IDENTIFYING ORDERED STRATA: EVIDENCE, METHODS AND MEANING

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ABSTRACT

Identifying order or pattern in strata on the basis of qualitative interpretation forms the basis for much current sedimentological and sequence stratigraphic analysis. Order can be usefully defined as some arrangement of facies or unit thickness that has a discernable trend or pattern that is unlikely to occur by chance because it requires some particular systematic process to form. Coarsening, fining, thickening, or thinning-upward trends, and arrangement of strata into cycles are examples of order. Qualitative interpretations of order often demonstrate little more than an implicit assumption of order. This paper defines a robust yet simple-to-apply quantitative method to identify order in strata and to indicate when order cannot be reliably demonstrated. The method is based on two calculated metrics, the Markov metric *m* derived from analysis of a vertical facies succession, and the runs metric *r* derived from analysis of observed thicknesses of stratal units. Most importantly, both metrics can be compared with equivalent metrics calculated for disordered strata composed of many randomly shuffled versions of the same lithological units. Probability values can then be calculated from the comparison between observed and randomly shuffled cases, and these *p* values indicate the degree of evidence present for order in the observed strata. Several test examples using synthetic strata show that the *m* and *r* values can define and identify different degrees and types of stratal order, and that the metrics are robust for both stationary and non-stationary successions with a range of different lengths and numbers of distinct facies. Analysis of four outcrop examples, two siliciclastic and two carbonate, demonstrate that ordered facies successions and thickness trends may be less common than typically assumed; none of the four examples analyzed show trends in thickness, and only the examples from the Book Cliffs, which represent a bedset scale composite of observations, show evidence for facies order. The examples demonstrate how a quantitative analysis can lead to better understanding of strata, either ordered or disordered, can provide better insight into the validity of current stratigraphic interpretations and models. Absence of order in many of the analyzed 1D vertical successions may also indicate that we need to focus more on longer-term trends and analysis of 2D and 3D stratal geometries.

INTRODUCTION

A central element in observation and interpretation of outcropping and subsurface strata is identification of order. In the context of a vertical succession of strata, order can be usefully defined as some arrangement of facies or unit thickness that has a discernable trend or pattern that is unlikely to occur by chance but instead requires some particular systematic process to form. Commonly identified examples of order are coarsening- or fining-upward trends, thickening- or thinning-upward trends, or an upwards progression of facies of whatever form (e.g., lithofacies, biofacies, etc.). These are assumed to indicate some systematic change in particular depositional conditions such as flow velocity or depositional water depth. A stratigraphic cycle is particular form of order composed of a series of connected events, for example depositional facies, which follow a particular trend and then return to a particular starting point (Schwarzacher, 1975; Goldhammer, 2003; Weedon, 2003).

When patterns are identified in strata they can be linked to various processes such as autogenic delta lobe avulsion, or shoreline progradation and retrogradation. Changes in stacking patterns are often assumed to be a response to allogenic forcing by changes in relative sea level, climate, or source-area tectonics, and this process response link provides the foundation for much stratigraphic interpretation. For example, following the sequence stratigraphic model, repetition of similar trends in a vertical succession of strata are often interpreted to indicate repetitive, cyclical autogenic and allogenic processes (see Goldhammer, 2003, for a succinct review) that stack strata into identifiable, organized packages such as cyclothems, parasequences, sequences and megasequences (e.g., Van Wagoner et al., 1990; Catuneanu, 2006; Catuneanu et al., 2009).

These models of processes and controls have come to dominate interpretations of sedimentary rock to the extent that, in some cases, order seems to be an *a priori* assumption that drives interpretation rather than being something requiring explicit demonstration by evidence (e.g., Olszewski and Patzkowsky 2003; Anderson 2004; Pomar et al. 2005; Myrow et al. 2012; Sena and John, 2013). Even if a periodic process such as Milankovitch-forced oscillations in eustatic sea level or ordered autocyclic processes (e.g. Burgess et al., 2001) has acted on a depositional system, other processes may also have acted to mask or remove this signal (Burgess, 2006; Jerolmack & Paola, 2010). As a consequence, one might expect to find examples of ordered strata preserving a strong allogenic signal but, equally, one might expect to see many examples where such a signal is absent and strata lack any kind of measurable order. As Helland-Hansen (2009) says "the stratigraphic stratal stacking patterns do not always show clear cyclicity … and if they do, the cyclicity may often deviate from the norm of an ideal base-level cycle".

From these observations it is perhaps surprising that, with some notable exceptions (e.g. Lehrmann & Rankey, 1999) it remains rare for an outcrop study to report an absence of pattern of order in a vertical succession of strata. The reason for this could be that most strata really are ordered due to dominant periodic allogenic forcing or organized autogenic dynamics. However, there is a strong argument that qualitative methods do not actually provide strong evidence for cyclicity in strata (Wilkinson et al., 1997) and, in the absence of careful testing for order versus disorder, a qualitative interpretation of order often demonstrates little more than an implicit assumption of order. It could also be that disordered strata are somehow not selected for study, though this seems more unlikely. Most likely, in many studies various elements of these different possibilities are combined. Another possibility is that order is manifest only in stratigraphic surfaces and not in the strata between surfaces (e.g., Weibel, 2004). Progress is certainly being made understanding the significance of stratigraphic surfaces (e.g., Martin et al. 2009; Martin et al. 2011), but claiming that surfaces are the only manifestation of order would rescind much of the sequence stratigraphic model and method that clearly does assume trends in lithofacies between surfaces (e.g., Catuneanu 2006; Catuneanu et al. 2009; Miall 2010).

To make progress and start to address some of the issues highlighted above we need robust yet simple-to-apply tools to identify a high probability of order in strata and, just as importantly, to indicate when order cannot be reliably demonstrated. This work presents and evaluates a method to identify ordered strata based on analysis of facies succession using transition probability matrices, and analysis of thickness trends using runs analysis. In both cases a metric is calculated that is indicative of the level of order present in the strata, and this is then compared with a random model via a Monte Carlo method to indicate how likely it is that such order could arise by chance. The focus is on outcrop-scale, core, or well-log-scale analysis, so one-dimensional vertical successions of a few tens to a few hundreds of facies units in length and with three or more distinct facies present are suitable for this analysis. It is assumed that the identification of facies is reproducible and as objective as possible.

METHOD

A simple but robust non-parametric method for calculating the degree of order present in strata is presented here. Matlab code for all these methods is available from the CSDMS software repository at *http://csdms.colorado.edu/wiki/Model:OrderID*, and the methods are also implemented as part of the graphic sedimentary-log generation tool SedLog, available at *http://www.sedlog.com/*. The method has two components, one based on analysis of an observed facies succession, the other based on analysis of observed thicknesses of stratal units. Both components are intended to determine the degree of evidence for order present in the strata. A key element in determining this is to have a simple but effective metric for order versus disorder for both facies and thicknesses, and an appropriate random model to compare each metric against. In this respect particularly this method is an important step forward compared to previous methods, for example Davis (2002), because the random models used for comparison are more robust, and the process of comparison less arbitrary. It is important to test a new method using both synthetic and real data examples. Six synthetic examples have been used to develop the metrics, representing a range from ordered to disordered strata, and including symmetric and asymmetric cycles, various frequencies of each facies, and inclusion of significant surfaces. The method has also been tested with four outcrop examples spanning a range of different depositional systems.

Imprecise terminology can be a problem when discussing randomness in strata. For example, an absence of measureable order does not necessarily demonstrate that something was produced by random processes. For this reason the term “random” is used here only to refer to the pseudo-random numerical method (see Press et al., 2002 for a full explanation) used to calculate synthetic strata. When referring to strata, either synthetic or outcrop, the terms ordered, partially ordered, and disordered are used. In the synthetic cases, partially ordered strata are those produced by some random shuffling of an originally perfectly ordered example. Random shuffling is carried out by selecting two facies at random from the succession, and swapping them. This is repeated *n* times. For a value of *n* close to or exceeding the number of units in the succession, any order present in the original strata should have been removed, or if it persists, it does so by chance. Disordered strata are those that have been generated directly with a pseudo-random-number generator, or by random shuffling with enough swaps to remove any order originally present. When discussing outcrop examples, in cases where there is no evidence for order, it may be misleading to say that "the strata are random", for the reason stated above; more correct is to simply say that there is no evidence for order based on the applied metrics. The strata should not then be interpreted as ordered or cyclical (which is a particular form of ordered), but there is nothing to preclude further testing with other methods, including 2D and 3D methods if appropriate.

Synthetic Successions

Perfectly ordered strata with symmetric cycles

The perfectly ordered example consists of five facies arranged in asymmetric thickening-upwards cycles (Fig. 1A). The total length of the succession is 15m and it consists of 50 facies units (Table 1) with an equal frequency of each distinct facies.

Partly Ordered Strata – Asymmetric Cycles

This succession was generated by making ten random swaps of units in the 50 unit long perfectly ordered asymmetric 5-facies cycle succession described above. Partly shuffling the strata with 10 swaps in a 50 unit succession introduces a random element to the strata while preserving some of the original order. Visual inspection of the strata (Fig. 1B) shows that one perfect asymmetric cycle remains, and trends in both facies and thickness are still present over three or four units at several positions within the strata, but the cyclicity has been disrupted by the random swaps. Total length of the succession is 15m and it consists of 50 facies units (Table 1) with an equal frequency of each distinct facies.

Disordered Strata

This succession consists of a sequence of random numbers with facies coded one to five and each unit with a random thickness between 0.1 and 1 m (Fig. 1C); using random numbers in this way is the simplest method to generate a disordered succession. The strata have been generated by a random process, but, importantly, some trends in facies and thickness are still present by chance, for example between 8 and 10 m (Fig. 1C). Total succession length is 12.93 m, and it consists of 50 facies units (Table 1) with an equal frequency of each distinct facies.

Perfectly Ordered Strata - Symmetric Cycles

The perfectly-ordered-strata example consists of five facies arranged in symmetric thickening- and then thinning-upwards cycles (Fig. 1D). Total length of the succession is 14.5 m and it consists of 49 facies units (Table 1) with an equal frequency of each distinct facies.

Disordered Strata with an Exponential Facies Frequency

Like the disordered-strata example described above, this succession consists of a sequence of random numbers representing five distinct facies, with thicknesses ranging randomly between 0.1 and 1 m. Facies are randomly selected, but their frequency follows an exponential distribution, such that facies one is approximately twice as common as faces two, and so on (Fig.

1E). This is significant because evidence suggests that many strata show an approximately exponential distribution of facies frequency (Wilkinson et al., 1996; Wilkinson et al., 1999; Burgess, 2008). The total length of the succession is 248.7 m, and it consists of 100 facies units (Table 1).

Disordered Strata with Exposure Surfaces

This succession is the disordered strata example described above with a sixth facies added to represent exposure surfaces (Fig. 1F). Facies six is assigned zero thickness, and occurs atop all occurrences of facies five. The total length of the succession is 12.93 m and it consists of 65 facies units (Table 1) with an equal frequency of each distinct facies.

An Order Metric for a Facies Succession

Transition Probability Matrices

In this method, calculation of the degree of order present in a facies succession is based on construction of a transition probability matrix. A transition probability matrix is a matrix of values, calculated from a succession of strata, representing the probabilities of all upward-younging transitions from one particular facies (represented in the matrix rows, e.g., Figure 1G) to another facies (represented in the matrix columns, e.g., Fig. 1G). For example, in the transition probability matrix (Fig. 1H) constructed from a partly ordered succession (Fig. 1B) the probability of transition from facies 2 to facies 4 is 0.1 and is recorded in $T_{2,4}$. Note that this is an embedded method (Davis, 2002); only transitions between different facies are recorded and transitions with the same facies are ignored, so the values on the diagonal of the matrix T_{ii} will always be zero. Also note that facies should be numbered consecutively from one upwards to avoid spurious empty cells in the matrix.

The distribution of values in a transition probability matrix can be diagnostic of the degree of order present in the strata. Perfectly ordered strata show systematic transitions such that any facies will always pass into only one other facies (Fig., 1A and G). In contrast, strata that lack any order will have an equal probability of transitions from each facies to any other facies assuming equal frequency of all facies (Fig., 1B and H). There are many other possible permutations between perfectly ordered and disordered (e.g. Fig. 1C and I) and it is useful to consider the structure of the transition probability matrix for perfectly ordered strata, perfectly disordered strata, and an intermediate case. Perfectly ordered strata lead to a matrix with values at unity in diagonal lines of cells parallel to the matrix diagonal and zero in all other cells (Fig. 1G). In contrast the disordered case shows identical values in all cells with no diagonal structure present except in the matrix diagonal itself. The intermediate partially ordered case (Fig. 1H) shows higher values in a diagonal line of cells parallel to the matrix diagonal. In this intermediate case transition probabilities range from zero in, e.g., $T_{4,1}$, meaning that no transitions occur between facies 4 and facies 1, and 0.67 in $T_{4,5}$ meaning that 67% of transitions from facies 4 go to facies 5. This case is clearly different from the strata in Figure 1A and 1C and represents an example where some order is present, in the sense that it is possible to state what, given one facies, is the most likely next facies to occur in the succession. This diagonal structure present in matrices created from strata where order is present can be used to create a Markov metric that has the value between one, for perfectly ordered strata, and zero, for perfectly disordered strata.

