Essays on Macroeconomics of Immigration

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Declaration of Authorship

I, *Takehiro Kiguchi* hereby declare that this thesis and the work presented in it is entirely my own. Where I have consulted the work of others, this is always clearly stated.

Abstract

This thesis analyzes the dynamic macroeconomic effects of immigration. Chapter 1 provides an overview of this thesis.

Chapter 2 empirically examines the effects of immigration by interpreting shocks to unanticipated changes in working population as immigration shocks and identifying the shocks using a VAR with sign restriction. We find that immigration shocks are not associated with rises in non-residential investment or short-run reductions in average wages. We also show how a neoclassical growth model with a CES production function where migrant labor and capital are complements to skilled domestic labor and substitutes to each other can produce responses closer to those in the VAR.

Chapter 3 examines theoretically the macroeconomic effects of immigration on labor market outcomes, especially labor share, for alternative assumptions on the bargaining power of workers using a New Keynesian model with labor market frictions and heterogeneous unemployed workers. Unemployed workers are heterogeneous in the sense that some of them are short-term unemployed (insiders), the others are long-term unemployed (outsiders). We find that, when immigrants enter as outsiders and reduce the bargaining power of workers, labor share of national income shows a hump-shaped decline, which is in line with empirical evidence by a VAR analysis.

Chapter 4 analyzes theoretically the macroeconomic impacts of a policy of increasing immigration in response to an unexpected increase in its debt to GDP ratio. We find that there is indeed a potential for a policy to boost economic activity of increased debt without increasing the present value of budget deficits if the expected tax revenue from future increased population growth is spent effectively on productive public capital at the correct time.

Acknowledgment

First of all, I wish to express my deepest appreciation to my supervisor and co-author of Chapter 2, Professor Andrew Mountford, for providing invaluable guidance, help and encouragement through the various stages of this thesis. I am also grateful to my second supervisors, Professor Dan Anderberg and Dr. Jesper Bagger, for their valuable advice. I would like to thank to Professor Markus Brückner and Professor Evi Pappa for providing their Matlab codes to adopt their codes to my model, and to seminar participants at the Department of Economics Annual PhD Conference in 2012 and 2014 at Royal Holloway for their helpful comments and feedback. Financial support from the Japan-IMF Scholarship Program is gratefully acknowledged. Finally, I would like to express very special thanks to my family, Hiroshi, Reiko, Ayumi and Akane, Wang Jun, Koji Nishimoto, and Naoko Takeya for their love and warm support. Needless to say, all remaining errors are mine.

Contents

List of Figures

List of Tables

Chapter 1

Overview of Thesis

This thesis consists of three essays that analyze the dynamic macroeconomic effects of immigration. In spite of there being a large literature on the microeconomic impacts of immigration, the studies on the macroeconomic effects of immigration are sparse. This should come as a surprise, considering current active debates on immigration policy in advanced countries. The following three chapters attempt to fill this gap and contribute to our better understanding of the macroeconomic implications of immigration.

As a framework for the analysis of macroeconomic impacts of immigration, we rely on a dynamic stochastic general equilibrium (DSGE) model throughout this thesis. Nowadays, medium-scaled DSGE models have become central tools for macroeconomic analysis in central banks and policy institutions.[1] The DSGE models have the strength that they are not exposed to the Lucas critique, since individual behavior is derived from intertemporal optimization problems by forward-looking agents who have rational expectations. The fully microfounded nature also enables us to conduct welfare analysis.

Furthermore, the increased popularity of DSGE models lies in their ability of incor-

[1] For example, the IMF's DSGE model, "GIMF", is described in Kumhof et al. (2010). The DSGE model used at FRB of New York, "FRBNY DSGE model", is presented in Del Negro et al. (2013).

porating many features and structural shocks, and its usefulness for forecasting. See seminal works by Christiano, Eichenbaum and Evans (2005), and Smets and Wouters (2003, 2007). This thesis introduces immigration shocks into DSGE models and analyzes its implications. In Chapter 2, we regard immigration as a shock to working population using a stochastic neoclassical growth model. In Chapter 4, we introduce immigration and public capital into a neoclassical growth model with distortionary taxes in order to look at the dynamic effects of immigration on debt dynamics, investment and growth.

Recent studies incorporate Diamond-Mortensen-Pissarides type of labor market frictions (Diamond, 1982; Mortensen and Pissarides, 1994) into an otherwise a standard New Keynesian (NK) DSGE models.[2] In line with this strand of research, Chapter 3 looks at the dynamic macroeconomics effects of immigration on labor market dynamics such as labor income share and unemployment in a NK model with search and matching frictions in the labor market.

Helpful insights for the state of the art macroeconomics were also gained outside Royal Holloway. For DSGE models with search, I owe a lot to the lectures series given at London School of Economics in 2013 by Professor Robert Shimer of University of Chicago. For NK DSGE models, I developed a deeper intuition from lecture series by Professor Lawrence Christiano of Northwestern University, given at the International Monetary Fund in 2014 where I worked as a summer intern.

In the following, I briefly summarize the content and results of each chapter.

[2] Merz (1995) and Andolfatto (1996) are the first to introduce search frictions into a standard RBC model.

1.1 Identifying Immigration Shocks

Chapter 2, coauthored with Andrew Mountford, provides the empirical evidence on the impact of shocks to immigration on macroeconomy. Standard macroeconomic theory predicts that unexpected increases in the population should lead to increases in investment per capita so that the capital stock returns to its balanced growth path capital-to-labor ratio. In Chapter 2, we show, using vector autoregression (VAR) analysis, that such a relationship is not found in the post war U.S. data.

One feature of the empirical analysis is that it takes account of the fact that the working population differs from other macroeconomic variables in that much of its movement can be predicted years ahead since records of birth and mortality rates are publicly available. Thus, a large proportion of the changes in the working population (aged 16-64) can be anticipated 16 years ahead of time. We therefore correct for such anticipated changes in working population and find that unanticipated changes in the working population corresponds closely with immigration levels. We then interpret shocks to unanticipated changes in working population as immigration shocks and identify immigration shocks using a VAR with sign restriction, following Uhlig (2005), and Mountford and Uhlig (2009). We find that immigration shocks are not associated with rises in non-residential investment, or short-run reductions in average wages. We also show how a neoclassical growth model with CES production function where (unskilled) migrant labor and capital are complements to skilled domestic labor and substitutes each other can replicate the responses obtained from VARs.

1.2 Effects of Immigration Shocks on Labor Market Dynamics

Motivated by the empirical evidence found in Chapter 2 that labor share does decline in response to an immigration shock, as well as the observation that the United States experienced the decline of the labor share over the las three decades, Chapter 3 theoretically examines the macroeconomic effects of immigration on labor share. An increase in the ability of immigrant labor may reduce the bargaining power of workers and therefore reduce labor share.

In Chapter 3, we use a model of Brückner and Pappa (2012), which is a New Keynesian model with labor market frictions and heterogeneous unemployed workers. The presence of search frictions allow us to analyze the bargaining power of workers and unemployment. Unemployed workers are heterogeneous in the sense that one is short-term unemployed (insiders), the other is long-term unemployed (outsiders). This heterogeneity in unemployed workers may well be helpful in discussing the impact of immigration on the macroeconomy since immigration could be thought of as an exogenous shock to the numbers of outsiders.

We find that, when immigrants enter as outsiders and reduce the bargaining power of workers, labor share of national income shows a hump-shaped decline, which is in line with empirical evidence by a VAR analysis. This suggests that the importance of the role of the worker's bargaining power in investigating the dynamic macroeconomic impacts of immigration, to which no role is given in the standard New Keynesian model. We also consider alternative scenarios where immigrants arrives as employed workers or insiders and find that immigration adversely effects, or directly competes with, the sector they enter, but benefits other sector(s).

1.3 Effects of Immigration Shocks on Debt Dynamics

Recently, Ben-Gad (2012) has shown how immigration creates an incentive for current native population to support higher deficits because the cost of financing them can be partially shifted to future immigrants. In Chapter 4, we reverse his argument and investigate whether government deficits create an incentive to admit more immigrants.

We theoretically analyze the macroeconomic effects of immigration in response to an unexpected increase in its debt to GDP ratio. It is intuitive that an increase in immigration will reduce the rate of debt per capita and the debt to GDP ratio, *ceteris paribus*, since the number of taxpayers increases. However, an increase in immigration will also reduce the domestic private capitals per person if other things are equal. This is what is called a “capital dilution effect”. This capital dilution effect may cause a slowdown in the growth of GDP per capita, which in turn potentially may lead to a rise in the level of debt.

In Chapter 4, we extend the model of Uhlig (2010) and Trabandt and Uhlig (2011) to allow for immigration and public capital in order to examine the macroeconomic impacts of immigration shocks in response to an unexpected increase in its debt to GDP ratio. We find that there is indeed a potential for such a policy to boost economic activity without increasing the present value of government debts if the expected tax revenue from future increased population growth is spent effectively on productive public capital at the correct time. However, there is also scope for depressing the economy further if the dilution of the domestic capital stock by increased population is not properly managed. For example, the direct government investment in private capital or less productive public capital is not useful in mitigating capital dilution.

Chapter 2

The Macroeconomics of Immigration*

2.1 Introduction

While there have been a lot of recent studies on the microeconomic impacts of immigration there has been less attention focused on the implications of immigration for the macroeconomy. According to U.S. Census Bureau and Current Population Survey (CPS) data, immigration has been a significant part of the U.S. population growth over recent decades. In 1970 about 9.6 million (4.7%) of the U.S. total population was foreign born to non-U.S. nationals, by 2010 this number had risen to nearly 40 million or 12.9% of the U.S. total population. In this paper we examine the effect of shocks to working population on the macroeconomy using the techniques of macroeconomic time series analysis. The analysis shows that, consistent with the standard neoclassical growth model, GDP per capita and consumption per capita temporarily fall in response to a positive shock to the

*This is joint work with Andrew Mountford, who performed an empirical analysis.

working population. However, non-residential investment per capita does not rise and real wages do not fall in the short run following an unexpected increase in the working population and as would also be predicted by the standard growth model.

The paper shows that a neoclassical growth model with a constant elasticity of substitution (CES) production function where migrant labor is a substitute for capital but a complement to skilled domestic labor can produces responses to an immigration shock much closer to those of the VAR. In particular, it can produce responses where investment falls in response to an immigration shock and where the wage response of most agents is initially positive due to the complementarity of immigrant labor with most domestic labor. Thus, the VAR results and the macroeconomic growth model both lend support to the findings of the microeconomic literature that immigrant labor is a much closer substitute for native unskilled labor than native skilled labor. See e.g., Ottaviano and Peri (2012) for the U.S. economy, Manacorda, Manning and Wadsworth (2012) for the U.K. economy, and Ortega and Verdugo (2014) for the French economy.

One feature of the empirical analysis is that it takes account of the fact that increases in the working population differ from other macroeconomic variables in that much of its movement can be predicted years ahead. Birth and mortality data are publicly available and so a large proportion of the changes in the working population can be anticipated 16 years ahead of time. As the work of Ramey (2011a) and Auerbach and Gorodnichenko (2012) detail, correcting for anticipated changes in the variables of a VAR is necessary to remove potential biases from the analysis. We therefore correct for such anticipated changes in population in the data and find, very intuitively, that unanticipated changes in the working population correspond quite closely with immigration levels. We therefore interpret shocks to unanticipated changes in the working population as immigration shocks.

The analysis and results of the paper are of interest for two distinct reasons. Firstly, the key state variable in balanced growth models is the capital-labor ratio and while the literature has paid a lot of attention to the determinants of individual labor supply,[1] much less attention has been given to the determinants of the size of the working population, although see Doepke, Hazan and Maoz (2012) for a notable exception. This paper attempts to redress this imbalance by focusing on the macroeconomic effects of immigration which is one of the key determinants of changes in the labor force. Secondly, there is a large microeconomic literature on the effects of immigration on the labor market. One of the key puzzles of this literature was the finding that immigration has only a small effect on aggregate wages, with only the wages of the least skilled workers being adversely affected by immigration.[2] This paper, using a very different methodology and different, macroeconomic, data provides macroeconomic support for this analysis by also finding that immigration shock is not empirically associated with short-run decreases in aggregate wage rates.

The paper is organized as follows. In Section 2.2 we present and discuss the raw data. In Section 2.3 we present results from the VAR analysis and in Section 2.4 we discuss to what extent the standard macroeconomic growth model can be adapted to explain these results.

[1] For surveys of the literature see e.g., Uhlig (1999) or Christiano, Eichenbaum and Evans (2005).

[2] See e.g., Dustmann, Frattini and Preston (2008), as well as the introductions to Ottaviano and Peri (2012), and Manacorda, Manning and Wadsworth (2012) and Ortega and Verdugo (2014) for surveys of this literature.

Figure 2.1: Live Births 16 years previously are a major and predictable influence on the Working Population

2.2 Trends in Population Growth and Immigration

This section presents the data we will be using below in our VAR analysis. One contribution of this section is to compute an unanticipated change in population variable by removing anticipated changes in the working population caused by publicly recorded changes in the birth and mortality rates. This is important and interesting for two reasons. Firstly because controlling for predictable changes in variables in a VARs is necessary to remove bias, as the work of Ramey (2011a) and Auerbach and Gorodnichenko (2012) detail. Secondly when we do construct this series it corresponds quite closely to immigration level which is intuitive. Thus in the VAR section below we interpret the shocks to unanticipated population as immigration shocks.

Figure 2.1 plots the changes in the rate of growth of the working population and the number of live births 16 years previously. It shows that they are highly related, although not perfectly correlated. VAR analysis assumes that the errors in the VAR are orthogonal to information contained in the past values of the variables in the VAR, see e.g., Canova (2007). Thus, it is necessary to remove this predictable element from the population series, see for example the discussion in Ramey (2011a) and Auerbach and Gorodnichenko (2012). We do this by constructing an unanticipated change in population series, $WPop_t^U$, to correct for these predictable effects, using the following formula;

$$\begin{aligned} WPop_t^U &= WPop_t - WPop_t^A \\ \text{where } WPop_t^A &= (1 - \delta_{t-1}^{65} - mort_{t-1}^{16-64})WPop_{t-1} + (1 - mort_{t-16}^{1-15})Births_{t-16} \end{aligned}$$

where the $WPop_t$ is the series for working population in the U.S. is taken from Cociuba, Prescott and Ueberfeldt (2009). $WPop_t^A$ is the anticipated working population in time which is equal to the previous year's working population minus an estimate of the proportion aged 64 who will retire, δ_{t-1}^{65}, and an estimate of the mortality rate of the working population plus the births from 16 years previously also adjusted for mortality. The data used is all freely and publicly available on the Internet. That for mortality rates and birth rates are taken from 5 yearly samples from the CDC/NCHS *National Vital Statistics* and data on the age distribution is taken from decennial census data. Linear interpolation is used to generate annual numbers for mortality rates.

