**The Goldilocks Principle: Applying the Exclusive Disjunction to Fuzzy Sets**

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**Abstract**

Qualitative Comparative Analysis (QCA), a technique by which the tools of Boolean algebra are applied to equifinal causal conditions, is gaining popularity amongst scholars. This paper draws upon a distinction largely overlooked by the QCA literature: the difference between inclusive- and exclusive-or (OR and XOR). Extending the work of Hackett (2015), I argue that XOR should be included amongst the tools of QCA, explain why XOR is more easily applied to crisp- than fuzzy-set QCA, and provide two original techniques for applying XOR to fuzzy sets: mechanical and calibrated. With the calibrated technique, the application of the exclusive-or is related to substantive knowledge of the cases with two threshold values: (1) how large two fuzzy set values need to be in order to violate a prior commitment or overshoot a target outcome, and (2) how similar two values need to be in order to violate the rule: ‘A or B, but not both’. This paper improves the capacity of QCA expressions to mirror natural language closely, formalize conversational implicature, and deal with mutually exclusive clusters of sufficiency conditions. It includes a helpful step-by-step guide for QCA practitioners.

In his work on the methodology of American Political Development, John Gerring argues:

‘Concepts must resonate with standard usage patterns within natural language and within the language region of interest in order to be useful in social science research...Propositions, like concepts, must be clear in order to be convincing. We must know precisely what it is that a writer is arguing before we can begin to evaluate its claim to truth.’ (Gerring 2003, 85)

This paper aims to make the tools of Qualitative Comparative Analysis (QCA) clearer and more sensitive to standard usage patterns within natural language. It argues that QCA should incorporate a new tool into its standard Boolean algebra – the exclusive disjunction (XOR) – because XOR captures a common way of speaking, an essential element of political life, and a distinctive relationship between equifinal causal conditions.

In normal usage the word ‘or’ has two meanings: inclusive and exclusive. Inclusive ‘or’ describes the relation ‘A or B or both’ and can also be written ‘A and/or B’; exclusive ‘or’ describes the relation ‘A or B but not both’. QCA currently uses only Boolean conjunction AND (\*) and *inclusive* disjunction OR (+) to express combinations of necessary and sufficient conditions for outcomes. A recent argument put forward in Hackett (2015) is that QCA should also adopt the Boolean XOR (⊕), the exclusive disjunction, for three reasons: (i) confusion between inclusive and exclusive ‘or’ leads to a loss of logical clarity, (ii) incorporating the exclusive disjunction mirrors natural language closely and extends our understanding of equifinality: the notion that different causal paths can lead to the same outcome, and (iii) exclusivity is a defining element of political life, in which trade-offs must often be made and priorities decided upon under conditions of competition between societal groups for scarce resources (Hackett 2015). This paper expands upon Hackett’s work by explaining how the Boolean XOR can be applied in the case of *fuzzy set* Qualitative Comparative Analysis (fsQCA), and not just crisp set Qualitative Comparative Analysis (csQCA).

This paper has three parts. The first, establishes that the ‘exclusivity barrier’ – that which prevents two or more explanatory conditions together producing an outcome of interest – is multiply realizable and dependent upon theoretical assumptions, contextual factors, and the domain over which the argument ranges. The second section, drawing upon examples from across social science, shows how the XOR can be used to express sufficiency relations between mutually-exclusive equifinal causal conditions and target categories in the mid-range of some scale. It also examines published fuzzy-set QCA analyses to demonstrate the value of using XOR rather than the ambiguous OR. The final section offers two methods for applying the exclusive disjunction to fuzzy sets: mechanical and calibrated. The mechanical approach, familiar from Hackett (2015), simply breaks the XOR down into its components to calculate the value of the set membership of the whole. The new calibrated approach provides the tools for researchers to incorporate crucial case-specific knowledge in determining how *similar* and how *large* the set memberships need to be in order to violate the rule ‘A or B *but not both’*. I demonstrate the calibrated approach with recent data, show how to calibrate mid-range target outcomes on an interval scale, and explore the use of the exclusive disjunction with expressions containing three or more equifinal pathways. A glossary of technical terms used is available in the Appendix.

**The Exclusivity Barrier**

This paper examines the use of the exclusive disjunction to describe equifinal explanatory paths to an outcome, of the form A⊕B 🡪 C. It does *not* consider the use of the exclusive disjunction on the dependent variable or outcome condition side of the equation (A 🡪 B⊕C), because scholars are typically most interested in delineating the various causal paths that lead to a single outcome of interest. This paper is concerned with the relationship between explanatory pathways. Although A⊕B can denote a constitutive relation (as the first section makes clear) this paper does not deal with constitutive claims but rather with *causal* arguments made using the exclusive disjunction.

If two conditions are joined by the XOR – A or B, but not both – then they cannot both occur at the same time and still produce the outcome of interest. By ‘cannot’ I do not mean that the two conditions simply do not happen to co-occur as a matter of empirical fact and are never found together in observed cases – as is often the case under conditions of limited diversity (Ragin and Sonnett 2005). Nor does the exclusive disjunction merely assert a linguistic relationship between two statements, but rather a relationship between two conditions or states of affairs. The use of the exclusive disjunction asserts that there is a barrier – either logical or practical – that prevents conditions A and B from co-occurring whilst producing the outcome of interest. The term ‘cannot’ has different meanings depending upon how conditions A and B, the outcome, and the sufficiency relationship linking explanatory and outcome conditions, are conceived. These differences result from the fact that QCA has multiple uses. ‘🡪’ may denote a constitutive or a causal relationship.

