Household Income and Wealth (SHIW), which allows us to link a measure of present-bias—elicited using an experimental matching task—to individuals' marital status. The evidence suggests a strong correlation between present-biasedness and divorce: among exponential discounters, the proportion of divorcees is 7.5%; in contrast, the share of divorcees among hyperbolic discounters is 56% higher and amounts to 11.7%.[3]

Arguing more indirectly, the notion that family decisions are made with a present bias is consistent with stylized facts that cannot be easily reconciled with standard models. First, social scientists and practitioners have long emphasized that a substantial fraction of divorced people express regrets about their choice to separate, even many years later. For example, US surveys conducted in New Jersey and Minnesota suggest that between 40 and 46 percent of currently divorced people have at least some regrets about their divorce (Waite *et al.*, 2002), and studies from the UK and Australia report similar figures (Wolcott and Hughes, 1999; Medved, 2017).[4] Second, in various cultures and religions

[3]In Appendix A, we provide details on the underlying data and the classification of survey respondents as exponential and hyperbolic discounters, respectively. There, we also further explore the results in Figure 1 in a simple regression framework and discuss additional survey evidence.

[4]Of course, this evidence has to be treated with some caution, as divorced couples may romanticize the time of marriage and may undervalue or ignore the problems the marriage has had. That said, using a sample of couples from the National Survey of Families and Households

around the world, it is common to observe marriage contracts that include clauses preventing unrestricted divorces (e.g., the Islamic "mehr", the Jewish "ketubah", or "covenant" marriages). As of yet, mainstream family economics has not dealt in any systematic way with the possibility that such contracts—by making it difficult to obtain a divorce—may offer a useful commitment device for couples. This seems important not least because much of the western world has recently seen a development in the opposite direction: the introduction of no-fault divorce substantially facilitated obtaining a divorce.

One of our contributions is to show how one can systematically think about these issues by looking at family decisions through the lens of a model of dynamically inconsistent household behavior. Moreover, we argue that this perspective generates a novel efficiency rational for family-related policy interventions, especially regarding measures regulating marriage and divorce. Indeed, although it has been recognized that a household can exhibit dynamically inefficient behavior when the decisions that family members take influence the household's balance of power with a time lag (e.g., Konrad and Lommerud, 2000; Lundberg and Pollak, 2003; Basu, 2006; Rasul, 2006; Mazzocco, 2007), the policy implications of behavioral distortions arising from present bias have not yet been explored.

At a general level, we envisage a setting in which hyperbolic-discounting couples engage in household production activities, thereby accumulating family-specific capital over time. The example of family-specific capital we have in mind are children, and so our notion of household production centers around the repeated choice how to allocate time between parental investments in children and working in the labour market. Couples are periodically exposed to a shock to match quality. Couples whose match quality deteriorates may choose to divorce, and this is more likely to happen if past investments in family-specific capital have been low. We view children as pure household public goods when parents are married. However, once a couple divorce, their children become an impure public good in that one or both parents will no longer be able to fully enjoy the value of the time investments. Married spouses make their resource allocation choices cooperatively whereas once divorced they act non-cooperatively. Eventually the children leave the parental home and the couple's investments in family-specific capital cease.

In our main model, married partners jointly have time-inconsistent preferences and act like an entity. We make this "unitary" simplification in order to highlight the impact of present-bias on the interplay between family-specific investments and marital stability in the cleanest possible fashion. In particular, the forces we highlight would also show up in alternative model specifica-

who were all self-reported unhappily married and shared comparable demographic characteristics, Waite et al. show that those who divorced or separated achieved lower subsequent psychological well-being—as measured by higher rates of depressive symptoms and alcohol use—than those who did not. If the within-marriage level of (un)happiness was homogenous within the sample, this would be inconsistent with divorce decisions being Pareto efficient and would be consistent with frequent feelings of divorce-regret.

tions, but most conceivable alternative approaches would give rise to additional sources of inefficiencies that we want to abstract away from. Examples include free-riding or "commons" problems in non-cooperative household models (Konrad and Lommerud, 1995; Hertzberg, 2016), limited commitment problems (Lommerud, 1989; Lundberg and Pollak, 2003; Mazzocco, 2007), and preference heterogeneity (Jackson and Yariv, 2015).[5] As a robustness check, we provide a brief analysis of non-cooperative behaviour in Section 8, where we highlight which results rely on the "unitary" assumptions adopted in the main model.

Our main theoretical results are derived from a stylized three-period model. In the first period, married couples decide how to allocate their time between household production and working in the labor market. At the beginning of the second period, each couple is exposed to a match quality shock and decide whether to continue marriage or to divorce. They then once again allocate their time between labor market activities and home production. In the third period, no household production takes place, but individuals still benefit from the stock of previously accumulated family-specific capital. In this setting, we compare the laissez-faire allocation with the first-best choices. In characterizing the first-best, we follow O'Donoghue and Rabin (2006) and treat individuals' preference for short-term gratification as an error. Several important insights follow from this comparison; here we mention three. First, a present-biased couple terminate their marriage for a larger set of match quality realizations than would be optimal (i.e., they "overdivorce"). Why is that the case? Present-biasedness implies that they overreact to negative temporary match quality shocks as they place too little weight on the value of continued marriage in terms of its positive impact on future investments in and enjoyment of family-specific capital. Second, time-inconsistent preferences induce a procrastination problem in which couples *ex-ante* underinvest in family-specific capital, a result that holds for both sophisticated and naive couples. Third, sophisticated married couples on the one hand recognize their tendency to over-divorce in the future and that this distortion can be mitigated by increasing their current investments; this gives them a strategic investment incentive. Naive married couples on the other hand hold overly optimistic beliefs about their future marriage survival chances and hence also about their future enjoyment of any current investments; this over-optimism bias mitigates their underinvestment. If the over-optimism bias of naive couples dominates the incentive effect of sophisticated couples, then the former will invest more in family-specific capital than they latter. This provides an analogy to the findings of Herweg and Mller (2011), who show that being aware of one's own self-control problems may reduce a persons performance on a task.

We further show that sophisticated couples – but not naive ones – would

[5] Besides being a simplification, our assumption of homogeneity of time preferences between spouses is not necessarily unrealistic. For example, a recent empirical study by Arrondel and Frémeaux (2016) shows that spouses are, due to marital sorting, very similar in their savings preferences, even when individual characteristics are controlled for.

like to have a commitment device preventing them from obtaining a divorce "too easily". Various cultures and religions around the world do offer versions of such commitment devices. In the US, the so-called "covenant" marriage has been a legally distinct kind of marriage in Louisiana, Arkansas and Arizona since the 1990s. The marrying spouses agree to sign a statement declaring that a covenant marriage is for life. They voluntarily choose to limit the grounds for divorce to adultery, felony and abuse (Spaht, 1998). If one of the spouses desires a divorce, the couple is first required to attend marital counseling. Thus, divorce becomes a costly option. Muslim marriages also feature a commitment device in the form of the so-called "mehr". A mehr is composed of two parts: the "muqaddam" which is paid by the groom upon marriage to honor his bride, and the deferred "mu'akhkhar" component which is paid to the woman in the event of divorce (Blenkhorn, 2002). The latter makes divorce an expensive endeavor for husbands in particular. A very similar role is played the so-called "ketubah" in the Jewish religion (Hardin, 1988). While standard models of family decision-making have difficulties in rationalizing why these devices exist, we show that sophisticated couples may have an incentive to *ex-ante* opt for such contracts. In addition, we establish that this incentive is particularly pronounced in environments in which divorce rate is naturally low.[6]

The final question we address is: what interventions are necessary to implement the first-best choices? We show that a suitably designed policy can completely undo the inefficiencies in the behavior of time-inconsistent couples by means of earnings and divorce taxes that vary over the marital life-cycle. In our setting, the earnings tax implicitly subsidizes household production; in each period, it corrects for the share of the future returns of family-specific investments not internalized by present-biased couples. The divorce tax corrects for the fact that couples overreact to negative match-quality shocks. In order to fully understand the properties of the efficiency-restoring policy scheme, we further report results from a generalized T-period version of the model, calibrated to the US economy. This exercise reveals that even a modest degree of present-bias has a quantitatively important impact on divorce hazards. The efficiency-restoring divorce tax is an inverted-U function of marriage duration, reaches its maximum when children are in their teens, and declines thereafter. The effect of present bias on investments in family-specific capital turns out to be quantitatively smaller than its impact of divorce decisions, and the efficiency-restoring earnings tax is relatively flat with respect to marriage duration.

We proceed as follows. Section 2 presents our stylized three-period model. Section 3 analyzes sophisticated couples' family-specific investments and divorce decisions over time. Section 4 characterizes the first-best allocation and

[6] An alternative argument for why higher divorce costs may be efficiency-enhancing is provided by limited commitment within marriage to future compensating transfers. This has recently been explored by Gemici and Laufer (2014) who argue that higher divorce costs may encourage efficient specialization by married couples. However, based on counterfactual simulations of their estimated model, they conclude that "changing divorce costs does not change expected lifetime utility substantially" (p. 19).

describes the inefficiencies arising due to present bias. Section 5 characterizes privately optimal marriage contracts. How the first-best allocation can be decentralized through policy is described in Section 6. Section 7 contains concluding remarks. All technical proofs are contained in Appendix B.

2. A Stylized Three-Period Model

We consider a three-period model with time indexed by $t = 1, 2, 3$. The economy is populated by men and women, and at the outset of the model, at $t = 1$, each individual is exogenously matched with a partner of the opposite sex. Later on, at $t = 2$, a given couple may divorce and we then let $k = m, d$ indicate marital status. In every period, men and women are endowed with one unit of time which they can allocate between household production and labor market activity. Time spent in household production allows the couple to build up their family-specific capital. The example we have in mind is children, and so our notion of household production centers around parental time investments in their children.

2.1. Payoffs

Marriage. The utility of a married individual i in period t is given by

$$u_{it}^{m} = x_{it}^{m} + G_{t}^{m}, \tag{1}$$

where x_{it}^{m} is the individual's private consumption and G_t^m is the accumulated family capital good. We index the partners by $i = s, h$ (for "she" and "he" respectively). In the second period, the couple are further exposed to a match quality shock, denoted θ, which enters additively if they stay married. For simplicity we assume that both partners perceive the match quality shock in the same way. The shock is assumed to be temporary: it affects the payoffs in the second period but does not persist into the third period.[7] An adverse match quality shock may, as outlined below, trigger a divorce.

Divorce. A divorced individual's utility in period t is given by

$$u_{it}^{d} = x_{it}^{d} + \lambda_i G_{t}^{d}. \tag{2}$$

[7]The assumption that the partners perceive the match quality shock the same way could be relaxed to allow for individual shocks. A couple's divorce decision would then be based on the sum of the individual match quality shocks. The assumption that the match quality shock is transitory can also be relaxed to allow it to persist into the final period. With a pure temporary shock, a present-biased couple place too little weight on the future gain from continued married relative to the current temporary match quality; in contrast, with a fully persistent shock, they place a too high relative weight on the current cost of investment relative to the persisting match quality. The assumption of a pure transitory shock adopted here makes the results from the three period model directly comparable to the full dynamic model in Appendix C where it is assumed that a couple experience a temporary shock in every period.

Thus, the payoffs of divorcees differ from those of their married counterparts in two ways. First, the match quality shock θ becomes irrelevant. Second, partner i may no longer enjoy the family capital good at a rate of unity but rather at a rate $\lambda_i \in [0, 1]$. For notational convenience, we define $\Lambda \equiv \lambda_s + \lambda_h$ and assume that at least one of the two partners enjoys the family capital good at a lower rate after divorce, $\Lambda < 2$.[8]

Intertemporal Preferences. Following Laibson (1997), individuals' intertemporal preferences are characterized by (quasi-)hyperbolic discounting also known as (β, δ)-preferences. The advantage of this preference structure is that it nests the standard (exponential) discounting model as a special case. An individual's intertemporal preferences at time t are represented by

$$V_{it} = u_{it} + \beta \sum_{\tau=t+1}^{3} \delta^{\tau-1} u_{i\tau}, \tag{3}$$

where $\delta \in [0, 1]$ represents long-run time-consistent discounting, while $\beta \in [0, 1]$ is a *bias for the present* (O'Donoghue and Rabin, 1999b). The latter reflects whether the individual is an exponential discounter ($\beta = 1$) or a hyperbolic discounter ($\beta < 1$). Hyperbolic discounters have a higher discount rate over short horizons than over long horizons. Hence, there exists a conflict between current preferences and those in the future. In particular, for $\beta \in (0, 1)$ the discount factor between the second and third period is $\beta\delta$ when viewed from the second period, while it is δ when viewed from the first period. Thus, preferences are time inconsistent, and individuals face a self-control problem. Individuals can be either sophisticated or naive. The former foresee their time inconsistency and undertake steps to manage it while the latter do not perceive their self-control problem and wrongly expect themselves to behave time consistently in the future. In our analysis we will concentrate on sophisticated couples. At the end of Section 4, we will discuss the behavior of naive couples. To simplify notation, we follow the vast majority of the literature and set $\delta = 1$ in the three-period model. In our calibration exercise, we use values of δ as suggested in the literature.

We assume that, while married, a couple behave *cooperatively* and maximize their joint intertemporal utility, $V_t \equiv V_{st} + V_{ht}$. After they divorce, they act *non-cooperatively*, each maximizing his or her own individual intertemporal utility (see e.g. Weiss and Willis, 1985).

2.2. Timing, Household Production, Earnings and Consumption

Period 1. All couples are married throughout the first period and decide how to allocate their unit time endowment between household production (g_{i1}^m) and labor market activity (ℓ_{i1}^m) in order to maximize their joint intertemporal

[8]This captures the idea that when parents are divorced, children become *impure* public goods to the extent that neither parent has "full access" to them due to court-imposed custodial arrangements (see, e.g., Francesconi and Muthoo, 2011).

utility. A production function $v(\cdot,\cdot)$ translates the time-inputs in household production into the family capital good G_1^m. In particular

$$G_1^m = v\left(g_{s1}^m, g_{h1}^m\right). \tag{4}$$

We assume that $v(\cdot,\cdot)$ is a strictly increasing and strictly concave, and that $v(0,0)=0$ and $\lim_{g_i\to 0} v_i(g_s,g_h)=\infty$ where v_i is the first-order partial derivative with respect to the input of partner $i=s,h$. The second-order partial derivatives are denoted by v_{ij}. Our setup allows the spouses' time inputs to be either substitutes ($v_{ij}<0$), independent ($v_{ij}=0$) or complements ($v_{ij}>0$).

Period 2. At the beginning of the second period, the couple's match quality shock θ is realized, representing a non-economic gain or loss from marriage (see, e.g., Fan, 2001). We assume that θ is drawn from a distribution $F(\cdot)$ with support $(-\infty,\infty)$ and associated density $f(\cdot)$. After θ is realized, the spouses first decide cooperatively whether to continue their marriage or to divorce. After that, they decide how to allocate their time. The level of family-specific capital enjoyed, G_2^k, is that carried forward from the first period, G_1^m, plus the amount added through household production in the current period:

$$G_2^k = G_1^m + v\left(g_{s2}^k, g_{h2}^k\right) \quad \text{for } k=m,d. \tag{5}$$

Note that the level of household production in this period will typically vary with the couple's chosen marital status, $k=m,d$.[9] Also note that, due to the additive specification in (5), the level of household production at $t=2$ will be independent of capital carried forward from $t=1$. This not only simplifies the analysis but also helps ensure continuity of behavior and the existence of an equilibrium.