Figure 2.2 displays the time series for the constructed unanticipated changes in the working population and also two immigration series. One corresponds to the numbers of new permanent resident status individuals from the U.S. Census Bureau and the other is net international migration series from OECD. All series are plotted as a percentage of the working population. Both immigration series are nearly identical, and the series

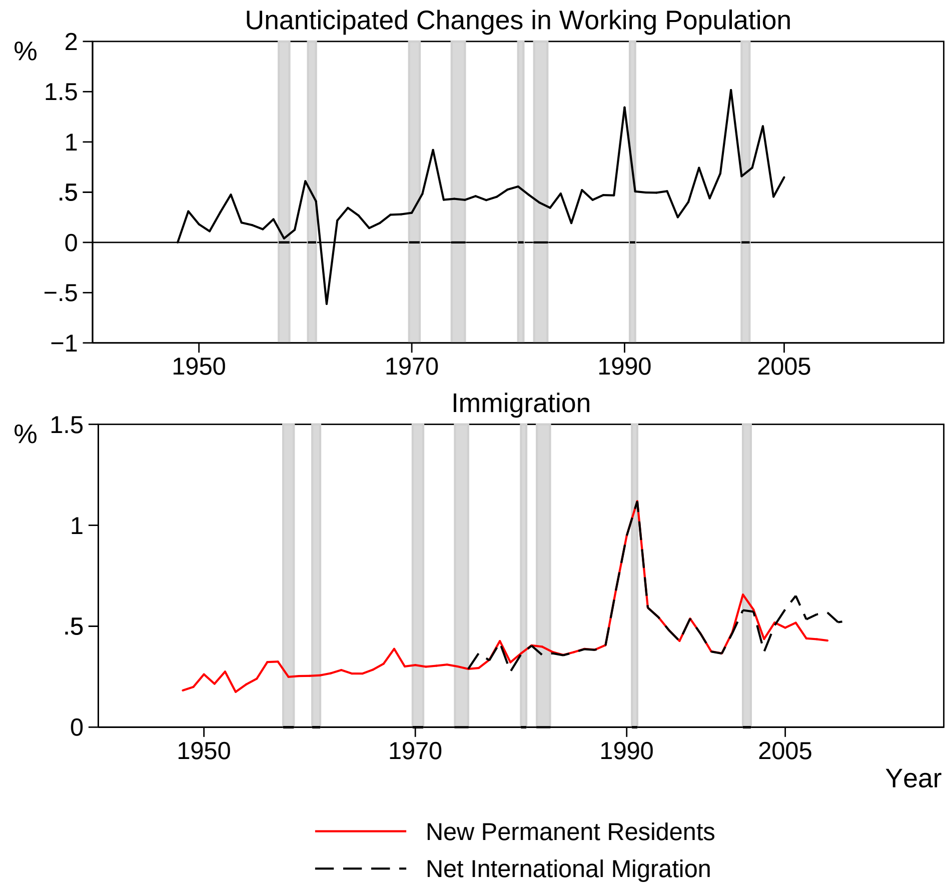


Figure 2.2: Unanticipated Changes in Population and Immigration Series. Shaded areas are NBER recessions.

for the unanticipated changed in working population and two immigration series have a similar pattern in that they all show a gradual rise from the 1950's to the 1980's and then a large increase in the latter period of the 1980's and a second peak around 2000 although the size of this last peak does differ. The first large peak was caused by the Immigration Reform and Control Act of 1986 which allowed significant numbers of formerly temporary workers to apply for permanent resident status after a period of three years.[3] The passing of the act also coincided with a period of high Mexican unemployment and so caused many temporary workers who would otherwise have returned to their country of origin to remain

[3] This is known as the Special Agricultural Workers provision.

in the United States and become permanent residents. It is estimated that 2.3 million Mexicans took advantage of this possibility, see Durand, Massey and Parrado (1999). The gradual track to permanent residency also explains why the peak of the new permanent residents series occurs after that for the changes in working population series.

The series are also similar in scale. Over the sample period 1950-2005 the cumulative unanticipated changes in the working population is approximately 38.2 million with 17.8 million occurring since 1990. The corresponding numbers for the new permanent residents series are 31.9 million and 15.7 million. One should not expect a perfect correspondence between these two figures since one can attain new permanent resident status and not be part of the working population and vice versa. However the similarity between the two series is reassuring. Figure 2.2 also plots the NBER business cycle dates with the recessions shaded in gray. It is noticeable that the response of the unanticipated changes in working population is more volatile and reactive to recessions than the series for new permanent residents which is intuitive.

2.3 VAR Analysis

2.3.1 Description of the VAR

We use an 8 dimensional VAR with annual data from 1950 to 2005 for the following variables; GDP, private consumption, non-residential investment, residential investment, hours worked, real wages and the two immigration series, the numbers of new permanent residents and the constructed unanticipated population variable described above.[4] All

[4]The series used the series for gross domestic product personal consumption expenditures, nonresidential fixed investment and Residential Fixed Investment taken from Bureau of Economic Analysis' NIPA table 1.1.5 all deflated by the GDP deflator from Table 1.13. The wage series is the Nonfarm Business Sector: Real Compensation Per Hour, series COMPRNFB, from the Bureau of Labor Statistics and the working population and hours worked series come from Cociuba, Prescott and Ueberfeldt (2009).

Table 2.1: Identifying Sign Restrictions

	GDP Cons	Non-Res Invest	Hours	Unantipated Working Pop.
Business Cycle	+	+	+	
Population				+

This table shows the sign restrictions on the impulse responses for each identified shock. 'Non-Res Inv' stands for Non-Residential Investment. A "+" means that the impulse response of the variable in question is restricted to be positive for two years following the shock, including the year of impact. A blank entry indicates, that no restrictions have been imposed.

variables are real and, with the exception of the wage series, expressed as per capita of the working population. The VAR has 2 lags, no constant or a time trend, and uses the logarithm for all variables except for the population variables where we have used the level.

The VAR in reduced form is given by

$$Y_t = \sum_{i=1}^{2} B_i Y_{t-i} + u_t, t = 1, \cdots, T, \quad \mathbb{E}[u_t u_t'] = \Sigma$$

where Y_t are 8×1 vectors, 2 is the lag length of the VAR, B_i are 8×8 coefficient matrices and u_t is the one step ahead prediction error.

2.3.2 Identification

The problem of identification is to translate the one step ahead prediction errors, u_t, into economically meaningful, or 'fundamental', shocks, v_t. In this paper we identify shocks using the sign restriction approach of Uhlig (2005), and Mountford and Uhlig (2009). Identification in this methodology amounts to identifying a matrix A, such that $u_t = Av_t$ and $AA' = \Sigma$. Each column of A represents the immediate impact, or impulse vector, of a one standard error innovation to a fundamental shocks. Each column is identified as the vector which minimizes a criterion function, $\Psi(a)$, based on the impulse responses of some of the variables in the VAR to a particular shock's impulse vector, a. If we define $r_{ja}(k)$ as the impulse response to the impulse vector a of the jth variable at horizon k then the criterion function, $\Psi(a)$, is

$$\Psi(a) = \sum_{j \epsilon J_{S,+}} \sum_{k=0}^{1} f\left(-\frac{r_{ja}(k)}{s_j}\right) + \sum_{j \epsilon J_{S,-}} \sum_{k=0}^{1} f\left(\frac{r_{ja}(k)}{s_j}\right),$$

where f is the function $f(x) = 100x$ if $x \geq 0$ and $f(x) = x$ if $x \leq 0$, s_j is the standard error of variable j, $J_{S,+}$ is the index set of variables, for which identification of a given shock restricts the impulse response to be positive and $J_{S,-}$ is the same for variables restricted by identification to be negative. Since we use annual data we only restrict the signs of the impulses for two periods, i.e., for the two years after the shock. When multiple shocks are identified there is an additional constraint on the minimization that the identified shock be orthogonal to previously identified shocks, as detailed in Mountford and Uhlig (2009).

In this paper we use two identification schemes. We first only identify the unanticipated population/immigration shocks and then we identify two shocks, first a business cycle shock and then the unanticipated population/immigration shock. Table 2.1 provides a description of the identifying sign restrictions for these shocks. The advantage of the penalty function approach is that, by rewarding larger responses of the correct sign, it gives the shock identified first the greatest opportunity to explain the variation in the data. Thus when the unanticipated population/immigration shock is identified second it is restricted to explaining the variation in the data left over after the variation explained by the business cycle shock has been taken out. As well as a robustness exercise this

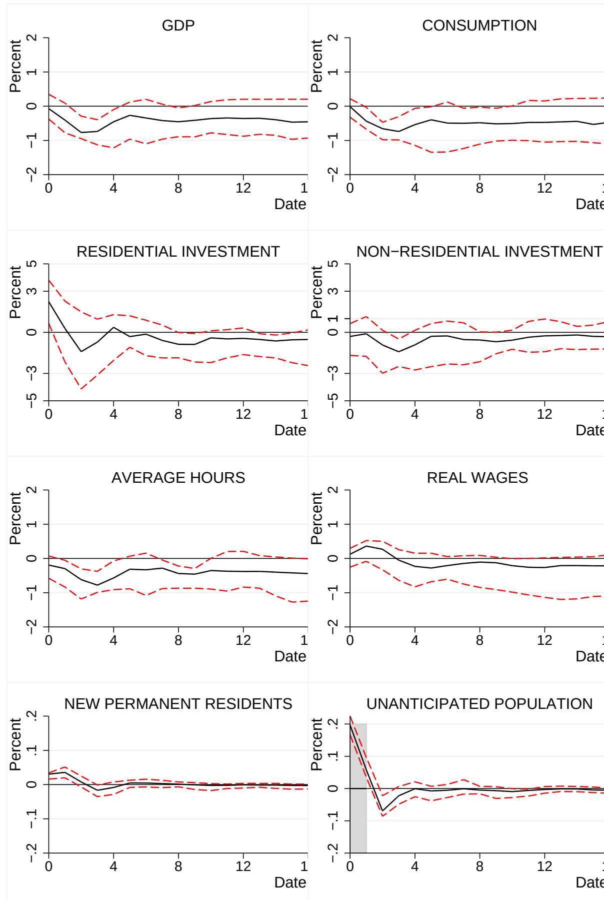


Figure 2.3: Impulse Responses to an Immigration Shock Ordered First.

identification scheme is interesting in its own right, as it should also pick up temporary variations in immigration which may be associated with business cycle fluctuations.

2.3.3 Empirical Results

The impulse responses for these fundamental shocks are shown in Figures 2.3 through 2.5, where we have plotted the impulse responses of all our 8 variables. The shocks are identified for each draw from the posterior and the 16th, 50th and 84th quantiles plotted, calculated at each horizon between 0 and 16 years after the shocks. The impulses restricted by the identifying sign restrictions are identified by the shaded area in the figures.

The Immigration Shock Ordered First

The impulse responses of the immigration shock, which is the shock to the unanticipated working population variable in the VAR, are plotted in Figure 2.3. They show that, as would be predicted by a standard growth model, output and consumption temporarily fall in response to the immigration shock. However, although there is an increase in residential investment on impact, there is not a positive response from non-residential investment which is the response predicted by a growth model after an unexpected increase in its labor force, see Figure 2.8 below. Indeed the median response of non-residential investment is always negative. With respect to the labor market real wages do not change significantly with the median response being initially positive before coming negative while average hours worked falls. Again this is not the pattern of responses that would be predicted by a standard growth model where wages fall on impact after an unexpected increase in its labor force. Finally note that the response of the new permanent residents to the immigration shock is intuitive. It is much smoother than the responses of the unanticipated population variable which is intuitive and consistent with the view that the unanticipated working population variable will contain more temporary immigrants than the new permanent residents series.

The scale of the response is a higher than is intuitive. An unexpected increase in the working population of 0.2% is associated with a fall in the median response of GDP per working population of greater than 0.2%. However, the confidence bands show that the response of GDP per working population is only significantly different from zero two and three years after the shock which is more intuitive.

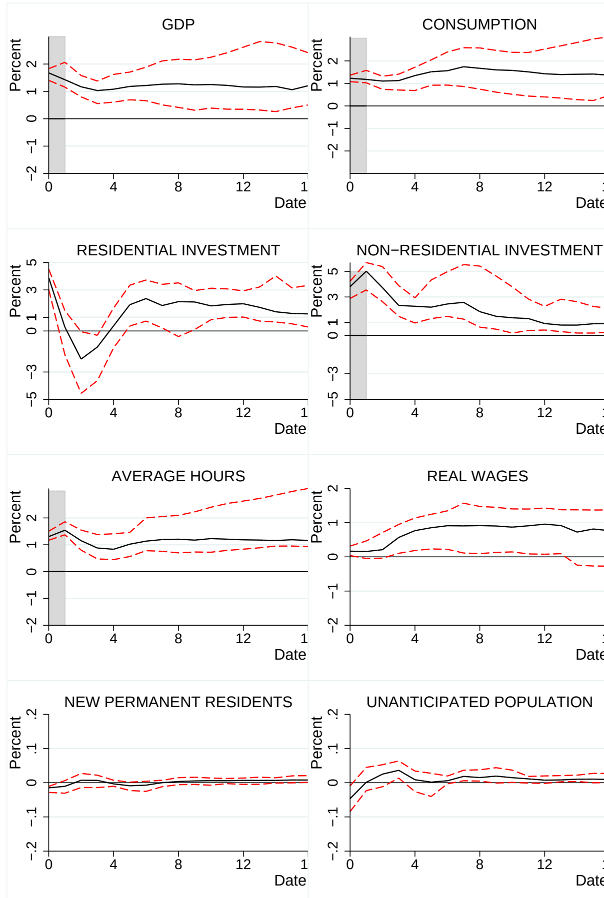


Figure 2.4: Impulse Responses to a Business Cycle Shock Ordered First.

The Immigration Shock Ordered Second

In this section, we present the impulse responses of the immigration shock when it is identified second after a business cycle shock. This is important to do for two reasons. Firstly, because identification methods are never definitive and so there is always a suspicion that the variation attributed to one identified shock may actually be due to another shock. In macroeconomics, the business cycle shock is commonly felt to be an important source of variation and so as a robustness check it is interesting to see whether the responses to the immigration shock change significantly once the business cycle variation is accounted for.[5]

[5]See Mountford and Uhlig (2009) for more discussion of this.

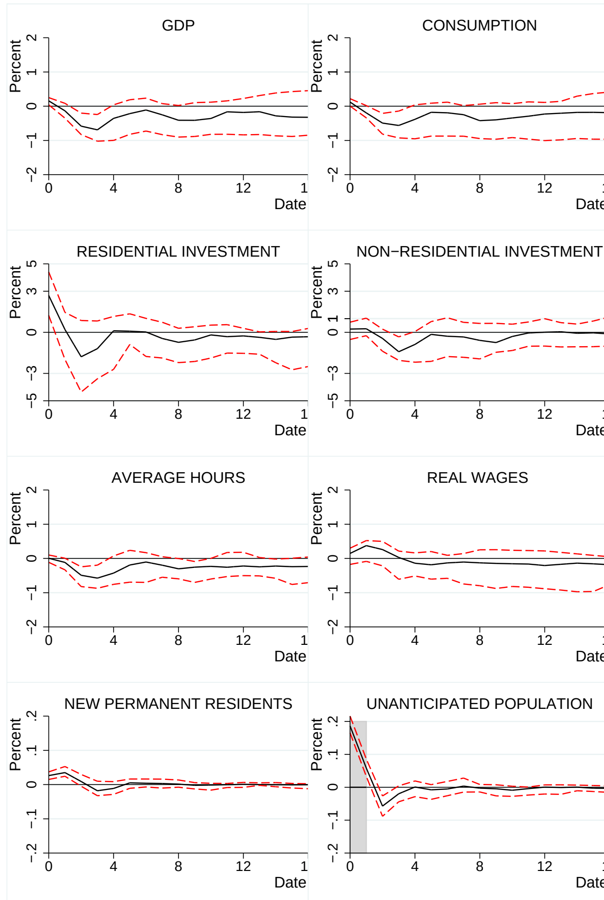


Figure 2.5: Impulse Responses to an Immigration Shock Ordered Second After a Business Cycle Shock

Secondly, it is often thought that immigration reacts to the business cycle and that while the stage of the business cycle should not matter for permanent immigrants, temporary migrants may be affected by the state of the business cycle.