When the barrier preventing A and B from together producing the target outcome, Y, is a logical one – A and B never occur together because B=a and A=b – the need for the exclusive disjunction simply follows from the properties of logical contradiction and is of limited interest for social scientists. However, a practical barrier preventing A and B together producing Y can take many forms: theoretical, normative, methodological, operational, a violation of a prior commitment, overshooting a target mid-range category, and many others. It should be noted that the items on this list are *not* mutually exclusive: there can be multiple reasons why a particular relationship between two equifinal explanatory conditions is exclusive. Table 1 provides a selection of different meanings of the word ‘cannot’, as deployed by social scientists and policymakers.

[TABLE 1 ABOUT HERE]

As Table 1 shows, scholarship and policymaking are characterised by trade-offs between alternative values, conditions, methods, policies, and ideas. These trade-offs are shaped by the domain in which decisions are made and the resources available to devote to each alternative, which determine the set of possible choices. Decisions are also shaped by the weighting the decision-maker gives to various values and prior commitments, and the forecasted outcomes. These considerations are central to social scientists’ research design and evaluation of evidence.

‘Research design involves fundamental trade-offs. Methodological advice needs to be framed in light of basic trade-offs among: (a) alternative goals of research, (b) the types of observations researchers utilize, and (c) the diverse tools they employ for descriptive and causal inference. A methodological framework that does not centrally consider trade-offs is incomplete’(Brady and Collier 2004, 26).

Public policymaking also requires hard choices between alternatives that cannot co-occur (Strom and Muller 1999). The use of the exclusive disjunction to describe the relation between causal conditions requires us to posit a ‘difficult’ counterfactual: To assert that the outcome *would not* have occurred *if* the two explanatory conditions had obtained together.

**The Equifinality of Exclusive Explanatory Paths**

Hackett showed that there are three situations in which the exclusive disjunction can fruitfully be applied to QCA analysis. In each situation two causal conditions, A and B, would each individually produce the outcome of interest, Y, but both together would not. The first is where A and B are logically incompatible with one another, the second is where A and B together violate some prior constraint, and the third is where A and B together would ‘overshoot’ the target outcome Y, Y being an intermediate, mid-range category on some scale. Here I focus on the third scenario. This third situation might be termed a ‘Goldilocks category’ (a middle category on some scale, between ‘too much’ and ‘too little’). Goldilocks categories are applicable across political science but particularly in public policy, wherever researchers wish to express a sufficiency relation between a set of causal conditions and a mid-range target outcome. Table 2 gives eight examples of possible mid-range outcomes in social scientific research. These examples are radically simplified and intended only to illustrate the use of the exclusive disjunction. The posited relationships, derived from published social scientific research, need not be valid because they are used here purely for illustrative purposes. They take the form of an exclusive relationship between explanatory conditions and a distinctive ‘middle-way’ target category:

[TABLE 2 ABOUT HERE]

The ‘Goldilocks’ analogy implies that mid-range outcomes are the normatively attractive options, and several of the outcomes in Table 2 are indeed desirable, for example: resource management, sustainable growth, and a balance of judicial independence and accountability. However, there are several intermediate outcomes in the fifth column of Table 2 that are *not* normatively desirable but do present questions of great interest to social scientists. We might term these hybrid, partial or midway scenarios ‘The Tyranny of Half Solutions’ (Campbell 2011). For example:

* Why do certain countries become stuck in the ‘Middle Income Trap’?
* What explains the persistence of sub-optimal policy solutions that do not maximize beneficiary welfare, such as the failure of a social insurance alternative to Medicaid in the United States?
* Under what circumstances do vulnerable populations achieve only a ‘dependent exit’ from homelessness, rather than a sustainable ‘independent exit’?
* Why do some countries fail to democratize fully?
* How can the competing values of representation, proportionality and stability be balanced using the electoral system?

In order to examine the conditions under which one of these mid-range outcomes can be guaranteed, and to express a sufficiency relation between conditions A and B and the target outcome in the fifth column, the exclusive disjunction must be deployed. By contrast, the logical conjunction, A\*B, would guarantee the outcome in the *sixth* column: by hypothesis in these cases A AND B always overshoots the target outcome. The inclusive disjunction, A+B, could produce outcomes in either fifth or sixth columns, so it cannot express a sufficiency relation between these explanatory conditions and the outcome of interest. The truth of A OR B does not depend upon only one of A and B being true: A OR B can be understood as ‘A and/or B’. If the inclusive disjunction is used then we can only guarantee *not* to end up in the *fourth* column, in which the absence of both A and B results in the target outcome failing to obtain because neither of its individually-sufficient equifinal causal conditions is present.

The XOR is particularly suitable to describe relations between explanatory conditions and outcomes with fuzzy set data because researchers using fsQCA often delineate specific intermediate categories of interest. Using fuzzy sets requires researchers to identify, through the process of calibration, distinctive clusters of cases at various intermediate levels of set membership. For example, Marr’s work on efforts to move vulnerable people off the streets involves the identification of several distinctive intermediate options that have some, but not full, membership of the set of ‘successful exits’ from homelessness (Marr 2012). Between an ‘independent exit’, in which a person has been continuously in housing and paying their own rent for a sustained period (set membership 1.0), and being ‘mostly on the streets’ (set membership 0.0), there are various intermediate categories: a ‘dependent exit’, in which a person is living continuously in subsidized accommodation or with friends and family (set membership 0.67), or living in an emergency shelter or housing (set membership 0.33).