The Markov Order Metric

For each diagonal in the transition probability matrix T a sum can be calculated and from these sums a minimum and maximum value can be found, so

$$max_{diag} = \arg max \left(\sum_1^F diag(T_j) + diag(T_{F-j})\right) \quad 1$$

$$min_{diag} = \arg min \left(\sum_1^F diag(T_j) + diag(T_{F-j})\right) \quad 2$$

where *j* is the offset value from the main diagonal, *F* is the number of facies and also the number of rows and the number of columns in the matrix, *arg min* and *arg max* are mathematical functions to find the minimum and maximum values in a series, and *diag* is a function to find the elements in a diagonal with offset *j* from the main diagonal (see green cells and blue lines in Fig 1G). Note that the Markov order metric *m* can then be calculated as

$$m = max_{diag} - \ min_{diag} \ . \quad 3$$

Consideration of the *m* values calculated for the six synthetic successions described above indicates that the Markov metric has potential to be effective at characterizing the degree of order present in the strata (Table 1). The Markov value *m* is 1 for the perfectly ordered asymmetric cycle strata (Fig. 1A and 1G), 0.2 for the disordered strata example (Fig. 1C and 1I), and has an intermediate value of 0.44 for the partly ordered strata with asymmetric cycles (Fig. 1B and 1H). For the perfectly ordered strata with symmetric cycles (Fig. 1D and 1J) the *m* value is 0.5, so lower than the 1.0 value for ordered asymmetric cycles, but still higher than the 0.2 for disordered strata. In a disordered succession with an exponential distribution of facies occurrence (Fig. 1E and 1K), the *m* value is 0.17. This demonstrates that with this method an exponential facies distribution does not give high values of m despite some high probabilities in the TP matrix. A final example shows that if otherwise disordered strata include frequently occurring exposure surfaces (Fig. 1F and 1L) the *m* value is increased to 0.34. This increase demonstrates that the *m* value has potential at least to identify an element of order present in the form of exposure surfaces when these are recorded as a facies.

An Order Metric for Stratal Thickness

Runs Analysis

A run is defined by Davis (2002) as uninterrupted occurrences of the same state within a series. Runs can be demonstrated with repeated coin tosses that create a series of states, where a state is one of the two possible outcomes from tossing a coin. For example, with time running from left to right

T H H H H H T T H T,

shows the outcome from a series of coin tosses in which there is a run of five heads (H). This approach can be used to analyze thickness of stratal units if, instead of a heads and tails, two states are defined where thickness either increases relative to the previous unit or decreases relative to the previous unit. For examples, a succession of thicknesses of facies units in meters younging to the right

0.2 0.4 0.6 0.8 1.0 0.2 0.4 0.6 0.8 1.0,

could be classified as either increasing or decreasing thickness thus

I I I I D I I I I

where I stands for increasing thickness and D stands for decreasing thickness, and three runs are present, two 4 states in length and one only one state in length. This example could also be encoded as two series, one recording position in a run of increasing thickness and one recorded position in a run of decreasing thicknesses, thus

I *1 2 3 4 0 1 2 3 4*

D	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*0*

This has the advantage of making the length of each upward-thickening or -thinning run of facies units explicit.

The above are examples of ordered succession where a clear thickening upward trend is present in the strata, occurring as two thickening-upward cycles, as shown in Figure 1D. In contrast

0.87 0.80 0.35 0.57 0.86 0.13 0.61 0.86 0.26 0.25

is a succession of thicknesses from a random-number generator which would classify as

I	*0*	*0*	*1*	*2*	*0*	*1*	*2*	*3*	*0*
D	*1*	*2*	*0*	*0*	*1*	*0*	*0*	*0*	*1*

It is difficult to make generalizations based on single random examples, but this succession illustrates the point that, in general, successions of facies units that occur by chance tend to have shorter runs. This observation forms the basis for a metric for ordered versus disordered arrangements of facies thickness.

The Runs Order Metric

We can use the classification of runs introduced above to summarize the sequence of runs present in any succession. A summary runs order metric *r* can be derived from this classification where

$$r = \left(\frac{\sum_1^n I_j}{n}\right) + \left(\frac{\sum_1^n D_j}{n}\right) \qquad 4$$

In this formula *n* is the number of units in the succession, *j* is the unit number within the succession, ranging from 1 to *j*, I_j is the *j*th element in the increasing thickness upwards series, and D_j is the *j*th element in the decreasing-thickness-upwards series.

Calculating the runs order metric for some of the six synthetic strata examples described above illustrates why this formulation was chosen. For the perfectly ordered asymmetric cycles succession (Fig. 1A) the runs order metric value is 2.18 (Table 1). In contrast, for the disordered strata example (Fig. 1B) the runs order metric value is 1.48 (Table 1). These two cases demonstrate the more general behavior of the runs order metric. In the disordered example thickening and thinning runs are short, typically only one or two units in length, and about half of the succession is composed of thickening runs, and half of thinning runs. As a consequence for disordered strata the sum of thickening or thinning runs falls in the range $0.5 < \sum_1^n I_j < 1$ so the final runs order metric for disordered strata is typically $1 < r < 2$. In ordered successions like the perfectly ordered asymmetrical cycle example longer runs occur, so the values of $\sum_1^n I_j$ and $\sum_1^n D_j$ are higher, and the final value of the runs order metric value for strata containing thickening or thinning trends is typically in the range $2 < r < 3$. Higher values are possible with longer runs of increasing or decreasing thickness. Partial shuffling of ordered strata (Fig. 1B) leads to an *r* value of 1.5, so slightly higher than the completely-disordered-strata example.

Perfectly ordered strata with symmetric cycles (Fig. 1D) demonstrates the importance of counting the position of a unit within a run. If this was not done, this example would give an *r* value similar to the disordered case because there are equal numbers of thickening and thinning runs present. However, because the metric takes into account the length of the runs, the *r* value in this case is 2.45, similar to the value for the perfectly ordered asymmetric example (Table 1).

The Importance of Comparative Disordered Models

Analysis of the Markov metric *m* and the runs thickness metric *r*, presented above, suggests that these metrics could be a useful summary of the degree of order present in a vertical succession of strata. For example, calculating the Markov and runs order metrics and for outcrop strata and finding that $m = 0.8$ and $r = 2.2$ might be considered good evidence for order in the strata. However, what is essential as a robust test for the presence or absence of order is to be able to construct a null hypothesis of no order, and then to compare a given succession against a range of strata similar in some respects but known to be disordered. If the comparison suggests that the null hypothesis of no order is highly unlikely to be true, then the typical sequence stratigraphic interpretations of ordered strata generated by periodic forcing may be a good way to proceed. Note though that this interpretation is somewhat complicated by model results that suggest autocyclic process are also capable of generating ordered strata, e.g., Burgess et al. (2001); Burgess (2006). However, if the null hypothesis cannot be rejected, typical sequence stratigraphic interpretations are unlikely to be valid, and alternative explanations would need to be found (e.g., Wilkinson et al., 1997; Wilkinson and Drummond, 2004).

Random shuffling of the observed strata, combined with Monte Carlo modelling (Kalos and Whitlock, 2009) provides a good way to generate a range of similar but disordered strata that can serve as a comparison with the observed succession. For each of 5000 realizations, the observed succession of strata is “randomized” by shuffling. Shuffling consists of randomly selecting two units within the succession, swapping them, and repeating this process *n* times, where *n* is the number of units in the section. In order to avoid problems creating transitions between the same facies, if swapping two units would lead to juxtaposition of the same facies the swap is not carried out and another two units are selected randomly to swap until a swap is possible. For each realization the *m* and *r* values are calculated and from this a probability density function for each variable is constructed by Monte Carlo modelling (e.g., Fig. 2B and 2C). This can then be plotted graphically along with the values of the *m* and *r* values calculated from the observed section. The final step in the process is to calculate from the distribution of shuffled realizations a probability that the observed succession could have occurred by chance.

The purpose of shuffling in this way is to ensure that a succession of the same length and with the same frequency of each different facies is maintained but any order present in the strata is, on average, removed. However, the "on average" caveat is important because even after *n* shuffles in an *n* unit long succession, some order may remain due to chance juxtaposition of particular facies. For this reason the process of shuffling the section *n* times is repeated 5000 times and the frequency distribution of all *m* and *r* values can then be used to compare with the *m* and *r* value calculated for the original succession. Note that using only a small number of shuffles relative to the number of units in the succession will tend to preserve some of whatever order was originally present in the strata, and that progressively increasing the number of shuffles will tend to create progressively more disordered strata with progressively lower values of *m* and *r*.

Calculation of 5000 examples of shuffled sections *n* units long, each with *n* swaps, defines a probability density function (PDF). The *m* and *r* values calculated from observed strata can then be compared with this PDF to give an indication of how much order may be present in the observed strata. In a perfectly ordered example (Fig. 2A-E) the observed *m* and *r* values lie well outside the limits of the PDF. In contrast, in the perfectly disordered case (Fig. 2K-O) the *m* and *r* values are both within the limits of the PDF. The position of the observed *m* and *r* values relative to the PDF can be quantified by calculating a *p* value that indicates the probability of the *m* or *r* value occurring within the range of values in the PDF as follows

$$p_m = \int_m^1 P_m(x)\,dx$$

$$p_r = \int_r^\infty P_r(x)\,dx$$

where P_m is the PDF for the Markov *m* value, p_m is the probability of finding an *m* value equal to or greater than the observed value within P_m, P_r is the PDF for the runs analysis *r* value, and p_r is the probability of finding an *r* value equal to or greater than the observed value within P_r. Some statisticians have argued that assigning specific significance to the *p* values, for example using confidence levels, is unwise because variation of the significance with *n* make this potentially inconsistent across different sample sizes (see arguments presented from a Bayesian point of view in, for example Lindley 2014). Sample-size issues are discussed below, but a simple and pragmatic approach uses the *p* value to summarize the strength of evidence for or against the null hypothesis but avoids use of specific confidence levels; values of p_r and p_m less than 0.01 are very strong evidence for ordered strata, values of p_r and p_m between 0.10 and 0.05 provide only weak evidence against the null hypothesis of disordered strata, and values of p_r and p_m greater than 0.1 provide no evidence against the null hypothesis of disordered strata.

Returning to the six examples of synthetic strata described above (Fig. 1), considering the p_m and p_r values produced for these cases is instructive. Values for *p* from these cases are given in Table 1 and plotted in Figures 2 and 3. In summary the six cases show that both p_m and p_r can be used to distinguish ordered from disordered strata. In the case of perfectly ordered asymmetric cycles (Fig. 1A and Fig. 2A-C), or perfectly ordered symmetric cycles (Fig. 1D and Fig. 3A-C), values of m and r calculated from the strata lie well outside the PDF produced by Monte Carlo modelling of the data, and both p_m and p_r are zero. For the facies succession in the partly ordered cyclical strata (Fig. 1B) the m value is still well outside the comparative disordered succession PDF (Fig. 2E and 2F), and p_m is effectively zero, confirming that the partly shuffled succession does still contain identifiable order that is highly unlikely to occur by chance. In contrast, considering thickness, the degree of disorder introduced by the shuffling is enough to bring the value of r into the positive tail of the shuffled PDF, leading to a p_r value of 0.009. Considering how much of the order is preserved in the partly shuffled strata (Fig. 2E and 2F), and considering the a large proportion of the shuffled successions generated lower r values, it would still be sensible to interpret this p_r value to be good evidence for order. However, this does show that the r statistic is more sensitive to disorder than the m statistic; $r > 2$ is likely only in very well-ordered strata.

As we might expect, the disordered-strata example (Fig. 1C) generates values for both m and r that lie within the shuffled PDF (Fig. 2H and I), and the resulting p_m value is 0.24 and p_r is 0.1 (Table 1). These values indicate that the strata could quite plausibly have arisen by random processes, as indeed they did, so in this case there is no evidence for order. The p_m and p_r values for the disordered strata with an exponential facies frequency distribution (Fig. 1E and Fig. 3E and F) are 0.144 and 0.411 respectively (Table 1) confirming that the m and r statistics can correctly indicate disordered strata even with a dominant facies. Finally, for disordered strata with exposure surfaces (Fig. 1F) p_m is 0.110 and p_r is 0.107 (Fig. 3H and I), compared to p_m = 0.237 and p_r = 0.097 for the disordered strata without exposure surfaces. This demonstrates that frequent occurrence of exposure surfaces related to a specific facies increases the level or order apparent in the strata.

Dependence of the Order Metrics on Section Length and Number of Different Facies

A key desirable property of metrics to identify order in strata is that they are applicable to various examples with a range of numbers of different facies and a range of succession lengths. Wilkinson et al. (1996) suggested that much interpreted cyclicity in peritidal carbonates was based on too few lithofacies per cycle to be properly distinguishable from what could occur by chance, so ideally the metrics should be not be too sensitive to section length or number of different facies, and any sensitivity should be easily explained and accounted for in interpretations. Calculating values for m and r for strata composed of perfectly ordered asymmetric cycles, and perfectly disordered strata with section lengths ranging from 5 to 100 units, and 2 to 20 different facies (Fig. 4) allows analysis of how m and r values are sensitive to total succession length and total number of different facies. Both m and r show a good distinction between the perfectly ordered and the perfectly disordered strata for sections 5 units or more in length. The cycle length in each section is 10 units, so section shorter than 10 units in length consist of one incomplete cycle, but this does not prevent distinct m and r values for these short successions of ordered and disordered strata. Increasing the section length produces a steady maximum value of both m and r for sections longer than 10 units. For the disordered strata, m decreases asymptotically from a mean of ≈ 0.3 to ≈ 0.1, with decreasing standard deviation (Fig. 4A). The r value for the disordered strata has a steady value of ≈ 1.3 for sections longer than 10 units (Fig. 4B). Looking at sensitivity to the number of different facies in a 100-unit-long succession, both m and r values successfully distinguish the perfectly ordered and perfectly disordered strata for sections with four or more different facies (Fig. 4C and 4D). For perfectly ordered strata, values of *m* are stable at 1 for sections with 3 or more facies, and value of *r* increase linearly reaching a value of □□9.5 for 20 different facies units. For the perfectly disordered strata, mean values of *m* and the standard deviation from 1000 random sections decrease from 4 to 10 facies and are stable at □0.1 for 10 or more facies. Mean values of *r* approach 1.4 for 20 distinct facies, and the standard deviation remains fairly constant. In summary, analyzing how the Markov order metric *m* and the runs order metric *r* vary with different number of facies and in sections of different lengths (Fig. 4) indicates that the *m* and *r* metrics provide a robust distinction between ordered and disordered strata for a section with at least four different facies, and enough units to define at least one complete cycle. Variation in cases with fewer than five different facies in a succession (Fig. 4B) supports this point. Note that this agrees with the observations in Wilkinson et al. (1996), who showed that interpretations of order in carbonate strata are problematic when based on identification of cycles composed of four or fewer lithofacies.