Period 3. We assume that divorced individuals cannot remarry and so continue to live as divorced. We explicitly think of the third period as the stage in a family's life cycle when the children have grown up and left home. Hence, we assume that no further time is devoted to investment in the children. Instead, individuals spend their entire time endowment on labor market activity. However, both married and divorced couples continue to benefit from the stock of family-specific capital accumulated in the first two periods. Thus,

$$G_3^k = G_2^k \quad \text{for } k=m,d. \tag{6}$$

Earned Income and Consumption. Partner i's earnings in period t and marital state k are given by $y_{it}^k=\omega_i\ell_{it}^k$, where ω_i is the wage rate. Given that

[9] In our stylized three-period model, we assume for simplicity that the family capital good is perfectly durable. However, our results would qualitatively also go through in a richer specification with a quasi-durable family public good which is subject to some positive rate of depreciation. The extended version of our model presented in Appendix C allows for such capital depreciation. The additive form in (5) also assumes perfect substitutability between additions to the family capital stock at different times. We comment further on this assumption below.

a couple, while married, maximize their joint utility with each partner's utility being linear in consumption, the exact sharing rule within marriage has no bearing on the current problem and can be left unspecified. How consumption is shared after divorce, when the couple act non-cooperatively, however does matter. Hence we assume that partner i enjoys a share φ_i of the couple's total earnings, whereby $\varphi_s + \varphi_h = 1$. These shares can be interpreted as determined by the legal environment regulating divorce. We will further assume, for reasons that will become clear below, that each partner's consumption share matches his or her share of joint utility of the family capital good post divorce.

Partner i's consumption level as divorce can then be written as

$$x_{it}^d = \varphi_i \sum_{j=s,h} y_{jt}^d, \tag{7}$$

with

$$\varphi_i = \frac{\lambda_i}{\Lambda}. \tag{8}$$

A leading case is where the woman has, due to custody arrangements, a higher post-divorce enjoyment of the accumulated household public good and also, through child support arrangements, enjoys the larger share of joint income.

3. Laissez-Faire Allocation

We begin by characterizing the equilibrium with sophisticated behavior.

Period 3. As noted above, no decisions have to be taken in the third period. The couple's joint instantaneous utility in the married state and the individual utility in the divorced state are given by

$$u_3^m = \sum_{j=s,h} \omega_j + 2G_3^m, \quad \text{and} \quad u_{i3}^d = \varphi_i \sum_{j=s,h} \omega_j + \lambda_i G_3^d, \tag{9}$$

respectively, with G_3^k given by eq. (6).

Period 2. At the beginning of the second period the temporary match quality shock θ is realized and a couple first of all have to decide wether to stay married or to divorce. However, as the gain to marriage also depends on the time allocations made in the second period, we first consider these choices, both within marriage and after divorce.

The joint instantaneous utility as *married* is given by

$$u_2^m = \sum_{j=s,h} \omega_j \left(1 - g_{j2}^m\right) + 2G_2^m + 2\theta, \tag{10}$$

where G_2^m is defined in eq. (5). The couple maximize their joint intertemporal utility from marriage, $u_2^m + \beta u_3^m$, where u_3^m and u_2^m are defined in eqs. (9) and (10), respectively. The chosen time allocation, denoted $(\hat{g}_{s2}^m, \hat{g}_{h2}^m)$, is the unique solution to the following first-order conditions:

$$\frac{\omega_i}{v_i\left(\hat{g}_{s2}^m, \hat{g}_{h2}^m\right)} = 2\left(1 + \beta\right) \quad \text{for } i = s, h. \tag{11}$$

Thus, the marginal cost of each partner adding a further unit of family capital – the left hand side – is set equal to the joint marginal return, where the latter includes the current and discounted future marginal utility to each partner.

The instantaneous utility of partner i as *divorced* is given by

$$u_{i2}^{d} = \varphi_i \sum_{j=s,h} \omega_j \left(1 - g_{j2}^{d}\right) + \lambda_i G_2^{d}. \tag{12}$$

As noted above, divorced partners act non-cooperatively, with partner i maximizing his/her own intertemporal utility, $u_{i2}^{d} + \beta u_{i3}^{d}$, where u_{i3}^{d} and u_{i2}^{d} are defined in eqs. (9) and (12), respectively. The Nash equilibrium time allocation choices of the divorced spouses, denoted $\left(\hat{g}_{s2}^{d}, \hat{g}_{h2}^{d}\right)$, are the unique solution to the following first-order conditions:

$$\frac{\omega_i}{v_i\left(\hat{g}_{s2}^{d}, \hat{g}_{h2}^{d}\right)} = \frac{\lambda_i}{\varphi_i}\left(1+\beta\right) \quad \text{for } i = s, h. \tag{13}$$

Our assumption in eq. (8) ensures that the time allocation choices of divorced spouses coincide with that which would maximize their joint intertemporal utility. This allows us to abstract away from inefficiencies arising from non-cooperative post-divorce behavior when analyzing optimal policy design. It also implies that in either marital state, the equilibrium level of household production is produced at the lowest possible cost in terms of total foregone earnings.

It turns out to be convenient to define the difference in equilibrium foregone earnings across the two marital states as:

$$\hat{\mathcal{C}}_2 \equiv \sum_{i=s,h} \omega_i \left(\hat{g}_{i2}^{m} - \hat{g}_{i2}^{d}\right). \tag{14}$$

As a basis for our subsequent analysis, it is also useful to compare the level of family-specific capital generated through household production in the two possible marital states:

Lemma 1. *For any given level of G_1^m, a couple choose a higher level of household production at $t = 2$ if they remain married than if they divorce: $v\left(\hat{g}_{s2}^{m}, \hat{g}_{h2}^{m}\right) > v\left(\hat{g}_{s2}^{d}, \hat{g}_{h2}^{d}\right) > 0$. Thus, they also forego more earnings: $\hat{\mathcal{C}}_2 > 0$.*

Consider next the couple's decision whether to remain married or to divorce. This decision takes place *after* the realization of θ but *before* they choose how to allocate their time. A couple will divorce if doing so gives them a larger joint intertemporal utility than remaining married. Thus, a divorce occurs if

$$\hat{u}_2^{m} + \beta \hat{u}_3^{m} < \hat{u}_2^{d} + \beta \hat{u}_3^{d}, \tag{15}$$

where the $\hat{u}_t^{m}$'s are instantaneous joint utilities, within marriage, evaluated at $\hat{g}_{s2}^{m}$ and $\hat{g}_{h2}^{m}$, and where $\hat{u}_t^{d}$ is similarly defined as the instantaneous joint utilities, as divorced, evaluated at $\hat{g}_{s2}^{d}$ and $\hat{g}_{h2}^{d}$. Naturally, the couple will divorce if their match quality shock is sufficiently unfavourable. In particular, substituting using eqs. (9), (10) and (12), shows that the couple will divorce if θ falls below a threshold value, denoted $\hat{\theta}\left(G_1^m\right)$, the value of which will depend on the level of family-specific capital carried forward from the initial period:

$$\hat{\theta}\left(G_1^m\right) \equiv \hat{\mathcal{C}}_2/2 - (1+\beta)\left[\hat{G}_2^m - (\Lambda/2)\,\hat{G}_2^d\right], \tag{16}$$

where $\hat{G}_2^k = G_1^m + v\left(\hat{g}_{s2}^k, \hat{g}_{h2}^k\right)$. It is straightforward to establish that $\hat{\theta}\left(G_1^m\right)$ is a decreasing function of G_1^m:

$$\hat{\theta}'\left(G_1^m\right) = -(1+\beta)(1-\Lambda/2) < 0. \tag{17}$$

From the perspective of the first period, the probability of divorce, which can be written as $F(\hat{\theta}(G_1^m))$, is thus endogenous since it decreases with the family-specific capital the couple accumulates in that period.

Period 1. The partner's instantaneous joint utility in the first period is given by

$$u_1^m = \sum_{j=s,h} \omega_j \left(1 - g_{j1}^m\right) + 2G_1^m, \tag{18}$$

where G_1^m is given by eq. (4). The couple choose their time allocation to maximize their joint intertemporal utility,

$$V_1 = u_1^m + \beta\left\{\mathbf{E}_\theta\left(\hat{u}_2^m + \hat{u}_3^m \middle| \theta > \hat{\theta}\right)\left[1 - F(\hat{\theta})\right] + \left(\hat{u}_2^d + \hat{u}_3^d\right)F(\hat{\theta})\right\}, \tag{19}$$

correctly anticipating their own future behavior, and where $\hat{\theta} = \hat{\theta}(G_1^m)$ and $\mathbf{E}_\theta$ is the expectations operator, with expectations taken over θ. The first term in the curly brackets reflects a couple's continuation value upon entering the second period *conditional* on the marriage surviving and the second term is the corresponding value conditional on divorcing.

Lemma 2. *The time allocation chosen by a couple at $t = 1$, denoted $(\hat{g}_{s1}^m, \hat{g}_{h1}^m)$, is the unique solution to the following first-order conditions:*

$$\frac{\omega_i}{v_i\left(\hat{g}_{s1}^m, \hat{g}_{h1}^m\right)} = 2 + 2\beta\left\{2 - (2-\Lambda)\left[F(\hat{\theta}) - \chi f(\hat{\theta})\right]\right\} \quad \text{for} \quad i = s, h, \tag{20}$$

where $\hat{\theta} = \hat{\theta}(\hat{G}_1^m)$ defined as in eq. (16) with $\hat{G}_1^m = v\left(\hat{g}_{s1}^m, \hat{g}_{h1}^m\right)$, and where

$$\chi \equiv \frac{\left(1-\beta^2\right)\left(\hat{G}_2^m - (\Lambda/2)\,\hat{G}_2^d\right)}{2} > 0, \tag{21}$$

with $\hat{G}_2^k = \hat{G}_1^m + v\left(\hat{g}_{s2}^k, \hat{g}_{h2}^k\right)$ for $k = m, d$.

Decomposing eq. (20) reveals that a couple has *five* sources of (dis)incentives to invest time in the family capital good. First, an increase in G_1^m increases a couple's joint lifetime benefits from family-specific capital conditional on the marriage remaining intact in the second period. This acts as an incentive to make family-specific investments and is given by $2(1+2\beta)$. Second, the lowered enjoyment of the capital good in the case of divorce acts as a disincentive to invest and is captured by $-2\beta(2-\Lambda)F(\hat{\theta})$.

Third, an increased accumulation of family capital good in the first period expands downwards the set of match quality shocks at which they choose to remain married in the second period, thereby lowering the expected match quality. This "marital quality effect" acts as a disincentive to invest in the family capital good and is given by $-2\beta\hat{\theta}f(\hat{\theta})\hat{\theta}'$. Fourth, counteracting this effect is the direct utility gain that comes with an increase in the probability of the marriage remaining intact. This "endogenous divorce effect" increases incentives to make family-specific investments and is given by $-\beta f(\hat{\theta})\hat{\theta}'[2(2\hat{G}_2^m - \Lambda\hat{G}_2^d) - \hat{\mathcal{C}}_2]$. Subtracting the marital quality effect from the endogenous divorce effect using the expressions for $\hat{\theta}$ and $\hat{\theta}'$ derived in eqs. (16)-(17), we have

$$2\beta(2-\Lambda)\chi f(\hat{\theta}) > 0. \tag{22}$$

In words, given that the couple are present-biased, the positive "endogenous divorce effect" dominates the negative "marital quality effect". For reasons that will become clear shortly, we will refer to this term as the "sophistication effect". Note that this effect would vanish in the absence of present bias: $\beta \to 1$ implies $\chi \to 0$.

Finally, as before, the left hand side captures the marginal cost of generating an additional unit of the family-specific capital through an increase in the time input into household production by partner i.

4. First-Best Allocation and Inefficiency of Laissez-Faire under Present-Bias

We follow O'Donoghue and Rabin (2006) and treat individuals' preference for short-term gratification as an error, leading to a self-control problem when $\beta < 1$. When viewed from an *ex ante* perspective, the first-best allocation maximizes the exponentially discounted joint expected utilities. Since the structure of the problem remains the same, but with β set to unity throughout, we can directly summarize a couple's efficient behavior. In the second period, the first-best time allocations by marital status $k = m, d$, denoted $\left(\tilde{g}_{s2}^k, \tilde{g}_{h2}^k\right)$, satisfy

$$\frac{\omega_i}{v_i\left(\tilde{g}_{s2}^k, \tilde{g}_{h2}^k\right)} = \begin{cases} 4 & \text{if } k = m \\ 2\Lambda & \text{if } k = d \end{cases} \quad \text{for } i = s, h. \tag{23}$$

The efficient match quality threshold, given an arbitrary G_1^m, satisfies

$$\tilde{\theta}\left(G_1^m\right) \equiv \tilde{\mathcal{C}}_2/2 - 2\left[\tilde{G}_2^m - (\Lambda/2)\tilde{G}_2^d\right], \tag{24}$$

where $\tilde{\mathcal{C}}_2 = \sum_{i=s,h}\omega_i\left(\tilde{g}_{i2}^m - \tilde{g}_{i2}^d\right)$ and $\tilde{G}_2^k = G_1^m + v\left(\tilde{g}_{s2}^k, \tilde{g}_{h2}^k\right)$.

In the initial period, a couple's first-best time allocation, denoted $(\tilde{g}^m_{s1}, \tilde{g}^m_{h1})$, is the unique solution to:

$$\frac{\omega_i}{v_i\left(\tilde{g}^m_{s1}, \tilde{g}^m_{h1}\right)} = 6 - 2\left(2 - \Lambda\right) F(\tilde{\theta}) \quad \text{for } i = s, h, \tag{25}$$

where $\tilde{\theta} = \tilde{\theta}(\tilde{G}^m_1)$ with $\tilde{G}^m_1 = v(\tilde{g}^m_{s1}, \tilde{g}^m_{h1})$. Thus, the probability of divorce in the first-best allocation can be written as $F(\tilde{\theta}(\tilde{G}^m_1))$.

How does a bias for the present distort a couple's behavior as compared to the first-best choices that would have maximized their exponentially discounted joint intertemporal utility? Consider first a couple's second-period choices. The following result holds globally for any degree of present bias (i.e. for any $\beta < 1$):

Proposition 1 (Second-Period Choices). *For any given level of G^m_1, a present bias causes a couple at $t = 2$ to:*

(a) choose an inefficiently low level of household production in each marital state: $v\left(\hat{g}^k_{s2}, \hat{g}^k_{h2}\right) < v\left(\tilde{g}^k_{s2}, \tilde{g}^k_{h2}\right)$ for $k = m, d$.

(b) "over-divorce"—i.e., to break up for a larger set of match quality realizations than in the first-best allocation: $\hat{\theta}\left(G^m_1\right) > \tilde{\theta}\left(G^m_1\right)$.