Figure 2.4 displays the responses to the business cycle shock. These responses of the non-population variables are as expected and very similar to those in Mountford and Uhlig (2009). The responses of all the macro variables are positive and persistent. The population variables both show cyclical variation in response to the business cycle shock although with a lag. The immigration response is negative on the impact before rising and becoming significantly positive after three years after the shock. The new permanent residents series shows the same pattern but is a much smaller response and so insignificant for all horizons after impact.

Figure 2.5 shows the impulse responses of the immigration shock identified after the business cycle shock. What is striking is how similar the responses are to those in Figure 2.3. This means that the restriction to be orthogonal to the business cycle shock hardly binds at all which is consistent with the most of the variation in immigration not being influenced by the business cycle. The main differences between Figures 2.5 and 2.3 are that the error bands around consumption are tighter and the responses of real wages appears less negative in Figures 2.5. The scale of the response of GDP per working population is also slightly lower with the upper confidence band only falling by about 0.2% in response to an unexpected increase in the working population of 0.2% and again the confidence bands show that the response of GDP per working population is only significantly different from zero two and three years after the shock.

Adding Labor Share

In Figures 2.6 and 2.7, we substitute a labor share variable in to the VAR in place of the real wage rate. The variable is the Nonfarm Business Sector: Labor Share, series PRS85006173, from the Bureau of Labor Statistics. This is interesting as a robustness check and also because an increase in the ability of immigrant labor may reduce the bargaining power of labor and so reduce labor share, see Chapter 3. These Figures do indeed show evidence that over the medium term labor share does decline in response to an immigration shock. Thus, the medium term responses of wages and labor share do seem to differ from their short term effects. This is a large issue see for example the work of Duenhaupt (2011), Heathcote, Perri and Violante (2010) and Piketty and Saez (2003, 2006) and which is beyond the scope of this paper, but it is certainly an exciting area for

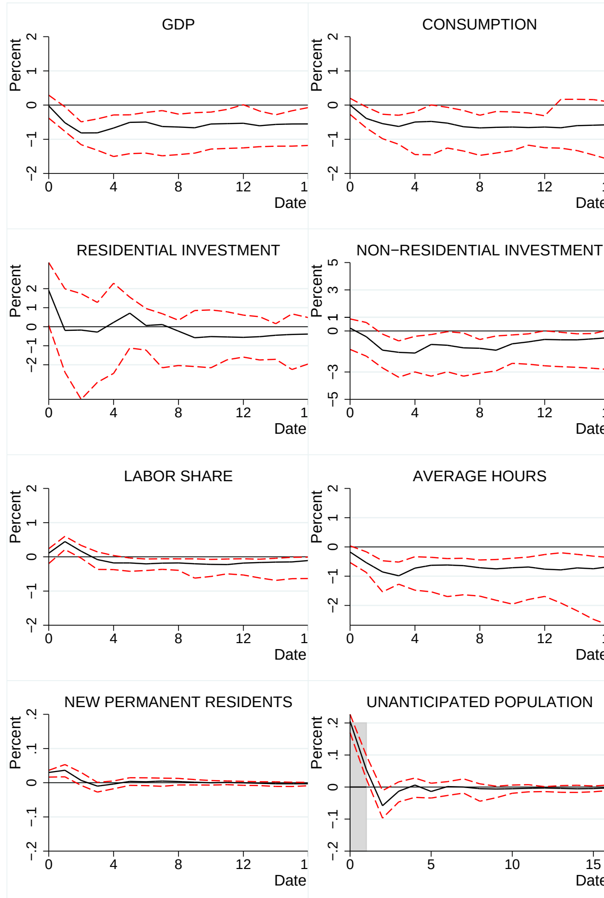


Figure 2.6: Impulse Responses to an Immigration Shock Ordered First With Labor Share.

further research.

2.4 A Growth Model with Immigration Shocks

The evidence discussed in Section 2.3 suggest that while increases in immigration are associated with temporary decreases in output and consumption per capita, as would be predicted by an exogenous shock to population in the standard neoclassical growth model, they are not associated with increases in non-residential investment which would also be expected in this case. To explain these results we use the findings of the recent labor economics literature which suggest that migrant labor is not a substitute for much of the

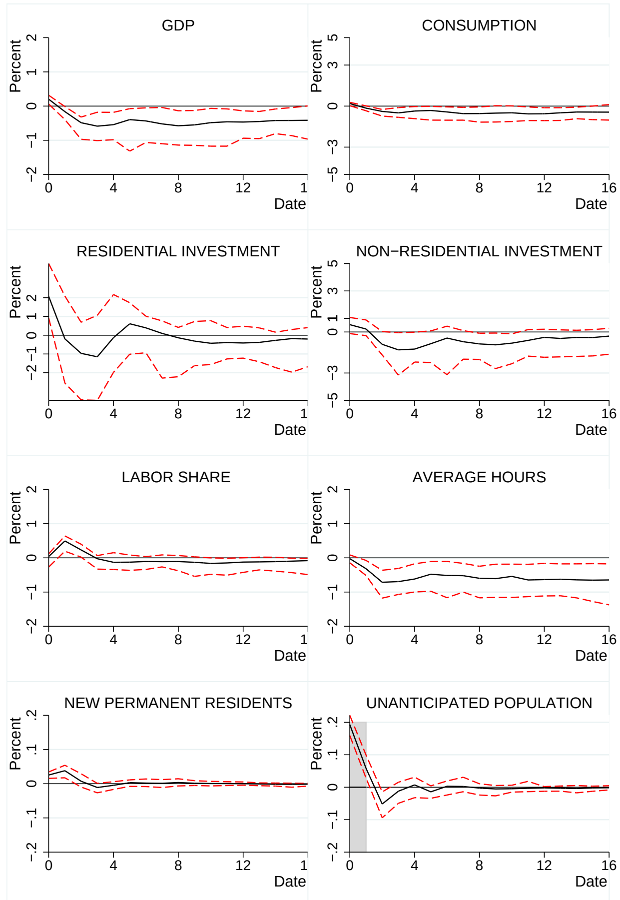


Figure 2.7: Impulse Responses to an Immigration Shock Ordered Second after a Business Cycle Shock with Labor Share.

domestic population, but a complement. For recent evidence on this see Ottaviano and Peri (2012) for the U.S., Manacorda, Manning and Wadsworth (2012) for the U.K., and Ortega and Verdugo (2014) for France. The intuition for these results is that migrants, perhaps because of poorer communication skills, tend to undertake unskilled work even when they possess skills themselves and this influx of unskilled labor allows business to expand without needing to invest in new machinery. The assumption that physical capital is complementary to skilled labor is well accepted and while the assumption that physical capital is a substitute for unskilled labor is less common it has support in the literature, see Cahuc and Zylberberg (2004).

We therefore adapt the standard neoclassical growth to allow for two types of labor, unskilled and skilled, and a household which is growing in size though time. To deal with this added complexity we will follow the literature and assume perfect risk sharing within the household, see e.g., Galí (2011) or Brückner and Pappa (2012). Intuitively, one can think of the household as a composite representative agent made up of a certain proportion of skilled and unskilled labor which can only be supplied together. This is a simplifying assumptions that allows the model to be solved in a standard way. There are clearly possible extensions of the model, such as allowing household members to differ in some dimension, as in Galí (2011) and Brückner and Pappa (2012), but this is not necessary for the purpose of this paper.

We follow the discrete time balanced growth model of Uhlig (2010) with the addition of stochastic population growth and two types of labor. We will use lower case letters to denote per capita terms. A representative household's utility function, U, has the following form

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t N_t u_t$$

where N_t is the number of agents in each household, $\beta \in (0,1)$ is the discount factor, and u_t is given by

$$u_t = \frac{(c_t \Phi(l_t))^{1-\eta} - 1}{1-\eta}$$

with c_t denoting per capita consumption, l_t denoting per capita labor supplied, $1/\eta > 0$ the intertemporal elasticity of substitution, and $\Phi(l_t)$ a strictly positive, decreasing, concave and thrice differentiable function. We will assume that $\Phi(l_t)$ is such that there is a constant Frisch elasticity of labor supply with respect to the wage rate, see Trabandt and Uhlig (2011). As mentioned above we will assume that the representative household is a composite of both types of labor, skilled and unskilled, which it can only supply together so that $l_t^s = l_t^u = l_t$, where l_t^s and l_t^u is the labor supply of skilled and unskilled household member's respectively. The proportion of skilled labor in the representative household's composite labor is the same proportion as in the economy, λ_t^s, where $\lambda_t^s = N_t^s/N_t$ and $N_t = N_t^s + N_t^u$. The representative agent's labor supply decision is to choose l_t subject to the weighted average wage, $w_t = w_t^s \lambda_t^s + w_t^u \lambda_t^u$. The household's budget constraint is in each period therefore

$$c_t N_t + x_t N_t = w_t l_t N_t + r_t k_t N_t,$$

where x_t is investment per person, k_t is the capital per person, w_t is the wage rate, and r_t is the capital rental rate. Capital accumulates via investment thus

$$k_{t+1} N_{t+1} = (1-\delta) k_t N_t + x_t N_t.$$

Production takes place under perfect competition and constant returns to scale according to a CES production function. We use the standardized function form as in Cantore, Ferroni, and León-Ledesma (2012),

$$y_t = \left[\alpha \left(\frac{k_t}{k} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(\frac{l_t \lambda_t^u}{l \lambda^u} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\theta\sigma}{\sigma-1}} \left(\frac{l_t \lambda_t^s}{l \lambda^s} \right)^{1-\theta},$$

where $y_t = Y_t/N_t$. The parameter $\sigma \in (0, \infty)$ denotes the degree of substitutability between capital and unskilled labor. Capital and unskilled labor are perfect complements as σ approaches 0, while they are perfect substitutes if $\sigma \to \infty$. As σ approaches 1, the production function becomes Cobb-Douglas. The parameter α denotes the capital intensity in production. Factor prices are determined by factors marginal products so that

$$
\begin{aligned}
r_t =& \frac{y_t}{k_t} \frac{\theta\alpha\left(\frac{k_t}{k}\right)^{\frac{\sigma-1}{\sigma}}}{\left[\alpha\left(\frac{k_t}{k}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(\frac{l_t\lambda_t^u}{l\lambda^u}\right)^{\frac{\sigma-1}{\sigma}}\right]}, \\
w_t^u =& \theta\frac{y_t}{l_t\lambda_t^u} \frac{(1-\alpha)\left(\frac{l_t\lambda_t^u}{l\lambda^u}\right)^{\frac{\sigma-1}{\sigma}}}{\left[\alpha\left(\frac{k_t}{k}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(\frac{l_t\lambda_t^u}{l\lambda^u}\right)^{\frac{\sigma-1}{\sigma}}\right]}, \\
w_t^s =& (1-\theta)\frac{y_t}{l_t\lambda_t^s}.
\end{aligned}
$$

The stochastic process population growth is

$$
\frac{N_t}{N_{t-1}} = \zeta_t^N,
$$

where ζ_t^N is a stationary stochastic process with mean ζ^N. Finally, the market clearing/feasibility constraint is given by

$$
c_t + x_t = y_t.
$$

2.4.1 Modeling Immigration Shocks

We model an immigration shock as a shock which leads to an increase in the proportion of unskilled labor in the economy, λ_t^u, as well an increase in the working population. We use a broad definition of skills and set $\lambda^s = 0.9$ which is justified by the fact that 90% of young people graduate from high school in the U.S., although clearly alternative

Table 2.2: The Log-linearized Equations of the Model

$\widehat{y}_t$	$=$	$\theta[\alpha\widehat{k}_t + (1-\alpha)(\widehat{l}_t + \widehat{\lambda}^u_t)] + (1-\theta)(\widehat{l}_t + \widehat{\lambda}^s_t)$	$\widehat{y}_t$	$=$	$(c/y)\widehat{c}_t + (x/y)\widehat{x}_t$
$\widehat{w}^u_t$	$=$	$\widehat{y}_t - (\widehat{l}_t + \widehat{\lambda}^u_t) - \alpha\left(\frac{\sigma-1}{\sigma}\right)\widehat{k}_t + \alpha(\frac{\sigma-1}{\sigma})(\widehat{l}_t + \widehat{\lambda}^u_t)$	$\widehat{w}^s_t$	$=$	$\widehat{y}_t - (\widehat{l}_t + \widehat{\lambda}^s_t)$
$\widehat{r}_t$	$=$	$\widehat{y}_t - \widehat{k}_t + (1-\alpha)(\frac{\sigma-1}{\sigma})\widehat{k}_t - (1-\alpha)(\frac{\sigma-1}{\sigma})(\widehat{l}_t + \widehat{\lambda}^u_t)$	$\widehat{w}_t$	$=$	$\frac{1}{\omega_S}\widehat{l}_t + \widehat{c}_t$
$\widehat{k}_{t+1}$	$=$	$\frac{(1-\delta)}{\zeta^N}\widehat{k}_t + \left[1 - \frac{(1-\delta)}{\zeta^N}\right]\widehat{x}_t - \widehat{\zeta}^N_{t+1}$	$\widehat{\lambda}_t$	$=$	$-\eta\widehat{c}_t - (1-\eta)\kappa\widehat{l}_t$
$\widehat{w}_t$	$=$	$\eta^s(\widehat{w}^s_t + \widehat{\lambda}^s_t) + (1-\eta^s)(\widehat{w}^u_t + \widehat{\lambda}^u_t)$	0	$=$	$\mathbb{E}_t[\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{R}_{t+1}]$
$\widehat{R}_t$	$=$	$(1-\beta(1-\delta))\widehat{r}_t$	$\widehat{\lambda}^u_t$	$=$	$-(\lambda^s/\lambda^u)\widehat{\lambda}^s_t$
$\widehat{\zeta}^N_t$	$=$	$\rho^N\widehat{\zeta}^N_{t-1} + \varepsilon^N_t$			

This table shows the equations of the log-linearized version of the model

interpretations and calibrations are possible. Given this if at the steady state the working population increases by $a\%$ and all of this increase is in unskilled workers then the new share of unskilled workers will rise to $\lambda^{u,new}_t = (100\lambda^u + a)/(100 + a)$ and so $\widehat{\lambda}^u_t = (100a(1-\lambda^u))/(100+a)\lambda^u$.[6] Thus an immigration shock in our model is a simultaneous shock to both population, ζ^N_t, and also to λ^u_t. The proportion of unskilled amongst immigrants can be varied so that the size of the response of $\widehat{\lambda}^u_t$ also varies. This is discussed below.