Measuring Disorder for Various Section Lengths and Number of Different Facies

Another key desirable property of metrics to identify order in strata is that they show a good correlation, positive or negative, with the degree of disorder present in the strata. In other words, perfectly ordered strata should produce a particular value, disordered strata a different value, and strata that are partly disordered should generate metric values somewhere in between. The *m* and *r* values calculated for the examples of perfectly ordered, partly ordered, and disordered synthetic strata (Fig. 1 and Table 1) suggest both metrics have this property, but it would be useful to know more about how this property might vary with the length of the section being examined, and the number of different facies in the section.

An effective way to determine this is to progressively increase the number of pairs of units randomly swapped in an initially perfectly ordered 100-unit section of asymmetric cycles. After each set of *k* swaps is complete, the *m* and *r* metrics are calculated. This process is repeated 1000 times for the same number of *k* swaps, and mean *m* and *r* values are calculated. The whole process is then repeated for a new number of swaps up to a total of 100 swaps, and in cases with different lengths of section, and different number of total facies (Fig. 5). In all cases 100 swaps should be enough, on average, to create a disordered section since the section is only 100 units long.

Results from these calculations show that for varying lengths of section the Markov metric *m* decreases in a linear manner from 1 towards zero for all section lengths greater than 5 units as the number of swaps increases (Fig. 5A). Longer sections maintain a degree of order with more swaps because in a longer section each swap introduces proportionately less disorder. As discussed earlier, values of *r* are more sensitive to even small amounts of disorder, so for all section lengths greater than 5 units, the *r* value decreases rapidly as the number of swaps increases (Fig. 5B) but longer sections maintain higher values of *r* with more swaps (Fig. 5B).

For strata with four or more different facies, with increasing numbers of swaps the Markov metric *m* also decreases in a linear manner from 1 towards zero (Fig. 5C). For the same number of swaps, e.g. 50, sections containing 4 facies show higher values of *m* than sections with 5 or more facies, but the degree of disorder present after a given number of swaps does not vary much for sections with 6 or more different facies (Fig. 5C). Note that the swapping method does not work for sections with only two or three different facies because in these cases it is not possible to swap different facies units without producing repeats of the same facies in adjacent positions in the succession after the swap, and the swapping algorithm does not permit this because same-to-same transitions are not permitted in the Markov analysis. Maximum *r* values increase with increasing number of facies because more facies allow longer runs (Fig. 5D), but for all examples with four or more facies increasing swaps shows a decrease in the *r* value from values near 2 to values close to 1.

In summary, analyzing the degree or disorder arising from increasing numbers of swaps (Fig. 5) indicates that the *m* and *r* metrics provide a robust distinction between ordered and disordered strata for sections with at least 4 different facies and enough units to define at least one complete cyclothem.

Nature is Messy - How to Select Facies Codes for This Method

In all of the ordered or partly-ordered synthetic examples given above, facies are coded according to position counting up through an asymmetrical or symmetrical cycle (e.g., Fig. 1), and this gives the best possible result in calculating the *m* statistic (e.g., Fig. 2). In synthetic examples it is possible to select facies codes in this way because the examples have been created to demonstrate cyclicity, so we know everything about the cyclicity and can select an appropriate facies coding. However, in natural examples where the nature of cyclicity is not known *a priori* it may be that the facies codes cannot easily be ordered to best reflect any cyclicity present. Since this may be a commonly encountered challenge when analyzing outcrop strata, it is important to understand what effect this has on this statistical-analysis method.

To explore this issue, three synthetic successions have been constructed and *m* and p_m values calculated for each (Table 1 and Fig. 6). The first succession contains five facies arranged as perfectly ordered asymmetrical cycles (Fig. 6A). In the previous example of synthetic asymmetrical cycles (Fig. 1A) the facies sequence was 1 2 3 4 5, then repeat; in this case each cycle consists of the facies sequence 2, 3, 5, 4, 1, then repeat. Examination of the transition probability matrix for this succession (Fig. 6B) shows a value of 1 in each row, representing the probability of going to the next facies in the cycle, and all other values in the row are zero. The key difference in this case is that, unlike the previous ordered example (Fig. 1G), the probability 1 values are not arranged along a diagonal in the matrix. Consequently the *m* value for this succession is 0.4, not 1.0. This reflects the lack of clustering of high probability values onto a matrix diagonal, which in turn reflects the fact that the facies are not coded according to their position in a cycle. However, most importantly, despite the low *m* value for this case, the p_m value is 0.00 (Table 1) because the *m* value of 0.4 still lies well outside the PDF of *m* values produced by random shuffling of the strata. This demonstrates that even in cases where cyclicity is present, but not recognized or assumed *a priori* and used to guide facies coding, this *m* statistic technique should still identify the order present in the strata despite the lower value of *m*.

The next succession contains seven facies, of which four are arranged in repeating asymmetrical cycles. The remaining three facies occur randomly and less frequently in the succession (Fig. 6F, H, K, and M). Asymmetric cycles consist of four facies in the sequence 1, 7, 3, and 5. Because these facies are arranged in well-ordered cycles the probability of transition following this sequence is high, and this is reflected in the TP matrix (Fig. 6B). However, as in the previous example, because the cyclical facies do not count up incrementally (see the histogram for facies frequencies, Fig. 6G), these high probabilities are not arranged along a single diagonal in the matrix as has been the case in the previous strongly ordered examples. Consequently the *m* value for this succession is 0.33 (table 1), lower than might be expected for ordered strata based on the previous examples discussed above. Despite this low value of *m* the value of p_m is 0.004, because the original succession lies on the extreme tail of the PDF produced by random shuffling of the strata (Fig. 6D), showing that the *m*-value technique can identify the order present in this more complicated case.

The third and final example in this section is a succession formed from asymmetric cycles with facies arranged in regularly rising continuous values 1, 2, 3, and 4 (Fig. 6K and M). Other rare random facies are interspersed randomly, leading to a TP matrix which has the strongest diagonal element of these three cases, but some off-diagonal high values too. The *m* value for these strata is 0.39, which is actually slightly lower than the previous example. Nevertheless, the P_m value is again 0.000, providing strong evidence for order.

In summary, these three examples demonstrate that while the choice of facies codes and how these relate to any cyclicity present in the strata does have an effect on calculated *m* values, this does not prevent this method from correctly identifying order present in synthetic successions. It may be possible to develop a method to allow the facies coding to be selected that will produce a maximum value of *m* using an inverse optimization technique, but this requires further investigation.

A Point about Stationarity

In statistical analysis, a stationary process has the property that the mean, variance, and autocorrelation structure do not change over time. Dealing with non-stationary data can be problematic with parametric methods where calculations are made on the assumption of a single value for the mean, since this may not apply equally well to all subsets of non-stationary data. The method presented here has the advantage of being non-parametric, so non-stationarity is less of an issue. If there are possibly significant changes in the properties of strata in a vertical succession, the best approach is to first analyze the whole succession. Results from this composite analysis can then be compared with results from analysis of particular intervals that may have different properties, for example different frequencies of occurrence of particular facies that lead to different structures in the TP matrix. An example of this issue and how this method can address it is given in the example of Upper Cretaceous carbonate platform-margin strata below.

OUTCROP APPLICATIONS OF THE METHOD

Pennsylvanian Siliciclastic High-Frequency Sequences, Illinois

Pennsylvanian (Upper Carboniferous) strata around the world are often presented as examples of well-developed cyclicity (e.g., Olszewski and Patzkowsky, 2003), and many ideas about cyclic strata were developed from strata of this age in continental USA, particularly Illinois (Weller, 1930) and elsewhere (Opluštil and Cleal, 2007). More recently Wilkinson et al. (2003) reanalyzed Pennsylvanian strata from Illinois using lithofacies thickness distributions and basic Markov analysis, and found that they “exhibit no compelling evidence of high-frequency cyclic order, either as regularity in the recurrence or in the compositional ordering of lithofacies elements”. In the ensuing discussion and reply, Weibel (2004) stated that “Because they are delimited by unconformities, cyclothems are allostratigraphic units” and therefore “stratal order is inconsequential, except perhaps for the lithofacies associated with unconformities." However, saying that strata show no evidence of order between unconformities is not consistent with much of the sequence stratigraphic method that attempts to recognize trends in the lithofacies, nor is it consistent with assumptions of regional to global control on the formation of these strata (e.g., Catuneanu, 2006; Catuneanu et al. 2009; Miall, 2010), as Wilkinson et al. (2004) pointed out in reply. Wilkinson et al. (2004) also make the point that there is no evidence to demonstrate that the unconformities bounding the units defined by Weller (1930) are in fact regionally extensive surfaces; equally plausible is that they are locally developed channel-base erosion surfaces. These are by now familiar arguments in stratigraphic debate. The aim here is to apply the techniques for *m* and *r* statistical analysis to one of the classic sections from Illinois and compare results with the original interpretation by Weller (1930) and subsequent analysis carried out by Wilkinson et al. (2003).

Section number 5 is one of the longer sections from the Wanless (1957) data, measured with bed-by-bed logging from a section near the Sangamon river in Illinois. The section is 68.9 m thick and composed of 52 lithofacies units with a mean thickness of 1.33 m. There are eleven distinct lithofacies recognized in the strata, including clean sandstones, sandy shales, shale, coal, and both freshwater and marine limestones (Table 2). The *m* value calculated from the strata is 0.187 and the *r* value is 1.44. In both cases these values lie within the range that can occur in sections constructed by random shuffling of the facies unit (Fig. 7); p_m is 0.605 and the p_r is 0.115, both substantially higher than 0.01, showing that the analysis provides no evidence of order in the strata. Aside from demonstrating how this analysis can be performed on outcrop data, this example also supports the results from Wilkinson et al. (2003), suggesting that, as they stated, these “classic Pennsylvanian successions in west-central Illinois exhibit no compelling evidence of high-frequency cyclic order”.

Campanian Progradational Siliciclastic Strata, Book Cliffs, Utah

Late Cretaceous Campanian alluvial, coastal, and shelfal foreland-basin deposystems of the Book Cliffs, Utah and Colorado, U.S.A., have been a focus of sequence stratigraphic research for many years, and work done here has been the basis for many of the outcrop-scale sequence stratigraphic models (e.g., Van Wagoner et al. 1990, Catuneanu, 2006). More recently Hampson et al. (2014) have analyzed these strata in the context of the source-to-sink concept to define a mass balance and to begin to understand, for the first time, the controls on the strata. Stratigraphic sections presented in Hampson et al. (2014) represent a compilation of previously published descriptions from various sources (e.g. Kamola and Van Wagoner, 1995, Charvin et al. 2010).

Two sections are used in this analysis. The first is from the Castlegate outcrop near Helper, which is Entry 7 in Appendix A of Hampson et al. (2014). The section is 585 m thick, with 99 lithofacies units composed of 9 different facies (Table 2) and a mean facies unit thickness of 5.91 m. Section data were compiled from data in Robinson and Slingerland (1998) and Campion et al. (2010). The second section is from the Tusher Canyon outcrop, which is Entry 10 in Appendix A of Hampson et al. (2014). This section is 401 m thick, with 41 facies units composed of 9 different facies, with a mean unit thickness of 9.780 m. Data were compiled from Van Wagoner (1995) and Hampson (2010). In contrast with the bed-by-bed logging at a single location in the Pennsylvanian example above, both these Book Cliffs log examples represent a coarser bedset scale composite of observations from several measured section and well logs in a 3 km^2 area (Fig. 8A and F). Lithofacies from the logged sections are shown in Table 2.

Both sections show an upward change from marine to terrestrial facies representing a progradational stacking pattern (Hampson et al., 2014). The Castlegate section (Fig. 8A) is more proximal than the Tusher Canyon outcrop (Fig. 8F). Both sections show good evidence for order in their facies successions, with p_m values of 0.00 in both cases, showing that the calculated m values of 0.495 and 0.388 lie well outside the range of m values produced by random shuffling of the facies units (Fig. 8D and I). In contrast, there is no evidence in either section for order in the thickness of facies units; values of p_r are 0.95 and 0.91 respectively.

Results from the statistical analysis support an interpretation of a distinct coarsening- and shallowing-upward trend in the facies related to a progradational stacking pattern in the strata consistent with interpretations made in Van Wagoner et al. (1990) and Hampson et al. (2014). Lack of evidence for order in lithofacies thicknesses suggests that while coarsening and shallowing trends produce demonstrable order, these trends are not reflected in the observed stratal thicknesses. This perhaps suggests either that variation in rate of accommodation creation is not a key control on thicknesses in this case, or that rate of accommodation creation did not vary in any ordered way.

This example demonstrates how this statistical technique can provide quantitative support for sequence stratigraphic interpretations. An interesting issue arising from these results is how much the resolution of the data, in this case relatively coarse, influences the results. Facies successions from offshore through distal lower shoreface to proximal lower shoreface to upper shoreface are developed over a vertical extent of a few tens of meters over a depositional-dip extent of 5-10 km, so the results may be broadly similar if taken from a single vertical section (Hampson, pers. comm., 2014) but this requires further testing. It may also be illuminating to investigate how the degree of order present in the strata varies in different locations down depositional dip and along depositional strike.

Upper Cretaceous Carbonate Platform Margin Strata, Northern Spain

Carbonate strata in the Pyrenees Mountains of northern Spain record the evolution of several Late Cretaceous carbonate-platform systems. One example of platform-margin strata in this area was described by Pomar et al. (2005), who interpreted Santonian carbonate strata on the flanks of the San Corneli anticline to be "simple sequences and parasequences according to internal lithofacies arrangement and inferred sea-level cyclicity". They interpret "persistent occurrence of these lithofacies" to show that strata are "grouped in two facies assemblages" that "allow definition of two types of carbonate shelves: rudist buildups and calcarenite wedges" and that these also have "recurring patterns of lithofacies arrangement". This facies interpretation was then used as the basis for a sequence stratigraphic interpretation that defined depositional sequences within the strata which were interpreted to "be explained by means of changes in accommodation related to high-frequency sea-level paracycles and cycles that allow identification of parasequences and simple sequences". All the observations and interpretation of recurring patterns and facies grouping were made on a purely qualitative basis without any quantitative analysis of the arrangement of lithofacies. It is therefore instructive to apply the Markov and runs analysis methods to these strata to see if the qualitative interpretations are supported by quantitative analysis.