That present-biasedness causes a couple to underinvest in the family-specific capital is hardly surprising given that they place too little weight on its future benefits. Perhaps more interesting is the over-divorcing result. On the one hand, divorce has two "short-term gratification" components: it allows the couple to avoid a temporary unfavorable match quality shock and they can enjoy more current consumption as they will forego less earnings to produce the household capital good. On the other hand, divorce has two "long-term" effects, the payoff consequences of which persist into the final period: it reduces the amount of family-specific capital that the couple accumulate and it also reduces their total enjoyment from their family capital good. Present-biased couples "over-divorce" because they overreact to the short-term gratification components of divorce relative to the two permanent effects.[10]

The impact of present bias on a couple's first-period investment is less obvious. Two opposing effects can be identified. As in the second period, a present bias directly causes the couple place too little weight on future relative to current payoffs which generates a tendency for underinvestment. However, being sophisticated, the couple are aware that their present bias will make them too prone to divorce in the following period. This provides them with a strategic investment motive: by investing more in the first period, they can reduce their divorce risk in the second period. Formally, the strategic investment motive

[10]The results in Proposition 1 continue to hold with capital depreciation between periods as long as the depreciation is not complete.

is captured by the sophistication effect in eq. (22) which is not present in the first-order condition characterizing the efficient investment level.[11]

We now ask whether the direct present bias effect dominates the sophistication effect or *vice versa*. In order to derive a clear-cut result, we will consider how the *introduction of present bias* into the couples' preferences – that is, a marginal reduction in β away from unity – affects the level of first-period household production.

Proposition 2 (First-Period Household Production). *At* $t = 1$*, the* introduction *of present bias causes a couple to choose an inefficiently low level of household production:* $v\left(\hat{g}_{s1}^{m}, \hat{g}_{h1}^{m}\right) < v\left(\tilde{g}_{s1}^{k}, \tilde{g}_{h1}^{k}\right)$.

Thus, the introduction of present bias will cause couples to exit the first period with an inefficiently low level of family-specific capital. A present bias therefore increases the equilibrium divorce risk through two channels: it does so *directly* by making the couple more prone to divorce for any given level of family capital, and also *indirectly* by reducing the accumulation of family-specific capital in the initial period.

One caveat is that the above result is only a "local" one as β is arbitrarily close to unity. There are sufficient conditions under which the result can be generalized to hold for any degree of present bias. One particularly simple condition concerns the probability density function f:

Corollary 1. *Suppose that the match quality shock* θ *is drawn from a uniform probability density function:* $\theta \sim \mathcal{U}(\underline{\theta}, \overline{\theta})$*. At* $t = 1$*, a couple will then choose an inefficiently low level of household production for any degree of present bias.*

The results above have naturally been stated in terms of a couple's total level of household production rather than in terms of the spouses' individual time inputs. As noted earlier, for any chosen level of household production, the spouses' time inputs minimize the cost in terms of total foregone earnings.[12] This means that, as long as neither partner's time is an "inferior" input, statements on the levels of household production carry over to individual time inputs. Neither partner's time input can be inferior if the household production function either exhibits complementarity or independence (i.e., when $v_{sh} \geq 0$). If the time inputs are substitutes (i.e., when $v_{sh} < 0$), one partner's time input may be inferior.

[11] As noted above, we have assumed that there are no complementarities between investments across time. The possibility of such complementarities has received some attention in the literature (e.g. Cunha and Heckman, 2007). Any such complementarities could also generate strategic incentives for early investments. For instance, a couple may strategically invest in the human capital of a very young child in order to raise the marginal return to investment – and hence their own future investment – in the child's human capital at a higher age.

[12] Note that this holds not only in marriage, but also after divorce due to the assumption in eq. (8).

So far, we have focused on sophisticated behavior by couples who are aware of their self-control problem. The literature contrasts this to naive behavior by individuals who – at any moment in time – acts on a preference for instant gratification, but fail to foresee that they will do so in the future.

Both sophisticated and naive couples will tend to underinvest in the family capital due to their preferences for instant gratification. However, it is not immediately clear whether the investment behaviour of naive couples will be more or less inefficient than that of sophisticated couples.

To see this, note that there are two implications of sophistication. On the one hand, the strategic *incentive effect* means that a sophisticated couple will invest in part in order to reduce their own future propensity to divorce. On the other hand, there is a *pessimism effect* stemming from their correct expectation.[13]

The pessimism of a sophisticated couple has to be understood in relation to the divorce expectation of a naive couple. To see this, note that for any given G_1^m built up in the first period, a naive couple will expect themselves to make an efficient divorce decision in the following period, thus expecting to divorce at rate $F(\tilde{\theta}(G_1^m))$. However, when the second period arrives they will divorce at the higher rate $F(\hat{\theta}(G_1^m))$.[14] Hence the naive couple will naturally be over-optimistic about their marriage survival chances, and they therefore perceive a larger marginal value to investment. Indeed, the first order condition characterizing the initial period investment by a naive couple, denoted $(\bar{g}_{s1}^m, \bar{g}_{h1}^m)$, can be written as

$$\frac{\omega_i}{v_i\left(\bar{g}_{s1}^m, \bar{g}_{h1}^m\right)} = 2 + 2\beta\left[2 - (2-\Lambda)\,F\left(\tilde{\theta}\left(\bar{G}_1^m\right)\right)\right] \quad \text{for } i = s, h, \tag{26}$$

where $\bar{G}_1^m = v\left(\bar{g}_{s1}^m, \bar{g}_{h1}^m\right)$. Comparing (26) to (25) we see that the presence of the present-bias factor β implies that the naive couple will under-invest relative to the first best. However, whether the optimism-bias of a naive couple is stronger or weaker than the incentive effect of a sophisticated couple—i.e., whether the former type of couple invest more than the latter in the initial period—is *a priori* not clear. The following result provides a condition that separates the two possibilities.

Proposition 3 (Naivity v. Sophistication). *At $t = 1$, a naive couple will choose a higher (lower) level of household production than a sophisticated couple if and only if*

$$\frac{F(\hat{\theta}) - F(\bar{\theta})}{\hat{\theta} - \bar{\theta}} > (<) \kappa f(\hat{\theta}), \tag{27}$$

[13]The notion that sophistication brings often couteracting incentive- and pessimism-effects is known in the literature (O'Donoghue and Rabin, 2002), particularly from the context of addiction where consumption decisions are inter-temporally linked. In our context, the effects arise since the initial investment decision is linked to the future divorce decision.

[14]They will also expect themselves to invest $\left(\tilde{g}_{s2}^k, \tilde{g}_{h2}^k\right)$ in marital state $k = m, d$ whereas, when period 2 actually arrives, they will invest $\left(\hat{g}_{s2}^k, \hat{g}_{h2}^k\right)$.

where $\kappa \simeq (1+\beta)/2 \leq 1$.

This result has an intuitive explanation. On the one hand, if $f(\hat{\theta})$ is large, an increase in G_1^m leads to a large increase in the equilibrium marriage survival rate, implying a strong strategic investment motive for a sophisticated couple. The left hand side of eq. (27), on the other hand, effectively measures the rate at which a naive couple's misperception of their own future divorce behaviour generates marriage survival optimism which, if large, implies a large optimism bias.

In practice, the left hand side of eq. (27) is, to a first order approximation, equal to $f(\hat{\theta})$ and, for modest present bias (β close to unity) κ will also be close to unity. This suggests that the investment behaviour of naive and sophisticated couple can be expected to be similar, which has been confirmed in numerical examples.

5. Privately Optimal Marriage Contracts

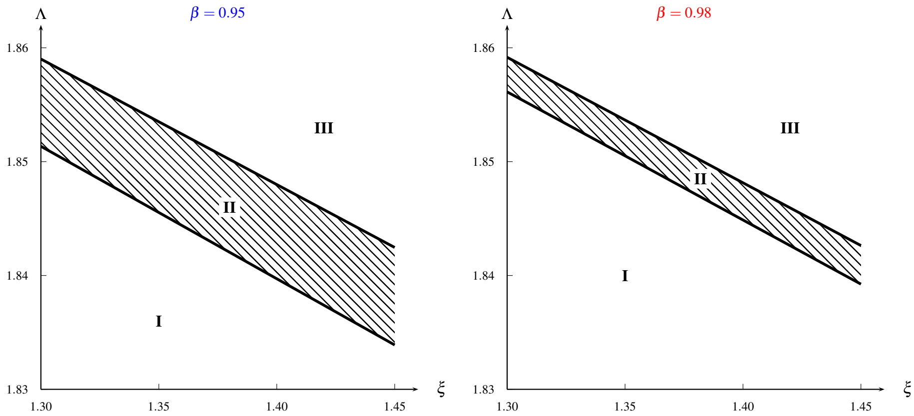
In the Introduction we highlighted how, in various cultures and religions around the world, couples enter into marriage on contractual terms that make divorce a costly option. While standard models of household behavior have difficulties in rationalizing this type of marriage contracts, our theory suggests that couples who expect themselves to behave with a present bias may find them privately optimal. To see this, we now add an additional *ex ante* stage, $t=0$, to the model.

We will think of this *ex ante* stage as the time of marriage, before any family investments commence, and we will explore if a marrying couple can be made better off with a marital contract obliging them to pay a positive monetary penalty ζ in the event of divorce. Framed differently, we may also think of a society deciding on opting for legislation that makes divorce more costly. The remainder of the game then follows the same structure as in the baseline model, but now includes the divorce penalty.

Consider the couple's joint intertemporal utility as viewed from the *ex ante* stage when a divorce penalty ζ is in operation. This can be written as:

$$V_0 = \beta \left\{ \hat{u}_1^m + \left[\mathbf{E}_\theta \left(\hat{u}_2^m + \hat{u}_3^m \middle| \theta > \hat{\theta} \right) \left[1 - F(\hat{\theta}) \right] + \left(\hat{u}_2^d - \zeta + \hat{u}_3^d \right) F(\hat{\theta}) \right] \right\}, \tag{28}$$

and where the match quality threshold level $\hat{\theta}$ depends on ζ not only directly but also indirectly via its impact on the couple's choice of first period level of household production. We will assume that a couple responds to the introduction of positive divorce penalty by increasing their first period level of household production, $\partial \hat{G}_1^m / \partial \zeta \big|_{\zeta=0} > 0$. This can be shown to hold for the case of a uniformly distributed match quality $\theta \sim \mathcal{U}(\underline{\theta}, \overline{\theta})$, but can also be expected to hold much more generally. We then obtain:



(A) *Privately Optimal Marriage Contracts in the (Λ, ξ)-space* (B) *A reduction in present bias*

FIGURE 2: *A Welfare Improving Divorce Penalty*

Proposition 4 (Welfare-Improving Divorce Penalty). *If*

$$F(\hat{\theta}) < \frac{2\left\{\left[2 + (2-\Lambda)\, f(\hat{\theta})\chi\right]\left(1-\beta^2\right)\left.\frac{\partial \hat{G}_1^m}{\partial \zeta}\right|_{\zeta=0} + \chi f(\hat{\theta})\right\}}{(1+\beta)\left[1 + 2\,(2-\Lambda)\,(1-\beta)\left.\frac{\partial \hat{G}_1^m}{\partial \zeta}\right|_{\zeta=0}\right]} \tag{29}$$

holds at the laissez-faire equilibrium, then introducing *a positive divorce penalty $\zeta > 0$ into the marriage contract will increase the couple's joint ex ante expected utility.*

The following provides an intuition. The introduction of a divorce penalty mitigates the over-divorce distortion in the second period as well as the extent of underinvestment in the first period, thus modifying a present-biased couple's behavior in the efficient direction.[15] However, a divorce penalty also comes with a first-order cost effect which is proportional to the risk of divorce. Hence, if β is only marginally below unity, a divorce penalty cannot improve a couples' *ex-ante* utility as their behavior is already close to efficient, implying that the first-order cost effect dominates.[16] But if the degree of present bias is more substantial and the baseline divorce risk is low, the impact of the divorce penalty on future behavior can dominate the cost effect.

This is illustrated in Figure 1 based on a model specification that gives rise to closed form solutions. In particular, it is assumed that $\theta \sim \mathcal{U}(-2, 0)$ and that

[15]Note that when, when $\zeta = 0$, V_0 is proportional to the joint utility that is maximized by the efficient allocation.

[16]Formally, in the limit where $\beta \to 1$, the right hand side of (29) goes to zero. To see this, recall that χ also goes to zero, see eq. (21).

the household production function is given by $v(g_{st}, g_{ht}) = \sqrt{g_{st}} + \sqrt{g_{ht}}$. Under this assumption, the cost of achieving a level of household production v can be written as $C(v) = \frac{1}{2}\xi v^2$, where ξ is an increasing function of ω_s and ω_h.

In Panel A, we set $\beta = 0.95$, and then examine couples' incentives to accept a divorce penalty as part of their marital contract in the (Λ, ξ)-space.[17] There are three regions to consider. Region I is irrelevant for the purpose of our analysis, in that for parameter values in this region the equilibrium divorce rate is zero. In Region II, the efficiency-enhancing effect of a divorce penalty dominates the first-order cost effect. Thus, it will be privately optimal for couples to accept a marriage contract making divorce a costly option. Notice that Region II represents parameter values for which the equilibrium divorce rate is relatively low. Put differently, the gains from marriage over divorce are relatively large, either because the opportunity cost of home production is low, or because the rate at which couples can enjoy the family capital good after a divorce is low. For (Λ, ξ)-combinations in Region III, the equilibrium divorce rate is larger. As a consequence, the efficiency-enhancing effect of a divorce penalty is too small relative to the first-order cost effect, and couples are better off to leave divorce as costless option. Panel B presents results for a comparative-statics exercise in which the degree of present bias is lowered. As would be expected, this has the effect of reducing the set parameter values under which couples benefit from a divorce penalty.

Overall, the main message here is that present-biased couples may have an incentive to make divorce a costly option, and that this incentive is particularly pronounced in environments in which the equilibrium divorce rate is naturally low.

6. Efficiency-Restoring Policy

A marital contract that involves a divorce cost may allow a present-biased couple to get *closer* to their first-best behavior, but will not achieve full efficiency. We now show that full efficiency can be restored through policy provided there is a sufficient set of policy instruments.[18]

6.1. Policy in the Three-Period Model

The instruments required directly correspond to the decisions taken by a couple over the course of their marital lifecycle, most notably their time allocations and potential divorce decision. Hence let τ_t be a proportional tax on labor

[17] We will use the same time preference parameters in the extended model developed in Appendix C and discussed in Section 6.

[18] While we focus here on policy that restores full efficiency based on long-run exponentially discounted preferences, it should be noted that sophisticated couples would support some form of policy also based on their preferences as of $t = 1$ when they are β-discounting the future. For instance it is straightforward to verify that a sophisticated couple at $t = 1$, recognizing their future tendency to over-divorce, would support the introduction of divorce fee at $t = 2$ with the expected revenue from that fee returned in lump-sum fashion at the beginning of that period.

earnings in period t, and let η be a divorce tax payable by a couple who choose to divorce. All expected tax proceeds are given to the couple as a lump-sum transfer at the outset of each period, before the couple take any decisions. We then have the following:

Proposition 5 (Efficiency-Restoring Policy). *The first-best allocation can be implemented by a policy scheme with the following elements:*

(a) At $t = 2$, there are taxes on labor earnings and on divorce given by:

$$\hat{\tau}_2 = \frac{(1-\beta)}{2} \quad \text{and} \quad \hat{\eta} = (1-\beta)\left[2\tilde{G}_2^m - \Lambda \tilde{G}_2^d - \tilde{\mathcal{C}}_2/2\right], \tag{30}$$

where $\tilde{G}_2^k = \tilde{G}_1^m + v\left(\tilde{g}_{s2}^k, \tilde{g}_{h2}^k\right)$ and $\tilde{\mathcal{C}}_2 = \sum_{i=s,h} \omega_i \left(\tilde{g}_{i2}^m - \tilde{g}_{i2}^d\right)$.