2.4.2 Log-Linearizing around the Balanced Growth Path

In order to be able to log-linearize around the steady state, we need to detrend variables on the balanced growth path. Following Uhlig (2010), we will denote log-deviations by hats so that $\widehat{c}_t = \log(c_t) - \log(c) \approx (c_t - c)/c$ where c is the steady state value of c_t. Thus noting that $\widehat{\lambda}^u_t = -(\lambda^s/\ \lambda^u)\widehat{\lambda}^s_t$ and that $w_t = w^s_t\lambda^s_t + w^u_t\lambda^u_t$ and so $\widehat{w}_t = \eta^s(\widehat{w}^s_t + \widehat{\lambda}^s_t) + (1-\eta^s)(\widehat{w}^u_t - \widehat{\lambda}^s_t)$ where $\eta^s = w^s\lambda^s/w$ and $w = w^s\lambda^s + w^u\lambda^u$, we can write the log-linearized equations of the model as in Table 2.2.[7]

[6]Thus, a 1% increase in population will increase the share of unskilled agents from 0.10 to 11/101 = 0.1089 which is a percentage increase of $(100 \times 0.9)/(101 \times 0.1) = 90/10.1 = 8.9\%$.

[7]See Section A.3 in Appendix A for the details of derivations.

Choosing Parameters

On the balanced growth path, we have $l_{t+1} = l_t = l$ and $c_{t+1} = c_t$ for any period of time t. Hence, the first order-conditions imply that $1 = \mathbb{E}_t[\beta(1 + r - \delta)]$, and in the steady state we have

$$r = \frac{1}{\beta} - (1 - \delta). \tag{2.1}$$

Following Uhlig (2010) and Trabandt and Uhlig (2011), we set the depreciation rate to be $\delta = 0.07$, and the intertemporal elasticity of substitution is set at 0.5, hence $\eta = 2$. The discount rate $\beta = 0.998$ hence $R \equiv (1 + r - \delta) = 1/\beta = 1/0.998 = 1.002$. From above we know that $r = \theta\alpha y/k$ and so given calibrated value for θ, and α and given that $r = 1/\beta - (1 - \delta)$, we will have an expression for y/k . Thus if $\theta = 0.4$ and $\alpha = 0.9$ then $y/k = 0.072/0.36 = 0.2$, hence $k/y = 4.99$. The capital accumulation equation in the steady state gives $x/y = (k/y)[\zeta^N - (1 - \delta)] = 4.99[1.012 - 0.93] = 0.41$, which is similar to that in Trabandt and Uhlig (2011) and which implies that $c/y = 0.59$. Given these values, $\kappa = (1 - \alpha\theta)(y/c) = 1.085$ and $1/\omega^S = 1/\phi - \kappa(1 - 1/\eta) = 0.458$, where we calibrate the Frisch elasticity, ϕ, to be unity following Uhlig (2010). There is no estimate of the degree of substitutability between capital and unskilled labor, so we set it to be $\sigma = 0.5$, assuming that they are mild complements. We will explore impulse response functions for alternative values of σ.

2.4.3 Impulse Responses

We discuss the impulse responses from two kinds of a shock. In Figure 2.8 we present what we call an immigration shock where both the rate of population growth and the proportion of unskilled workers in the economy have a positive shock in the manner described in Section 2.4.1. In Figure 2.9 by way of contrast we present the impulse of a pure population shock which is a positive shock to the rate of population growth with no change in the proportion of unskilled workers in the economy.

The immigration shock in Figure 2.8 has many features in common with the VAR impulses responses in Figures 2.3 and 2.5. Notably GDP per capita and and consumption per capita both decline before recovering back towards their balanced growth paths. This is also the case for the pure population shock as shown in Figure 2.9. However, the similarity does not carry over to the response of investment where in the pure population shock, in Figure 2.9 investment rises immediately in response to the population shock as the economy wide capital to labor ratio falls. In contrast in response to an immigration shock in Figure 2.8, investment falls as the increased in unskilled labor substitutes for capital in the production function. This investment dynamics is robust to higher values of σ. The response of unskilled wage rates also differs greatly between the two cases with unskilled wages falling sharply in response to an immigration shock while skilled wages initially rise in response to the immigration shock before falling slightly.

The responses to the immigration shock in Figure 2.8 are much closer to the VAR responses of Figure 2.3 and Figure 2.5 than those of Figure 2.9. However, they are clearly not a perfect fit. The most notable discrepancy is that aggregate wages still fall in response to an immigration shock. It is interesting to note however that the skilled wage does initially rise in response to an immigration shock before falling which is indeed the qualitative response of the wage variable in the VAR. Note that in our calibration 90% of labor is skilled labor and so if the wage variable in the VAR - Nonfarm Business Sector wage rate - has a greater skill component than the economy as a whole, or if immigrants wages do not make it onto the official wage data then the equivalent variable to the VAR in this model would indeed be the skilled wage responses. However, we do not model an informal sector in this paper and so again we leave this as a potential fruitful avenue for further research.

2.5 Conclusion

The paper has presented macroeconomic evidence on the effects of immigration on the macroeconomy. It has shown empirically that immigration shocks are not associated rises in non-residential investment or short run reductions in average wages. It has shown how a standard growth model with a CES production function where migrant labor and capital are complements to skilled domestic labor can produce responses closer to those of the VAR than a skill-neutral shock to the working population. Thus using a very different empirical and theoretical methodology, as well as macroeconomic data, this paper has provided empirical support for the microeconomic finding that immigrant labor is complementary to, rather than a substitute for, most native labor.

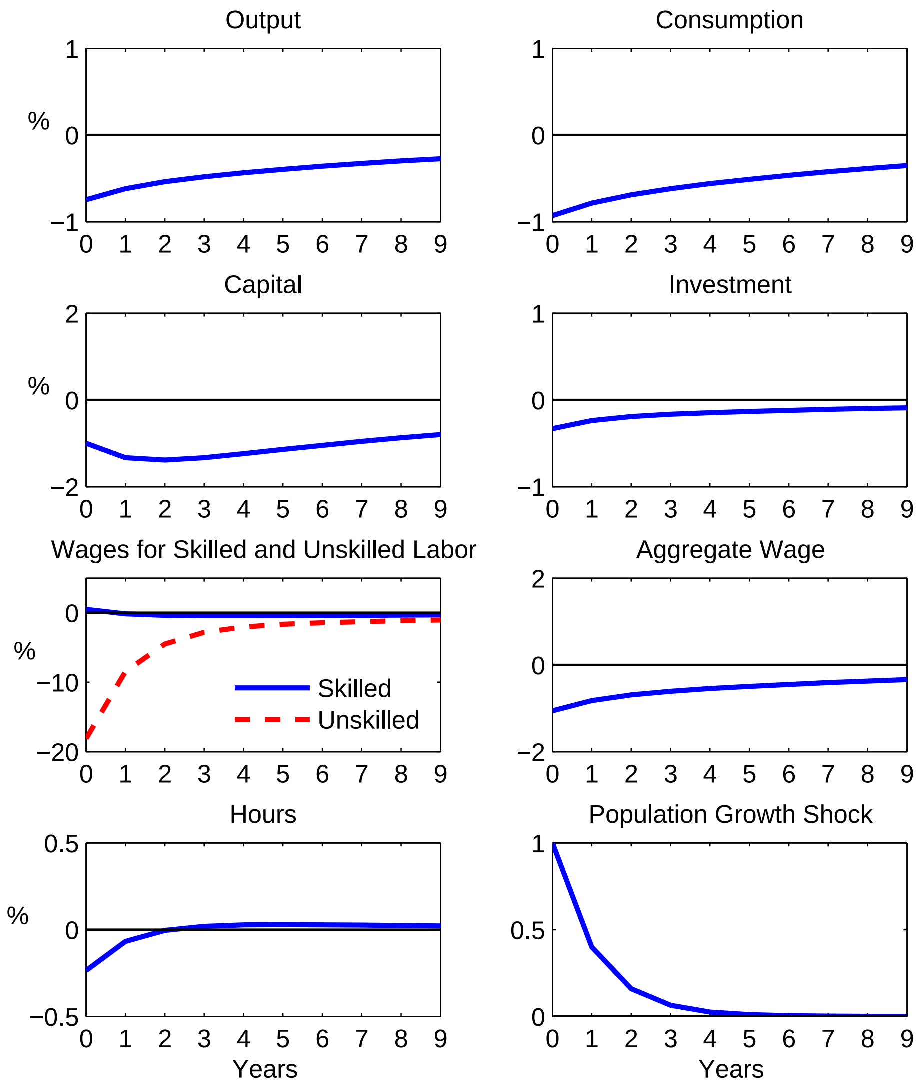


Figure 2.8: Impulse Responses to an Immigration Shock – a shock where population rises and the proportion of unskilled workers in the labor force also rises.

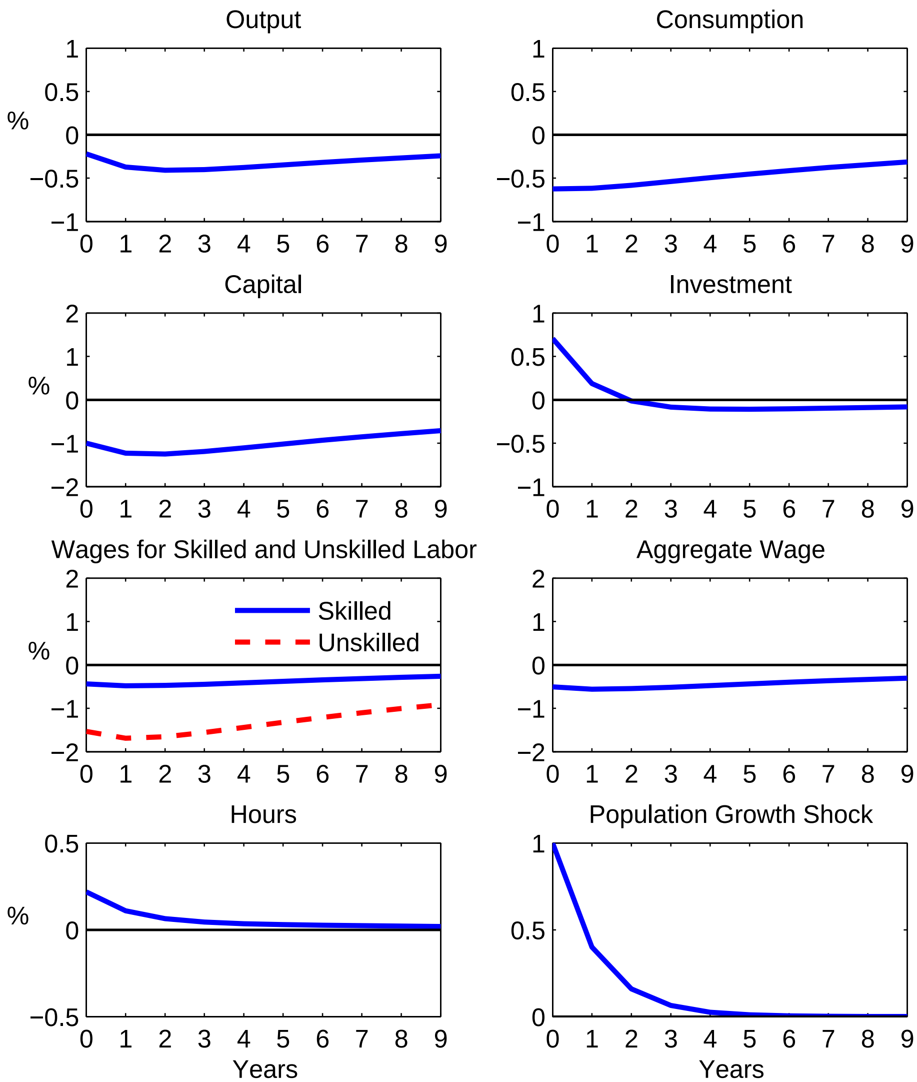


Figure 2.9: Impulse Responses to a pure Population Shock – a shock where population rises and the proportion of skiiled and unskilled workers is unchanged.

Appendix A

Appendices to Chapter 2

A.1 Log-Linearizing around the Balanced Growth Path

This appendix reports the details of how we derived the first-order conditions and the log-linearized equations as well as the steady state conditions.

A.2 Derivations of the First-Order Conditions

A.2.1 Household's Problem

Setting up the Lagrangian:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[N_t \frac{(c_t \Phi(l_t))^{1-\eta} - 1}{1-\eta} - \lambda_t (c_t N_t + k_{t+1} N_{t+1} - (1-\delta) k_t N_t - w_t l_t N_t - r_t k_t N_t) \right].$$

The first-order conditions with respect to c_t, l_t and k_{t+1} are given by

$$
\begin{aligned}
[c_t] :& \ N_t c_t^{-\eta}(\Phi(l_t))^{1-\eta} - \lambda_t N_t = 0 \\
&\Leftrightarrow \lambda_t = c_t^{-\eta}(\Phi(l_t))^{1-\eta} \\
[l_t] :& \ N_t c_t^{1-\eta}(\Phi(l_t))^{-\eta}\Phi'(l_t) + \lambda_t w_t N_t = 0 \\
&\Leftrightarrow c_t^{1-\eta}(\Phi(l_t))^{-\eta}\Phi'(l_t) + \lambda_t w_t = 0 \\
[k_{t+1}] :& \ -\lambda_t N_{t+1} + \beta \mathbb{E}_t[\lambda_{t+1}(1-\delta+r_{t+1})N_{t+1}] = 0 \\
&\Leftrightarrow 1 = \mathbb{E}_t\left[\beta\left(\frac{\lambda_{t+1}}{\lambda_t}\right)R_{t+1}\right]. \qquad \text{(A.1)}
\end{aligned}
$$

where $R_t := (1 + r_t - \delta)$. Now combining the first two conditions implies that

$$
w_t = -\frac{c_t \Phi'(l_t)}{\Phi(l_t)}. \qquad \text{(A.2)}
$$

Thus, in the steady state we have

$$
\frac{wl}{c} == -\frac{\Phi'(l)l}{\Phi(l)} =: \kappa. \qquad \text{(A.3)}
$$

Note that in the steady state Equation (A.1) implies that

$$
1 = \beta(1-\delta+r) \Leftrightarrow r = \frac{1}{\beta} - (1-\delta).
$$

A.2.2 Firm's Problem

Factor prices are determined by the marginal products so that

$$
\begin{aligned}
r_t =& \theta \left[\alpha \left(\frac{k_t}{k} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(\frac{l_t \lambda_t^u}{l \lambda^u} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\theta\sigma}{\sigma-1}-1} \alpha \left(\frac{1}{k} \right)^{\frac{\sigma-1}{\sigma}} k_t^{\frac{\sigma-1}{\sigma}-1} \left(\frac{l_t \lambda_t^s}{l \lambda^s} \right)^{1-\theta} \\
\Leftrightarrow \quad =& \theta \frac{y_t}{\left[\alpha \left(\frac{k_t}{k} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(\frac{l_t \lambda_t^u}{l \lambda^u} \right)^{\frac{\sigma-1}{\sigma}} \right]} \alpha \left(\frac{k_t}{k} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{1}{k_t} \right) \\
\Leftrightarrow \quad =& \frac{y_t}{k_t} \frac{\theta \alpha (\frac{k_t}{k})^{\frac{\sigma-1}{\sigma}}}{\left[\alpha \left(\frac{k_t}{k} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(\frac{l_t \lambda_t^u}{l \lambda^u} \right)^{\frac{\sigma-1}{\sigma}} \right]}.
\end{aligned}
$$