Similarly, Engeli and Allison’s work on national fertility and embryo regulation policies identifies a distinctive ‘intermediate’ category between ‘restrictive’ and ‘permissive’ policies (Engeli and Allison 2013). Countries in this category tend to impose relatively light regulation but have some restrictions on the usage, storage or disposal of embryos. Although neither of these scholars attempts to formalize the relationship between the explanatory conditions and these intermediate categories specifically, any attempt to do so would benefit from use of the XOR, as shown in Table 2. In order to express a sufficiency relationship between two or more equifinal clusters of explanatory conditions and a distinctive intermediate target of interest, the XOR can be deployed.

Given that social scientific theories sometimes –even often – involve policy choices between incompatible alternatives (explanatory conditions) or focus on distinctive compromise positions between two undesirable results (outcome conditions), the XOR is a valuable addition to the Boolean algebra of the QCA toolkit. It allows researchers to specify sufficiency relations between equifinal causal pathways and mid-range target outcomes. Competition between hostile forces and the production of compromise ‘middle-way’ outcomes are essential parts of the practice of politics.

**Exclusive Paths in Fuzzy Set Research**

Scholars using fsQCA often describe relations between exclusive equifinal explanatory conditions because QCA assumes multiple conjunctural causation (Rihoux 2006). Hackett (2015) employs examples from crisp-set QCA to demonstrate the uses of the exclusive disjunction, but users of *fuzzy-set* QCA also describe relations between clusters of equifinal causal conditions that are exclusive, rather than inclusive, in nature. For example, italicization of the words ‘either...or’ in Arvind and Stirton’s article on the adoption of the Napoleonic code suggests that the authors intend the resulting formula to express an exclusive relationship between INUS conditions:

‘In other words, the absence of Napoleonic control, feudalism, strong institutions and nativism are necessary but not sufficient conditions for non-adoption of the code. In addition to these factors, *either* the absence of liberalism and anti-French sentiment *or* liberalism and territorial diversity were necessary conditions for the non-adoption of the Code.’ (Arvind and Stirton 2010) [italics in the original]

In Ordanini and Maglio’s work on the determinants of successful service innovations, pathways are also described in exclusive terms. These authors identify two distinct INUS conditions: first, a ‘formal top-down innovative process’ – combining a pro-active management orientation with the lack of a reactive management orientation – and secondly, a ‘different innovation strategy largely driven by customer needs, both existing and potential...whose successful implementation requires a deep participation of many external actors’ – a reactive, not pro-active, management orientation. Ordanini and Maglio’s results have a curvilinear shape: either a top-down or a bottom-up approach leads to successful service innovations, but a mixture is usually unsuccessful.

‘...In the case of radical efforts hierarchically generated and managed by the top managers, proactive inclination should be associated with an absence of responsive orientation, otherwise the innovation efforts might become too prone to the needs of existing customers, failing to gain a market share.’ (Ordanini and Maglio 2009, 617–8)

In Arvind and Stirton’s work, the absence of liberalism and anti-French sentiment XOR liberalism and territorial diversity are necessary for non-adoption of the Napoleonic code. In Ordanini and Maglio’s work, a top-down XOR bottom-up approach is sufficient for successful service innovation. Both sets of authors frame the relationship between INUS conditions in an exclusive way, but QCA currently does not provide the logical tools to specify that the disjunction is exclusive. Including the XOR amongst the QCA toolkit would enable more accurate logical expression of relationships that are described ambiguously using the inclusive Boolean OR.

As Table 1 shows, exclusivity between equifinal pathways is shaped by context, domain, prior commitments, values and goals. Sometimes the two explanatory conditions identified are exclusive in some domains but not others. For example, Damonte’s consideration of the trade-off between environmental and economic policy goals involves the OECD ‘decoupling index’, defining the ratios of environmental depletion levels to economic affluence levels for the same country at two time points:

1 – [(depletiont /affluencet ) / (depletiont-1 /affluencet-1)].

‘Its values span from +1, when the perfect decoupling occurs and both environmental and economic goals are effectively achieved, to –1, when the two processes maintain the exact same trend and one of the two goals is achieved at the expense of the other’ (Damonte 2014). The relationship between environmental and economic policymaking objectives is inclusive insofar as the decoupling index is positive and exclusive insofar as the decoupling index is negative. In Damonte’s formulation, most approaches to environmental policymaking involve zero-sum trade-offs between environmental and economic policy goals: ‘frontier economics’ and ‘environmental protection’ approaches prioritize economic development at the expense of conservation, while ‘deep ecology’ and ‘eco-development’ approaches prioritize conservation over economic policy objectives. (The relationship between these two policy goals is only inclusive when taking the ‘resource management’ perspective). For the first four approaches, XOR could be used to express the exclusivity of the alternative paths.