The Rio Carreu section as measured in Pomar et al. (2005) is 163 m thick, with 61 stacked facies units composed of 6 distinct facies, and a mean thickness of 2.69 m (Fig. 9A). The original facies coding from Pomar et al. (2005) has been modified here to ensure that their interpreted facies trends are represented by a progression of facies codes (Table 2) to ensure the best possible quantification of any cyclicity present. Quantitative analysis shows that the Markov metric *m* is 0.236, which lies well within the range of *m* values that can occur in randomly shuffled versions of the strata, leading to a p_m value of 0.214 (Table 3 and Fig. 9D). This is despite a high probability of transition from facies 2 and 3 to facies 1. Similarly, there is little evidence for order in thickness values; *r* is 1.15 and p_r is 0.952.

Somewhat ironically, the clean carbonate rudist- and coral-rich facies (facies 3 to 6 here, facies 1 to 4 in Pomar at el., 2005, see table 2), highlighted by Pomar et al. (2005) as being particularly cyclic and related to rudist buildups, show the least evidence for order based on the transition probability matrix (Fig. 9B). This is probably in part due to low frequency of occurrence of some of these facies (Fig. 9C), but if so, the point is still that there is not enough evidence in this section to say that these strata are arranged in any particular pattern. Yet this is exactly what Pomar et al. (2005) did argue, despite the lack of any quantitative evidence. They also stated that “many high frequency shallowing-upward cycles in carbonates are often truncated and incomplete”. This is essentially the same as saying that the cycles do not exist in any evidence-based form, unless you make an *a priori* assumption of external forcing. There is no particular justification for this assumption given an equally plausible alternative interpretation of a disordered facies mosaic that responds to any external forcing in a more complex manner than simple sequence stratigraphic models assume (e.g., Wilkinson and Drummond, 2004; Wright and Burgess, 2005; Burgess, 2006; Purkis and Vlaswinkel, 2012). In summary, the interpretation of the Rio Careu section in Pomar et al. (2005) is a good example of a model-driven sequence stratigraphic interpretation of cyclicity in carbonate strata that is not supported by quantitative evidence. It could be argued that based on detailed fieldwork and observations on two-dimensional sections the interpretations of stratal order are justified. However, based on the vertical section data presented in the paper, there is little justification for an interpretation of cyclicity.

The Rio Carreu vertical succession is also an interesting case for another reason. Particular facies occur with higher or lower frequency in specific parts of the section. For example, the coral-sponge-rudist sheetstone, the coral-rudist mixstone, and the rudist-bearing grainstone (facies 2, 3, and 4 in Pomar et al. (2005), facies 4, 5, and 6 here, Table 2) occur almost exclusively only between 50 and 80 m, whereas facies one to three are the only lithologies present from 80 m upwards. This is an example of statistical non-stationarity, as discussed above and in other carbonate examples (e.g., Wilkinson and Drummond, 2004). It is important to consider how non-stationarity impacts this analysis method. One way to do this is to reanalyze the strata over smaller thickness ranges for which the statistical properties are less variable and compare with results from the whole succession.

Markov metric *m* is 0.236, which lies well within the range of *m* values that can occur in randomly shuffled versions of the strata, leading to a p_m value of 0.214. Values of p_r and p_m less than 0.01 are very strong evidence for ordered strata, values of p_r and p_m between 0.10 and 0.05 provide only weak evidence against the null hypothesis of disordered strata, and values of p_r and p_m greater than 0.1 provide no evidence against the null hypothesis of disordered strata

Analysis of the lowest 79 m of Rio Carreu strata gives an *m* value of 0.242 and a p_m value of 0.450 (Table 3, Figure 9I), providing no evidence against the null hypothesis of disordered strata, which is the same result as obtained from analysis of the whole section. Analysis of the upper 93 m gives an *m* value of 0.427 and a p_m value of 0.032 (Table 3, Fig. 9G), which are quite different from the values obtained for the whole succession. The p_m value lies within the upper tail of the distribution of p_m values likely to occur by chance (Fig. 9G), and provides some evidence against the null hypothesis. However, with only three facies in the upper part of the succession, a degree of apparent order is inevitable and therefore not particularly instructive; analysis of the upper section in isolation demonstrates a degree of non-stationarity but does not reveal anything that was not clear from analysis of the whole succession.

Silurian Barn Hills Formation, Utah

Silurian strata of the Barn Hills Formation in Utah were described in Harris and Sheehan (1996) and analyzed quantitatively in Lehrmann and Goldhammer (1999). The described section consists of 109 units with a total thickness of 180 m with 10 distinct facies and a mean facies unit thickness of 1.69 m (Fig. 10a). Based on a Markov chain analysis, Lehrmann and Goldhammer (1999) claimed that the facies succession in these strata are non-random at 95% confidence and they used this section and other Silurian strata to define characteristic vertical stacking patterns interpreted with a sequence stratigraphic model. The strata have been reanalyzed here based on the logged section presented as Figure 13 in Lehrmann and Goldhammer (1999). Facies 6 and 8 were not presented in this section, so for this analysis facies codes 7 and 9 to 11 have been renumbered (Table 2).

The calculated *m* value for the Barn Hills succession is 0.212, and random shuffling of the strata to calculate a PDF of *m* values gives a p_m value of 0.0902. This represents an *m* value that is in the highest 10% of values generated by random shuffling, and in a test with a confidence interval at 80% this would be sufficient to reject the null hypothesis and say that the strata are ordered. A more careful interpretation is that there is some weak evidence for order in the strata, as indicated by some values of 0.5 and higher in the transition probability matrix. However, many of these are related to the most frequent facies, and there is only weak clustering of high transition probabilities on the diagonals of the matrix, so the evidence for development of cycles in this succession is weak. The *r* value calculated for these strata is 1.385 and the p_r value is 0.132, indicating no evidence for thinning- or thickening-upwards patterns in the strata.

An interesting question to ask is why the Markov-chain analysis in Lehrmann and Goldhammer (1999) shows evidence for ordered strata when analysis using the method presented here does not. The difference arises because the methods are quite different, especially in terms of the random models used in each case. Lehrmann and Goldhammer (1999) used the method given in Davis (2002), which is a rather convoluted approach to calculate embedded transition probabilities in a theoretical succession with the same number of distinct and independent facies. The approach used here is non-parametric, makes more use of the information contained in the transition probability matrix, and uses a random model more directly based on the observed data, so that issues like a small number of more frequently occurring facies do not unduly influence the results.

DISCUSSION

A simple periodic external signal controlling stratal architecture is a ubiquitous element in sequence stratigraphic interpretations. Periodic external forcing is assumed to create ordered strata. Ordering could take many forms, for example trends in depth-dependent fauna (e.g., Spence et al., 2004; Spence and Tucker, 2007) but more commonly described patterns are coarsening- or fining-upward trends, thickening- or thinning-upward trends, or a particular upwards progression of lithofacies, all of which are unlikely to occur by chance. If evidence for the presence of such order can be demonstrated, then there may well be an argument for external forcing of the strata, but too often such interpretations are made on the basis of only weak evidence (e.g. Anderson, 2004; Weibel, 2004; Pomer et al., 2005; Myrow et al., 2012).

The methods presented in this work have been designed to provide robust yet simple-to-derive and simple-to-interpret measurements of the degree of order present in strata. They should allow sedimentary geologists to quickly and easily test a hypothesis of ordered strata. Quantitative tests of order are important because, with certain caveats, they tend to be more objective and reproducible. In contrast, making interpretations without quantitative evidence often leads to subjective, error-prone, non-reproducible results that are a poor basis from which to build a firm scientific consensus (Baddeley et al., 2004). It is also important to note that consensus built on qualitative interpretation made via collective application of a model to interpret strata (e.g., sequence stratigraphy) is not at all the same thing as consensus built on more objective quantitative evidence. In many cases, simply being able to interpret strata in terms of a model, for example to provide qualitative identification of sequences, is taken as evidence in support of that model. More accurately, this is simply circular reasoning, as pointed out by Miall and Miall (2004). A quantitative test to identify whether order is present in the strata by rigorous quantitative comparison with disordered strata has the potential to break this circle and allow more progress in understanding the strata being studied.

There are, however, some caveats associated with this method. Classifying strata into a small number of discrete facies and then logging using these facies classes is probably a more subjective exercise than we care to admit. Application of a quantitative test for order cannot address this subjectivity if it is inherent in the data as collected. Logging strata using a small number of discrete facies also has the potential to mask much complexity, for example subtle variations in lithology across gradational boundaries, and the problem of how to define the thickness of facies units with gradational boundaries. Application of quantitative tests for order cannot address this issue either. Essentially the quantitative techniques presented here are only as good as the facies and thickness data they are used on. However, even with flawed data, it is still better to test for order using a quantitative method rather than state order is present based only on a qualitative interpretation.

Related to this issue of facies classification, the two siliciclastic examples given above demonstrate how the scale and detail of observation may influence the result when testing for the presence of order in the facies succession, even with a quantitative method. The method presented here works with both detailed and less detailed observation of strata, but recording every variation in lithology on a centimeter-by-centimeter scale through a succession may give quite different indications of order compared to a less detailed, more averaged record of the strata. This appears to be especially true with the thickness of lithological units, and it raises an interesting question: is the difference in order quantified at different scales of observation simply an artefact of the method of observation, or does it demonstrate that order is absent bed-by-bed but present when considering transitions between larger-scale stratigraphic units? It has been argued by various workers (e.g., Wilkinson et al., 1996; Wilkinson et al., 1997) that at a bed-by-bed scale a high degree of variability is present, while at a larger scale in the strata trends in facies transitions are more apparent. In statistical terms, this can mean that strata are non-stationary over longer length scales (e.g., Wilkinson and Drummond, 2004). This is usually explained as a consequence of random or complex events being manifest at the small scale, for example beds provided by individual floods or storms or migration of organisms into an area, while longer-term trends in relative sea level or climate produce a pattern in the strata when looking at longer stratigraphic intervals. If the method presented here is now applied to strata at different scales in a variety of studies, as illustrated in the two siliciclastic examples above, it is possible to begin to create the necessary database to address this question in a quantitative manner. Future work will address this.

Exposure surfaces, and other significant surfaces such as maximum transgressive surfaces (Catuneanu, 2006), are often invoked as evidence for order in strata, especially when such evidence is lacking in the facies or thickness data. For example, Weibel (2004) stated that "stratal order is inconsequential, except perhaps for the lithofacies associated with unconformities". This statement is inconsistent with much previous work claiming to identify exactly such stratal order, as pointed out in response by Wilkinson et al. (2004). A more thoughtful consideration of the significance of surfaces in an analysis of order was provided by Lehrmann and Rankey (1999), who showed that while statistical evidence for order may be absent in simple facies analysis of a 1D vertical section, including surfaces increased the evidence for order, and considering the lateral extent of those surfaces increased it yet further. The example of disordered strata with exposure surfaces discussed above replicates the

Lehrmann and Rankey (1999) result, showing increased order when exposure surfaces are included. It is certainly always sensible to include, if possible, a consideration of lateral extent of both facies in surfaces in any outcrop analysis, because clearly this can yield important additional evidence for order. However, it is important to consider the implications of this point. If there is no evidence for order in the 1D vertical facies succession alone, but the order becomes apparent when surfaces are included in the analysis, what does this mean? One interpretation of this result, in a carbonate setting at least, is that the depositional system being studied is a facies mosaic, with no intrinsic order in the facies arrangement, and order only emerges from surfaces generated by periodic external forcing. This is not a particularly radical view (e.g., Wilkinson et al. 1999; Wright and Burgess, 2005), but it is quite different from the view presented by typical sequence stratigraphic interpretations (e.g., Pomar et al. 2005; Bosence et al. 2009), indicating that perhaps the sequence stratigraphic models underpinning these interpretations will need to be modified or discarded.

As already mentioned, none of the examples studied in this work show evidence for order in thickness of facies units, even at the larger scale of observation. Although thickening- and thinning-upward trends are commonly cited in interpretations of strata, this selection of examples shows that there may be little quantitative evidence to support these interpretations, at least at the bed scale. When considering bed-set, parasequence, and high-frequency sequence scale thickness variations in carbonate strata, previous quantitative analysis (Lehrmann and Goldhammer, 1999) used several statistical techniques and showed that, depending on the technique applied, between a half and a quarter of 44 outcrop examples showed some evidence for order at a bedset or parasequence and high-frequency sequence scale. The contrast between the lack of evidence for order found at the bed-by-bed scale in this study versus Lehrmann and Goldhammer (1999) may indicate that the same distinction between short-term random events and longer-term externally forced patterns discussed above in the context of facies transitions also applies to stratal thicknesses. Further work on a wider range of examples is needed to test this. If further analysis shows that bed-scale thickness trends are indeed rare, we may need to rethink some key elements of current sequence stratigraphic models that predict such trends in 1D vertical successions, and focus more on 2D and 3D relationships and on longer-term trends that may be present between thicker packages of strata . Further work applying this method to many examples may also establish a relationship between order in facies unit thicknesses and order in the facies succession itself, which might prove particularly helpful in advancing our understanding of ancient depositional systems.

A complication in many interpretations of order in strata is the multiple possible origins of any order identified. Although the dominant often unquestioning application of the sequence stratigraphic and cyclostratigraphic models means that alternatives to simple periodic external forcing are only rarely considered, ordered strata is often just as easy to explain by autocyclic processes (e.g., Lehrmann and Goldhammer, 1999; Burgess, 2006). Generally if there is not good evidence of abnormal subaerial exposure of subtidal strata, then an autocyclic and allocyclic explanation should be considered as equally plausible; favoring one or the other in the absence of diagnostic evidence is not helpful, yet examples of unjustified interpretations of allocyclic processes are numerous (Miall, 2010). Accumulation of quantitative data demonstrating the degree of order or disorder present in strata demonstrated to be allocyclic or autocyclic may help identify which is the more plausible interpretation in cases where other diagnostic evidence is absent. This quantitative approach, combined with construction and further testing of multiple hypotheses to explain the observations, rather than asserting that just one qualitative interpretation is the most favored explanation for strata, would be a big step forward.