(b) At $t = 1$, there is a tax on labor earnings given by:

$$\hat{\tau}_1 = \frac{2(1-\beta)\left[2-(2-\Lambda)F(\tilde{\theta})\right]}{6-2(2-\Lambda)F(\tilde{\theta})}. \tag{31}$$

A number of points are worth noting. First, the efficiency-restoring earnings tax is gender-neutral: even if spouses differ in their labor market and household productivities, they face the same tax rate on their earnings at any moment in time. Second, the earnings tax $\hat{\tau}_t$ that the couple face when married varies over time, and it does so for two reasons. On the one hand, family investments made in the first period are enjoyed over a longer time horizon than those made in the second period. On the other hand, they are made under the risk of future divorce. The earnings tax $\hat{\tau}_2$ is here also independent of marital status; this is due to the assumption of a zero divorce risk in the final period. Third, in the current three period model, a couple only face a divorce decision once. In a more general environment the divorce tax $\hat{\eta}$ will also be marriage-duration dependent. Fourth, while our focus is on an efficiency-restoring policy, even a non-optimal policy can be welfare-improving. For instance, the *introduction* of a divorce fee η on its own can improve the welfare of a sophisticated couple. The logic follows the case of privately imposed divorce costs, noting that the lump-sum return of the expected proceeds from the fee removes the expected cost to the couple.

6.2. An Extended Calibrated Model

To gain further insight into the potential magnitudes of the distortions generated by present bias and of the tax policies required to restore efficiency the model can usefully be extended to a longer horizon. Here we report some basic insights from an extension to $T = 40$ periods starting at the time of marriage, with investments occuring in the first $T_0 = 20$ periods.[19] T was chosen to capture the time between average age at first marriage in the US which is about

[19]The details of the extended model are presented in Appendix C.

27 for women and 29 for men in the latest statistics (US Census Bureau) and retirement age, while T_0 was chosen to correspond to the average age at which children leave full time education (OECD, 2008).

To calibrate the model to the US economy, we then use a combination of parameter estimates from the literature and a set of key empirical stylized facts that we match. The latter set includes inter alia (i) the percent of marriages still intact after 10 periods (68 percent according to Copen et al., 2012), (ii) the gender pay ratio (82 percent according to BLS, 2014), (iii) fraction of available time allocated to labour market work by married men and women respectively (set at 67 and 36 percent respectively)[20], (iv) the impact of children growing up and moving out on divorce hazard (an increase of 75 percent following Hiedemann, Suhomlinova and O'Rand, 1998, and Walker and Zhu, 2004), and (v) the impact of divorce on children's outcomes (set at 15 percent following Piketty, 2003, Gruber, 2004, and Björklund and Sundström, 2006).[21]

The calibration highlights two key features in particular. First, while it was argued above that the tax on earnings will vary with marriage duration, in practice this duration-dependence is negligible. The reason is simple: since the couple's full horizon T extends well beyond their investment phase T_0, the variation in horizon within the investment phase plays a minor role. Indeed, both within marriage and after divorce, the efficiency-restoring tax on earnings is essentially flat over time and close to $(1-\beta)\,\gamma\delta$ (where γ is the capital carry-forward rate). This is illustrated in the left panel of Figure 3 which plots the efficiency-restoring earnings tax by marital status over the T_0 years of the investment phase, with the red line indicating the value of $(1-\beta)\,\gamma\delta$.

Second, the calibrated model shows that even a level of present-bias, $\beta = 0.95$, that is modest from the perspective of the literature can generate a substantial increase in the incidence of divorce: in the calibrated model the fraction of couples that divorce within 10 years is increased by a about a third relative to the efficient fraction. This marked impact on divorce behavior also implies a more substantial and strongly time-varying efficiency-restoring divorce tax. Indeed, we find that the efficient divorce tax, illustrated in the right panel of Figure 3, is inverted U-shaped and is at its largest during the children's early teenage years. At that stage we find that the efficiency-restoring divorce tax corresponds to about 10 percent of the annual earnings of a married couple.

It should be noted that the model is calibrated without any direct divorce costs beyond η. In practice divorce costs can be substantial. Indeed, for the US, many sources estimate the average cost of divorcing ranges from less than \$1,000 for uncontested divorce to \$15,000 for divorces involving mediation and/or liti-

[20]This follows figures from the the Bureau of Labor Statistics (BLS, 2014) and the American Time Use Survey (ATUS, 2014)

[21]The parameter values obtained from the literature includes (i) the discount rate δ, set at 0.95 following Prescott (1986) and others, (ii) the present-bias parameter β, set at 0.95 following Augenblick, Niederle and Sprenger (2013) and Andreoni and Sprenger (2012), and (iii) a capital depreciation rate, $1-\gamma$, set at 2.5 percent following Manuelli, Seshadri and Shin (2012).

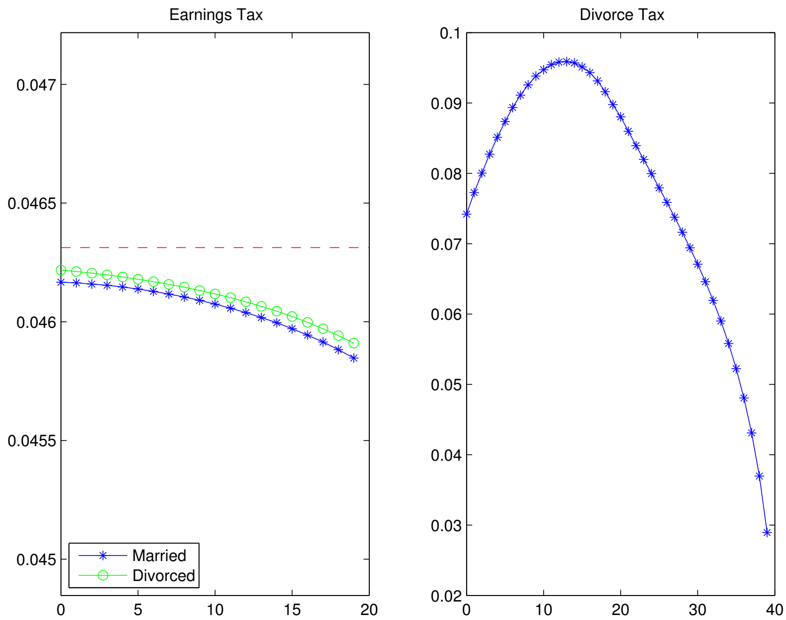


Figure 3: Efficiency-restoring earnings and divorce taxes by marriage duration.

gation.[22]. In contrast, average household income for married couples (under age 65) is around \$120,000 (Table HINC-02, US Census Bureau, 2016). This suggests that actual divorce cost may potentially come close the efficiency-restoring divorce fees, and may even exceed these for couples who are either at the initial or final stages of their investment phase.

Overall, the results presented here provide a new perspective for research into the optimal taxation of couples. The existing literature has mainly focused on the design of gender-specific tax schedules (e.g., Boskin and Sheshinski, 1983; Alesina *et al.*, 2011; Immervoll *et al.*, 2011; Meier and Rainer, 2015). A conventional result that emerges from these studies is that the spouse with the higher labor supply elasticity – typically the secondary earner – should be taxed at a lower rate. With time-inconsistent couples, there will be additional drivers in the design of optimal income tax schedules that call for potentially *gender-neutral* but *marital-life-cycle-dependent* earnings taxes for married spouses.

7. The Role of Cooperative Behavior

What role does the assumption of cooperative household behavior, which implies that couples act like an entity, play for our results? Fundamentally, it

[22]See e.g. http://www.divorcestatistics.info or https://www.huffingtonpost.com/cheryl-and-joe-dillon/how-much-does-divorce-cos_b_6143024.html.

ensures that a couple internalize any external effects onto each other that arise from their investment and divorce decisions. In Appendix D, we show what would be the implications of replacing the assumption of cooperative behavior with a basic non-cooperative Nash equilibrium assumption.

There would be two key implications. First, the model effectively becomes one of private contributions to a public good (Bergstrom *et al.*, 1986). Hence each partner tends to devote inefficiently little time to investing in the household public good as he/she neglects the benefit to the partner. Second, even with a commonly experienced match quality shock θ, the partners will generally not agree on the divorce decision. Assuming a unilateral divorce regime, the marriage will continue if and only if both partners prefer this to divorce. Hence, in equilibrium, one partner will effectively be "decisive" with respect to this decision, and any strategic investments—by either partner—will focus on strengthening the decisive partner's incentives to remain married.

In terms of policy, the public good aspect of the private investments will generally motivate a base earnings tax rate of 1/2 in any period and state where investments are made. The exact efficiency-restoring tax rate at time $t = 2$ will further generally be both marital status and gender-dependent (in the divorced state) except in the case where both spouses enjoy the capital stock equally after divorce, $\lambda_s = \lambda_h$. If $\lambda_s \neq \lambda_h$, the spouse with the relatively lower λ will face the relatively higher efficiency restoring earnings tax rate. The efficiency-restoring divorce fee will have to be tailored to the incentives of the decisive spouse, but otherwise has a similar structure to the main model. At time $t = 1$ the efficiency-restoring earnings tax rate is generally gender-specific except under symmetric post-divorce enjoyment of the public good, $\lambda_s = \lambda_h$.[23]

Hence, as expected, non-cooperation introduces further inefficiencies that need to be corrected by policy. Moreover, asymmetries between the partners will generally break the gender-neutrality that characterized policy in the main model.

8. Concluding Remarks

The dominant view among family economists is that the basic dynamics of household decision-making can be explained within a framework in which family members behave fully rationally over time. This paper departs from this view and takes some steps towards connecting the economic theory of the family with behavioral research on time-inconsistent preferences. At the center of our story are hyperbolic discounting couples who make two sets of decisions. First, they engage in home production activities, thereby accumulating family-specific capital over time. Second, the probability of divorce is endogenously determined by the level of accumulated family-specific capital itself. We show that present-biased preferences induce couples to underinvest in family-specific capital and to

[23] Further gender symmetry is also required with respect to differences in investments across marital states. See the Appendix for details.

over-divorce. From a policy perspective, our model gives a sense of how earnings and divorce taxes that vary of the marital life cycle can undo the inefficiencies in the behavior of time-inconsistent couples. Moreover, it provides a rationale for the existence of marriage contracts that serve as barriers to hasty divorces.

We have presented the most parsimonious model we could construct in order to highlight our main ideas. Thus, our approach leaves open many interesting directions for future theoretical research. The model assumes that individuals do not consume leisure, nor does it allow couples to purchase market goods as substitutes for the their own time input into home production. It would be interesting to allow for either or both. Another possible extension would be to recognize the possibility of heterogeneity in time preferences, even with married couples. Although there is a small literature on joint intertemporal choice in environments with heterogenous time preferences (Jackson and Yariv, 2015; Lizzeri and Yariv, 2017), in the household context, it would further raise the question of how sorting into marriage occurs. Hence such an extension would be most naturally accompanied by endogenizing marriage formation.

Last but not least, while our contribution is a theoretical exercise, future work should attempt to assess the empirical relevance of present bias for intra-household decisions. In this regard, an interesting, albeit difficult, task would be to formulate an estimable model that allows inferences about the fractions of time-consistent and time-inconsistent households from observational data.

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Appendix A: Some Correlational Evidence on Present-Biasedness and Divorce (Intended for Online Publication)

In Figure 1, we have provided some correlational evidence on the link between present-biasedness and divorce. To that end, we have drawn upon the 2000 Bank of Italy Survey of Household Income and Wealth (SHIW) and followed the methodology proposed by Eisenhauer and Ventura (2006) to characterize survey respondents as either hyperbolic or exponential discounters. In essence, the 2000 SHIW contains two experimental matching tasks in which respondents indicate the amount of an immediate reward that makes it just as attractive as a larger future reward:

> q_1 = "Suppose that you were informed that you had won the sum of 10 million lire,[24] payable for certain in a year's time. How much would you be prepared to pay (maximum amount) to receive the 10 million immediately?"
>
> q_2 = "And if the 10 million lire winnings were available in 2 years' time, how much would you be prepared to pay (maximum amount) to receive the 10 million immediately?"

The answers to these questions can be used to calculate (β, δ) in a simple model of quasi-hyperbolic discounting. In particular, assuming linear utility, we can estimate the present bias parameter β according to

$$\beta = \frac{(10,000,000 - q_1)^2}{10,000,000(10,000,000 - q_2)},$$

and classify an individual as a hyperbolic discounter if $\beta < 1$, and as an exponential discounter if $\beta \geq 1$.[25]

The 2000 SHIW also contains a fairly rich set of individual background variables including some basic information on marital status. In particular, respondents indicate whether they are married, divorced, single or widowed. For the purpose of linking present-biasedness to divorce, we restrict our sample to individuals who, at the time of the survey, were (i) either married or divorced

[24] As noted by Eisenhauer and Ventura (2006), the 2000 SHIW was conducted before Italy converted to the Euro and, at the time of the survey, 10 million lire were worth roughly US$5000.

[25] Eisenhauer and Ventura (2006) also use this specification to classify individuals as hyperbolic or exponential discounters. However, their leading specification takes the case of logarithmic utility and accounts for preexisting income (y) and wealth (w) by estimating β according to

$$\beta = \frac{[\ln(y + w + 10000000 - q_1)]^2}{\ln(y + w + 10000000)\ln(y + w + 10000000 - q_2)}.$$

The results we present below are qualitatively robust to this alternative characterization of hyperbolic and exponential discounters.

Table A.1
Present-Biasedness and Divorce

Dependent Variable: Married/Divorced (0/1)	(1)	(2)	(3)	(4)
Observations=1,067; Mean Exponential Discounters=0.075				
Hyperbolic Discounter	0.042**	0.038*	0.042**	0.040**
	(0.021)	(0.020)	(0.020)	(0.020)
p-value	0.042	0.054	0.034	0.049
R-squared	0.004	0.113	0.147	0.152
Basic Controls		Yes	Yes	Yes
Regional Controls			Yes	Yes
Education				Yes

Notes: OLS regressions of a dummy for whether an individual is married/divorced (0/1) on our time-preference indicator and a set of controls. Standard errors reported in parentheses. *Basic controls* include a respondent's gender and age. *Regional controls* include dummy variables for a respondent's region of residence (20 groups) and the population density of his or her municipality (4 groups). *Education* includes dummy variables for a respondent's education (6 groups: none, elementary school, middle school, high school, bachelor's degree, post-graduate qualification). *** (**) (*) indicates significance at the 1% (5%) (10%) level.

and (ii) between 25 and 50 years of age.[26] Our final sample includes 1,067 survey respondents. In line with the evidence reported in Eisenhauer and Ventura (2006), the share of hyperbolic discounters in our sample amounts to 21.6%. The share of divorced individuals in our sample is 8.4%.