Similarly, Stanko and Olleros describe a zero-sum trade-off between innovativeness and profitability amongst firms that outsource their innovation, but also find that the exclusivity of the relationship between innovativeness and profitability can be reversed under conditions of industry growth or when clustering is present but labour mobility is limited. Given the following set of conditions – a lack of industry growth or[[1]](#footnote-1) high labour mobility and a lack of clustering – the two strategies available to the firm are mutually exclusive. Firms must choose between outsourcing innovation activities to maximize profit, or[[2]](#footnote-2) keeping innovation activities in-house at higher expense but with more support for bold initiatives and fewer incremental ‘off-the-peg’ product solutions (Stanko and Olleros 2013). As with the other examples the relationship between pathways to business success is exclusive rather than inclusive, although in these cases exclusivity is *explicitly* made conditional upon the domain of reference. Of course, all claims to exclusivity are dependent upon the domain for which the claim is made.[[3]](#footnote-3) XOR rather than OR should be used when formulating logical expressions to describe the relationship between these pathways and the outcome of interest.

Determining whether the inclusive OR or the exclusive XOR should be used to describe equifinal INUS conditions requires knowledge of cases and context. In Gjølberg’s examination of the origins of corporate social responsibility (CSR), for example, the author identifies two hypotheses that seem mutually exclusive. A ‘globalist’ hypothesis is that a company’s CSR efforts are a function of the dictates of the global market place; by contrast, an ‘institutionalist’ hypothesis states that such efforts are a function of institutional factors in the national political-economic system. However on closer examination the relationship between these two clusters of characteristics is found to be inclusive, rather than exclusive:

‘...It is important to note that the globalist and the institutionalist pathways are not mutually exclusive. Some of the top ranking countries score highly in both combinations, namely Sweden, Switzerland and The Netherlands. Empirically, this can be explained by the argument put forth by Katzenstein (1985, 2003) that small, open economies tend to develop strong political-economic institutions. However, one can also claim that they are mutually supportive: that TNCs located in strongly embedded societies simultaneously have a susceptibility to anti-globalization pressures, while also having the ability to respond effectively to these pressures due to the comparative institutional advantage provided by their political-economic environment. Thus, there is reason to believe there is a multiplication effect between globalist and institutionalist factors’ (Gjølberg 2009).

As Table 1 shows, the barrier that prevents two or more explanatory conditions (A and B) together producing the outcome of interest is multiply realizable and dependent upon the domain over which the argument ranges. Accordingly, determining whether the relationship between A and B takes the form ‘A and/or B’ (OR), or ‘A or B but not both’ (XOR) requires contextual and case-specific knowledge (here: the nature of small, open economies and transnational corporations). The fact that Gjølberg found it necessary to state explicitly that the two pathways in her work are *not* mutually exclusive suggests a need to distinguish between ambiguous inclusive-or and the exclusive-or. XOR provides the means of eliminating this logical ambiguity.

**Applying the Exclusive Disjunction to Fuzzy Sets**

Hackett (2015) showed that the exclusive disjunction can easily be applied to crisp-set QCA, in a similar manner to the existing tools of Boolean algebra: OR and AND. When cases may only be said to have or lack membership in a particular set (set membership 1 or 0), there are two simple rules for applying the exclusive disjunction:

*XOR Rule 1:* If the values of conditions A and B are different, the A⊕B value is 1

*XOR Rule 2:* If the values of conditions A and B are the same, the A⊕B value is 0

These two XOR rules are similar to the rules for AND and OR (Goertz and Mahoney 2012).

*AND Rule 1:* If the minimum of the A and B values is 1, the A\*B value is 1

*AND Rule 2:* If the minimum of the A and B values is 0, the A\*B value is 0

*OR Rule 1:* If the maximum of the A and B values is 1, the A+B value is 1

*OR Rule 2:* If the maximum of the A and B values is 0, the A+B value is 0

The rules for AND and OR (‘take the minimum’, ‘take the maximum’) can be transplanted to fuzzy set analysis easily but this route is harder for XOR. It is harder to apply the XOR to fsQCA sets than to crisp set Qualitative Comparative Analysis (csQCA) because it is harder to judge when a case has ‘both’ A and B, ‘neither’ A or B, or ‘just one’ of A and B, where ‘A’ and ‘B’ represent causal conditions in which a case may have any degree of membership between 0 and 1. An obvious way to apply XOR to fuzzy sets would be to have the A⊕B value take the higher of the two fuzzy set values (as with OR) except where the values of A and B are precisely the same, whereupon the value of A⊕B would be 0. But this method cannot do because fuzzy set scores are unlikely to be precisely the same for both conditions, especially where there are a large number of qualitative breakpoints or the fuzzy sets are continuous. It would be strange to argue that, for example, a case with 0.94 membership in set A and 0.95 membership in set B does not violate the rule ‘A or B but not both’ because the set memberships are ‘not the same’, while a case with 0.95 membership in both A and B *does* violate the ‘but not both’ rule. Even if a case’s degrees of membership in two causal conditions are precisely the same, the level might be too low for exclusivity to bite. For example, if two cases have just 0.2 membership in sets A and B, we might be reluctant to state that this low membership score shows the cases are really members of both A and B, thereby violating the rule ‘A or B but not both’.