Note that in the steady state this implies $r = \theta \alpha y / k$. Similarly,

$$
\begin{aligned}
w_t^u =& \theta \left[\alpha \left(\frac{k_t}{k} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(\frac{l_t \lambda_t^u}{l \lambda^u} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\theta\sigma}{\sigma-1}-1} (1-\alpha) \left(\frac{1}{l \lambda^u} \right)^{\frac{\sigma-1}{\sigma}} (l_t \lambda_t^u)^{\frac{\sigma-1}{\sigma}-1} \left(\frac{l_t \lambda_t^s}{l \lambda^s} \right)^{1-\theta} \\
\Leftrightarrow \quad =& \theta \frac{y_t}{\left[\alpha (\frac{k_t}{k})^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (\frac{l_t \lambda_t^u}{l \lambda^u})^{\frac{\sigma-1}{\sigma}} \right]} (1-\alpha) \left(\frac{l_t \lambda_t^u}{l \lambda^u} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{1}{l_t \lambda_t^u} \right) \\
\Leftrightarrow \quad =& \theta \frac{y_t}{l_t \lambda_t^u} \frac{(1-\alpha)(\frac{l_t \lambda_t^u}{\lambda^u l})^{\frac{\sigma-1}{\sigma}}}{\left[\alpha (\frac{k_t}{k})^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (\frac{l_t \lambda_t^u}{l \lambda^u})^{\frac{\sigma-1}{\sigma}} \right]}.
\end{aligned}
$$

Note that in the steady state this implies $w^u = \theta(1-\alpha) y / (l \lambda^u)$. Finally,

$$
\begin{aligned}
w_t^s =& (1-\theta) \left[\alpha \left(\frac{k_t}{k} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(\frac{l_t \lambda_t^u}{l \lambda^u} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\theta\sigma}{\sigma-1}} \left(\frac{l_t \lambda_t^s}{l \lambda^s} \right)^{-\theta} \frac{1}{l \lambda^s} \\
\Leftrightarrow \quad =& (1-\theta) \frac{y_t}{l_t \lambda_t^s}.
\end{aligned}
$$

Note that in the steady state this implies $w^s = (1-\theta) y / (l \lambda^s)$. Given these steady-state values the aggregate wage is $w = w^s \lambda^s + w^u \lambda^u = (1 - \alpha\theta)(y/l)$ in the steady state, which in turn implies that the parameter κ, defined in (A.3), can be also expressed as

$$\kappa = \frac{wl}{c} = (1-\alpha\theta)\frac{y}{c}.$$

A.3 Derivations of Log-linearized Equations

In this section, we describe the details of derivations of log-linearized equations. Here, hats over variables indicate percent deviation from the steady state, e.g., $\widehat{c}_t = \log(c_t) - \log(c) \approx (c_t - c)/c$.

A.3.1 Labor Supply

Taking the first-order Taylor expansion of Equation (A.2) around the steady state gives

$$0 = \Phi'(l)c\frac{(c_t - c)}{c} + \Phi(l)w\frac{(w_t - w)}{w} + [c\Phi''(l) + w\Phi'(l)]l\frac{(l_t - l)}{l}.$$

Dividing through by $\Phi'(l)c$ gives and remembering $\widehat{c}_t = (c_t - c)/c$,

$$\widehat{c}_t = -\frac{\Phi(l)w}{\Phi'(l)c}\widehat{w}_t - \frac{[c\Phi''(l)l + w\Phi'(l)l]}{\Phi'(l)c}\widehat{l}_t$$

From the first order conditions we know $\frac{\Phi(l)w}{\Phi'(l)c} = -1$ and that $\kappa \equiv -\frac{\Phi'(l)l}{\Phi(l)} = wl/c$ and defining $1/\omega_S \equiv \kappa + \Phi''(l)l/\Phi'(l)$ then we can write

$$\widehat{w}_t = \frac{1}{\omega_S}\widehat{l}_t + \widehat{c}_t.$$

These results follow from the functional form of $\Phi(l_t)$ as shown by Trabanadt and Uhlig (2011). They show that the function Φ must have the form

$$\Phi(l_t) = \left(1 - \kappa(1-\eta) l_t^{1+\frac{1}{\phi}}\right)^{\frac{\eta}{1-\eta}}$$

then,

$$\begin{aligned}
\Phi'(l_t) =& -\frac{\eta}{1-\eta}\left(1 - \kappa(1-\eta) l_t^{1+\frac{1}{\phi}}\right)^{\frac{\eta}{1-\eta}-1} \kappa(1-\eta) l_t^{\frac{1}{\phi}}\left(1+\frac{1}{\phi}\right) \\
\Phi''(l_t) =& \frac{\eta}{1-\eta}\left(\frac{\eta}{1-\eta}-1\right)\left(1 - \kappa(1-\eta) l_t^{1+\frac{1}{\phi}}\right)^{\frac{\eta}{1-\eta}-2} \kappa^2(1-\eta)^2 l_t^{\frac{2}{\phi}}\left(1+\frac{1}{\phi}\right)^2 \\
& -\frac{\eta}{1-\eta}\left(1 - \kappa(1-\eta) l_t^{1+\frac{1}{\phi}}\right)^{\frac{\eta}{1-\eta}-1} \kappa(1-\eta) l_t^{\frac{1}{\phi}-1}\left(1+\frac{1}{\phi}\right)\frac{1}{\phi}.
\end{aligned}$$

Hence,

$$\frac{\Phi''(l_t) l_t}{\Phi'(l_t)} = \frac{1}{\phi} - \left(\frac{\eta}{1-\eta}-1\right)\left(1 - \kappa(1-\eta) l_t^{1+\frac{1}{\phi}}\right)^{-1} \kappa(1-\eta) l_t^{1+\frac{1}{\phi}}\left(1+\frac{1}{\phi}\right),$$

and

$$\kappa \equiv -\frac{\Phi'(l_t) l_t}{\Phi(l_t)} = \frac{\eta}{1-\eta}\left(1 - \kappa(1-\eta) l_t^{1+\frac{1}{\phi}}\right)^{-1} \kappa(1-\eta) l_t^{1+\frac{1}{\phi}}\left(1+\frac{1}{\phi}\right).$$

Thus,

$$\frac{\Phi''(l_t) l_t}{\Phi'(l_t)} = \frac{1}{\phi} - \kappa\frac{\left(\frac{\eta}{1-\eta}-1\right)}{\frac{\eta}{1-\eta}} = \frac{1}{\phi} - \kappa\left(2-\frac{1}{\eta}\right).$$

Thus,

$$
\begin{aligned}
\frac{1}{\omega_S} \equiv \kappa + \frac{\Phi''(l)l}{\Phi'} =& \kappa + \frac{1}{\phi} - \kappa\left(2 - \frac{1}{\eta}\right) \\
=& \frac{1}{\phi} - \kappa\left(1 - \frac{1}{\eta}\right),
\end{aligned}
$$

which is the same as in Uhlig (2010).

A.3.2 Lagrange Multiplier

Taking the first-order Taylor expansion of $\lambda_t = c_t(\Phi(l_t))^{1-\eta}$ gives,

$$
\begin{aligned}
\lambda\frac{(\lambda_t - \lambda)}{\lambda} =& -\eta c^{-\eta}\Phi(l)^{1-\eta}\frac{(c_t - c)}{c} + (1-\eta)c^{-\eta}\Phi(l)^{-\eta}\Phi'(l)l\frac{(l_t - l)}{l} \\
\Leftrightarrow \lambda\widehat{\lambda}_t =& -\eta\lambda\widehat{c}_t + (1-\eta)c^{-\eta}\Phi(l)^{1-\eta}\frac{\Phi'(l)l}{\Phi(l)}\widehat{l}_t \\
\Leftrightarrow \widehat{\lambda}_t =& -\eta\widehat{c}_t - (1-\eta)\kappa\widehat{l}_t,
\end{aligned}
$$

where we use the steady state relatihonship of $\lambda = c^{-\eta}\Phi(l)^{1-\eta}$ and $\kappa = \Phi'(l)l/\Phi(l)$.

A.3.3 Intertemporal

Now log-linearizing (A.1) gives

$$
\begin{aligned}
1 =& \mathbb{E}_t\left[\beta R\exp(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{R}_{t+1})\right] \\
\Leftrightarrow 0 =& \mathbb{E}_t\left[\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{R}_{t+1}\right]
\end{aligned}
$$

A.3.4 Capital Return

Log-linearizing around its steady states gives

$$
\begin{aligned}
R_t &= (1 + r_t - \delta) \\
\Leftrightarrow R \exp(\widehat{R}_t) &= 1 - \delta + r \exp(\widehat{r}_t) \\
\Leftrightarrow \widehat{R}_t &= \frac{r}{R} \widehat{r}_t \\
\Leftrightarrow \quad &= (1 - \beta(1 - \delta)) \widehat{r}_t,
\end{aligned}
$$

where the last equality use the stedy state relationship that $R = 1 + r - \delta$ and $R = \beta^{-1}$.

A.3.5 Capital Accumulation

The capital accumulation equation can be rewritten in terms of per-capita variables as follows

$$
\begin{aligned}
k_{t+1} N_{t+1} &= (1 - \delta) k_t N_t + x_t N_t \\
\Leftrightarrow k_{t+1} \frac{N_{t+1}}{N_t} &= (1 - \delta) k_t + x_t \\
\Leftrightarrow k_{t+1} \zeta_{t+1}^N &= (1 - \delta) k_t + x_t
\end{aligned}
$$

In the steady state,

$$
k \zeta^N = (1 - \delta) k + x \Leftrightarrow \frac{x}{k} = \zeta^N - 1 + \delta \tag{A.4}
$$

Log-linearizing gives

$$\begin{aligned}
k\zeta^N \exp(\widehat{k}_{t+1} + \widehat{\zeta}^N_{t+1}) &= (1-\delta)k\exp(\widehat{k}_t) + x\exp(\widehat{x}_t) \\
\Leftrightarrow k\zeta^N(\widehat{k}_{t+1} + \widehat{\zeta}^N_{t+1}) &= (1-\delta)k\widehat{k}_t + x\widehat{x}_t \\
\Leftrightarrow \widehat{k}_{t+1} + \widehat{\zeta}^N_{t+1} &= \frac{(1-\delta)}{\zeta^N}\widehat{k}_t + \frac{x}{k}\left(\frac{1}{\zeta^N}\right)\widehat{x}_t
\end{aligned}$$

Substituting (A.4),

$$\widehat{k}_{t+1} = \frac{1-\delta}{\zeta^N}\widehat{k}_t + \left[1 - \frac{(1-\delta)}{\zeta^N}\right]\widehat{x}_t - \widehat{\zeta}^N_{t+1}.$$

A.3.6 Share of Skilled and Unskilled Labor

$$\begin{aligned}
N_t &= N_{s,t} + N_{u,t} \\
\Leftrightarrow 1 &= \frac{N_{s,t}}{N_t} + \frac{N_{u,t}}{N_t} = \lambda^s_t + \lambda^u_t.
\end{aligned}$$

In the steady state,

$$1 = \lambda^s + \lambda^u.$$

Thus, log-linearizing

$$
\begin{aligned}
1 &= \lambda^s \exp(\widehat{\lambda^s}_t) + \lambda^u \exp(\widehat{\lambda}^u_t) \\
\Leftrightarrow \quad &= \lambda^s(1 + \widehat{\lambda}^s_t) + \lambda^u(1 + \widehat{\lambda}^u_t) \\
\Leftrightarrow 0 &= \lambda^s \widehat{\lambda}^s_t + \lambda^u \widehat{\lambda^u}_t \\
\Leftrightarrow \widehat{\lambda}^u_t &= -\frac{\lambda^s}{\lambda^u}\widehat{\lambda}^s_t = -\frac{\lambda^s}{1-\lambda^s}\widehat{\lambda}^s_t.
\end{aligned}
$$

A.3.7 Production Function

Log-linearizing the production function:

$$
y_t = \left[\alpha\left(\frac{k_t}{k}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(\frac{l_t\lambda^u_t}{l\lambda^u}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\theta\sigma}{\sigma-1}} \left(\frac{l_t\lambda^s_t}{l\lambda^s}\right)^{1-\theta},
$$

it is easiest to define a new function f_t:

$$
f_t = \alpha\left(\frac{k_t}{k}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(\frac{l_t\lambda^u_t}{l\lambda^u}\right)^{\frac{\sigma-1}{\sigma}},
$$

and so $f = \alpha(\frac{k}{k})^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(\frac{\lambda^u l}{\lambda^u l})^{\frac{\sigma-1}{\sigma}} = 1$ in the steady state, and log-linearizing of f_t gives:

$$
\begin{aligned}
f\exp(\widehat{f_t}) &= \alpha\left(\frac{k\exp(\widehat{k_t})}{k}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(\frac{l\lambda^u\exp(\widehat{l_t}+\widehat{\lambda}^u_t)}{l\lambda^u}\right)^{\frac{\sigma-1}{\sigma}} \\
\Leftrightarrow 1 + \widehat{f_t} &= \alpha\exp\left[\left(\frac{\sigma-1}{\sigma}\right)\widehat{k_t}\right] + (1-\alpha)\exp\left[\left(\frac{\sigma-1}{\sigma}\right)(\widehat{l_t}+\widehat{\lambda}^u_t)\right] \\
&= \alpha\left[1 + \left(\frac{\sigma-1}{\sigma}\right)\widehat{k_t}\right] + (1-\alpha)\left[1 + \left(\frac{\sigma-1}{\sigma}\right)(\widehat{l_t}+\widehat{\lambda}^u_t)\right] \\
\Leftrightarrow \widehat{f_t} &= \left(\frac{\sigma-1}{\sigma}\right)[\alpha\widehat{k_t} + (1-\alpha)(\widehat{l_t}+\widehat{\lambda^u_t})].
\end{aligned}
$$

. Thus, log-linearizing of y_t gives:

$$
\begin{aligned}
y_t =& f_t^{\frac{\theta\sigma}{\sigma-1}} \left(\frac{l_t \lambda_t^s}{l\lambda^s}\right)^{1-\theta} \\
\Leftrightarrow \widehat{y}_t =& \left(\frac{\theta\sigma}{\sigma-1}\right) \widehat{f}_t + (1-\theta)(\widehat{l}_t + \widehat{\lambda}_t^s) \\
=& \theta[\alpha\widehat{k}_t + (1-\alpha)(\widehat{l}_t + \widehat{\lambda}_t^u)] + (1-\theta)(\widehat{l}_t + \widehat{\lambda}_t^s).
\end{aligned}
$$

A.3.8 Capital Rental Rate

Using f_t defined above,

$$
r_t = \frac{y_t}{k_t} \frac{\theta\alpha\left(\frac{k_t}{k}\right)^{\frac{\sigma-1}{\sigma}}}{\left[\alpha\left(\frac{k_t}{k}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(\frac{l_t\lambda_t^u}{l\lambda^u}\right)^{\frac{\sigma-1}{\sigma}}\right]} = \theta\alpha\frac{y_t}{k_t}\frac{\left(\frac{k_t}{k}\right)^{\frac{\sigma-1}{\sigma}}}{f_t}.
$$