Figure 1 in the main body provides some first evidence on the link between present-biasedness and divorce. In Table 1, we further explore the evidence in this figure in a simple regression framework. Specifically, we regress a dummy D for whether an individual is married ($D = 0$) or divorced ($D = 1$) on our time-preference dummy T indicating whether an individual is an exponential discounter ($T = 0$) or a hyperbolic discounter ($T = 1$). The coefficient in Column (1) corresponds to the raw gap in means from Figure 1. In Column (2), we include a respondent's gender and age as basic controls. In Column (3), we additionally control for a respondent's region of residence and the population density of his or her municipality. Finally, in Column (4), we also control for a respondent's education. Throughout all specifications we find that hyperbolic discounters have significantly higher likelihood to be divorced than exponential discounters. The point estimates suggest that being a hyperbolic discounter is

[26] There are two reasons for this age restriction. First, in the 2000 SHIW, there are no reported cases of divorce among individuals up to an age of 25. Second, individuals older than 50 years at the time of the 2000 SHIW belong to cohorts born prior to 1950. In comparison to individuals born from the 1950s onwards, these cohorts experienced relatively low rates of marital instability, which leaves us with very little variation to exploit.

associated with 3.8 to 4.2 percentage points higher divorce rate. Compared to the divorce rate of 7.5% among exponential discounters, this correspondents to a 51% to 56% higher divorce incidence.

To check the robustness of this finding, we have also explored surveys that include qualitative measures of present bias. For example, the Dutch National Bank (DNB) Household Survey contains a battery of questions of the following type: Do you only care about the immediate consequences of your actions? Do you find it more important to do work that gives short-run rather than long-run results? Which time-horizon do you use in your decision-making? We used several of this type of questions to construct qualitative measures of present-biasedness, and mostly found positive correlations between these measures and the incidence of divorce. The results are available upon request.

Appendix B: Proofs (Intended for Online Publication)

Proof of Lemma 1

While married, a couple choose their time allocation cooperatively. A consequence of this is that, whatever level of total household production they choose, their associated time allocation will minimize the cost in terms of foregone earnings. Thus consider the cost minimization problem

$$C(v) \equiv \min_{g_s, g_h} \left\{ \sum\nolimits_i \omega_i g_i \middle| v(g_s, g_h) \geq v \right\}, \tag{B1}$$

where v is the total level of household production. The solution of this problem has standard properties: the marginal rate of technical substitution is set equal to the wage ratio, $\omega_s/\omega_h = v_s(g_s, g_h)/v_h(g_s, g_h)$, and the marginal cost satisfies

$$C'(v) = \frac{\omega_s}{v_s(g_s, g_h)} = \frac{\omega_h}{v_h(g_s, g_h)}. \tag{B2}$$

Strict concavity of $v(\cdot)$ implies a strictly increasing marginal cost, $C''(\cdot) > 0$.

Consider now the household production problem at $t = 2$ for a couple that chose to remain married. Using the cost function, we can focus on the chosen level of production level, denoted $\hat{v}_2^m$, which maximizes the couple's joint objective function,

$$\max_{v_2^m} \left\{ 2\theta + (1+\beta)\left[\sum\nolimits_i \omega_i + 2\left(G_1^m + v_2^m\right)\right] - C\left(v_2^m\right) \right\}. \tag{B3}$$

and hence satisfies the first order condition,

$$2(1+\beta) - C'\left(\hat{v}_2^m\right) = 0, \tag{B4}$$

while the second order condition $-C''\left(v_2^m\right) < 0$ is satisfied due to convexity of the cost function. The first order conditions (11) follow from (B4) and (B2).

After a divorce, the couple no longer choose their time allocation cooperatively. However, the assumption on φ_i in (8) implies that the couple's chosen time allocation continue to minimize the total cost of their chosen level of production, $\hat{v}_2^d$: the individual first order conditions in (13), along with (B2), implies that $\hat{v}_2^d$ satisfies

$$\Lambda(1+\beta) - C'\left(\hat{v}_2^d\right) = 0, \tag{B5}$$

which, together with (B2) and (8), gives the first order condition in (13).

Comparing eqs. (B4) and (B5), and using that $C(\cdot)$ is strictly convex it is clear that $\hat{v}_2^d < \hat{v}_2^m$ as $\Lambda < 2$. From the cost minimization problem in (B1), it then also follows that a couple forego more earnings if still married than if divorced.

Proof of Lemma 2

As the couple is married at $t = 1$, they choose their time allocation cooperatively, thus minimizing the cost of production for their chosen level of household production. Their joint objective function $V_1 = V_{s1} + V_{h1}$ can then be written as a choice of G_1^m,

$$\begin{aligned} V_1 = & \sum\nolimits_i \omega_i + 2G_1^m - C\left(G_1^m\right) \\ & + \beta\Big\{\Big(1 - F(\hat{\theta})\Big)\left[2\left[\sum\nolimits_i \omega_i + 2\hat{G}_2^m\right] - C\left(\hat{v}_2^m\right)\right] \\ & + 2\int_{\hat{\theta}}^{\infty} \theta f(\theta)\mathrm{d}\theta + F(\hat{\theta})\left[2\left[\sum\nolimits_i \omega_i + \Lambda\hat{G}_2^d\right] - C\left(\hat{v}_2^d\right)\right]\Big\}, \end{aligned} \tag{B6}$$

where $\hat{\theta} = \hat{\theta}(G_1^m)$ and $\hat{G}_2^k = G_1^m + \hat{v}_2^k$ with $G_1^m = v\left(g_{s1}^m, g_{h1}^m\right)$. The first order condition characterizing $\hat{G}_1^m$ thus becomes

$$\begin{aligned} C'(\hat{G}_1^m) = & 2 + 2\beta\left\{2 - (2 - \Lambda)\, F(\hat{\theta})\right\} \\ & + 2\beta f(\hat{\theta})\hat{\theta}'\left\{\frac{\hat{\mathcal{C}}_2}{2} - 2\left[\hat{G}_2^m - \frac{\Lambda}{2}\hat{G}_2^d\right] - \hat{\theta}\right\}. \end{aligned} \tag{B7}$$

We can now substitute for $\hat{\theta}$ and $\hat{\theta}'$ using eqs. (16)-(17) and simplify the right hand side. This gives

$$C'(\hat{G}_1^m) = 2 + 2\beta\left\{2 - (2 - \Lambda)\left[F(\hat{\theta}\left(\hat{v}_1^m\right)) - \chi f(\hat{\theta}(\hat{G}_1^m))\right]\right\}, \tag{B8}$$

with χ defined as in (21). The result then follows immediately from (B2).

Proof of Proposition 1

Part (a) follows immediately from (B4) and (B5) which show that a lower β implies a lower $C'\left(\hat{v}_2^k\right)$ and hence a lower level of household production $\hat{v}_2^k$ in each marital state k. Thus, more present bias causes a couple to reduce the level of total household production at $t = 2$ in both marital states.

To establish part (b), note that, in terms of total household production levels, the divorce threshold characterized in (16) can be rewritten as

$$\hat{\theta} = \frac{C(\hat{v}_2^m) - C(\hat{v}_2^d)}{2} - (1 + \beta)\left[(G_1^m + \hat{v}_2^m) - \frac{\Lambda}{2}\left(G_1^m + \hat{v}_2^d\right)\right]. \tag{B9}$$

Differentiating we obtain

$$\begin{aligned} \frac{\partial\hat{\theta}}{\partial\beta} = & \left[(1 + \beta)\,\frac{\Lambda}{2} - \frac{C'\left(\hat{v}_2^d\right)}{2}\right]\frac{\partial\hat{v}_2^d}{\partial\beta} - \left[(1 + \beta) - \frac{C'\left(\hat{v}_2^m\right)}{2}\right]\frac{\partial\hat{v}_2^m}{\partial\beta} \\ & - \left[(G_1^m + \hat{v}_2^m) - \frac{\Lambda}{2}\left(G_1^m + \hat{v}_2^d\right)\right]. \end{aligned} \tag{B10}$$

The first two terms vanish due to eqs. (B4) and (B5), leaving

$$\frac{\partial \hat{\theta}}{\partial \beta} = -\left[(G_1^m + \hat{v}_2^m) - \frac{\Lambda}{2} \left(G_1^m + \hat{v}_2^d \right) \right] < 0, \tag{B11}$$

where the sign follows from the facts that $\hat{v}_2^m > \hat{v}_2^d$ (see Lemma 1) and $\Lambda < 2$. Next note that $\hat{\theta}(G_1^m)$ converges to $\tilde{\theta}(G_1^m)$ as β approaches unity. Since $\hat{\theta}$ additionally decreases with β, any reduction in β away from unity raises the divorce threshold $\hat{\theta}(G_1^m)$ above $\tilde{\theta}(G_1^m)$. Present-biased couples therefore divorce for a larger set of match quality realizations than in the first-best solution.

Proof of Proposition 2

As noted in proof of Lemma 2 the first order condition characterizing the couple's choice of first period level of household production satisfies, $C'(\hat{v}_1^m) = \mathcal{B}(\hat{v}_1^m)$, where the marginal benefit $\mathcal{B}(\hat{v}_1^m)$ is given by the right hand side of (B8), and with χ defined as in (21). The cost function on the left hand side does not depend on β but the marginal benefit does. Hence we now characterize the impact of β on $\mathcal{B}$ for a given value of v_1^m, noting that this impact can both be direct (in terms of β appearing directly in $\mathcal{B}$) or via future choices. For notational convenience, we suppress the arguments of $\hat{\theta}$. We then obtain that

$$\begin{aligned} \frac{\partial \mathcal{B}}{\partial \beta} =& 2\left\{ \left[2 - (2-\Lambda)\left(F(\hat{\theta}) - \chi f(\hat{\theta}) \right) \right] \right. \\ & \left. -\beta(2-\Lambda)\left[f(\hat{\theta})\frac{\partial \hat{\theta}}{\partial \beta} - \left(f(\hat{\theta})\frac{\partial \chi}{\partial \beta} + \chi f'(\hat{\theta})\frac{\partial \hat{\theta}}{\partial \beta} \right) \right] \right\}. \end{aligned} \tag{B12}$$

We focus on evaluating this expression in the limit where $\beta \to 1$. This allows us to consider how the introduction of present bias—i.e, a marginal reduction in β away from unity—affects $\mathcal{B}$. We first note from eq. (21) that $\chi = 0$ in the limit where $\beta \to 1$. Moreover, from eqs. (B11) and (21) it follows that

$$\left.\frac{\partial \hat{\theta}}{\partial \beta}\right|_{\beta \to 1} = \left.\frac{\partial \chi}{\partial \beta}\right|_{\beta \to 1} = -\left[(G_1^m + \hat{v}_2^m) - \frac{\Lambda}{2}\left(G_1^m + \hat{v}_2^d \right) \right] < 0. \tag{B13}$$

It now follows immediately that,

$$\left.\frac{\partial \mathcal{B}}{\partial \beta}\right|_{\beta \to 1} = 2\left[2 - (2-\Lambda) F(\hat{\theta}) \right] > 0. \tag{B14}$$

Thus, a decrease in β from unity—i.e., the introduction of present bias in couples' preferences—reduces the marginal benefits from first-period household production. Given the convexity of the cost function C, it thus follows that a couple that are (marginally) present biased will choose a level of household production that is below the first best level.

Proof of Corollary 1

Suppose that the total private gains from marriage are drawn from a uniform probability density function: $\theta \sim \mathcal{U}(\underline{\theta}, \overline{\theta})$. Under this assumption,

$$F(\hat{\theta}) = \frac{\hat{\theta} - \underline{\theta}}{\overline{\theta} - \underline{\theta}}, \quad \text{and} \quad \chi f(\hat{\theta}) = \frac{\chi}{\overline{\theta} - \underline{\theta}}. \tag{B15}$$

Subtracting and using (21) we obtain

$$F(\hat{\theta}) - \chi f(\hat{\theta}) = \frac{\hat{\hat{\theta}} - \underline{\theta}}{\overline{\theta} - \underline{\theta}}, \tag{B16}$$

where

$$\hat{\hat{\theta}} \equiv \frac{C(\hat{v}_2^m) - C(\hat{v}_2^d)}{2} - (1+\beta)\left(\frac{3-\beta}{2}\right)\left[(v_1^m + \hat{v}_2^m) - \frac{\Lambda}{2}\left(v_1^m + \hat{v}_2^d\right)\right]. \tag{B17}$$

The first-order condition characterizing v_1^m is as before $C'(v_1^m) = \mathcal{B}(\hat{v}_1^m)$, where the marginal benefit can now be written as

$$\mathcal{B}(\hat{v}_1^m) = 2 + 2\beta\left[2 - (2-\Lambda)\left(\frac{\hat{\hat{\theta}} - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right)\right]. \tag{B18}$$

Differentiating $\mathcal{B}$ with respect to β we obtain

$$\frac{\partial \mathcal{B}}{\partial \beta} = 2\left[2 - (2-\Lambda)\left(\frac{\hat{\hat{\theta}} - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right)\right] - 2\beta\left(\frac{2-\Lambda}{\overline{\theta} - \underline{\theta}}\right)\frac{\partial \hat{\hat{\theta}}}{\partial \beta}. \tag{B19}$$

To establish the result, we need to show that $\partial\mathcal{B}/\partial\beta > 0$ for any $\beta \in (0,1)$ (and for any given value of v_1^m). The first term on the r.h.s. of (B19) is positive. Thus, a sufficient condition for $\partial\mathcal{B}/\partial\beta > 0$ is that $\partial\hat{\hat{\theta}}/\partial\beta < 0$. Differentiating eq. (B17) with respect to β we obtain,

$$\begin{aligned}\frac{\partial \hat{\hat{\theta}}}{\partial \beta} =& \left[\frac{C'(\hat{v}_2^m)}{2} - \psi(1+\beta)\right]\frac{\partial \hat{v}_2^m}{\partial \beta} - \left[\frac{C'(\hat{v}_2^d)}{2} - \psi(1+\beta)\frac{\Lambda}{2}\right]\frac{\partial \hat{v}_2^d}{\partial \beta} \\ & - (1-\beta)\left[(v_1^m + \hat{v}_2^m) - \frac{\Lambda}{2}(v_1^m + \hat{v}_2^d)\right],\end{aligned} \tag{B20}$$

where we define $\psi \equiv (3-\beta)/2 > 1$. We can substitute for the marginal cost using eqs. (B4) and (B5) and also use that these same equations imply that

$$\frac{\partial \hat{v}_2^m}{\partial \beta} = \frac{2}{C''(\hat{v}_2^m)} \quad \text{and} \quad \frac{\partial \hat{v}_2^d}{\partial \beta} = \frac{\Lambda}{C''(\hat{v}_2^d)}, \tag{B21}$$

We further restrict our attention to production functions giving rise to a cost function satisfying $C''' \leqslant 0$.[27] Thus,

$$\frac{\partial\hat{\hat{\theta}}}{\partial\beta} = -\frac{(1+\beta)(\psi-1)}{2}\left[\frac{4}{C''(\hat{v}_2^m)} - \frac{\Lambda^2}{C''(\hat{v}_2^d)}\right] - (1-\beta)\left[(v_1^m + \hat{v}_2^m) - \frac{\Lambda}{2}\left(v_1^m + \hat{v}_2^d\right)\right] < 0, \tag{B22}$$

where the sign follows from the facts that $\hat{v}_2^m > \hat{v}_2^d$ (See Lemma 1) and $2 > \Lambda$. Thus, if θ is drawn from a uniform probability density function, the couple will underproduce family-specific capital for any degree of present bias.