Finally, an argument sometimes made by workers keen to invoke periodic external forcing, even in the absence of any evidence, is that order may be present in a form too complex for quantitative methods to identify. Cyclostratigraphers often take this approach, assuming that a signal must be present if only they can decipher it. An argument in support of this may be that describing something as disordered or indistinguishable from random is just another way of saying that it is too complicated for any order to be identified (e.g., Burgess, 2006). If such examples of complex allocyclic strata do exist, either the external forcing or the sedimentary response to the forcing, or both, is so complex that the strata produced are indistinguishable from a random succession. If the external forcing is not the simple type of signal so typically invoked by sequence stratigraphic and cyclostratigraphic models, this perhaps indicates that as well as thinking carefully about what we take as good evidence or order, we need to step back and consider the nature of external forcing we are searching for.

CONCLUSIONS

1. Order can be usefully defined as some arrangement of facies or unit thickness that has a discernable trend or pattern that is unlikely to occur by chance but instead requires some particular systematic process to form. Examples of order are coarsening-, fining-, thickening- or thinning-upward trends, and arrangement of strata into cycles.
2. Identifying order or pattern in strata on the basis of qualitative interpretation alone is not helpful if it is based more on *a priori* assumption than on sound evidence. To make progress in better understanding the behavior of depositional systems we need robust yet simple-to-apply tools to identify order in strata and, just as importantly, to indicate when order cannot be reliably demonstrated.
3. A simple but robust method for calculating the degree of order present in a vertical succession of strata has been presented here. The method has two metrics, the Markov metric *m*, based on analysis of an observed facies succession, and the runs metric *r*, based on analysis of observed thicknesses of stratal units. Each metric can be easily calculated for an observed vertical succession and compared with equivalent metrics calculated many times for randomly shuffled versions of the same strata. *V*alues of *p* can then be calculated from the comparison between observed and randomly shuffled cases, and these *p* values indicate the degree of evidence present for order in the observed strata.
4. Several synthetic strata examples show that the Markov and run metric values can successfully identify different degrees and types of order present in strata, and are robust for successions with a range of different lengths and different numbers of distinct facies.
5. Four outcrop examples, two siliciclastic and two carbonate, demonstrate how the method can be successfully applied, including to non-stationary strata. The analyses demonstrate that ordered facies successions and thickness trends may be less common than typically

assumed; none of the four examples analyzed show trends in thickness, and only the examples from the Book Cliffs, which represent a bedset-scale composite of observations, show evidence for facies order. This perhaps indicates that order will tend to be more apparent at larger scales.

6. Analyzing more examples of outcrop and subsurface strata using this method, especially systematic analysis of larger datasets composed of multiple vertical sections, has the potential to significantly enhance stratigraphic interpretation and move sedimentary geology forward to a new phase where quantitative analysis is a standard and discussions of order versus disorder are based on sound evidence. This should in turn improve our understanding of the nature of stratigraphic successions, and will hopefully provide better insight into the validity or otherwise of current stratigraphic models. It may also indicate that we need to focus less on bed-by-bed trends in 1D vertical successions and focus instead on longer-term trends and 2D and 3D geometries.

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Figure 1. Vertical successions (A to F) showing different degrees of order with the transition probability (TP) matrix (G to L) calculated for each succession. TP matrices are color coded according to the probability shown in each cell, from red (low probability) to green (high probability). Section A and TP matrix G is an example of a perfectly ordered facies succession. Strata are arranged in 5-unit asymmetric cycles, and the order is reflected in the TP matrix, which has a dominant diagonal structure shown with green cells (offset j = 1, see Equation 1) and blue arrows (offset j=2, see Equation 1) and gives a Markov statistic *m* value of 1. Section B and TP matrix H represent an intermediate situation, where a 100-unit ordered succession has been shuffled with 30 swaps. Some order is still present, leading to a Markov order *m* value of 0.56, intermediate between the perfectly ordered (A and G) and the random realization examples (C and I). Section C and TP matrix I represents disordered strata where there is equal probability of changing to any other facies. The resulting *m* value is relatively low at 0.2. Section D and TP matrix J show perfectly ordered strata arranged in symmetrical cycles, leading to a TP matrix with relatively high values concentrated in two diagonals and a Markov order metric value of 0.5. Section E shows disordered strata with a dominant facies, and the resulting TP matrix K shows high transition probabilities concentrated in a single column due to the dominant facies. Section F shows disordered strata with facies 6 representing exposure surfaces that always develop atop facies 5. The TP matrix (L) shows that this introduces an element of order to the strata.

Figure 2. A-C Analysis results from the perfectly ordered asymmetric cycles synthetic strata analyzed A = facies frequency histogram; B = calculated *m* value shown as a vertical red line (observed), along with a histogram (blue) showing the probability density function of *m* values arising from the 5000 iterations of random shuffling of the strata. Note that the observed *m* value lies well outside the range of the PDF, reflecting a p_m value of 0.00. C = the calculated *r* value as a vertical red line(observed), along with the histogram (blue) showing the probability density function of *r* values arising from the 5000 iterations of random shuffling of the strata. The observed *r* value also lies well

outside the range of the PDF, reflecting a p_r value of 0.00. D-F results from analysis of the partly ordered asymmetric cycles; D to F show the same plots as for perfectly ordered asymmetric cycles (A-C). The observed *r* value also lies well outside the range of the PDF reflecting a p_r value of 0.00. G-I Results from analysis of the disordered synthetic strata G to I show the same plots as for perfectly ordered asymmetric cycles (A-C). The observed *m* and *r* values now lie within the range of the PDF showing that the succession could occur by chance, as indeed it did.

Figure 3. A-C Facies frequency histogram (A), observed and randomly shuffled *m* values (B), and observed and randomly shuffled r values (C) for perfectly ordered strata with symmetric cycles. D-F Facies frequency histogram (D), observed and randomly shuffled *m* values (E), and observed and randomly shuffled r values (F) for disordered strata with a dominant facies. G-I Facies frequency histogram (G), observed and randomly shuffled *m* values (H), and observed and randomly shuffled r values (IO) for disordered strata with an exposure surface facies.

Figure 4. Results from tests of the sensitivity of the Markov order metric and the runs order metric to the number of different facies and to total section length. Metrics have been calculated for two stratal models, a range of different section lengths and a range of numbers of distinct facies. The results suggest that sensitivity to both section length and number of facies is low for both metrics.

Figure 5. Color-coded plots of *m* and *r* showing how these values vary with a particular number of swaps for different lengths of section. Each color-coded value of *m* and *r* is calculated from an initial perfectly ordered 100-facies-unit section of asymmetric cycles. After each set of *k* swaps is complete, the *m* and r metrics are calculated. This process is repeated 1000 times for the same number of *k* swaps, and mean *m* and *r* values calculated. The whole process is then repeated for a new number of swaps up to a total of 100 swaps, and in cases with different lengths of section and different number of total facies. In all cases 100 swaps should be enough, on average, to create a disordered section.

Figure 6. A-E Synthetic section (A) created to investigate the consequences of different ways of coding facies. In the top example (A) asymmetric cycles consist of five facies in the sequence 2, 3, 5, 4, 1. B is the transition probability matrix calculated for these strata showing high transition probabilities in one cell per row as expected for cyclical strata, but these cells are not arranged on the matrix diagonals, C is the facies frequency histogram, D is the observed and randomly shuffled *m* values, and E is the observed and randomly shuffled *r* values for the 1, 7, 3, 5 cycle strata. F-J Synthetic section (F) has asymmetric cycles formed from facies arranged in regularly rising values 1, 3, 5, and 7. G is the transition probability matrix calculated for these strata, H is the facies frequency histogram, I is the observed and randomly shuffled *m* values, and J is the observed and randomly shuffled *r* values for the 1, 3, 5, and 7 cycle strata. K-O Synthetic section (K) is strata composed of asymmetric cycles arranged in regularly rising continuous values 1, 2, 3, and 4. L is the transition probability matrix calculated for these strata, M is the facies frequency histogram, N is the observed and randomly shuffled *m* values, and O is the observed and randomly shuffled *r* values for the 1, 3, 5, and 7 cycle strata.

Figure 7. Pennsylvanian strata from section number 5 from Wanless (1957). The vertical section (A) was measured bed-by-bed from a section near the Sangamon River in Illinois. There are eleven distinct lithofacies recognized in the strata, including clean sandstones, sandy shales, shale, coal, and both freshwater and marine limestones (Table 2). The transition probability matrix (B), the facies frequency histogram (C), the observed and randomly shuffled *m* values (D), and the observed and randomly shuffled *r* values (E) all indicate no evidence for order in these strata.

Figure 8. Two Upper Cretaceous Campanian sections from the Book Cliffs, Utah and Colorado, U.S.A, from Hampson et al. (2014). The logs represent a composite of observations from several measured section and well-logs in a ≈ 3 km^2 area from previously published descriptions (Van Wagoner, 1995; Kamola and Van Wagoner, 1995; Robinson and Slingerland, 1998; Campion et al. 2010; Charvin et al.

2010; Hampson et al., 2014). The section in Part A is from the Castlegate Outcrop near Helper which is Entry 7 in Appendix A of Hampson et al. (2014). The section is composed of 9 different facies (Table 2). B is the transition probability matrix calculated for these strata, C is the facies frequency histogram, D is the observed and randomly shuffled *m* values, and E is the observed and randomly shuffled *r* values. The lower section (F) is from the Tusher Canyon outcrop, which is Entry 10 in Appendix A of Hampson et al. (2014). This section is composed of 9 different facies (Table2). G is the transition probability matrix calculated for these strata, H is the facies frequency histogram, I is the observed and randomly shuffled *m* values, and J is the observed and randomly shuffled *r* values. In both cases there is evidence for order in the facies succession but not in the stacked thicknesses.

Figure 9. Upper Cretaceous Santonian carbonate strata from the Rio Carreu section in the San Corneli anticline near Tremp, Spain. Strata are composed of 6 distinct facies (Table 2). Elements B to E are calculated for the complete succession shown in part A. B is the transition probability matrix, C is the facies frequency histogram, D is the observed and randomly shuffled *m* values, and E is the observed and randomly shuffled *r* values. From both facies and thickness analysis of the complete succession, there is no evidence of order. Separate analysis of the strata from 80 m upwards shows high transition probabilities (F), but these can occur by chance (G) because there are only three facies. Separate analysis of the lower section shows no transition probabilities higher than 0.5 (H) leading to an *m* value within the range likely to occur in randomly shuffled strata (I).

Figure 10. Silurian strata of the Barn Hills formation, Utah, USA, described in Harris and Sheehan (1996) and analyzed quantitatively in Lehrmann and Goldhammer (1999). The strata (A) consist of ten distinct facies (Table 2). B is the transition probability matrix calculated for these strata, C is the facies frequency histogram, D is the observed and randomly shuffled *m* values, and E is the observed and randomly shuffled *r* values. From both facies and thickness analysis, there is only very weak evidence of order.

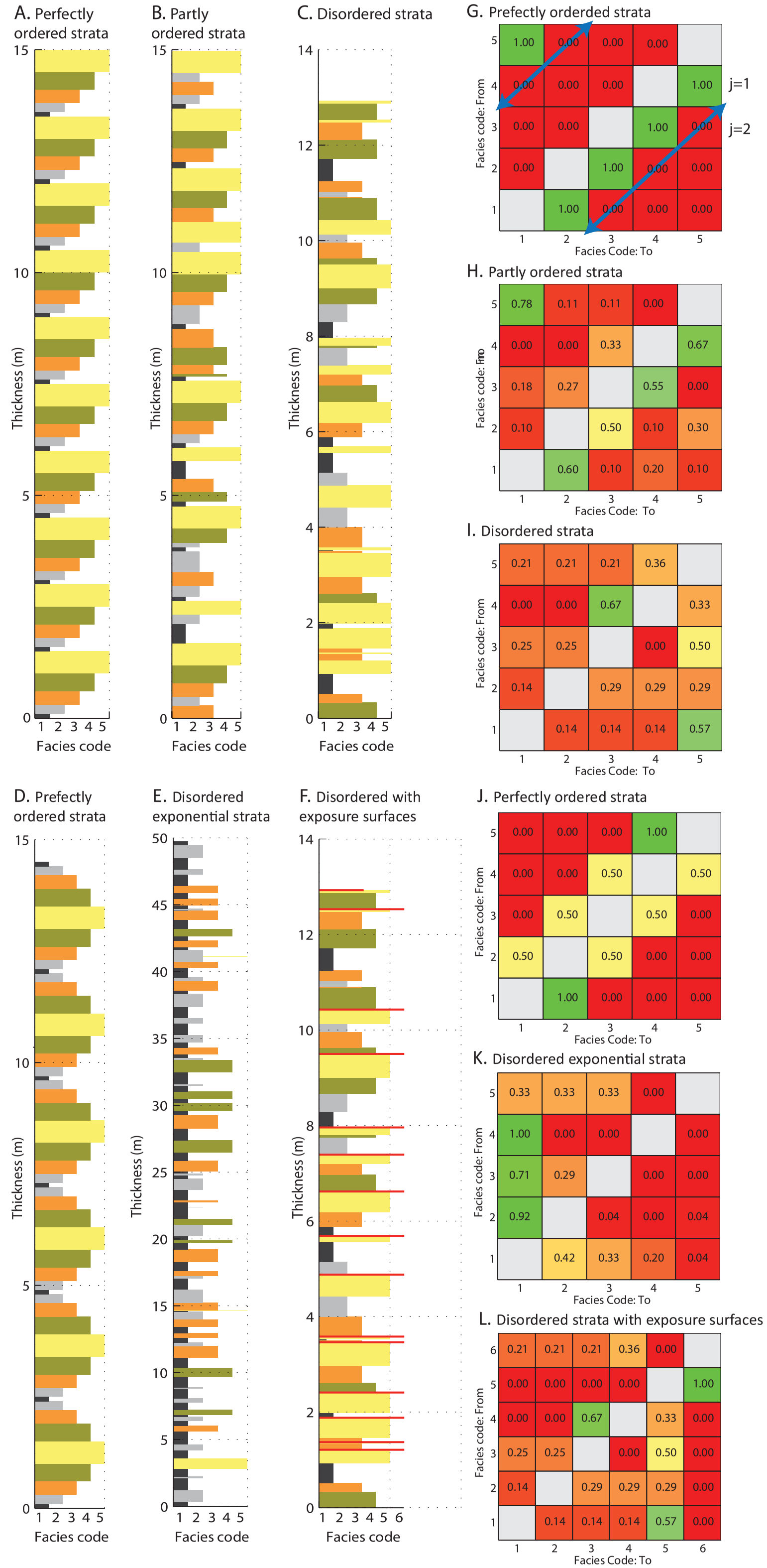

A. Perfectly ordered strata
B. Partly ordered strata
C. Disordered strata
D. Prefectly ordered strata
E. Disordered exponential strata
F. Disordered with exposure surfaces
Thickness (m)
Facies code
G. Prefectly orderded strata
H. Partly ordered strata
I. Disordered strata
J. Perfectly ordered strata
K. Disordered exponential strata
L. Disordered strata with exposure surfaces
Facies code: From
Facies Code: To
j=1
j=2