Thus log-linearizing gives

$$
\begin{aligned}
\widehat{r}_t &= \widehat{y}_t - \widehat{k}_t - \widehat{f}_t + \left(\frac{\sigma-1}{\sigma}\right)\widehat{k}_t \\
\Leftrightarrow \quad &= \widehat{y}_t - \widehat{k}_t - \left(\frac{\sigma-1}{\sigma}\right)[\alpha\widehat{k}_t + (1-\alpha)(\widehat{l}_t + \widehat{\lambda}_t^u)] + \left(\frac{\sigma-1}{\sigma}\right)\widehat{k}_t \\
\Leftrightarrow \quad &= \widehat{y}_t - \widehat{k}_t + (1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)\widehat{k}_t - (1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)(\widehat{l}_t + \widehat{\lambda}_t^u)
\end{aligned}
$$

A.3.9 Wages for Unskilled Labor

Similarly, using f_t,

$$
w_t^u = \frac{y_t}{l_t\lambda_t^u} \frac{\theta(1-\alpha)\left(\frac{l_t\lambda_t^u}{l\lambda^u}\right)^{\frac{\sigma-1}{\sigma}}}{\left[\alpha\left(\frac{k_t}{k}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(\frac{l_t\lambda_t^u}{l\lambda^u}\right)^{\frac{\sigma-1}{\sigma}}\right]} = \frac{y_t}{l_t\lambda_t^u}\frac{\theta(1-\alpha)\left(\frac{l_t\lambda_t^u}{l\lambda^u}\right)^{\frac{\sigma-1}{\sigma}}}{f_t}.
$$

Thus, log-linearizing

$$
\begin{aligned}
\widehat{w}_t^u &= \widehat{y}_t - (\widehat{l}_t + \widehat{\lambda}_t^u) - \widehat{f}_t + \left(\frac{\sigma - 1}{\sigma}\right)(\widehat{l}_t + \widehat{\lambda}_t^u) \\
\Leftrightarrow \quad &= \widehat{y}_t - (\widehat{l}_t + \widehat{\lambda}_t^u) - \left(\frac{\sigma - 1}{\sigma}\right)[\alpha \widehat{k}_t + (1-\alpha)(\widehat{l}_t + \widehat{\lambda}_t^u)] + \left(\frac{\sigma - 1}{\sigma}\right)(\widehat{l}_t + \widehat{\lambda}_t^u) \\
\Leftrightarrow \quad &= \widehat{y}_t - (\widehat{l}_t + \widehat{\lambda}_t^u) - \alpha \left(\frac{\sigma - 1}{\sigma}\right)\widehat{k}_t + \alpha \left(\frac{\sigma - 1}{\sigma}\right)(\widehat{l}_t + \widehat{\lambda}_t^u).
\end{aligned}
$$

A.3.10 Wages for Skilled Labor

Log-linearizing of $w_t^s = (1-\theta)y_t/(l_t\lambda_t^s)$ gives

$$
\widehat{w}_t^s = \widehat{y}_t - (\widehat{l}_t + \widehat{\lambda}_t^s).
$$

A.3.11 Aggregate Wages

Log-linearizing aggregate wages:

$$
\begin{aligned}
w_t =& w_t^s \lambda_t^s + w_t^u \lambda_t^u \\
w \exp(\widehat{w}_t) =& w^s \lambda^s \exp(\widehat{w}_t^s + \widehat{\lambda}_t^s) + w^u \lambda^u \exp(\widehat{w}_t^u + \widehat{\lambda}_t^u) \\
\Leftrightarrow w(1 + \widehat{w}_t) =& w^s \lambda^s (1 + \widehat{w}_t^s + \widehat{\lambda}_t^s) + w^u \lambda^u (1 + \widehat{w}_t^u + \widehat{\lambda}_t^u) \\
\Leftrightarrow \widehat{w}_t =& \eta^s (\widehat{w}_t^s + \widehat{\lambda}_t^s) + (1 - \eta^s)(\widehat{w}_t^u - \widehat{\lambda}_t^s),
\end{aligned}
$$

where $\eta^s := w^s \lambda^s / w$.

A.3.12 Resource Constraint

The feasibility constraint is given by

$$y_t = c_t + x_t.$$

Thus, log-linearizing

$$\begin{aligned} y\exp(\widehat{y}_t) &= c\exp(\widehat{c}_t) + x\exp(\widehat{x}_t) \\ \Leftrightarrow y(1+\widehat{y}_t) &= c(1+\widehat{c}_t) + x(1+\widehat{x}_t) \\ \Leftrightarrow \widehat{y}_t &= \frac{c}{y}\widehat{c}_t + \frac{x}{y}\widehat{x}_t. \end{aligned}$$

A.4 List of log-linearized Equations

Thus to summarize we can write the log-linearized equations of the model as

Table 2.3: The Log-Linearized Equations

Neoclassical Growth Model with CES Production and Immigration	
Household	
(1)	$\widehat{w}_t = \frac{1}{\omega_S}\widehat{l}_t + \widehat{c}_t$
(2)	$\widehat{\lambda}_t = -\eta\widehat{c}_t - (1-\eta)\kappa\widehat{l}_t$
(3)	$0 = \mathbb{E}_t\left[\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{R}_{t+1}\right]$
(4)	$\widehat{R}_t = (1-\beta(1-\delta))\widehat{r}_t$
(5)	$\widehat{k}_{t+1} = \frac{1-\delta}{\zeta^N}\widehat{k}_t + \left[1 - \frac{(1-\delta)}{\zeta^N}\right]\widehat{x}_t - \widehat{\zeta}^N_{t+1}$
(6)	$\widehat{\lambda}^u_t = -\frac{\lambda^s}{1-\lambda^s}\widehat{\lambda}^s_t$
Firm	
(7)	$\widehat{y}_t = \theta[\alpha\widehat{k}_t + (1-\alpha)(\widehat{l}_t + \widehat{\lambda}^u_t)] + (1-\theta)(\widehat{l}_t + \widehat{\lambda}^s_t)$
(8)	$\widehat{r}_t = \widehat{y}_t - \widehat{k}_t + (1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)\widehat{k}_t - (1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)(\widehat{l}_t + \widehat{\lambda}^u_t)$
(9)	$\widehat{w}_t = \eta^s(\widehat{w}^s_t + \widehat{\lambda}^s_t) + (1-\eta^s)(\widehat{w}^u_t - \widehat{\lambda}^s_t)$
(10)	$\widehat{w}^u_t = \widehat{y}_t - (\widehat{l}_t + \widehat{\lambda}^u_t) - \alpha\left(\frac{\sigma-1}{\sigma}\right)\widehat{k}_t + \alpha\left(\frac{\sigma-1}{\sigma}\right)(\widehat{l}_t + \widehat{\lambda}^u_t)$
(11)	$\widehat{w}^s_t = \widehat{y}_t - \widehat{l}_t - \widehat{\lambda}^s_t$
Resource Constraint	
(12)	$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{x}{y}\ \widehat{x}_t$
Exogenous Processes	
(a)	$\widehat{\zeta}^N_t = \rho_N\widehat{\zeta}^N_{t-1} + \epsilon^N_t$

Chapter 3

Bargaining and Immigration in a Macro Model

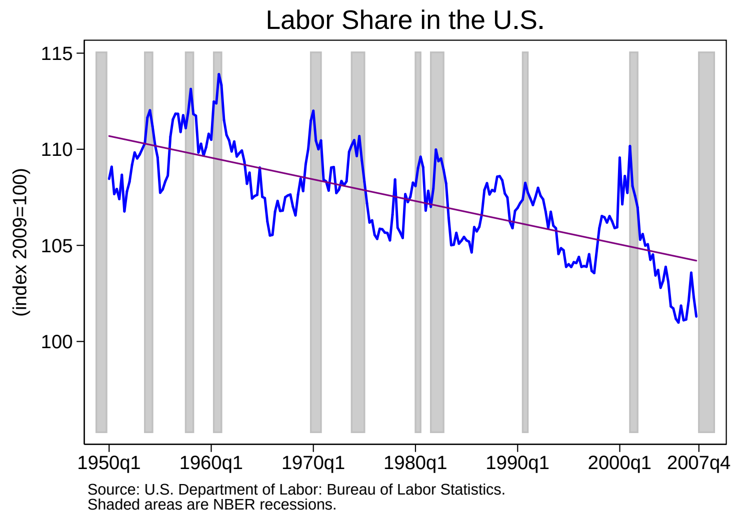
3.1 Introduction

The United States has experienced (at least) three changes in labor market dynamics over the last three decades. First, the labor share of national income has been falling, which has also been observed in many developed countries. Figure 3.1 shows the time series for labor income share in the United States from 1950 to 2007. The labor share looks stable in the first half of this period, which is consistent with Kaldor's (1957) stylized fact of economic growth and Blanchard's (1997) work. However, it took a downward turn around the 1990's and showed a sharp decline after 2000.[1] Figure 3.1 also displays a simple regression of labor share on time, which shows a clearly negative relationship. If we exclude the top one percent of income earners, the decline in labor share would be more severe, since the fraction of wages and salaries in top incomes have also been rising.

[1]The average of labor share from 1950 to 1987 is about 64%, while from 1987 to 2007 it is about 60%.

Empirical evidence for this can be found in Piketty and Saez (2003, 2006) and Atkinson, Piketty, and Saez (2011).

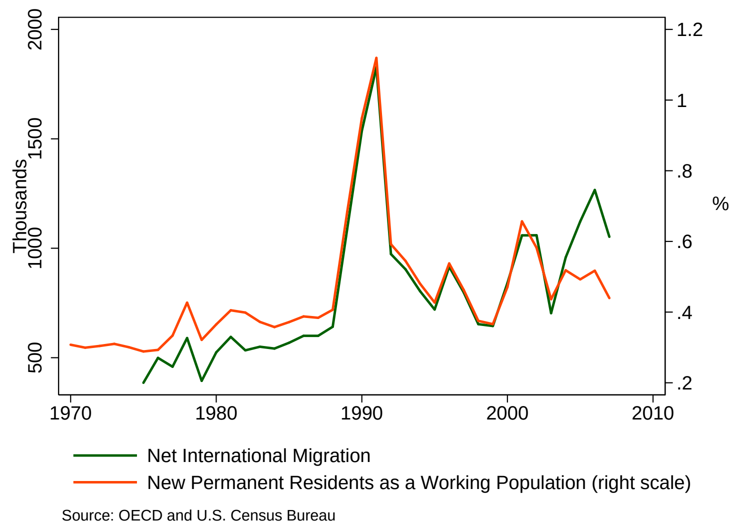
Figure 3.1: The Decline of Labor Share in the U.S.



The second change is the steady increase in the immigration share as discussed in Section 2.2 of Chapter 2. Figure 3.2 displays two time series for immigration share, which corresponds to the share of new permanent residents in the working population and net international migration. It is noticeable that both have a similar pattern and that the immigration flow keeps rising, with two peaks around the late 1980's and in the early 2000's. This significant increase in migration flow was generally caused on the part of generous immigration policies by the U.S. government. See Section 4.2 of Chapter 4 for a brief description of U.S. immigration policies.

Lastly, the bargaining power of workers has been declining. As shown in the top left panel in Figure 3.3, the ratio of the U.S. federal minimum wage to hourly earnings showed a decrease from 0.47 % in 1970 to 0.33 % in 2007. The decline was accompanied by a decrease both in union membership and in the net replacement ratios (ratio of

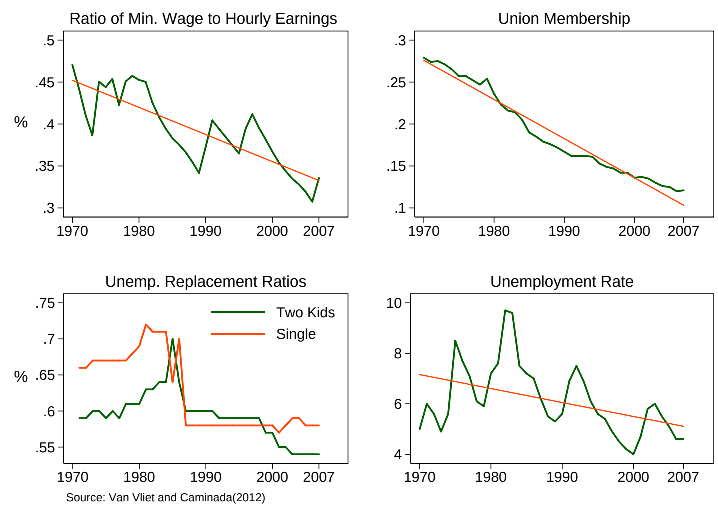
Figure 3.2: The Increase in Immigration in the U.S.



unemployment benefit to employment income) for a single person and one earner with two children, which is also shown in Figure 3.3. These facts indicate that the relative position of workers declined during this period, which can be interpreted as a decline of the bargaining power of workers.

In this paper, we seek to explore the possible connections between the three changes described above, and examine theoretically the macroeconomic effects of immigration on labor market outcomes, especially labor share, for alternative assumptions on bargaining power. We adopt a dynamic stochastic general equilibrium (DSGE) framework with the Diamond-Mortensen-Pissarides model of labor market frictions (Diamond, 1982; Mortensen and Pissarides, 1994) since a standard DSGE model fails to address the issues of labor market outcomes. This is because, in a standard model, the labor market is competitive and the wage rate equals the marginal product of labor. With the standard assumption of the Cobb-Douglass production function, this implies that labor share remains constant. Consequently, there is growing literature on policy analyses using a

Figure 3.3: The Decline in Bargaining Power of Workers in the U.S.



DSGE with search and matching model – see, e.g., Walsh (2005), Trigari (2006), Blanchard and Galí (2010). A textbook exposition of a DSGE model with labor market search can be found in Shimer (2010).

Specifically, this paper employs the model of Brückner and Pappa (2012) (the BP model, hereafter), which is a New Keynesian model with search frictions in the labor market, a labor force participation choice, and heterogeneous unemployed workers. In their model, there are two types of unemployed workers, insiders and outsiders, and they are different in that the former faces a more efficient matching function, and hence, the job finding rates for insiders are higher than those of outsiders. It is possible to interpret insiders as skilled workers and outsiders as unskilled workers. We think the distinction between inside and outside labor may well be helpful in discussing the impact of immigration on the macroeconomy because immigration could be thought of as an exogenous shock reaction to the numbers of outsiders (this assumption is relaxed in Section 3.4). This will lead to a reduction in the bargaining power of labor, and as a result, a fall in wage rates and labor income share. Indeed in Chapter 2 we identify immigration shocks using a VAR with sign restrictions and find that increases in immigrant labor do reduce labor share over the medium term.

Our paper is closest to a recent work by Chassamboulli and Palivos (2014), which analyzes the impact of immigration on labor market outcomes using a search and matching model. Though their model has many features in common with ours, such as differential search costs between natives and immigrants, they also allow for skill heterogeneity among natives, which enables them to address distributional issues. However, Chassamboulli and Palivos (2014) assume that the bargaining power of workers is constant and do not examine dynamics of labor share.

There are some recent studies that attempt to explain the decline in the labor share in the U.S. and the globe. Elsby, Hobijn and Şahin (2013) have shown that offshoring of the labor-intensive component of U.S. production to countries with lower labor costs is a major reason for the recent decline in the U.S. labor share. Karabarbounis and Neiman (2013) have pointed out that the decrease in the relative price of investment brought about by the evolution of information technology leads to a shift away from labor towards capital, and hence, the decline in the global labor share. This paper contributes to the literature by showing that the decline in worker's bargaining power caused by an increase in immigration could be one reason for the decline in labor share.