Proof of Proposition 3

Note that that the condition that characterizes the first period investment level $\bar{G}_1^m$ by a naive couple can be written as

$$C'\left(\bar{G}_1^m\right) = 2 + 2\beta\left[2 - (2-\Lambda)\,F(\tilde{\theta}(\bar{G}_1^m))\right], \tag{B23}$$

whereas the condition that characterizes the first period investment level $\hat{G}_1^m$ by a sophisticated couple can be written as in (B8). Since the cost function is the same, $\bar{G}_1^m \geq \hat{G}_1^m$ if and only if

$$F(\tilde{\theta}(\bar{G}_1^m)) \leq F(\hat{\theta}(\hat{G}_1^m)) - \chi f(\hat{\theta}(\hat{G}_1^m)), \tag{B24}$$

where χ is defined as in (21). Then define

$$\kappa \equiv \frac{\chi}{\hat{\theta}(\hat{G}_1^m) - \tilde{\theta}(\bar{G}_1^m)}. \tag{B25}$$

Defining also

$$\check{\theta}\left(G_1^m\right) \equiv \hat{\mathcal{C}}_2/2 - 2\left[(G_1^m + \hat{v}_2^m) - (\Lambda/2)\left(G_1^m + \hat{v}_2^d\right)\right], \tag{B26}$$

it follows that

$$\chi = \frac{(1+\beta)}{2}\left[\hat{\theta}(\hat{G}_1^m) - \check{\theta}(\hat{G}_1^m)\right], \tag{B27}$$

whereby

$$\kappa = \frac{(1+\beta)}{2}\left[\frac{\hat{\theta}(\hat{G}_1^m) - \check{\theta}(\hat{G}_1^m)}{\hat{\theta}(\hat{G}_1^m) - \tilde{\theta}(\bar{G}_1^m)}\right]. \tag{B28}$$

But, $\check{\theta}(\hat{G}_1^m) \simeq \tilde{\theta}\left(\bar{G}_1^m\right)$ since both involve exponential discounting only. Hence $\bar{G}_1^m \geq \hat{G}_1^m$ if and only if

$$\kappa\left[\hat{\theta}(\hat{G}_1^m) - \tilde{\theta}(\bar{G}_1^m)\right] f(\hat{\theta}(\hat{G}_1^m)) \leq F(\hat{\theta}(\hat{G}_1^m)) - F(\tilde{\theta}(\bar{G}_1^m)), \tag{B29}$$

with $\kappa \simeq (1+\beta)/2$

[27] Many commonly used production function yield cost functions that satisfy having $C''' \leq 0$. Examples include the Cobb-Douglas functions $v(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2}$ with $\frac{1}{2} \leqslant \alpha_1 + \alpha_2 < 1$ or the independent production functions $v(x_1, x_2) = x_1^{\alpha} + x_2^{\alpha}$ with $\frac{1}{2} \leqslant \alpha < 1$.

Proof of Proposition 4

From the perspective of period zero, and for a given divorce cost ζ, the couple's joint intertemporal utility, $V_0 = V_{0s} + V_{0h}$, can be written as

$$\begin{aligned}\frac{V_0}{\beta} =& \sum_i \omega_i + 2\hat{v}_1^m - C\left(\hat{v}_1^m\right) + 2\int_{\hat{\theta}\left(\zeta, \hat{v}_1^m(\zeta)\right)}^{\infty} \theta f\left(\theta\right) \mathrm{d}\theta \\ &+ \left[1 - F(\hat{\theta})\right] \left\{2\left[\sum_i \omega_i + 2\left(\hat{v}_1^m + \hat{v}_2^m\right)\right] - C\left(\hat{v}_2^m\right)\right\} \\ &+ F(\hat{\theta}) \left\{2\left[\sum_i \omega_i + \Lambda\left(\hat{v}_1^m + \hat{v}_2^d\right)\right] - \zeta - C\left(\hat{v}_2^d\right)\right\},\end{aligned} \tag{B30}$$

where the divorce threshold that the couple expect themselves to adopt at $t = 2$ can now be written as

$$\hat{\theta} = \frac{\hat{\mathcal{C}}_2 - \zeta}{2} - (1+\beta)\left[\left(\hat{v}_1^m + \hat{v}_2^m\right) - \frac{\Lambda}{2}\left(\hat{v}_1^m + \hat{v}_2^d\right)\right]. \tag{B31}$$

Note that we used that the marital status-specific time allocations at $t = 2$ do not depend on ζ (see 11 and 13), whereas that at $t = 1$ does: $\partial \hat{v}_1^m / \partial \zeta|_{\zeta=0}$ is generally non-zero. Indeed we will show below that it is strictly positive for the case of uniformly distributed match quality shocks. Note also that ζ affects the divorce threshold both directly and also indirectly via $\hat{v}_1^m$. As we will consider the incentives to *introduce* a divorce cost – that is, to increase ζ from zero – the couple's behaviour will be evaluated at $\zeta = 0$, thus corresponding to the laissez-faire equilibrium described above.

Differentiating (B30), evaluating at $\zeta = 0$, and collecting terms gives that

$$\begin{aligned}\frac{\partial V_0/\partial\zeta|_{\zeta=0}}{\beta} =& \left(2 - C'\left(\hat{v}_1^m\right)\right) \left.\frac{\partial \hat{v}_1^m}{\partial \zeta}\right|_{\zeta=0} \\ &+ 2f(\hat{\theta}) \left\{\frac{\hat{\mathcal{C}}_2}{2} - 2\left[\hat{G}_2^m - \frac{\Lambda}{2}\hat{G}_2^d\right] - \hat{\theta}\right\} \left[\frac{\partial\hat{\theta}}{\partial\zeta} + \frac{\partial\hat{\theta}}{\partial\hat{v}_1^m}\frac{\partial\hat{v}_1^m}{\partial\zeta}\right]_{\zeta=0} \\ &+ 2\left\{2 - F(\hat{\theta})\left(2 - \Lambda\right)\right\} \left.\frac{\partial \hat{v}_1^m}{\partial \zeta}\right|_{\zeta=0} - F(\hat{\theta}),\end{aligned} \tag{B32}$$

where $\left[\frac{\partial\hat{\theta}}{\partial\zeta} + \frac{\partial\hat{\theta}}{\partial\hat{v}_1^m}\frac{\partial\hat{v}_1^m}{\partial\zeta}\right]_{\zeta=0}$ is the total – direct plus indirect – effect of ζ on $\hat{\theta}$, evaluated at $\zeta = 0$. Note that, from (B31),

$$\frac{\partial\hat{\theta}}{\partial\hat{v}_1^m} = -\frac{(1+\beta)}{2}\left(2 - \Lambda\right) \quad \text{and} \quad \frac{\partial\hat{\theta}}{\partial\zeta} = -\frac{1}{2}. \tag{B33}$$

Using this, and also substituting for $\hat{\theta}$ using (B31) evaluated at $\zeta = 0$, and using also the definition of χ in (21) gives

$$\begin{aligned}\frac{\partial V_0/\partial\zeta|_{\zeta=0}}{\beta} =& \left[2 - C'\left(\hat{v}_1^m\right) + 2\left\{2 - \left(2 - \Lambda\right)\left[F(\hat{\theta}) - f(\hat{\theta})\chi\right]\right\}\right] \left.\frac{\partial \hat{v}_1^m}{\partial \zeta}\right|_{\zeta=0} \\ &- \left\{F(\hat{\theta}) - (1-\beta) f(\hat{\theta})\left[\hat{G}_2^m - \frac{\Lambda}{2}\hat{G}_2^d\right]\right\}.\end{aligned} \tag{B34}$$

Using then the characterization of the laissez-faire choice of first period investment in (B8) to substitute for $2 - C'(\hat{v}_1^m)$ and also using the definition of χ to substitute in the second term gives

$$\frac{\partial V_0/\partial\zeta|_{\zeta=0}}{\beta} = \left\{2 - (2-\Lambda)\left[F(\hat{\theta}) - f(\hat{\theta})\chi\right]\right\} 2(1-\beta)\left.\frac{\partial \hat{v}_1^m}{\partial \zeta}\right|_{\zeta=0} - \left[F(\hat{\theta}) - \frac{2}{(1+\beta)} f(\hat{\theta})\chi\right]. \tag{B35}$$

Note that, in the limit where $\beta \to 1$, $\partial V_0/\partial\zeta|_{\zeta=0} = -\beta F(\hat{\theta}) < 0$ (recall that χ limits to zero as well), implying that if the couple's present bias is sufficiently small, then they will not benefit from a contracted divorce cost. But for inframarginal degress of present bias, $\partial V_0/\partial\zeta|_{\zeta=0} > 0$ if and only if the right hand side of (B35) is positive. Solving for $F(\hat{\theta})$, this is equivalent to the stated condition.

We can next readily confirm that $\hat{v}_1^m$ is increasing in ζ under the assumption of uniformly distributed match quality shocks, $\theta \sim \mathcal{U}\left(\underline{\theta}, \overline{\theta}\right)$. With a divorce cost $\zeta > 0$, the first order condition characterizing $\hat{v}_1^m(\zeta)$ can be written as before as $C'(\hat{v}_1^m(\zeta)) = \mathcal{B}$, but where the marginal benefit can now be written as

$$\begin{aligned}\mathcal{B} =& 2 + 2\beta\left[2 - (2-\Lambda)F(\hat{\theta})\right] \\ &+ 2\beta f(\hat{\theta})\left\{\frac{\hat{\mathcal{C}}_2}{2} - 2\left[\hat{G}_2^m + \frac{\Lambda}{2}\hat{G}_2^d\right] - \frac{\zeta}{2} - \hat{\theta}\right\}\frac{\partial\hat{\theta}}{\hat{v}_1^m},\end{aligned} \tag{B36}$$

where $\hat{G}_2^k = \hat{v}_1^m(\zeta) + \hat{v}_2^k$. Consider now the direct effect of ζ on the marginal benefit $\mathcal{B}$, noting that part of this direct effect is the direct effect on $\hat{\theta}$. In doing we invoke the uniformity assumption which implies that $f(\theta) = f = 1/\left(\overline{\theta} - \underline{\theta}\right)$ is constant, and we further note that $\partial\hat{\theta}/\partial\hat{v}_1^m$ is unaffected by ζ (see eq. B33). Thus we obtain

$$\frac{\partial\mathcal{B}}{\partial\zeta} = -2\beta(2-\Lambda) f\frac{\partial\hat{\theta}}{\partial\zeta} - 2\beta f\frac{1}{2}\frac{\partial\hat{\theta}}{\hat{v}_1^m} - 2\beta f\frac{\partial\hat{\theta}}{\partial\zeta}\frac{\partial\hat{\theta}}{\hat{v}_1^m}. \tag{B37}$$

Invoking (B33) the last two terms cancel and we obtain that $\partial\mathcal{B}/\partial\zeta = 2\beta f(1-\Lambda/2) > 0$. Since ζ raises the marginal benefit to $\hat{v}_1^m$ it follows that, under uniform match quality shocks, $\partial\hat{v}_1^m/\partial\zeta > 0$.

Proof of Proposition 5

Consider first the couple's time allocation choice in the second period, starting with a couple that remain married. With a proportional tax τ_2 on earnings, the time allocation choice that maximizes their joint intertemporal utility satisfies

$$\frac{\omega_i(1-\tau_2)}{v_i(g_{s2}^m, g_{h2}^m)} = 2(1+\beta). \tag{B38}$$

Contrasting this to the characterization of the first best time allocation in (23), we see that the efficiency-restoring tax, denoted $\hat{\tau}_2$, satisfies

$$\frac{(1+\beta)}{(1-\hat{\tau}_2)} = 2. \tag{B39}$$

Solving for $\hat{\tau}_2$ gives (30).

In contrast, after a divorce each spouse chooses his or her own time allocation to maximize the own intertemporal utility. However, as before the assumption in (8) implies that their choices also maximize the joint utility. Hence the time allocation choices of a divorced couple satisfies

$$\frac{\omega_i (1-\tau_2)}{v_i (g^m_{s2}, g^m_{h2})} = \Lambda (1+\beta). \tag{B40}$$

Contrasting this to the characterization of the first best time allocation in (23), we see that $\hat{\tau}_2$ as defined in (30) restores efficiency of the time allocation also among divorced couples. With $\hat{\tau}_2$ set as in (30), it thus follows that $\hat{g}^m_{i2} = \tilde{g}^m_{i2}$ and $\hat{g}^d_{i2} = \tilde{g}^d_{i2}$.

Turning next to the divorce decision, the match quality threshold adopted by couples under the policy is given by

$$\hat{\theta} = \frac{(1-\hat{\tau}_2)\tilde{\mathcal{C}}_2 - \eta}{2} - (1+\beta)\left[(G^m_1 + \tilde{v}^m_2) - \frac{\Lambda}{2}\left(G^m_1 + \tilde{v}^d_2\right)\right], \tag{B41}$$

where we used that $\hat{\tau}_2$ induces the first best time allocation in each marital state (and that the lump-sum policy element α_2 does not affect the divorce decision). Setting this theshold equal to the first best threshold in (24) as solving for the efficiency-restoring tax $\hat{\eta}$ gives

$$\hat{\eta} = (1-\beta)\left[2(G^m_1 + \tilde{v}^m_2) - \Lambda\left(G^m_1 + \tilde{v}^d_2\right) - \frac{1}{2}\tilde{\mathcal{C}}_2\right], \tag{B42}$$

where we also made use of the expression for $\hat{\tau}_2$ in (30). Note that, unlike $\hat{\tau}_2$, $\hat{\eta}$ generally depends on the capital stock carried forward from the first period, G^m_1. The first period policy will induce the couple to choose $G^m_1 = \tilde{G}^m_1$; evaluating $\hat{\eta}$ at this first period choice, gives the expression for $\hat{\eta}$ in (30).

The lump-sum transfer given to the couple at the outset of the second period is the expected earnings tax revenue within the period plus the expected divorce fee:

$$\hat{\alpha}_2 = \hat{\tau}_2\left[(1-F(\tilde{\theta}))\sum\nolimits_i[\omega_i (1-\tilde{g}^m_{i2})] + F(\tilde{\theta})\sum\nolimits_i\left[\omega_i\left(1-\tilde{g}^d_{i2}\right)\right]\right] + F(\tilde{\theta})\hat{\eta}. \tag{B43}$$

Consider now the behavior of couples in the first period under policy, and with the second period policy set as outlined above. The couple's joint intertemporal utility can then be written as

$$
\begin{aligned}
V_1 = & \sum\nolimits_i \omega_i \left(1-\tau_1\right)\left(1-g_{i1}^m\right) + \alpha_1 + v\left(g_{s1}^m, g_{h1}^m\right) \\
& + \beta \left\{ \hat{\alpha}_2 + \left(1 - F(\tilde{\theta})\right) \left[\sum\nolimits_i \omega_i \left[\left(1-\hat{\tau}_2\right)\left(1-\tilde{g}_{i2}^m\right)+1\right] + 4\tilde{G}_2^m \right] \right. \\
& \left. + F(\tilde{\theta}) \left[\sum\nolimits_i \omega_i \left[\left(1-\hat{\tau}_2\right)\left(1-\tilde{g}_{i2}^d\right)+1\right] - \hat{\eta} + 2\Lambda \tilde{G}_2^d \right] + 2\int_{\tilde{\theta}}^{\infty} \theta f(\theta) \mathrm{d}\theta \right\}.
\end{aligned} \tag{B44}
$$

where we used that the second period policy induces first best behaviour. Substituting in $\hat{\tau}_2$, $\hat{\eta}$ and $\hat{\alpha}_2$, the first-order condition satisfied by the couple's first period effort choice reduces to

$$
\frac{(1-\tau_1)\omega_i}{v_i(\hat{g}_{s1}^m, \hat{g}_{h1}^m)} = 2 + 2\beta \left[2 - (2-\Lambda) F(\tilde{\theta}) \right]. \tag{B45}
$$

Using the first-order condition for the first-best solution [eq. (25)], the efficiency-restoring first-period earnings tax rate satisfies

$$
\frac{2 + 2\beta \left[2 - (2-\Lambda) F(\tilde{\theta}) \right]}{1-\hat{\tau}_1} = 6 - 2\left(2-\Lambda\right) F(\tilde{\theta}). \tag{B46}
$$

Solving for $\hat{\tau}_1$ gives the expression in (31). The first period lump sum transfer $\hat{\alpha}_1$ finally is simply the first period earnings by the couply given that tax rate $\hat{\tau}_1$ and the fact that they, under the policy, chooses the efficient first period time allocation.