Figure 1

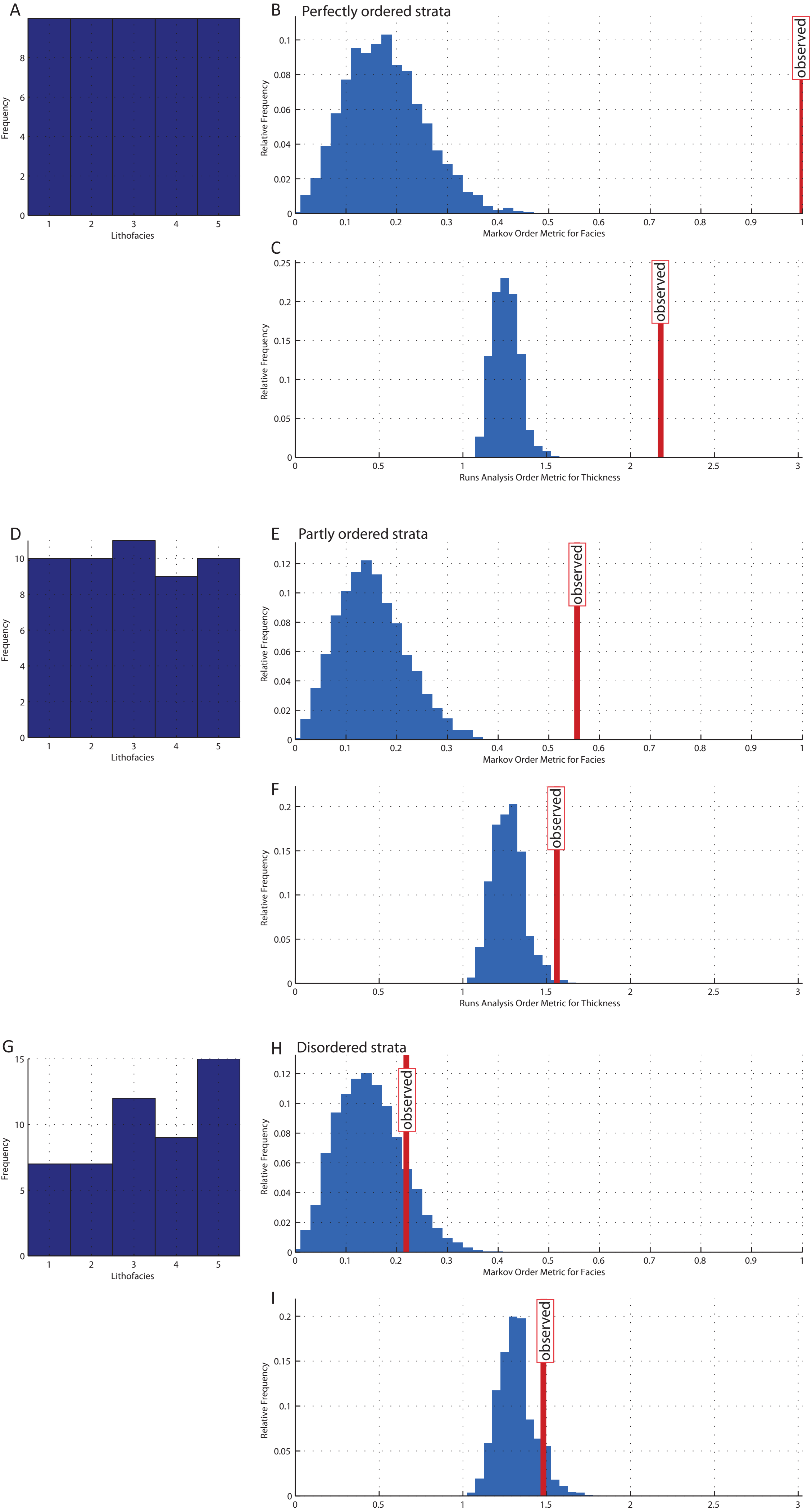

A
Frequency
Lithofacies
B Perfectly ordered strata
Relative Frequency
observed
Markov Order Metric for Facies
C
Relative Frequency
observed
Runs Analysis Order Metric for Thickness
D
Frequency
Lithofacies
E Partly ordered strata
Relative Frequency
observed
Markov Order Metric for Facies
F
Relative Frequency
observed
Runs Analysis Order Metric for Thickness
G
Frequency
Lithofacies
H Disordered strata
Relative Frequency
observed
Markov Order Metric for Facies
I
Relative Frequency
observed
Runs Analysis Order Metric for Thickness

Figure 2

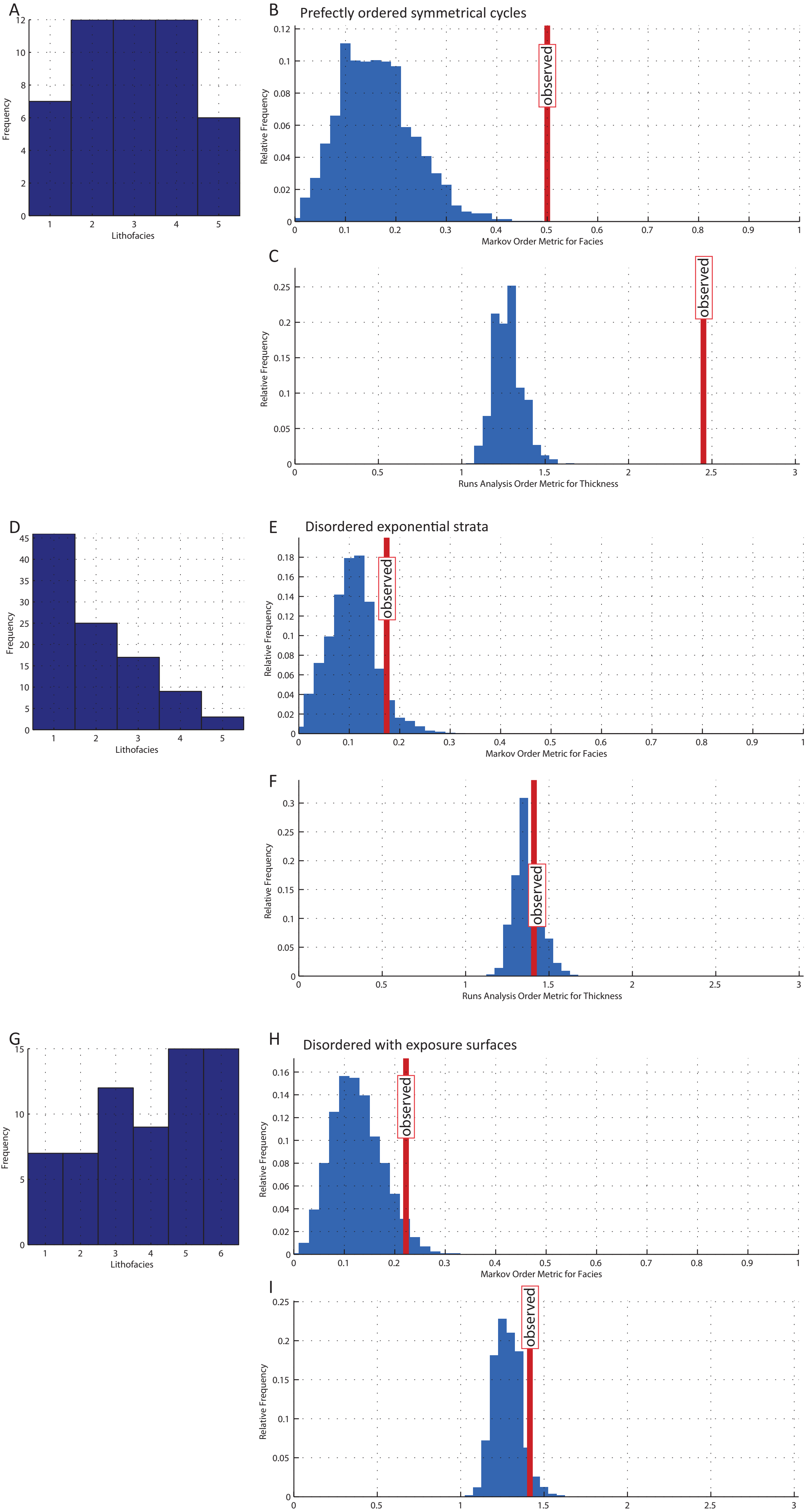
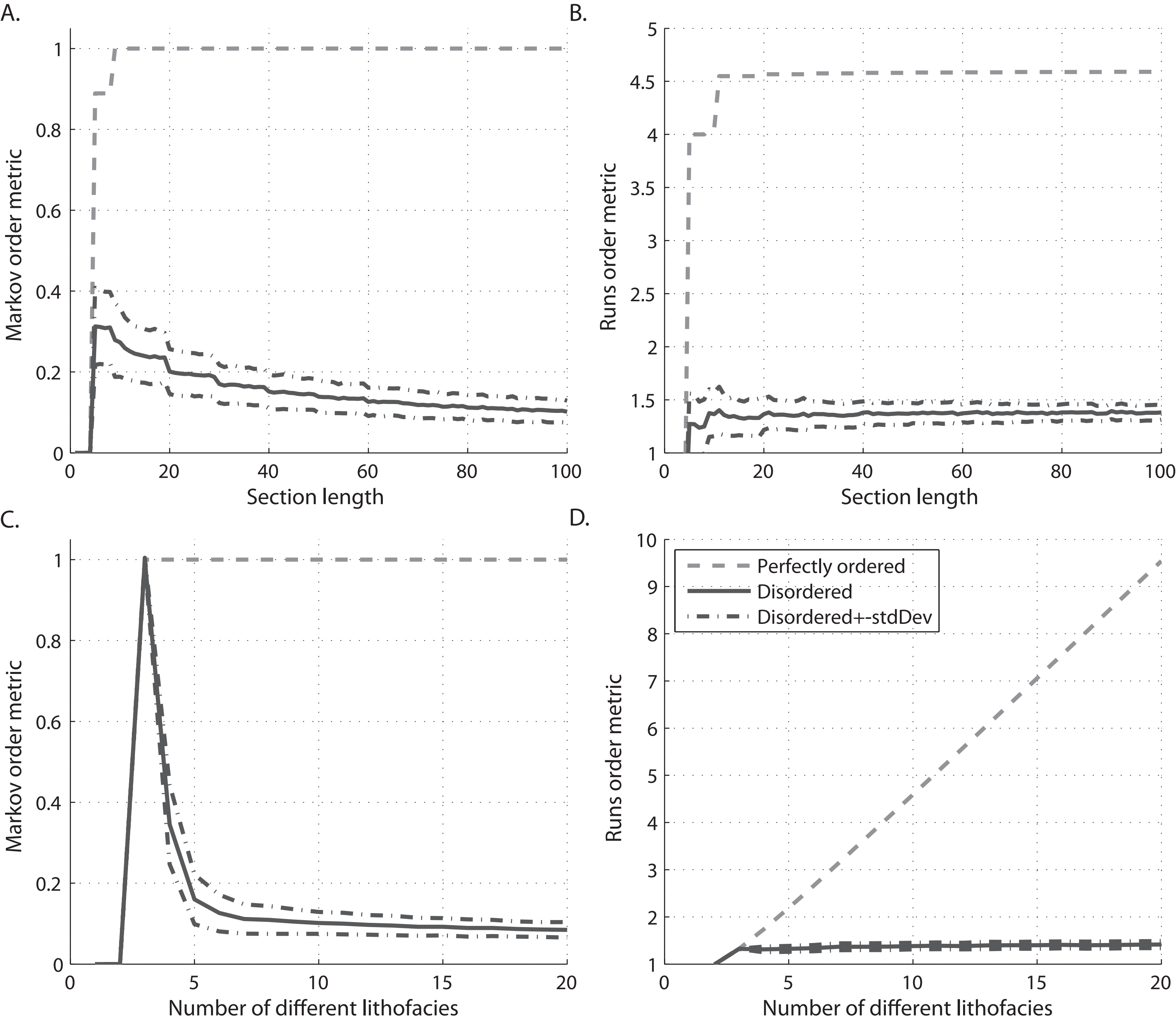

A
B
Prefectly ordered symmetrical cycles
C
D
E
Disordered exponential strata
F
G
H
Disordered with exposure surfaces
I
Frequency
Lithofacies
Relative Frequency
Markov Order Metric for Facies
Runs Analysis Order Metric for Thickness
observed

Figure 3



A.
Markov order metric
Section length
B.
Runs order metric
Section length
C.
Markov order metric
Number of different lithofacies
D.
Runs order metric
Number of different lithofacies
Perfectly ordered
Disordered
Disordered+-stdDev