The rest of this paper is organized as follows. In Section 3.2 we outline the theoretical model. Section 3.3 discusses the results of the simulation. Section 3.4 extends the assumption of the baseline and considers two cases where immigrants enter a host economy (i) with employment and (ii) as insiders. Section 3.5 concludes.

3.2 The Model

In this section, we extend the BP model to allow for immigration. The model consists of households, firms (intermediate and retail), and a government which conducts both monetary and fiscal policy. Each household consists of a continuum of infinitely-lived employed workers, two types of unemployed workers (the short term and the long term unemployed), and non-participants. Immigrations are assumed to be the exogenous shock to the number of outsiders in the baseline. We also consider alternative scenarios where immigrants arrives as employed workers or insiders. We can interpret employment-based immigration as the case where employers sponsor immigrant workers for green cards based on employment, while insider immigration can be viewd as immigration through a family member (i.e., family reunifications), depending on the assumption on the matching prospects.

Households supply labor services to the intermediate firms and earn wages when employed, while they search for jobs and earn unemployment benefit when unemployed, or enjoy leisure when not participated in a labor market. Intermediate firms hire workers in a frictional labor market, i.e., they increase their current workforce by posting vacancies, which is costly. They then produce intermediate goods by using capital and labor and sell the products to retailers, which differentiate them and sell to households in a competitive market. Adding retailers allows us to incorporate inertia in price setting. In the following, we explain the details of the model.

3.2.1 Immigration

At any time of t, the number of household members who are employed is denoted by E_t, the number of short term unemployed (we call them insiders) is denoted by U_t^I, the number of long-term unemployed (we call them outsiders) is denoted by U_t^O, and the number of non-participants (i.e., out of labor force) is denoted by L_t.

At the beginning of the period, we assume that there is the exogenous flow of immigrations into the host economy. Since newly immigrated people are likely to have less chances of finding jobs, we treat them as outsiders and denote it as Mig_t, which follows a stationary stochastic process. The total population in the domestic economy, N_t, is therefore given by

$$N_t = E_t + U_t^I + U_t^O + Mig_t + L_t,$$

or equivalently

$$1 = e_t + u_t^I + u_t^O + mig_t + l_t \tag{3.1}$$

where e_t, u_t^I, u_t^O, mig_t, and l_t are proportions in the total population. In period $t + 1$, the number of Mig_{t+1} are newly immigrated in the domestic economy, and therefore, the total population of the domestic economy evolves as

$$\begin{aligned} N_{t+1} &= N_t + Mig_{t+1} \\ \Leftrightarrow \frac{N_{t+1}}{N_t} &= \frac{1}{1 - mig_{t+1}} =: \zeta_{t+1}^N. \end{aligned} \tag{3.2}$$

3.2.2 Matching

The aggregate number of matches in the economy, M_t, is given by the sum of the constant-returns-to-scale matching function of insiders and that of outsiders, whose inputs are vacancies that firms create and unemployed workers.

$$\begin{aligned} M_t =& M_t^I(V_t, U_t^I) + M_t^O(V_t, U_t^O + Mig_t) \\ =& \rho_m^I V_t^\alpha (u_t^I N_t)^{1-\alpha} + \rho_m^O V_t^\alpha ((u_t^O + mig_t)N_t)^{1-\alpha} \end{aligned} \tag{3.3}$$

where V_t is the aggregate vacancy, and $\rho_m^I > \rho_m^O > 0$ is assumed. That is, insiders face more efficient matching technology than outsiders. The parameter $\alpha \in (0,1)$ is the elasticity of the matching function with respect to vacancies. The aggregate job finding rates for insiders and outsiders are defined respectively as

$$\gamma_t^{Ih} := \frac{M_t^I}{u_t^I N_t} \tag{3.4}$$

$$\gamma_t^{Oh} := \frac{M_t^O}{(u_t^O + mig_t)N_t} \tag{3.5}$$

and $\gamma_t^h := \gamma_t^{Ih} + \gamma_t^{Oh}$. The aggregate vacancy filling rate is

$$\gamma_t^f := \frac{M_t}{V_t} \tag{3.6}$$

Using the job finding rates defined above, the transition equation for employment is expressed as

$$\begin{aligned} E_{t+1} &= (1-\sigma)E_t + M_t^I + M_t^O \\ \Leftrightarrow \widetilde{e}_{t+1}N_t &= (1-\sigma)e_t N_t + \gamma_t^{Ih} u_t^I N_t + \gamma_t^{Oh}(u_t^O + mig_t)N_t. \end{aligned} \tag{3.7}$$

where $\sigma \in (0,1)$ is the exogenous job destruction rate, and $\widetilde{e}_t := E_t/N_{t-1}$ represents employment per person at the beginning of time t. Similarly, the transition for insiders is given by

$$\begin{aligned} U_{t+1}^I &= (1-\mu)U_t^I + \sigma E_t - M_t^I \\ \Leftrightarrow \widetilde{u}_{t+1}^I N_t &= (1-\mu)u_t^I N_t + \sigma e_t N_t - \gamma_t^{Ih} u_t^I N_t \end{aligned} \tag{3.8}$$

where $\mu \in (0,1)$ is the probability of becoming outsiders and $\widetilde{u}_t^I := U_t^I/N_{t-1}$.

3.2.3 Household

The household's total instantaneous utility function takes the form

$$u(c_t, l_t)N_t = \left(\frac{c_t^{1-\eta}}{1-\eta} + \Phi \frac{l_t^{1-\zeta}}{1-\zeta} \right) N_t, \tag{3.9}$$

where c_t is the consumption of each member of the household at time t, l_t, as is defined in (3.1), is the fraction of non-participants who enjoy leisure, $1/\eta$ is the intertemporal elasticity of substitution, ζ is the inverse of the Frisch elasticity of labor supply, $\Phi > 0$ is a preference parameter that measures the disutility from being in the labor market. As common in the macroeconomic literature, full risk sharing among household members is assumed so that they can insure themselves against income uncertainty and unemployment.[2] The household's problem is expressed as

$$\mathcal{J}(k_t, e_t, u_t^I, b_t) = \max_{c_t, k_{t+1}, b_{t+1}, e_{t+1}, u_{t+1}^I, u_t^O} \left(\frac{c_t^{1-\eta}}{1-\eta} + \Phi \frac{l_t^{1-\zeta}}{1-\zeta} \right) N_t$$
$$+ \beta \mathbb{E}_t \mathcal{J}(k_{t+1}, e_{t+1}, u_{t+1}^I, b_{t+1}) \tag{3.10}$$

subject to total population (3.1), the law of motion for employed workers (3.7) and that of insiders (3.8), the following budget constraint (3.11), and the capital accumulation equation with adjustment costs (3.12):

$$c_t N_t + i_t N_t + \frac{b_{t+1} N_{t+1}}{p_t R_t} \leq r_t k_t N_t + w_t e_t N_t + ben(u_t^I + u_t^O(1 + mig_t))N_t + \frac{b_t N_t}{p_t}$$
$$+ pro_t N_t - t_t N_t, \tag{3.11}$$
$$\widetilde{k}_{t+1} N_t = (1-\delta) k_t N_t + i_t N_t - \frac{\omega}{2} \left(\frac{\widetilde{k}_{t+1}}{k_t} - \zeta^N \right)^2 k_t N_t, \tag{3.12}$$

where i_t is investment, b_t is the government bond, R_t is the gross nominal interest rate, p_t is price level, w_t is real wage, r_t is the rental rate of capital, ben is unemployment benefits, pro_t is profits from firms, t_t is lump sum taxes, k_t is capital, $\delta \in (0,1)$ is the depreciation rate, ω captures the degree of adjustment costs, and $\widetilde{k}_t := K_t / N_{t-1}$.

As common in the literature,[3] we define the marginal value to the household of having one member employed rather than unemployed, and that of being insider unemployed by

[2] See, e.g., Merz (1995), Andolfatto (1996), Galí (2011) and Brückner and Pappa (2012).
[3] See, e.g., Ravn (2008), Shimer (2010), Monacelli, Gertler and Trigari (2010).

using the first-order conditions to the household's problem above as follows:[4]

$$\mathbb{V}_t^E = -\Phi l_t^{-\zeta} N_t + c_t^{-\eta} w_t N_t + (1-\sigma)\beta\mathbb{E}_t \frac{\mathbb{V}_{t+1}^E}{\zeta_{t+1}^N} + \sigma\beta\mathbb{E}_t \frac{\mathbb{V}_{t+1}^{UI}}{\zeta_{t+1}^N}, \tag{3.13}$$

$$\mathbb{V}_t^{UI} = -\Phi l_t^{-\zeta} N_t + c_t^{-\eta} ben N_t + \gamma_t^{Ih}\beta\mathbb{E}_t \frac{\mathbb{V}_{t+1}^E}{\zeta_{t+1}^N} + ((1-\mu) - \gamma_t^{Ih})\beta\mathbb{E}_t \frac{\mathbb{V}_{t+1}^{UI}}{\zeta_{t+1}^N} \tag{3.14}$$

The marginal value to the household of an employed worker consists of the disutility from being in the labor market, $-\partial u(c_t, l_t)N_t/\partial l_t = -\Phi l_t^{-\zeta} N_t$, the wage rates, w_t, multiplied by the marginal utility of wealth $c_t^{-\eta}$ and the total numbers in household members, N_t, and the continuation value, which is the value of being employed if the match is not terminated, which occurs with the probability $(1-\sigma)$, and the value of becoming an insider if it is destroyed, which occurs with the probability σ. The continuation value is discounted by the discounted factor, β, and adjusted by the expected population growth rate ζ_{t+1}^N. Similarly, the marginal value to the household of being an insider consists of the disutility from being in the labor market, and unemployment benefit, *ben*, and the continuation value. Note that an insider finds a job with the probability γ_t^{Ih}, as defined in (3.4), and remains an insider with the probability $((1-\mu)-\gamma_t^{Ih})$ since an insider becomes an outsider with the probability μ.

3.2.4 Intermediate Firms

Intermediate firms employ the aggregate household's labor E_t and aggregate capital, K_t, to produce goods. The production function is given by:

$$Y_t = F(K_t, E_t) = K_t^{\varphi}(E_t)^{1-\varphi}, \tag{3.15}$$

[4] See 3.5 in Appendix B for derivations

where $\varphi \in (0,1)$ is the elasticity of output with respect to capital. The value function of a firm with E_t currently employed workers is:

$$\mathcal{V}(E_t) = \max_{K_t, V_t} \; x_t F(K_t, E_t) - w_t E_t - r_t K_t - \kappa V_t + \mathbb{E}_t \Lambda_{t+1} \mathcal{V}(\underbrace{(1-\sigma)E_t + \gamma_t^f V_t}_{=E_{t+1}}) \quad (3.16)$$

where x_t is the relative price of intermediate goods, $\Lambda_{t+1} = \beta u_{ct+1}/u_{ct}$ is the stochastic discount factor, and $\kappa > 0$ is the cost of posting vacancies. The first-order conditions for K_t and V_t are;

$$[K_t]: \;\; \varphi \, x_t \frac{Y_t}{K_t} = r_t, \quad (3.17)$$

$$[V_t]: \;\; \frac{\kappa}{\gamma_t^f} = \beta \mathbb{E}_t \left(\frac{c_t}{c_{t+1}} \right)^{\eta} \mathbb{V}_{t+1}^F, \quad (3.18)$$

where $\mathbb{V}_t^F$ is the value of filling a vacancy which is defined as[5]

$$\mathbb{V}_t^F := (1-\varphi) x_t \frac{Y_t}{E_t} - w_t + (1-\sigma)\frac{\kappa}{\gamma_t^f}. \quad (3.19)$$

Therefore, the optimal vacancy condition (3.18), together with (3.19), states that the marginal cost of positing a vacancy should equal the expected marginal benefit, which is the marginal product of labor minus the wage plus the continuation value, knowing that the match can be terminated with probability σ.

[5]See Section B.2 in Appendix B for the derivation.

3.2.5 Bargaining over Wages

The Nash bargaining problem is to maximize the weighted sum of log post-match surpluses:

$$\max_{w_t} \;\; (1-\vartheta)\ln \mathbb{V}_t^E + \vartheta \ln \mathbb{V}_t^F \tag{3.20}$$

where $\vartheta \in (0,1)$ denotes the firms' bargaining power. The first-order condition with respect to w_t leads to the following Nash wage equation:

$$w_t = (1-\vartheta)\left[(1-\varphi)x_t\frac{y_t}{e_t} + \frac{\kappa\gamma_t^{Oh}}{\gamma_t^f}\right] + \vartheta[ben - c_t^{\eta}\sigma\beta\mathbb{E}_t\mathbb{V}_{t+1}^{UI}]. \tag{3.21}$$

In words, the equilibrium Nash bargained wage is the weighted average of the marginal product of labor plus the value to the firm of marginal job (κ/γ_t^f), multiplied by the vacancy filling rate for an outsider, and the outside option of being unemployed minus the expected value of becoming an insider next period if the match is destroyed.

3.2.6 Retailers and Price Setting

There is a continuum of monopolistically competitive retailers indexed by $i \in [0,1]$, which buy intermediate goods and differentiate them with a technology that transforms one unit of intermediate goods into one unit of retail goods. The relative price of intermediate goods, x_t, coincides with the real marginal cost that the retailers face. Final goods are expressed as the composite of individual retail goods Y_{it}:

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{\epsilon-1}{\epsilon}}\, di\right]^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$ is the elasticity of substitution between intermediate goods. Retail firms can optimize their price with a fixed probability $1 - \chi^p \in (0, 1)$ in any period following Calvo (1983), which will lead to the standard New Keynesian Phillips Curve:[6]

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda \widehat{x}_t$$

where π_t is the inflation rate of prices of retail goods, and $\lambda = (1 - \beta\chi^p)(1 - \chi^p)/\chi^p$. Hat over the marginal cost denotes the deviation from the steady state. Monetary policy follows an interest rate rule:

$$R_t = R \exp(\zeta_\pi \pi_t).$$

Government finances the expenditure on unemployment benefits and government spending by lump sum tax,

$$ben\ U_t + G_t = T_t.$$

Resource constraint is given by

$$Y_t = C_t + I_t + G_t + \kappa V_t.$$

3.2.7 List of Log-linearized Equations

We log-linearize per-capita equations around the steady states. We will denote log-deviations by hats over variables so that for a generic aggregate variable X_t, $\widehat{x}_t = \log(x_t) - \log(x) \approx (\widetilde{x}_t - x)/x$ where $x_t := X_t/N_t$ and x is the steady state value of x_t. The only exception is the inflation rate, π_t, which is expressed as a percentage deviation from the steady state of zero inflation, so that $\widehat{\pi}_t = \pi_t$. The log-linearized dynamics of the

[6]See, e.g., Galí (2008) for the derivation.

model is shown in Table 3.1.[7] Equations (1) to (27) determine 27 endogenous variables: $y_t, c_t, i_t, k_t, \widetilde{k}_t, r_t, e_t, \widetilde{e}_t, u_t^I, \widetilde{u}_t^I, u_t^O, l_t, w_t, R_t, x_t, \pi_t, m_t^I, m_t^O, v_t$, γ_t^{Ih}, γ_t^{Oh}, γ_t^{If}, γ_t^{Of}, γ_t^f, λ_{et}, λ_{ut}, ζ_t^N. Equation (a) represents an exogenous process.