Appendix D: A Calibrated Model (Intended for Online Publication)

In this Appendix, we extend the model to a T-period setting which we calibrate to the US economy. Doing so allows for a richer set of family paths and enables us to address key quantitative questions: what impact does an empirically relevant level of present bias have on family behavior and outcomes? And what structure and level of policy is required to restore efficiency?

The family-specific capital enjoyed in period t is, as before, $G_t + \upsilon\left(g_{st}, g_{ht}\right)$ where G_t is the amount of capital carried forward from the previous period. We now also allow for capital depreciation by setting

$$G_t = \gamma\left[G_{t-1} + \upsilon\left(g_{s,t-1}, g_{h,t-1}\right)\right], \tag{C1}$$

where $\gamma \in (0,1)$ is the capital carry-forward rate. In line with our interpretation of the capital good as investments in children, we assume that the investment process terminates at some $T_0 < T$. In each period, a couple experience a match quality shock θ_t, which is i.i.d. across periods and drawn from a distribution $F(\cdot)$. An adverse temporary shock may induce a couple to divorce and if they do so they remain divorced forever. Thus, there will be couples who find themselves divorced but still making investments and, conversely, couples who find themselves still married but with no further investments to make. We will focus on sophisticated behaviour.

The efficiency-restoring policy will involve (i) an earnings tax τ_t^k which generally varies with marital status, $k = m, d$, and marriage duration $t \leq T_0$, and (ii) a divorce tax η_t that also varies with duration $t \leq T$.[28]

The post-divorce earnings tax has a simple analytical solution:

$$\hat{\tau}_t^d = (1-\beta)\frac{\sum_{n=1}^{T-t}(\delta\gamma)^n}{1+\sum_{n=1}^{T-t}(\delta\gamma)^n} \quad \text{for} \quad t \leq T_0, \tag{C2}$$

which generalizes eq. (30). This shows that the earnings tax, at any duration t (within the investment phase), is proportional to the degree of present bias. It also varies with t as a longer marriage duration implies a shorter remaining horizon. However, given that there are several post-investment periods the ratio on the right hand side of eq. (C2) will be close to $\delta\gamma$ for any investment period $t \leq T_0$. Hence while $\hat{\tau}_t^d$ is decreasing over time due to the horizon effect, it is, for reasonable δ and γ, very close to $\delta\gamma(1-\beta)$ throughout a couple's investment phase.

A similar expression, though somewhat more involved, can be provided for the earnings tax facing married couples:

$$\hat{\tau}_t^m = (1-\beta)\frac{\gamma\delta\tilde{V}'_{t+1}}{2+\gamma\delta V'_{t+1}} \quad \text{for } t \leq T_0, \tag{C3}$$

[28]No earnings tax is imposed at $t > T_0$ since there are no time-allocation decisions to be made once the investment process has terminated. The divorce tax however is applicable across all periods. As in the three-period model, the efficiency-restoring policy also involves a transfer to each couple at the beginning of each period that corresponds to the expected total tax paid in that period.

where

$$\tilde{V}_t' = \sum_{n=0}^{T-t} (\delta\gamma)^n \left\{ 2 \prod_{j=0}^{n} \left[1 - F(\tilde{\theta}_{t+j}) \right] + \Lambda \left[1 - \prod_{j=0}^{n} \left[1 - F(\tilde{\theta}_{t+j}) \right] \right] \right\}, \tag{C4}$$

with $\tilde{\theta}_{t+j} = \tilde{\theta}_{t+j}(\tilde{G}_{t+j})$, captures the joint marginal value of capital, in the efficient allocation, to a couple entering period t as married.[29] This generalizes both eq. (31) and eq. (30) in the three-period model. It is straightforward to show that $\hat{\tau}_t^m \leq \hat{\tau}_t^d$, with the difference obtaining from the fact that $\Lambda < 2$ and positive future divorce risk. In particular, if either (i) $\Lambda \to 2$, or (ii) at t, there was no future divorce risk, then the earnings tax at t would coincide across marital status.[30] When we calibrate the model we obtain that $\Lambda/2 \geq 0.9$ (see below). From this it follows that the difference between $\hat{\tau}_t^m$ and $\hat{\tau}_t^d$ will be fairly small, and both will be close to $(1-\beta)\gamma\delta$ throughout the couple's investment phase.

We assume a normal distribution for the match quality shocks, $\theta_t \sim N(\mu_\theta, \sigma_\theta)$, and a simple additively separable iso-elastic specification for the household production function,

$$\upsilon(g_{st}, g_{ht}) = a g_{st}^b + a g_{ht}^b, \tag{C5}$$

where $a, b > 0$. We set $T = 40$, thus effectively capturing the time from the median age at first marriage until retirement age.[31,32] A number of parameters are set based on the literature, including β and δ. For the latter, a large macro literature, following Prescott (1986), have argued that a reasonable range for the annual discount rates is 2-7 percent so we fix a value in the middle of this range, $\delta = 0.95$. For β there have been recent findings of little or no present bias in experiments with monetary payments (e.g., Andreoni and Sprenger, 2012), but clear present bias in the experiments based on real effort tasks (Augenblick, Niederle and Sprenger, 2013). Furthermore, it is not clear whether joint decision-making by couples or other groups of individuals are more or less time-consistent than is individual decision-making (Jackson and Yariv, 2015). Contrasting choices made separately and jointly, Carlsson and Yang (2013) find no evidence that married couples behave systematically more time-consistent when making joint decisions than when making individual ones. The estimates of Augenblick, Niederle and Sprenger (2013) suggest values of β of around 0.9,

[29] Intuitively, an extra unit of capital increases a couple's joint utility by 2 in any remaining period in which the couple are still married and by Λ in any period in which they are divorced. The expression captures this while accounting also for discounting and capital depreciation.

[30] This explains why, in the three period model, we obtained that $\hat{\tau}_2^m = \hat{\tau}_2^d$ as we assumed zero divorce risk at $t = 3$.

[31] The median age at first marriage, according to the latest US Census Bureau, stands at 27.1 for women and 29.2 for men.

[32] We include a further 10 "retirement" periods during which a couple cannot divorce. The purpose of this is to prevent a sharp increase in the divorce hazard for a couple that approach T. Indeed, otherwise the future component of the gain to marriage would become quickly dominated by the match quality shocks.

Table C.1: Baseline Calibration of the Extended Dynamic Model

	Parameter/Stylized Fact	Value	Source
δ	Annual discount date	0.95	Literature
β	Present-bias factor	0.95	Literature
ω_h	Male wage rate	1.00	Normalized
ω_s	Female wage rate	0.82	BLS
γ	Capital carry-forward rate	0.975	Literature
a	Investment efficiency	0.042	Matched
b	Investment elasticity	0.70	Matched
λ_s	Female post-divorce utility	1	Fixed
λ_h	Male post-divorce utility	0.84	Matched
μ_θ	Match quality location	-0.01	Matched
σ_θ	Match quality spread	0.90	Matched
	Marriage survival at 10 years	0.68	NCHS
	"Empty nest" divorce effect	0.75	Literature
	Married males' labour market time	0.67	BLS, ATUS
	Married females' labour market time	0.36	BLS, ATUS
	Maximum impact divorce at 10 years	0.15	Literature

which would seem like a natural lower bound for the current exercise and hence we impose a conservative value of $\beta = 0.95$.

We set the length of the investment phase T_0 to 20 years, corresponding to the average school leaving age in the US (OECD, 2008). The "gender pay gap" is generally defined as the ratio of female to male median yearly earnings among full-time, year-round workers. In 2013, the female-to-male earnings ratio was 82 percent (BLS, 2014). Hence we normalize the male wage to unity, $w_h = 1$, and set the female wage to $w_s = 0.82$. Based on the literature, we set a modest degree of human capital depreciation, $\gamma = 0.975$ (Manuelli, Seshadri and Shin, 2012). We further assume a wife continues to fully enjoy the family-specific capital post divorce, $\lambda_s = 1$, but not the husband.

We calibrate the remaining parameters: the household production technology parameters a and b, the location and spread of the distribution of match quality shocks μ_θ and σ_θ, and the husband's post-divorce enjoyment of the capital good, λ_h. To do so we use empirical stylized facts on how couples with children allocate their time, on marriage survival rates, on the impact divorce have on kids' outcomes, and on the impact of children leaving on the divorce risk.

Combining data on labor force participation from the U.S. Bureau of Labor Statistics with time use data from the American Time Use Survey, we use as a stylized fact that married women and men with children in the household spend on average 64 percent and 33 percent of (non-leisure) time on household work respectively.[33] Turning to marriage survival rates, in the latest figures, the probability of a first marriage being intact after 10 years is 0.68.[34] The literature has also noted an "empty nest effect", i.e., an increase in the divorce risk when children leave the parental home. Hiedemann, Suhomlinova and O'Rand (1998) find that the empty nest effect increases the divorce hazard by 50 - 200 percent. Walker and Zhu (2004) find an effect at the lower end of this range for the UK. In our calibrated model, the divorce hazard is at its lowest when the kids are in their early teens, and we calibrate the model so that it generates an increase in the hazard of 75 percent from this point to two years after the termination of investments.

The most controversial stylized fact is the impact of divorce on kids' outcomes, represented in the model by the accumulated family capital. While there is a strong negative empirical *association* between divorce and children's outcomes, establishing the *causal effect* of divorce on kids outcomes has proven more difficult. Our reading of the literature is that the causal effects are overall limited in size, especially for kids who are older when the divorce occurs.[35] This effectively puts a lower bound on λ_m. In order to capture this in our calibrated model, we compare the capital stock, ten years post-divorce, for divorcing couples relative to couples who remain intact (by duration at divorce), and cap the negative effect of divorce at 15 percent. The final set of calibrated parameter values are shown in Table C.1. The top panel gives the parameter values imposed based on the literature. The middle panel gives the parameter values that were obtained by matching the model to the stylized facts listed in the bottom panel.

In Figure C.1, we consider the impact of present bias on marital outcomes. The left panel shows the fraction of couples still married by years since marriage. In addition to the matched fraction at 10 years since marriage (highlighted with a red marker), the model also closely matches the median marriage duration of 20 years. The figure highlights that even a modest degree of present bias can lead to significant over-divorcing. For example, at durations between 10 and 30 years since marriage, the fraction of couples that have divorced in the laissez-faire allocation is 30-40 percent larger than in the efficient allocation. The extent of overdivorcing also becomes apparent in the right panel which shows the equilibrium divorce hazard by marriage duration. The divorce hazard is naturally U-shaped: it decreases as family capital is gradually accumulated, but then increases again due to the shortening of the remaining time horizon and the gradual depreciation of the capital stock. It is thus at its lowest when

[33]BLS Report 1052 (2014, Tables 5, 6 and 21) give fraction of married women with children under the age of 18 who were in the labor force and the fraction working full time, along with the fraction of men who are in the labor force. Table A-6 from BLS ATUS statistics gives time spent in primary activities by mothers and fathers during the period 2009-2013 by gender and labour force status.

[34]See Copen *et al.* (2012)

[35]For example, see Gruber (2004), Piketty (2003), Björklund and Sundström (2006) and Francesconi *et al.* (2010).

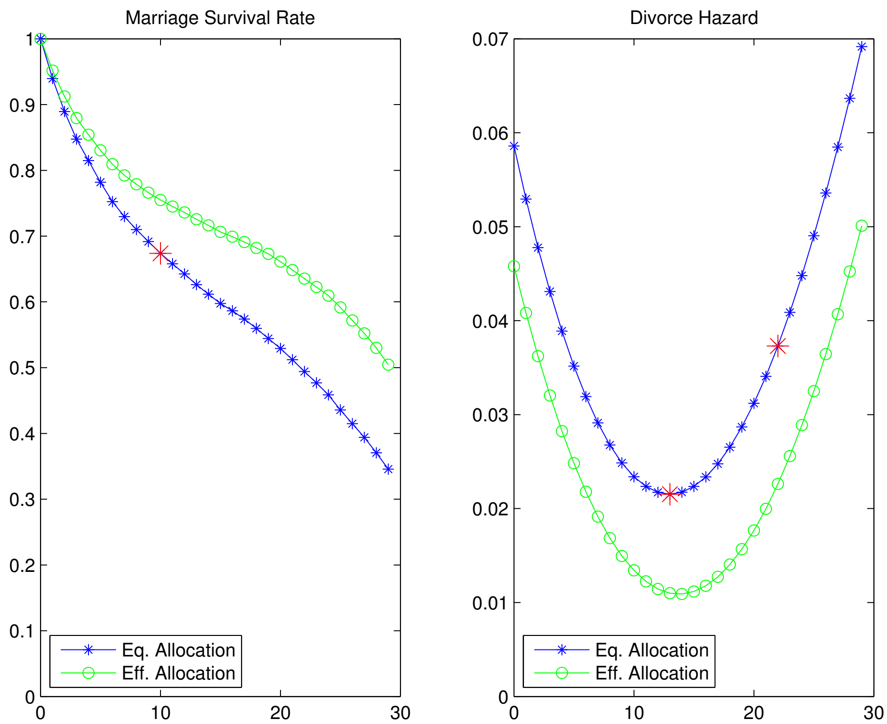

Figure C.1: Marriage survival rate and divorce hazard by marriage duration

the kids are in their teens, and at this stage the divorce hazard under present bias is about *double* the efficient hazard.[36]

In Figure C.2, we present our results for the time allocation choices by females and males over their investment phase. The two top panels compare the laissez-faire equilibrium with the efficient allocation. In general, the effect of present bias on couples' time allocation choices is quantitatively smaller than its impact on divorce decisions. In particular, a present bias reduces both female and male household time by roughly 15-17 percent as compared to the first-best, with relatively little variation by marriage duration. The bottom panel shows how divorced couples devote less time to household production. In proportional terms, divorced couples, both men and women, tend to devote about 20 percent less time to household production relative to married couples.

In Figure C.3, we focus our attention on how the stock of family-specific capital evolves with marriage duration. The left panel shows how the average capital stock in the present-biased equilibrium is lower than in the efficient allocation. Note that this is a combination of more couples being divorced and

[36]The red markers highlight the increase in the divorce hazard from when the child is aged 13 to aged 22 which was match to 75 percent.