There are two ways to apply the XOR to fuzzy set values: mechanical and calibrated. Hackett (2015) demonstrated a simple mechanical process by which XOR can be applied to fuzzy sets. It involves breaking down the XOR into its component parts: AND, OR and NOT, and finding the value of (A+B)\*NOT-(A\*B). This process applies the exclusive disjunction to fuzzy set values using several intermediate steps. Since A⊕B = (A+B)\*NOT-(A\*B), taking the inverse of the value for A\*B as an intermediate step it is possible to calculate the fuzzy set value of A⊕B. The mechanical operation of XOR to fuzzy sets has the advantages of being relatively swift, clear, and requiring no further knowledge on the part of the researcher. But it treats the equifinality of exclusive explanatory paths crudely. Wherever either the value of A+B is lower or A\*B is higher than the point of maximum ambiguity (0.5), or both, then the value of A⊕B will be lower than the point of maximum ambiguity (more out of the set than in). *Vice versa* where A+B is higher or A\*B lower than 0.5. The mechanical application has no means of conveying just how precarious the balance is, when the presence of two conditions makes it easy to overshoot a target outcome. For some exclusive disjunctions, even low levels of set membership might violate the rule ‘but not both’, while for others, only full membership of both sets will do so. In this section I offer a second way to apply the exclusive disjunction to fuzzy sets: a ‘calibrated’ method. The calibrated application of XOR uses context and case-specific knowledge to give content to this notion: at precisely which point does exclusivity start to bite?

One of the most important features of fsQCA is the deployment of contextual and case-specific expert knowledge to calibrate degrees of membership in fuzzy sets. This procedure involves the iterative process of grounding measures in conceptual frameworks, theory, and empirical reality, so QCA measures are not simply sample-specific but connected with how we conventionally think and speak about the world (Ragin 2008; Ragin 2006a; Schneider and Wagemann 2012). ‘QCA is both theory-driven and inductive’ (Rihoux and Grimm 2006). ‘Determining thresholds for sets requires either applying a well-specified standard of measurement of the concept at hand, or, when no such standard is available, establishing a theoretical and empirical basis for proposing one’ (Lilliefeldt 2012, 201). Theoretical and case knowledge is also required to decide whether to favour a more complex or parsimonious explanation: ‘The choice of explanations is theory and knowledge-dependent’ (Ragin and Sonnett 2005). The calibrated process requires a deeper understanding of the equifinality of exclusive explanatory paths than the mechanical process. With calibrated fuzzy set XOR, the application of the exclusive-or is related to substantive knowledge of the cases, theoretical knowledge, and relevant contextual knowledge, such that two threshold values are established:

(1) how *large* two values need to be in order to violate a pre-commitment or overshoot a desired outcome

(2) how *similar* the two values need to be in order to violate the rule: ‘but not both’.

This section explains how these two processes work. The classification of cases can be related to threshold values determined by substantive knowledge of the cases. Rather than arguing that *all* non-identical membership scores should be assigned the higher membership score, the researcher can determine just *how dissimilar* the scores in A and B sets should be to warrant set membership of A⊕B. If the scores are both high and similar (1.0 and 0.9, for example), then both set memberships may count as full memberships for the purposes of the exclusive disjunction and cases with such set-membership values would score 0 when XOR was applied. Cases with large gaps between the degree of membership in one set compared to another will probably be sufficiently dissimilar to warrant the OR-like XOR application, so their set membership in A⊕B would be equal to the value of the larger of the two set memberships. Rather than arguing that *all* identical membership scores should be assigned a zero score in A⊕B, the researcher should determine a *threshold fuzzy-set value* above which identical scores warrant zero scores in A⊕B but below which identical scores merit exactly the same treatment as with the Boolean OR. The calibrated application of XOR to fuzzy sets can be summarized with two pre-analysis steps and three rules:

*fsXOR Step 1:* A threshold value (I) based on substantive knowledge of the score calibration must be established to show *how similar* two non-identical values in sets A and B joined by XOR should be to warrant a zero-value in A XOR B.

*fsXOR Step 2:* A threshold value (II) based on substantive knowledge of the score calibration must be established to show *how large* two identical (or ‘near-identical’) values in sets A and B joined by XOR should be to warrant a zero-value in A XOR B.

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*fsXOR Rule 1:* If the values of A and B are non-identical and the threshold level of similarity between them (I) is not reached, then A⊕Q is equal to the higher of the values of A and B.

*fsXOR Rule 2:* If the values of A and B are identical but the threshold set membership size (II) is not reached, then A⊕B is equal to the values of A and B.

*fsXOR Rule 3:* If the values of A and B are identical – or non-identical but the threshold level of similarity between them (I) is reached – and the threshold membership size (II) is reached, then A⊕B is 0.

The Figure in the Appendix provides a step-by-step guide to applying the calibrated procedure, laying out these rules graphically. The researcher must determine the threshold difference (I) and the threshold membership size (II) based on substantive knowledge of what constitutes a tipping point. The decision as to which threshold values to use cannot be determined prior to the empirical investigation because all fuzzy set values must be calibrated so that they reflect important qualitative differences between target sets in the real world. Unlike standard statistical analysis, which is typically content to use sample-based threshold values such as the mean and standard deviation, configurational analysis requires thresholds to reflect differences of substantive importance that are not sample-specific (Ragin 2008).

To illustrate, reconsider some of the theories presented in Table 2: causal conditions for anocracy, monetary policy needed for sustainable growth, the level of vulnerability and dysfunction that results in only a ‘dependent exit’ from homelessness, or the degree of political and legal insulation required to maintain an appropriate level of judicial independence. These theories can all be formulated in terms of fuzzy sets. To determine the threshold values, the researcher would need to clarify his or her definition of the possible outcome variables: (1) ‘dictatorship’, ‘anocracy’ and ‘democracy’, (2) ‘overheated economy’, ‘sustainable growth’, ‘depressed economy’, (3) ‘homelessness’, ‘dependent exit’, ‘independent exit’, and (4) ‘too little’, ‘too much’, or ‘correct amount of’ judicial independence. Depending on his or her criteria for deciding where to place cases on the scale, and how much of a change in the causal conditions is required to move cases between categories, the researcher would specify the threshold values accordingly. If the target intermediate category is specified narrowly then threshold I would be set high and II low, making it relatively easy for the XOR ‘*but not both’* condition to be violated and the outcome variable score 0. If the category is relatively broadly specified then threshold I should be set low and II high, so it is harder to violate the XOR ‘*but not both’* condition and easier to reach the target category.