Figure 4

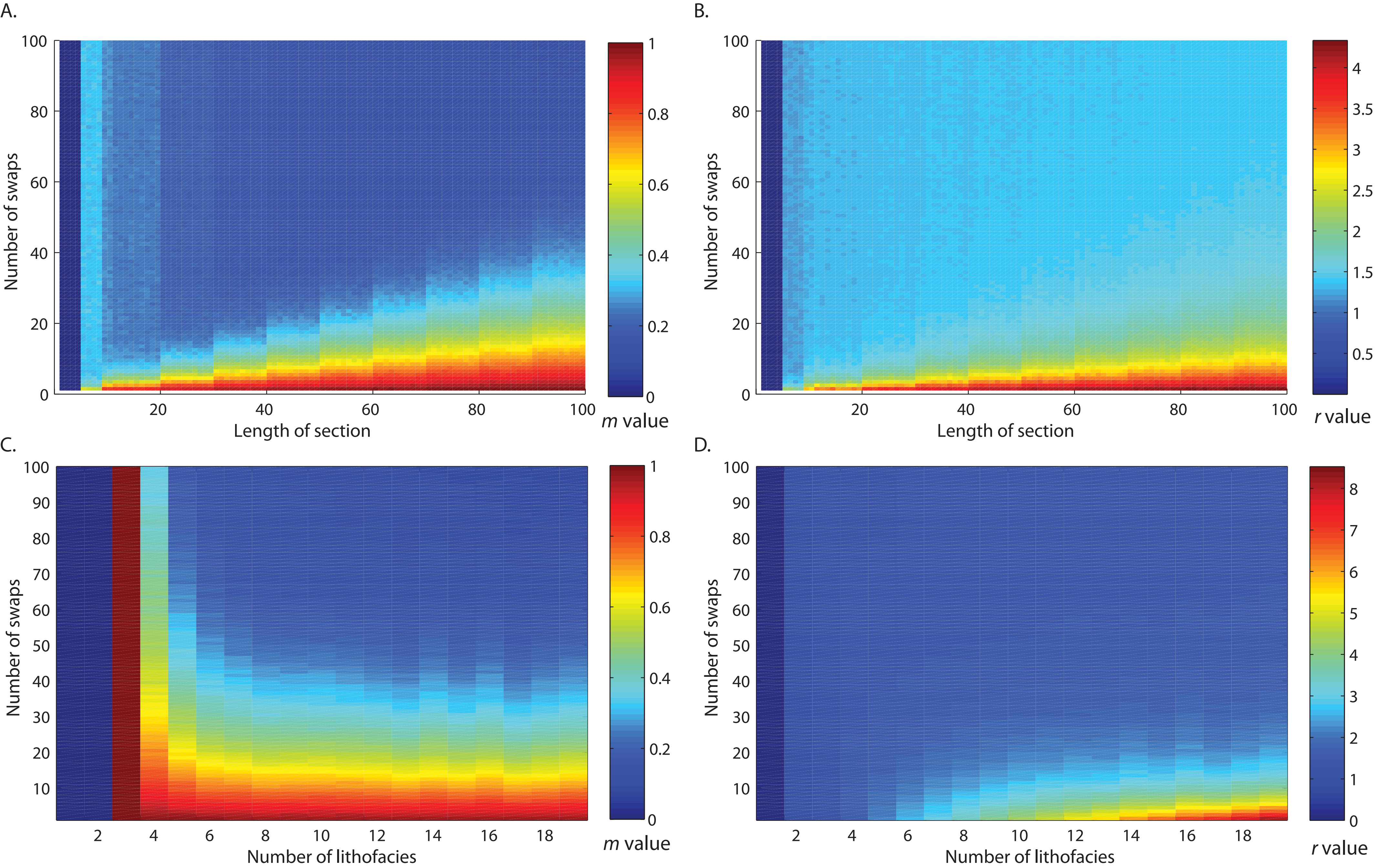

A.
B.
C.
D.
Number of swaps
Length of section
Number of lithofacies
m value
r value

Figure 5

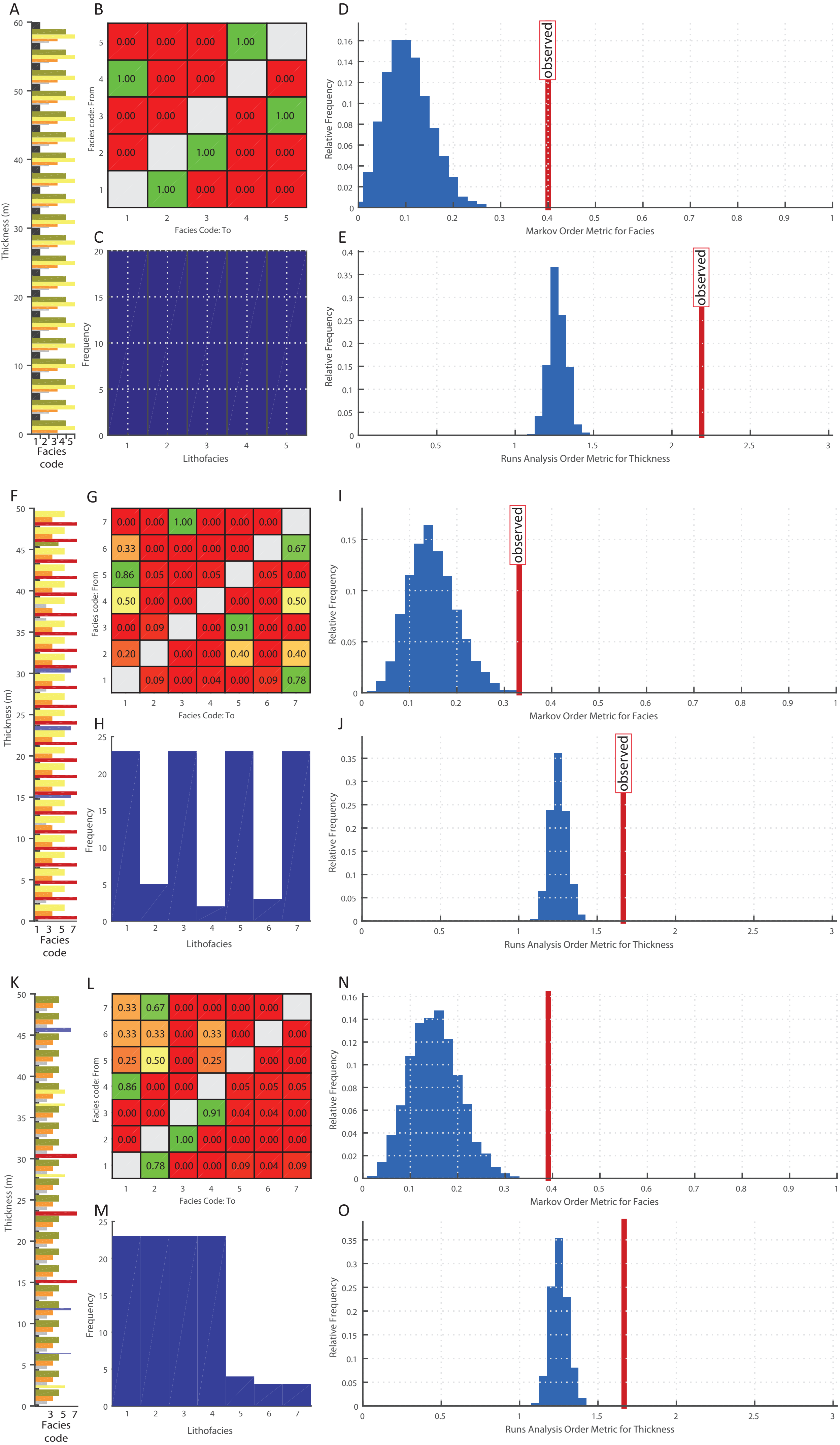

A
Thickness (m)
Facies code
B
Facies code: From
Facies Code: To
0.00 0.00 0.00 1.00
1.00 0.00 0.00 0.00
0.00 0.00 0.00 1.00
0.00 1.00 0.00 0.00
1.00 0.00 0.00 0.00
C
Frequency
Lithofacies
D
Relative Frequency
observed
Markov Order Metric for Facies
E
Relative Frequency
observed
Runs Analysis Order Metric for Thickness
F
Thickness (m)
Facies code
G
Facies code: From
Facies Code: To
0.00 0.00 1.00 0.00 0.00 0.00
0.33 0.00 0.00 0.00 0.00 0.67
0.86 0.05 0.00 0.05 0.05 0.00
0.50 0.00 0.00 0.00 0.00 0.50
0.00 0.09 0.00 0.91 0.00 0.00
0.20 0.00 0.00 0.40 0.00 0.40
0.09 0.00 0.04 0.00 0.09 0.78
H
Frequency
Lithofacies
I
Relative Frequency
observed
Markov Order Metric for Facies
J
Relative Frequency
observed
Runs Analysis Order Metric for Thickness
K
Thickness (m)
Facies code
L
Facies code: From
Facies Code: To
0.33 0.67 0.00 0.00 0.00 0.00
0.33 0.33 0.00 0.33 0.00 0.00
0.25 0.50 0.00 0.25 0.00 0.00
0.86 0.00 0.00 0.05 0.05 0.05
0.00 0.00 0.91 0.04 0.04 0.00
0.00 1.00 0.00 0.00 0.00 0.00
0.78 0.00 0.00 0.09 0.04 0.09
M
Frequency
Lithofacies
N
Relative Frequency
Markov Order Metric for Facies
O
Relative Frequency
Runs Analysis Order Metric for Thickness

Figure 6

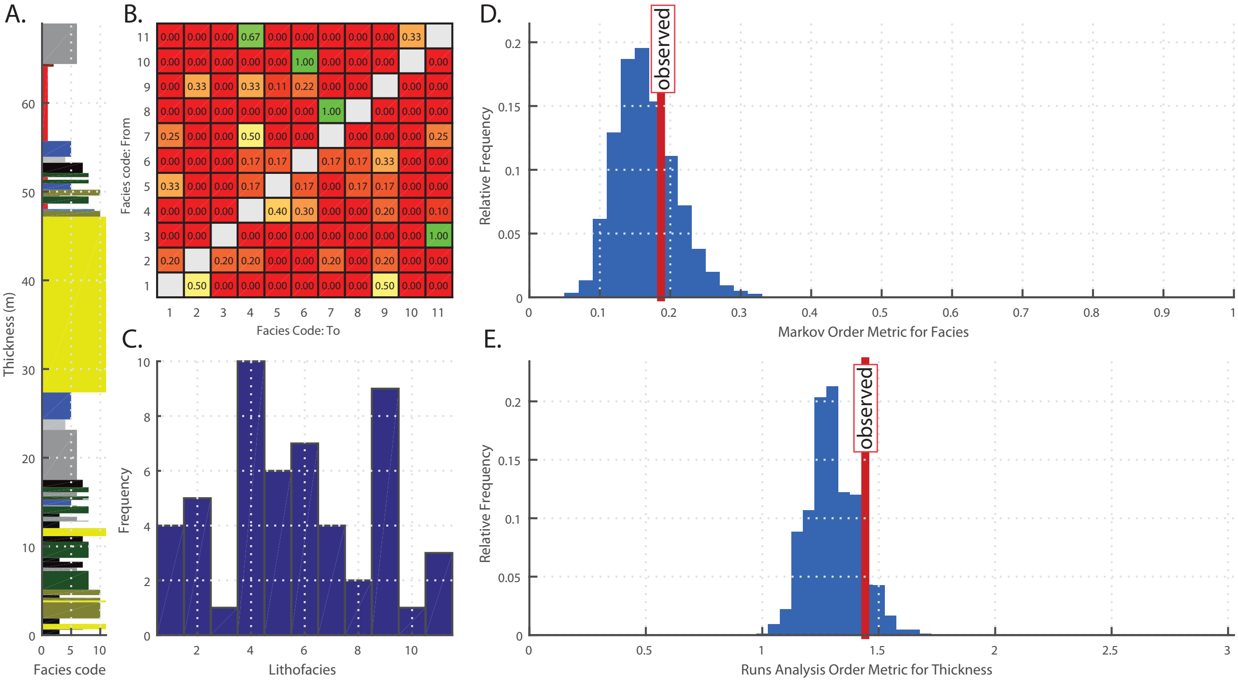

A.
B.
C.
D.
E.
Thickness (m)
Facies code
11 0.00 0.00 0.00 0.67 0.00 0.00 0.00 0.00 0.00 0.33
10 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00
9 0.00 0.33 0.00 0.33 0.11 0.22 0.00 0.00 0.00 0.00
8 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00
7 0.25 0.00 0.00 0.50 0.00 0.00 0.00 0.00 0.00 0.25
6 0.00 0.00 0.00 0.17 0.17 0.17 0.17 0.33 0.00 0.00
5 0.33 0.00 0.00 0.17 0.17 0.00 0.17 0.17 0.00 0.00
4 0.00 0.00 0.00 0.40 0.30 0.00 0.00 0.20 0.00 0.10
3 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00
2 0.20 0.20 0.20 0.00 0.00 0.20 0.00 0.20 0.00 0.00
1 0.50 0.00 0.00 0.00 0.00 0.00 0.00 0.50 0.00 0.00
Facies code: From
Facies Code: To
Frequency
Lithofacies
Relative Frequency
observed
Markov Order Metric for Facies
Runs Analysis Order Metric for Thickness