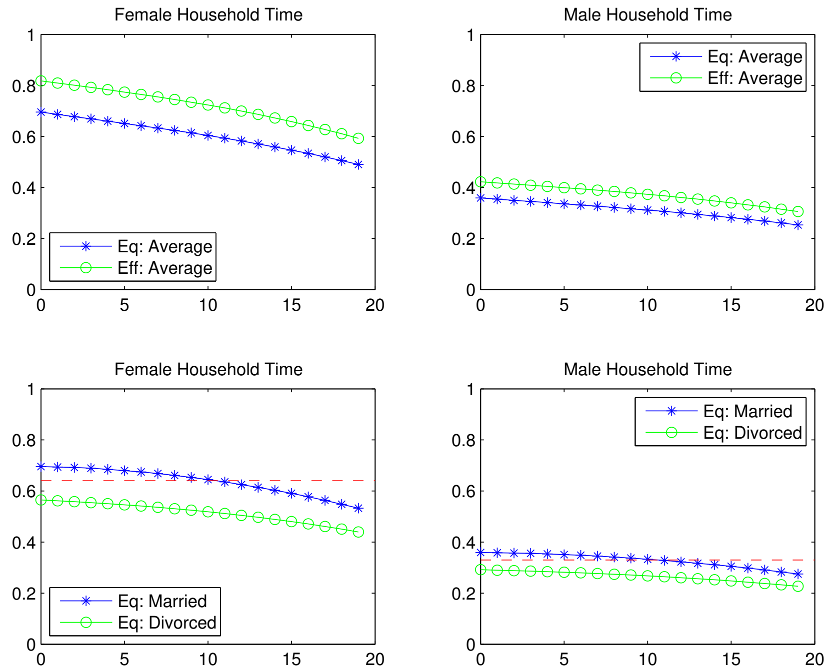

Figure C.2: Fraction of time devoted to household production by marriage duration.

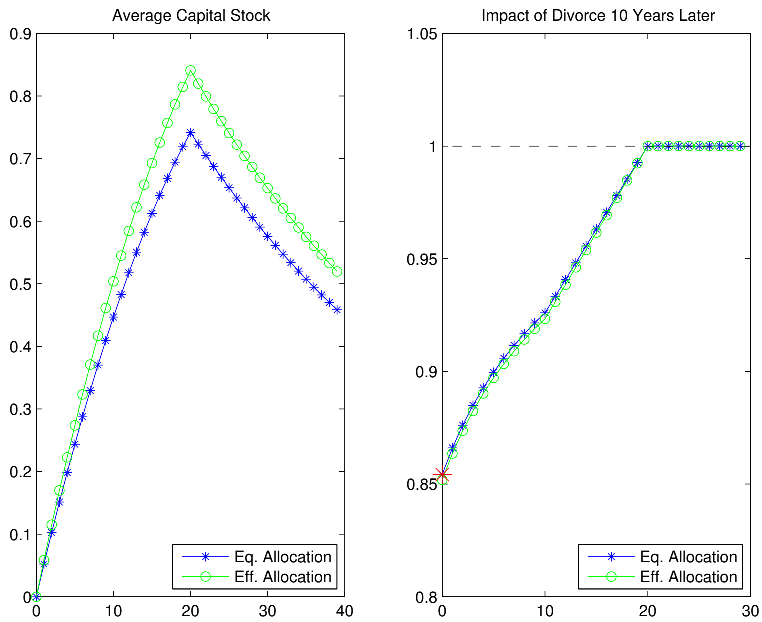

Figure C.3: Average accumulated capital stock and impact of divorce on accumulated capital 10 years on by marriage duration lower investments in each marital state. At any duration, the average capital stock accumulated by a present-biased couple is about 11 percent below the average capital stock they would have had in the efficient allocation. The right panel illustrates the capital stock at 10 year post divorce for couples who divorce at time t relative to the corresponding capital stock for couples who are still intact at $t+10$. Naturally the impact of divorce is larger the earlier it occurs. As noted above, we have calibrated the model so that this impact of divorce does not exceed 15 percent (highlighted by the red marker). Interestingly, present bias does not exacerbate the negative impact of divorce.

In Figure 3 in the main text, we turn to the efficiency-restoring policy in the calibrated model. The left panel shows the optimal earnings tax for married and divorced couples by years since marriage. As noted above, $\hat{\tau}_t^m < \hat{\tau}_t^d$ and both are decreasing over time. In addition, both are quantitatively very close to $(1-\beta)\,\gamma\delta$ (highlighted by the horizontal red line). More interesting is the efficiency-restoring divorce tax shown in the right panel. This is inverted U-shaped and is largest during children's early teenage years, i.e., when the gap between the laissez-fair and the efficient divorce hazard is at its largest. In quantitative terms, the maximum optimal divorce tax of 0.095 corresponds to roughly 10 percent of the annual earnings of a married couple during their investment phase. In contrast, the optimal divorce tax for couples who are well past their investment phase is relatively minor.

Appendix D: Non-Cooperative Behavior (Intended for Online Publication)

In this appendix, we replace the assumption of cooperative behavior in marriage with non-cooperative Nash equilibrium behavior. We focus on the simplest case with pure non-altruistic preferences. In this context it is also natural to focus on the case in which there are no income transfers between the partner, neither in marriage nor after divorce. Hence in this version of the model, each partner always consumes his or her own earnings. Throughout, we set $\delta = 1$.

8.1. Laissez-Faire

As no decisions are taken in the final period, individual utilities are unchanged from the main model. In period 2, conditional on continued marriage, individual utilities are as before and each spouse now maximize his/her own utility only. Hence the second-period time allocations in marriage are now characterized by the following first order conditions,

$$\frac{\omega_j}{\upsilon_i\left(\hat{g}_{s2}^m, \hat{g}_{h2}^m\right)} = 1 + \beta, \text{ for } i = s, h, \tag{D1}$$

which hold simultaneously in equilibrium.

In the divorced state—and assuming no income transfers—a corresponding characterization applies except that spouse i now only enjoys the family capital good at rate λ_i. Hence the second-period time allocations in the divorced state are characterized by

$$\frac{\omega_i}{\upsilon_i\left(\hat{g}_{s2}^d, \hat{g}_{h2}^d\right)} = \lambda_i\left(1 + \beta\right), \text{ for } i = s, h, \tag{D2}$$

holding simultaneously. As we assume that $\lambda_i \leq 1$ for both spouses with at least one inequality, weak strategic complementarity between the two inputs is sufficient to ensure that neither spouse invests more in divorce than in marriage.

We can now consider the divorce decision. Letting $\hat{G}_2^k = G_1^m + \upsilon\left(\hat{g}_{s2}^k, \hat{g}_{h2}^k\right)$ denote the laissez-faire equilibrium level of the capital stock at time $t = 2$ in marital state k it follows that spouse i will prefer continued marriage if and only if

$$\theta \geq \hat{\theta}_i\left(G_1^m\right) \equiv \omega_i\left(\hat{g}_{i2}^m - \hat{g}_{i2}^d\right) - \left(1 + \beta\right)\left(\hat{G}_2^m - \lambda_i \hat{G}_2^d\right). \tag{D3}$$

Note the $\hat{\theta}_i\left(G_1^m\right)$ generally differ across the spouses. Assuming a unilateral divorce regime, divorce occurs if θ falls below either of the two $\hat{\theta}_i\left(G_1^m\right)$. Hence define

$$\hat{\theta}\left(G_1^m\right) = \max_{i=s,h}\left\{\hat{\theta}_i\left(G_1^m\right)\right\}, \tag{D4}$$

for the couple and let $i^*\left(G_1^m\right)$ denote the "decisive spouse" – that is, the spouse with the higher critical match quality (given G_1^m). It then follows that

$$\hat{\theta}'\left(G_1^m\right) = \hat{\theta}_{i^*}'\left(G_1^m\right) = -\left(1 + \beta\right)\left(1 - \lambda_{i^*}\right) \leq 0. \tag{D5}$$

Turning to the initial period, the first order condition characterizing the time allocation of spouse $i = s, h$ can be written as

$$\begin{aligned}\frac{\omega_i}{\upsilon_i\left(\hat{g}_{s1}^m, \hat{g}_{h1}^m\right)} &= 1 + 2\beta\left[1 - F\left(\hat{\theta}\right)(1-\lambda_i)\right] \\ &\quad + \beta f\left(\hat{\theta}\right)\hat{\theta}'\left(G_1^m\right) \times \\ &\quad \left[\omega_i\left(\hat{g}_{i2}^m - \hat{g}_{i2}^d\right) - 2\left(\hat{G}_2^m - \lambda_i \hat{G}_2^d\right) - \hat{\theta}\left(G_1^m\right)\right].\end{aligned} \tag{D6}$$

Of course, in the Nash equilibrium (D6) holds simultaneously for the two spouses.

For the decisive spouse i^* this can be simplified, as in the main model, by substituting for $\hat{\theta}\left(G_1^m\right)$ and $\hat{\theta}'\left(G_1^m\right)$ to obtain that

$$\frac{\omega_{i^*}}{\upsilon_{i^*}\left(\hat{g}_{s1}^m, \hat{g}_{h1}^m\right)} = 1 + 2\beta\left\{1 - (1-\lambda_{i^*})\left[F\left(\hat{\theta}\right) - f\left(\hat{\theta}\right)\chi\right]\right\}, \tag{D7}$$

where

$$\chi \equiv \frac{\left(1-\beta^2\right)\left[\hat{G}_2^m - \lambda_{i^*}\hat{G}_2^d\right]}{2} > 0, \tag{D8}$$

captures the strategic investment incentives. The corresponding condition for the non-decisive spouse does not simplify as easily. We therefore consider two special cases.

Case 1: A Custodial Spouse who is also Decisive.

Suppose that $\lambda_{i^*} = 1$ and $\lambda_{-i^*} < 1$. In keeping with the interpretation of the family capital good as investments in children will refer to the former spouse as "custodial". Note from (D3) that a spouse who loses relatively less of the enjoyment of the family capital good at divorce tends to have a weaker incentives to remain married, but investment differences – across the spouses and across marital states – implies that it is not guaranteed that the spouse with the higher λ is also decisive. Hence we adopt that as an assumption here.

When the decisive spouse is also custodial, we see from (D5) that $\hat{\theta}'\left(G_1^m\right) = 0$ whereby it follows that the strategic investment incentives vanishes for *both* partners, leading to the reduced characterization of the first period investments,

$$\frac{\omega_i}{\upsilon_i\left(\hat{g}_{s1}^m, \hat{g}_{h1}^m\right)} = 1 + 2\beta\left[1 - F\left(\hat{\theta}\right)(1-\lambda_i)\right], \text{ for } i = s, h, \tag{D9}$$

where we also note that the square bracket further reduces to unity for the custodial spouse.

Case 2: Shared Custody

Suppose that $\lambda_s = \lambda_h = \lambda < 1$. Moreover, assume that

$$\omega_s\left(\hat{g}_{s2}^m - \hat{g}_{s2}^d\right) - \omega_h\left(\hat{g}_{h2}^m - \hat{g}_{h2}^d\right) = 0. \tag{D10}$$

This is an assumption on endogenous variables, but could be guaranteed by assuming complete symmetry across the partners in terms of wages and the investment technology. We highlight (D10) here to show that this is exactly the condition that implies symmetry across the partners. First note that, with shared custody and using (D10), we see from (D4) that both spouses agree on the divorce decision. In this case, the strategic investment incentive is present, but is symmetric across the spouses. In particular, the first period investments are characterized by

$$\frac{\omega_i}{\upsilon_i\left(\hat{g}_{s1}^m, \hat{g}_{h1}^m\right)} = 1 + 2\beta\left\{1 - (1-\lambda)\left[F\left(\hat{\theta}\right) - f\left(\hat{\theta}\right)\chi\right]\right\}, \text{ for } i = s, h. \quad \text{(D11)}$$

8.2. Efficiency-Restoring Policy

The characterization of the first best allocation is unchanged from the main model. Consider then the efficiency-restoring earnings tax at $t = 2$ in the married state. This tax now becomes

$$\hat{\tau}_2^m = \frac{3-\beta}{4}. \quad \text{(D12)}$$

Note that in the limiting case of no present-bias this reduces to $\hat{\tau}_2^m = 1/2$ which is now required in order to internalize the externality generated as each partner fails to take into account the benefit of the own investment to the spouse.

In the divorced state the efficiency-restoring earnings tax for spouse i becomes

$$\hat{\tau}_{i2}^d = \frac{\Lambda(1-\beta) + (1+\beta)\lambda_{-i}}{2\Lambda}, \; i = s, h. \quad \text{(D13)}$$

Unlike the tax in the married state, this tax is generally not gender-neutral, except in the case where there is equal custody, $\lambda_s = \lambda_h$, in which case the tax is not only gender-neutral but also invariant to marital status, $\hat{\tau}_{i2}^d = \hat{\tau}_2^m$ for $i = s, h$. Also, in the limiting case of no present-bias, the tax on spouse i reduces to $\hat{\tau}_{i2}^d = \lambda_{-i}/\Lambda$. Hence the efficiency-restoring tax is naturally higher on the spouse with the relatively lower post-divorce enjoyment of the family capital good.

The efficiency-restoring divorce fee (levied in equal shares on the two partners) in the general case satisfies

$$\begin{aligned}
&\omega_{i^*}\left[\tilde{g}_{i^*2}^m\left(1-\hat{\tau}_2^m\right) - \tilde{g}_{i^*2}^d\left(1-\hat{\tau}_{i^*2}^d\right)\right] - \frac{\hat{\eta}}{2} - (1+\beta)\left(\tilde{G}_2^m - \lambda_{i^*}\tilde{G}_2^d\right) \\
= \;& \frac{1}{2}\sum_{i=s,h}\omega_i\left(\tilde{g}_{i2}^m - \tilde{g}_{i2}^d\right) - 2\left(\tilde{G}_2^m - \frac{\Lambda}{2}\tilde{G}_2^d\right) \qquad \text{(D14)}
\end{aligned}$$

where, as before, $i^* = i^*\left(G_1^m\right)$ indicates the identity of the decisive spouse. In Special Case 2 with equal shared custody and with (D10) assumed to hold also at the first best allocation, the efficient divorce fee reduces to

$$\hat{\eta} = (1-\beta)\left(2\tilde{G}_2^m - \Lambda\tilde{G}_2^d\right) - \hat{\tau}_2^m\tilde{\mathcal{C}}_2, \quad \text{(D15)}$$

which directly corresponds to the efficiency-restoring divorce fee in the main model, except for the fact that $\hat{\tau}_2^m$ now also corrects for the externality generated by the spouses' investments.

The efficiency-restoring first period earnings tax is in general quite involved so we will focus here on the two special cases outlined above. Consider first the case of a custodial spouse who is also decisive, $\lambda_{i^*} = 1$ and $\lambda_{-i^*} < 1$. In this case the efficiency restoring tax for spouse i satisfies

$$1 - \hat{\tau}_{i1} = \frac{1 + 2\beta \left[1 - F\left(\tilde{\theta}\right)(1 - \lambda_i)\right]}{6 - 2(2 - \Lambda) F\left(\tilde{\theta}\right)}, \tag{D16}$$

whereby it follows that the tax should be higher on the non-custodial spouse than on the custodial spouse for reasons similar to above. In the second special case where custody is equally shared and (D10) holds the efficiency restoring tax is again gender neutral and satisfies

$$\hat{\tau}_1 = \frac{1 + 2(2 - \beta) \left[1 - (1 - \lambda) F\left(\tilde{\theta}\right)\right]}{6 - 4(1 - \lambda) F\left(\tilde{\theta}\right)}. \tag{D17}$$

As expected, in the limit with no present bias the tax reduces $\hat{\tau}_1 = 1/2$.