For example, in a global economy vulnerable to shocks (context), and a precarious national economy vulnerable to capital outflows (domain), the target outcome – sustainable growth – is relatively difficult to achieve. In such a context it would be easy to over-inflate the economy, or else fail to stimulate the economy enough, because the balance needed to achieve growth is relatively precarious. Sustainable growth might lie around the 0.5-0.6 mark on a hypothetical fuzzy set scale between 0.0 (depression) and 1.0 (over-inflated economy). Even if a case has full membership (1.0) of the ‘expansionist open market operations’ explanatory condition, and only 0.5 membership of the ‘low interest rates’ explanatory condition, setting threshold I relatively high (0.5) would make the exclusivity condition bite even at relatively dissimilar set memberships in the two explanatory conditions. The case would ‘overshoot’ the target outcome. In this precarious environment threshold II would be set relatively low (0.3, for example), so even cases with set memberships in the explanatory conditions of just 0.3 would fall foul of the ‘but not both’ condition and fail to reach the target category.

In a more robust economic environment, by contrast, economic policymakers might have greater latitude with respect to the use of monetary policy instruments so the target category could be specified relatively broadly. For example, set memberships of between 0.3 and 0.7 on the hypothetical fuzzy set scale between inflation and depression might qualify as ‘sustainable growth’. For the explanatory conditions, threshold I would be set relatively low and threshold II relatively high so that only very high and similar levels of set membership in the two conditions would trigger the exclusivity condition ‘but not both’. Cases might have full membership in the ‘expansionist open market operations’ condition (1.0) and be almost entirely in the set of ‘low interest rates’ (0.9) but if threshold I is 0.05 these cases cannot be said to ‘have both’ conditions, and hence the cases avoid triggering the ‘but not both’ condition. They can easily reach the target category. Similarly, if threshold II is set very high (for example, 1.0) then even cases with almost full membership of both explanatory conditions (for example, 0.9) could avoid triggering the exclusivity condition. In this environment the breadth of the target category, and its robustness, make it difficult to move between categories and easier to hit the target.

The same procedure can be applied to all of the fuzzy-set exclusive disjunction examples in Table 2. Since XOR applications deal with balancing-acts and intermediate categories, threshold setting depends on the ease of moving between categories and the narrowness of the target category. As in all QCA applications, researchers are required to consider carefully how they ‘constitute’ their populations of interest (Ragin 2006b, 636). As with Ragin’s indirect method of set calibration researchers are required to categorize cases in a qualitative manner, but it is possible to conduct robustness checks by varying the level of Thresholds I and II in order to see how the outcome changes. In order to prevent accusations of selection bias, at all times the threshold values should be set in a transparent way and backed up by substantive empirical knowledge, as fsQCA calibration requires (Wagemann and Schneider 2010).

**Calibrated Exclusive Disjunction in Practice**

To demonstrate the value of using the calibrated approach to the exclusive disjunction to evaluate fuzzy set data, I provide some exemplar data on monetary policy and inflation rates derived from the World Bank. Consider the relationship between inflation (the outcome variable) and two monetary policy instruments: interest rates and money supply manipulation (explanatory variables). Lowering interest rates and boosting the money supply tends to stimulate the economy, while raising interest rates and constricting the money supply constrains it. We can delineate three possible outcome sets:

1. The set of high inflation countries (even hyperinflation) with an overheated economy
2. The set of countries with very low inflation (even deflation) and a depressed economy
3. The set of countries with moderate inflation and a stable economy

Policymakers target the third set and attempt to avoid falling into either the high- or the very low-inflation categories. For simplicity, assume that the two policy instruments they can manipulate are the interest rate (I) and the money supply (M) through central bank rate-setting and policies such as quantitative easing. If policymakers reduce interest rates *and* expand the money supply rapidly (I\*M), they could overheat the economy and fall into category 1. If policymakers *neither* reduce interest rates *nor* expand the money supply (i\*m), the economy is constricted and they fall into category 2. But if they reduce interest rates *or* expand the money supply, *but not both* (I\*m *or* i\*M), then they should hit the moderate, stable inflation target category 3 (F). The posited relationships are:

Exclusive disjunction: I⊕M 🡪 F

Conjunction (presence of conditions): I\*M 🡪 f

Conjunction (absence of conditions): i\*m 🡪 f

Inclusive disjunction: I+M 🡪 ?

Deploying 2014 data derived from the World Bank and a variety of indicators on quantitative easing, I code eight OECD countries according to their membership in the set of ‘low interest rate’ countries (I), and the set of ‘expansive money supply’ countries (M). For the outcome condition (F), the delineation of a moderate target category requires that scores be calibrated such that there are two cross-over points and two ways in which countries can have full non-membership of the set: either over- or under-shooting the target. This relationship is described graphically in Figure 1 below.