Figure 7

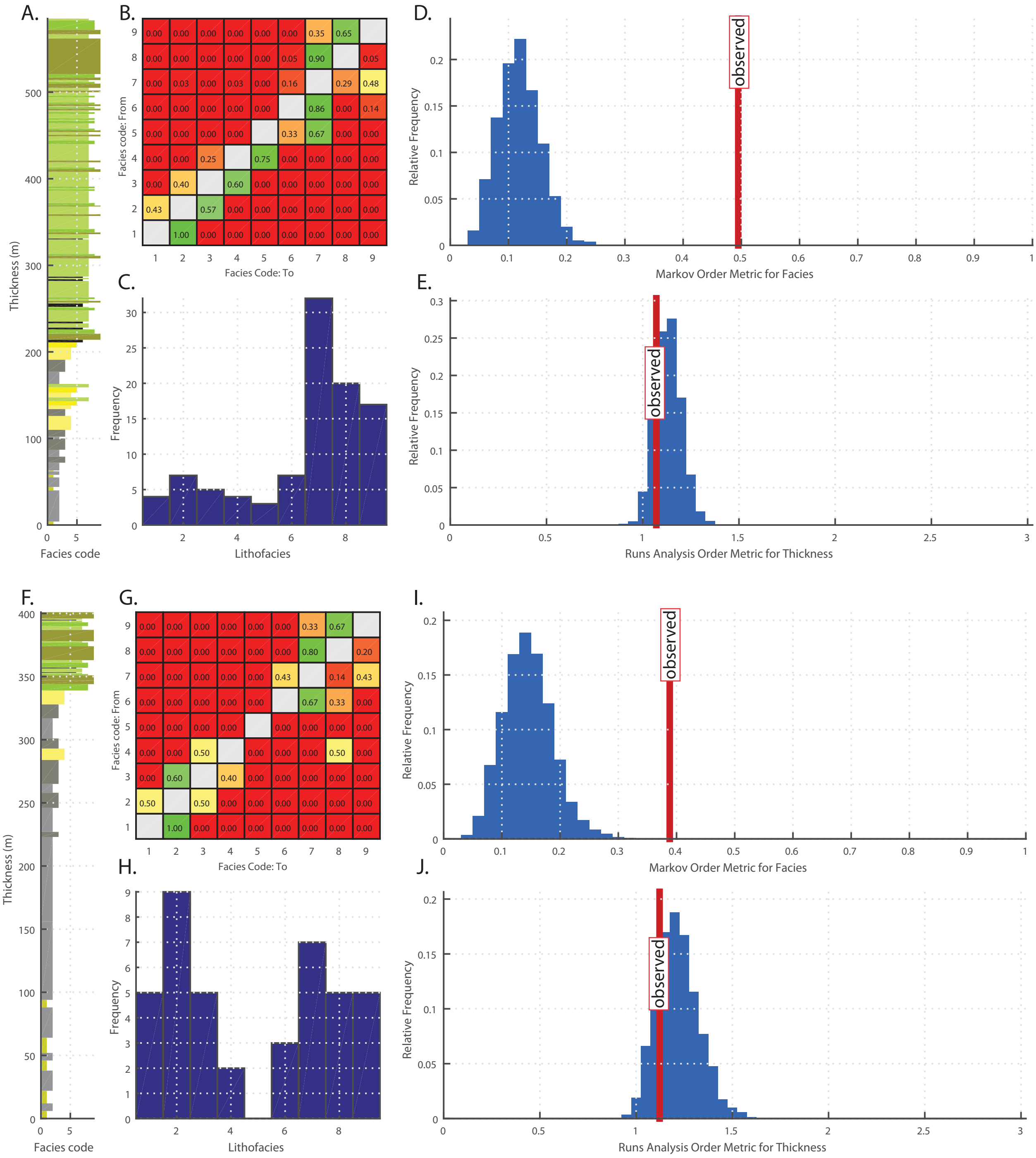

A.
B.
C.
D.
E.
F.
G.
H.
I.
J.
Thickness (m)
Facies code
Facies code: From
Facies Code: To
Frequency
Lithofacies
Relative Frequency
Markov Order Metric for Facies
Runs Analysis Order Metric for Thickness
observed

Figure 8

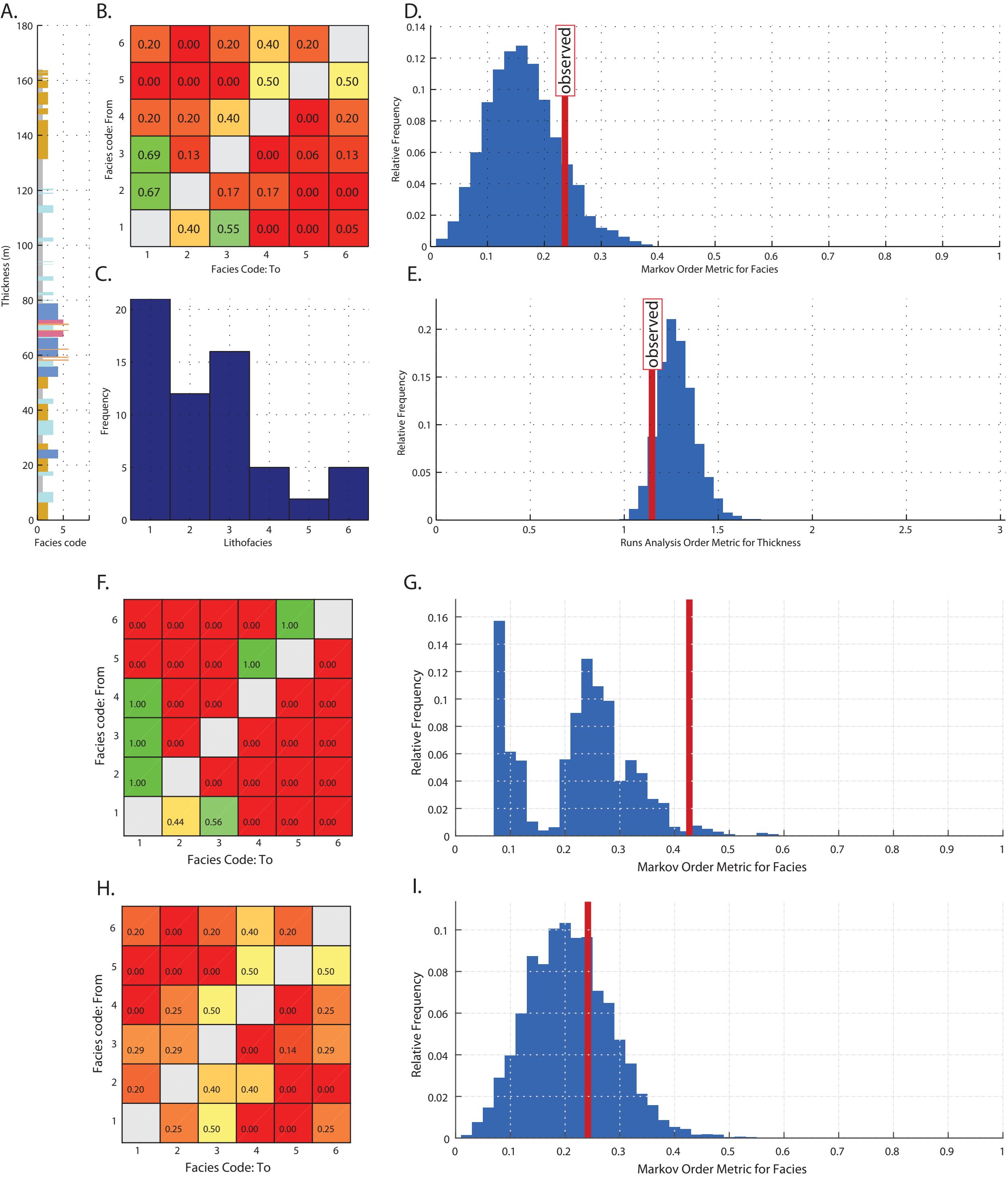

A.
B.
C.
D.
E.
F.
G.
H.
I.
Thickness (m)
Facies code
Facies code: From
Facies Code: To
Frequency
Lithofacies
Relative Frequency
Markov Order Metric for Facies
Runs Analysis Order Metric for Thickness
observed

Figure 9

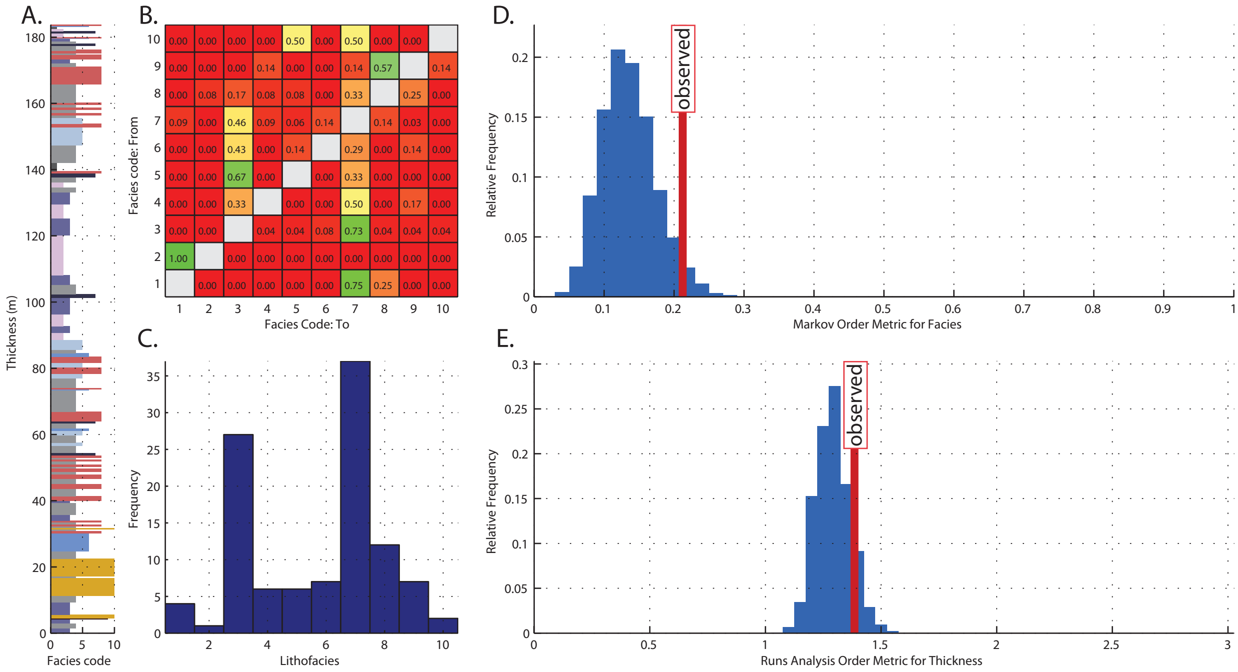

A.
B.
C.
D.
E.
Thickness (m)
Facies code
Facies code: From
Facies Code: To
Frequency
Lithofacies
Relative Frequency
Markov Order Metric for Facies
Runs Analysis Order Metric for Thickness
observed

Figure 10

Table 1. Basic statistics, Markov order metric, and runs order metric for the synthetic strata shown in Figures 1,2, 3, and 6.

Succession type	Number of facies units in the succession	Mean facies unit thickness (m)	Markov statistic *m*	Runs statistic *r*	Markov statistic p value	Runs statistic p value
Perfect asymmetric ordered cycles (Fig. 1A&G, Fig. 2A-C	50	0.30	1.00	2.18	0.000	0.000
Partly disordered (Fig. 1B&H, Fig. 2D-F)	50	0.30	0.56	1.56	0.000	0.009
Perfectly disordered (Fig. 1C &I, Fig. 2G-I)	50	0.259	0.22	1.48	0.237	0.097
Perfect symmetric ordered cycles (Fig 1D&J, Fig. 3A-C)	49	0.30	0.50	2.45	0.000	0.000
Disordered with exponential facies frequency (Fig. 1E&K, Fig. 3D-F)	100	0.497	0.17	1.41	0.144	0.411
Disordered with exposure surfaces (Fig. 1F&L, Fig. 3G-I)	65	0.20	0.22	1.42	0.110	0.107
Perfectly ordered but irregularly numbered cycles (Fig. 6A-E)	100	0.60	0.40	2.19	0.000	0.000
Partly disordered asymmetric cycles with irregularly numbered facies (Fig. 6F-J)	102	0.487	0.33	1.67	0.004	0.000
Partly disordered asymmetric cycles (Fig. 6K-O)	102	0.487	0.39	1.67	0.000	0.000

Table 2 Lithofacies code from the outcrop strata analyzed in Figures 7 to 10.

Pennsylvanian Illinois Figure 7		Campanian Book Cliffs, Figure 8			Santonian San Corneli anticline, Figure 9			Barns Hill, Figure 10		
Lithofacies	Code	Lithofacies	Hampson et al. (2014) Code	Code	Lithofacies	Pomar et al (2005) Code	Code	Lithofacies	Lehrmann & Goldhammer (1999) code	Code
Sandstone	1	Fluvial conglomerate and conglomeratic sst	F1	10	Coral sponge rudist sheetstone	1	3	Quartz sandstone	1	1
Sandstone and shale	2	Fluvial medium to fine cross-bedded sst	F2	9	Coral rudist mixstone	2	4	Karst	2	2
Sandstone and claystone	3	Fluvial thin-bedded fine sst	F3	8	Dense hippuritid pillarstone	3	5	Cryptalgal prism-cracked fenestral laminite	3	3
Underclay	4	Fluvial siltstone and mudstone	F4	7	Rudist-bearing grainstone	4	6	Laminated lime mudstone	4	4
Coal	5	Coal	F5	6	Benthic foraminifer rich grainstone	5	2	Stromatolitic wackestone to packstone	5	5
Shale	6	Marine medium cross-bedded sst	M1	5	Wackstone to mud-dominated packstone	6	1	Ripple cross-laminated wackestone packstone	7	6
Limestone	7	Marine fine HCS sst	M2	4				Bioturbated mudstone	9	7
Limestone and shale	8	Fine HCS sst with Interbedded siltstone and mudstone	M3	3				Bioturbated skeletal peloidal wackestone packstone	10	8
Claystone	9	Siltstones and mudstones	M4	2				Coral stromatoporoid wackestone to packstone	11	9

Shale	10	Fine gravity-flow sst with interbedded siltstone and mudstone	M5	1		Argillaceous wackestone	12	10
Not exposed	11							

Table 3 Results from the quantitative analysis of the outcrop examples

Section name	Total thickness (m)	Number of distinct facies	Mean facies unit thickness (m)	Markov statistic *m*	Markov probability p_m	Runs statistic *r*	Runs probability p_r
Pennsylvanian, Sangamon river, Illinois	69	11	1.33	0.187	0.590	1.44	0.117
Campanian, Helper, Book Cliffs, Utah and Arizona	585	9	5.91	0.495	0.000	1.07	0.947
Campanian, Tusher,Book Cliffs, Utah and Arizona	401	9	9.78	0.388	0.000	1.12	0.912
Santonian, Rio Carreu, Spanish Pyrenees	163	6	2.69	0.236	0.214	1.15	0.952
Santonian, Rio Carreu, upper section	93	6	2.58	0.427	0.032	1.03	0.991
Santonian, Rio Carreu, lower section	79	6	2.81	0.242	0.450	1.29	0.284
Silurian, Barn Hills, Utah	184	10	1.69	0.212	0.081	1.39	0.147