[FIGURE 1 ABOUT HERE]

Using two extra pieces of data it is possible to calibrate membership in a mid-level target set by Ragin’s direct method. Rather than three qualitative anchors at the thresholds for full non-membership (0.0), maximum ambiguity (0.5) and full membership (1.0), the researcher needs to delineate six thresholds: the target set boundaries (coded 1.0), the thresholds for full non-membership of the mid-level set both above and below the moderate category on an interval level scale (coded 0.0), and two points of maximum ambiguity (0.5), between the target category and the thresholds for full non-membership above and below the target. As with all fuzzy set calibration the intervals between these thresholds need not be symmetrical above and below the target category. Here, I utilize the target of 2-2.5% inflation as the criterion for full membership of the set of ‘moderate inflation’ countries. Full non-membership in the set of ‘moderate inflation’ countries is set at 0% inflation *and* 3% inflation, so countries below 0% or above 3% are fully out of the set. The crossover points of maximum ambiguity are set at 1% and 2.75% inflation. Calibration tables are available in the Appendix.[[4]](#footnote-4)

Table 3 shows the fuzzy set membership of eight countries in the sets of ‘moderate inflation’ (F), ‘low interest rates’ (I), and ‘expansive money supply growth’ (M).

[TABLE 3 ABOUT HERE]

The countries fall into four clusters: (1) Depressed Southern European countries with a sluggish economy, high real interest rates and a relatively constricted money supply, because the figures date to the period before the European Central Bank began its quantitative easing program (Italy, Spain and Greece). (2) Cases that weathered the recession relatively well, with moderate levels of inflation and relatively buoyant economies (Australia and Germany). (3) Countries with high inflation and loose, expansionist monetary policy (Mexico). (4) Cases with partial membership in the set of ‘moderate inflation’ countries, low interest rates and major quantitative easing programs (United States and United Kingdom).

With these simplified data, the assertion that I⊕M 🡪 F seems more accurate than either I+M 🡪 F or I\*M 🡪 F. Australia and Germany – with membership of I or M but not both – have the greatest degree of membership of the target set F. By contrast, Mexico, Italy, Spain and Greece have either I and M (Mexico), or neither I nor M (Italy, Spain and Greece), and all of these countries have full- or almost full-*non*membership of set F. I apply the mechanical XOR according to the formula I⊕M = (I+M)\*NOT-(I\*M), as shown in Table 4 below:

[TABLE 4 ABOUT HERE]

The mechanical exclusive disjunction describes the data quite well and certainly better than the conjunction or the inclusive disjunction, since either of the latter operators would predict that the overheated Mexican economy reaches (or may reach) the target ‘moderate inflation’ category. The Boolean operators AND, OR and XOR produce different results. For example, Australia and Germany have low membership of the configuration I\*M but high membership of the configuration I⊕M; Mexico has high membership of the configuration I+M but low membership of the configuration I⊕M.

However, although the mechanical application of XOR produces consistent, intuitive results largely aligned with expectations, the crudeness of the mechanical application is apparent in its treatment of the US and UK cases. Although these cases are ‘more out than in’ the configuration I⊕M (when calculated mechanically) because they lowered interest rates and engaged in quantitative easing, both countries are ‘more in than out’ of the target category F. There are two ways we could treat these cases: the first is simply to concede that these cases are inconsistent with the hypothesized relationship between the configuration I⊕M and the outcome F. The second is to utilize the calibrated XOR, which takes account of the domain of reference in order to refine the meaning of the causal configuration I⊕M. Whether to take this second route depends upon the nature of the cases and the causal conditions, and the plausibility of the posited argument as to why those particular cases do not fit the I⊕M 🡪 F relationship. Researchers should not merely re-read the data to account for his or her favored argument, but if the domain of reference clearly alters the meaning of ‘full membership’, and ‘nonmembership’ of the set of cases possessing the explanatory conditions, the calibrated XOR can provide the nuance and sophistication the mechanical XOR lacks.

The first part of this paper shows that claims to exclusivity of causal paths are shaped by the domain in which decisions are made and the resources available to devote to each alternative, which determine the set of possible choices. As open liberal market economies with large financial sectors, the US and UK were more exposed to the financial crisis than other countries. Given the depth of the recession (context) for this class of cases (financialized LMEs), it was harder for these countries to overshoot the desired inflation rate than it was for other countries to do so. In that context, the meaning of ‘low interest rates’ and ‘expansive monetary policy’ are particularly strict: only very low interest rates and very large QE programs qualify. Hence Threshold I (the degree of similarity between conditions I and M required for exclusivity to bite) is here set low (0.1), and Threshold II (the size of the membership in I and M at which cases are deemed to ‘have’ membership of I and M) is set high (0.8).

In accordance with Appendix Figure, the sequence of moves required to reach the scores generated in Column 6 of Table 4 is shown in Table 2 in the Appendix.The calibrated XOR produces set memberships in the configuration I⊕M of 1.0 for US, 0.8 for UK and 0.0 for Mexico, as opposed to 0.4, 0.3 and 0.2 respectively with the mechanical XOR application. In other words the calibrated procedure ‘promotes’ the US and UK to fully, or almost fully, in the set, while ‘demoting’ Mexico from mostly to completely out of the set. These revised scores more accurately reflect our substantive knowledge of the context in which these countries conduct monetary policy. What counts as a ‘low’ interest rate? How do low interest rates and quantitative easing interact? The calibrated XOR takes account of the way that domain shapes answers to these questions: a ‘low’ interest rate in the context of an open, financialized liberal market economy may not be the same as a ‘low’ interest rate in a less financialized, cooperative market economy, and such an economy may be less likely to overheat in response to expansion of the money supply.

**Exclusive Disjunction with Three or More Variables**

Where there are three or more pathways to an outcome, the procedure for applying XOR is more complex. If the substantive meaning of ‘P XOR Q XOR R’ is ‘Just *one* of P or Q, or R’ rather than ‘*Some but not all* of P and Q and R’ (Proposition ii below rather than Proposition i), then the logic of P⊕Q cannot simply be transplanted to P⊕Q⊕R.

1. P XOR Q XOR R ≠ ¬ (P \* Q \* R) \* ¬ (p \* q \* r)
2. P XOR Q XOR R = (P + Q + R) \* ¬ (P \* Q) \* ¬ (P \* R) \* ¬ (Q \* R)

The problem is that by the associative principle, P⊕Q⊕R is true in Boolean logic where just one *or all three* of the terms of the equation are true (Proposition iii below).

1. P⊕Q⊕R = **(P \* Q \* R)** + (P \* q \* r) + (p \* Q \* r) + (p \* q \* R)

In terms of the plain meaning of the exclusive disjunction outlined in the first two sections of this paper, this result seems counterintuitive. In natural language, and in the formulation of social scientific arguments of the kind shown in Table 2, the expression ‘just *one* of P or Q, or R’ seems a more natural extension of ‘P or Q but not both’ than ‘just one of P or Q or R, or else P, Q and R together’. Instead of the binary exclusive disjunction ⊕ three-pathway arguments require the Ternary XOR (Graphic), which is true just in case exactly one of its three arguments is true (Pelletier and Hartline 2007). When applying the exclusive disjunction to three or more variables it is necessary to break the equation into a series of binaries and evaluate each relation separately as shown in Proposition ii. The procedure is illustrated in Table 5.

[TABLE 5 ABOUT HERE]

In this example Case A has the lowest degree of membership in the configuration P XOR Q XOR R (0.2), Case B is at the point of maximum ambiguity as to its membership in the configuration (0.5), and Case C is mostly in the configuration (0.8). Since the procedure is based on evaluating a series of pairings, it would be possible to apply the calibrated exclusive disjunction instead of the mechanical procedure here as with the binary ⊕. The same thresholds would apply.

In terms of substantive applications, however, as the number of pathways rises the chances that these pathways are truly *exclusive* of one another in producing a target outcome become smaller. Equifinal relations consisting in three or more pathways may well include configurations joined by exclusive disjunctions and others by inclusive disjunctions. Researchers should not assert that there is an exclusivity barrier between pathways unless they can confidently posit a difficult counterfactual: to assert that the outcome *would not* have occurred *if* *any* of the explanatory conditions had obtained together. As the first part of this paper demonstrates, all claims to exclusivity require attention to cases, concepts and context.

**Conclusion**

Applying the exclusive disjunction to fsQCA does not simply improve this particular method’s ability to capture how we typically speak about the world, but provides additional insight into fundamental questions in philosophy of social science: How should we form and evaluate concepts? What additional analytic leverage can qualitative research bring to social scientific investigation? How is validity bounded by domain and context? This paper provides a means by which the Boolean algebra of QCA can be made to reflect natural language more closely, through the formalization of conversational implicature.[[5]](#footnote-5) I have shown that the use of the XOR requires close attention to domain, context, and actors’ prior commitments, values and goals, and is centrally concerned with how those actors rationalize and categorize the world. In social science, measurement claims are always context-specific, ‘intentionality is an integral part of causal analysis’ and ‘rationalization’ is a species of causal explanation (Davidson 1980; Geertz 1973; Gerring 2004). Making fsQCA more sensitive to linguistic usage and context provides the resulting explanations with greater qualitative insight and renders them less like standard statistical approaches or typical ‘covering law’ explanations in the natural sciences.

Unlike standard statistical approaches, fsQCA requires the deployment of rich case-specific and contextual information in order to justify the calibration of measures. The new calibrated method outlined in this paper involves the use of what Brady and Collier term ‘causal process observations’ (CPOs) – observations about context, process, or mechanism that provide an alternative source of insight into the relationships among the explanatory conditions, and between these conditions and the target outcome. ‘The strength of causal-process observations lies not in breadth of coverage, but depth of insight’ (Brady and Collier 2004, 24). Applying the exclusive disjunction to fuzzy sets utilizes the depth of case and context insight provided by qualitative data in order to understand multiple conjunctural causation.

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1. An inclusive ‘or’ [↑](#footnote-ref-1)
2. An exclusive ‘or’ [↑](#footnote-ref-2)
3. As Hackett (2015) notes, statements such as ‘Male XOR pregnant’ seems obviously exclusive amongst human beings but can be inclusive amongst members of the Sygnathidae family, such as seahorses and pipe fish. Domain matters. [↑](#footnote-ref-3)
4. Note that these figures are intended for illustration only. [↑](#footnote-ref-4)
5. Conversational implicature is something that can be worked out from *how* something was said, rather than simply *what* was said. See (Grice 1989). Usually there are contextual clues that indicate whether the ‘or’ used is exclusive or inclusive. See Grice’s 1989 work *Studies in the Way of Words* for discussion of both conversational and conventional implicature. [↑](#footnote-ref-5)