3.3 Simulation

3.3.1 Parameter Values

In the baseline calibration, we take the period in the model to correspond to a quarter and set the model parameters to fit the U.S. economy. The values are taken from Brückner and Pappa (2012). Table 3.2 summarizes the calibration. The new parameter introduced here, the steady-state immigration rate, mig, is set to 0.0057/4, following Ben-Gad (2012). The implies annual (gross) population growth rate is about 1.0057.

It is assumed that the discount factor $\beta = 0.99$ (implying an annualized steady-state real interest rate of approximately 4 percent), the relative risk aversion parameter $\eta = 2$, the capital share $\varphi = 0.3$, the Frisch elasticity of labor supply $\zeta = 4$, the elasticity of substitution $\epsilon = 6$ (implying a gross steady-state markup is equal to 1.2), the degree of price stickiness $\chi^p = 0.75$ (implying an average price duration of four quarters), and the capital depreciation rate $\delta = 0.01$ (implying annual depreciation rate of 4 percent), the capital adjustment cost $\omega = 2$, the coefficient on inflation in the interest rate rule $\zeta_\pi = 1.5$, and the steady-state value for government spending to output ratio $g/y = 0.18$. For these values, conventional values are used.

Total unemployment rate is set to 0.055, and according to CPS data, the share of the long-term unemployed in total unemployment is set to 0.16. We use the aggregate job finding rate $\gamma^h = 0.83$, following Shimer (2010). The aggregate vacancy filling rate γ^f

[7] See Section B.3 in Appendix B for the details of the derivations.

Table 3.1: The Log-Linearized Equations of the Model

Brückner and Pappa (2012) model with immigration

(1) $$\widehat{\widehat{e}}_{t+1} = \left(\frac{1-\sigma}{\zeta^N}\right)\widehat{e}_t + \left(\frac{m^I}{e\,\zeta^N}\right)\widehat{m}^I_t + \left(\frac{m^O}{e\,\zeta^N}\right)\widehat{m}^O_t$$

(2) $$\widehat{e}_t = \widehat{\widehat{e}}_t - \widehat{\zeta}^N_t$$

(3) $$\widehat{m}^I_t = \alpha\widehat{v}_t + (1-\alpha)\widehat{u}^I_t$$

(4) $$\widehat{m}^O_t = \alpha\,\widehat{v}_t + (1-\alpha)\left[\frac{u^O}{u^O+mig}\,\widehat{u}^O_t + \frac{mig}{u^O+mig}\,\widehat{mig}_t\right]$$

(5) $$\widehat{\gamma}^{Ih}_t = \widehat{m}^I_t - \widehat{u}^I_t$$

(6) $$\widehat{\gamma}^{Oh}_t = \widehat{m}^O_t - \frac{u^O}{u^O+mig}\,\widehat{u}^O_t - \frac{mig}{u^O+mig}\,\widehat{mig}_t$$

(7) $$\widehat{\widehat{k}}_{t+1} = \left(\frac{1-\delta}{\zeta^N}\right)\widehat{k}_t + \left(1-\frac{1-\delta}{\zeta^N}\right)\widehat{i}_t$$

(8) $$\widehat{k}_t = \widehat{\widehat{k}}_t - \widehat{\zeta}^N_t$$

(9) $$\widehat{\widehat{u}}^I_{t+1} = \left(\frac{1-\mu}{\zeta^N}\right)\widehat{u}^I_t + \sigma\left(\frac{e}{u^I\zeta^N}\right)\widehat{e}_t - \left(\frac{m^I}{u^I\zeta^N}\right)\widehat{m}^I_t$$

(10) $$\widehat{u}^I_t = \widehat{\widehat{u}}^I_t - \widehat{\zeta}^N_t$$

(11) $$0 = e\widehat{e}_t + u^I\widehat{u}^I_t + u^O\,\widehat{u}^O_t + mig\,\widehat{mig}_t + l\widehat{l}_t$$

(12) $$\frac{\eta}{\beta}\widehat{c}_t + \frac{\omega\zeta^N}{\beta}\widehat{k}_t = \mathbb{E}_t\left[\frac{\eta}{\beta}\widehat{c}_{t+1} - r\widehat{r}_{t+1} - \omega(\zeta^N)^2\widehat{\widehat{k}}_{t+2} + \frac{w\zeta^N}{\beta}\widehat{\widehat{k}}_{t+1} + \omega(\zeta^N)^2\widehat{k}_{t+1}\right]$$

(13) $$\frac{\gamma^{Oh}\lambda_e}{\gamma^{Oh}\lambda_e+c^{-\eta}ben}(\widehat{\gamma}^{Oh}_t + \widehat{\lambda}_{et}) - \frac{c^{-\eta}ben\eta}{\gamma^{Oh}\lambda_e+c^{-\eta}ben}\widehat{c}_t = -\zeta\widehat{l}_t$$

(14) $$\lambda_u\widehat{\lambda}_{ut} = \beta\mathbb{E}_t[-c^{-\eta}ben\eta\widehat{c}_{t+1} + \gamma^{Ih}\lambda_e\widehat{\lambda}_{et+1} + \lambda_u[(1-\mu)-\gamma^{Ih}]\widehat{\lambda}_{ut+1}$$
$$+\gamma^{Ih}[\lambda_e-\lambda_u]\widehat{\gamma}^{Ih}_{t+1} + \Phi l^{-\zeta}\zeta\widehat{l}_{t+1}]$$

(15) $$\lambda_e\widehat{\lambda}_{et} = \beta\mathbb{E}_t[-\eta c^{-\eta}w\widehat{c}_{t+1} + c^{-\eta}w\widehat{w}_{t+1} + (1-\sigma)\lambda_e\widehat{\lambda}_{nt+1} + \sigma\lambda_u\widehat{\lambda}_{ut+1} + \Phi l^{-\zeta}\zeta\widehat{l}_{t+1}]$$

(16) $$\widehat{c}_t = \mathbb{E}_t\widehat{c}_{t+1} - \frac{1}{\eta}(\widehat{R}_t - \mathbb{E}_t\widehat{\pi}_{t+1})$$

(17) $$\widehat{y}_t = \varphi\widehat{k}_t + (1-\varphi)\widehat{e}_t$$

(18) $$\widehat{\gamma}^{If}_t = \widehat{m}^I_t - \widehat{v}_t$$

(19) $$\widehat{\gamma}^{Of}_t = \widehat{m}^O_t - \widehat{v}_t$$

(20) $$\frac{\kappa}{\beta\gamma^f}\widehat{\gamma}^f_t + \frac{\kappa\eta}{\beta\gamma^f}\widehat{c}_t = \frac{\kappa\eta}{\beta\gamma^f}\mathbb{E}_t\widehat{c}_{t+1} + (1-\varphi)x\frac{y}{e}\mathbb{E}_t(\widehat{e}_{t+1} - \widehat{x}_{t+1} - \widehat{y}_{t+1}) + w\mathbb{E}_t\widehat{w}_{t+1}$$
$$+(1-\sigma)\frac{\kappa}{\gamma^f}\mathbb{E}_t\widehat{\gamma}^f_{t+1}$$

(21) $$w\widehat{w}_t = (1-\vartheta)(1-\varphi)x\frac{y}{e}(\widehat{x}_t + \widehat{y}_t - \widehat{e}_t) - \zeta\vartheta c^\eta\Phi l^{-\zeta}\widehat{l}_t - \vartheta c^\eta\sigma\lambda_u\widehat{\lambda}_{ut}$$
$$+\vartheta c^\eta\eta(\Phi l^{-\zeta} - \sigma\lambda_u)\widehat{c}_t$$

(22) $$\widehat{r}_t = \widehat{x}_t + \widehat{y}_t - \widehat{k}_t$$

(23) $$\gamma^f\widehat{\gamma}^f_t = \gamma^{If}\widehat{\gamma}^{If}_t + \gamma^{Of}\widehat{\gamma}^{Of}_t$$

(24) $$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \lambda\widehat{x}_t$$

(25) $$\widehat{R}_t = \zeta_\pi\pi_t$$

(26) $$\widehat{\zeta}^N_{t+1} = (mig/(1-mig))\widehat{mig}_{t+1}$$

(27) $$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \kappa\frac{v}{y}\widehat{v}_t + \frac{g}{y}\widehat{g}_t$$

Exogenous Processes

(a) $$\widehat{mig}_{t+1} = \rho_m\,\widehat{mig}_t + \epsilon^{mig}_{t+1}$$

is set equal to 2/3, and the participation rate is equal to $1 - l = 0.62$. The bargaining power of firms is set to 0.4, based on the estimates by Petrongolo and Pissarides (2001). We use the Hosios (1990) condition to pin down the matching elasticity, so $\alpha = \vartheta$.

The unemployment benefits *ben* and the average cost of hiring a worker κ are chosen to hit the target of 40 percent and 4.5 percent of the average quarterly wage of employed workers following Shimer (2005), and Hagedorn and Manovskii (2008) respectively.

Table 3.2: Parameter Values

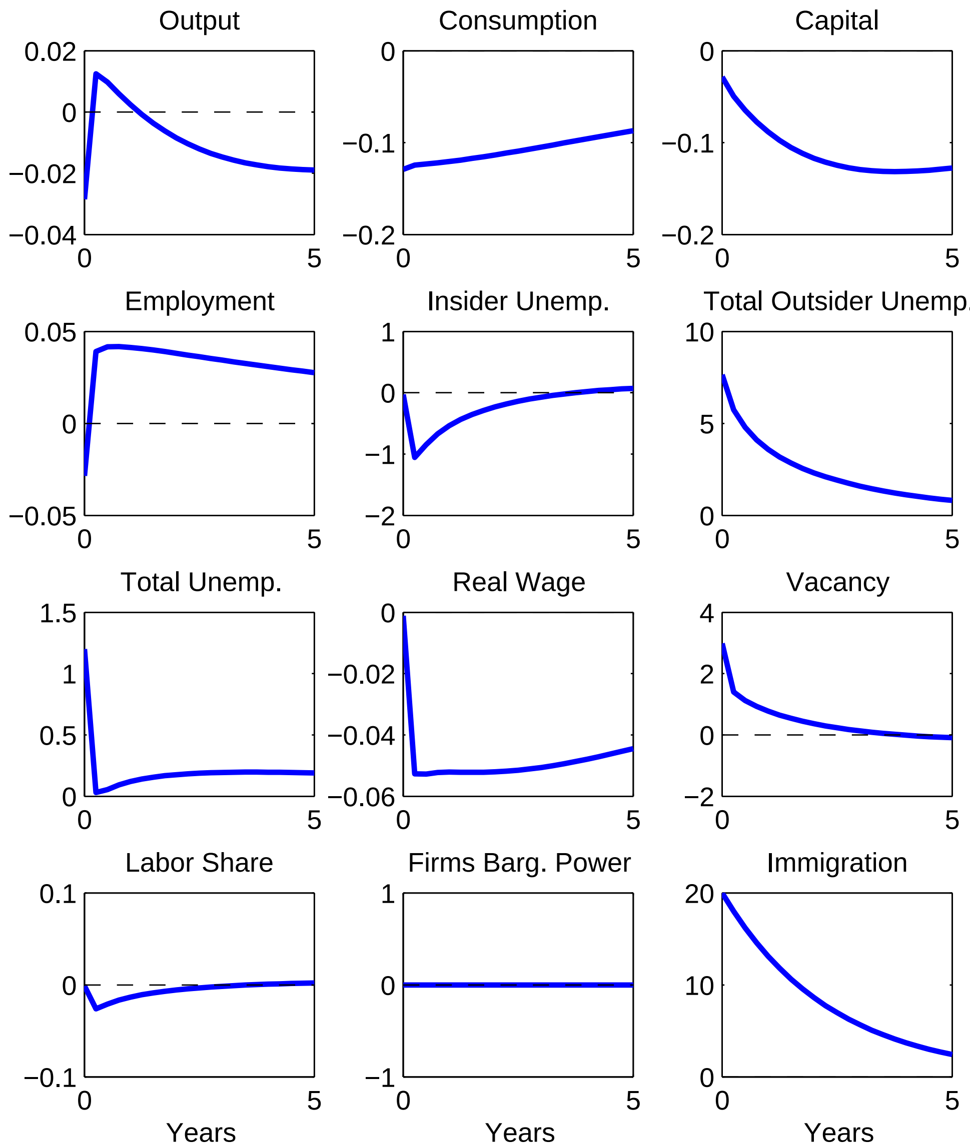
Parameter	Value	Description
m	0.0057/4	steady state immigration rate
$u/(n+u)$	0.055	total unemployment rate
u^O/u	0.16	share of outsiders in total unemployment
γ^h	0.83	aggregate job finding rate
γ^f	2/3	aggregate vacancy filling rate
$1-l$	0.62	participation rate
ϑ	0.4	relative bargaining power
α	0.4	elasticity of matching
ben/w	0.4	replacement rate
k/w	0.045	cost of vacancies as a % real wage
β	0.99	discount factor
φ	0.3	capital share
δ	0.01	capital depreciation rate
ζ	4	elasticity of labor supply
η	2	inverse of IES
ω	2	capital adjustment cost
$x = \epsilon/(\epsilon-1)$	1.2	gross steady state markup
χ^p	0.75	degree of price stickiness
g/y	0.18	gov cons to GDP ratio
ζ_π	1.5	Taylor rule coefficient on inflation

3.3.2 The Effects of Immigration Shocks

Figure 3.4 displays the short-run dynamics of twelve macroeconomic variables (output, consumption, capital, employment, insider unemployment, outsider unemployment, total unemployment, real wage, vacancy, labor share, bargaining power of firms, and immigration) predicted by the benchmark model in response to an immigration shock which corresponds to an increase of 20 percent in ϵ_{t+1}^{mig} in equation (a) in Table 3.1. There is no change in the bargaining power of firms since it is assumed to be constant in the baseline model. The figure shows that an immigration shock generates an instantaneous rise in outsider unemployment as we assume, which leads to a fall in the job finding rate for outsiders (not shown). According to (3.21), real wage is expected to decline reflecting the decline in job finding rate for outsiders, which creates an incentive for firms to post more job vacancies, since the marginal benefit of positing vacancies, and hence, the value of filling a vacancy increase. It is insiders that get the extra jobs, and hence, the unemployment rate of insiders decreases. As a result, both employment and the total unemployment increase. At the same time, an increase in immigration, or equivalently, an increase in population, will cause a reduction in private capital per person (which is called a "capital dilution effect"). Households reduce consumption and save more in order to increase investment and rebuild capital. As a consequence of the decrease in capital dominating the increase in employment, output gradually decreases after a temporary rise. Labor share decreases on the impact and reaches its lowest value about half a year after the shock.

We now turn to see the long-run dynamics in response to the same shock as above. As is emphasized by Uhlig (2010) in the context of effects of government spending, the long-run dynamics can be substantially different from the short-run dynamics. Figure 3.5 shows the effects of the same immigration shock over the 20 years rather than 5 years. In our model, the long-run impulse responses are qualitatively the same as those in the short run. What should be emphasized here is that immigration shocks have more persistent effects on capital, i.e., capital dilution effect is long-lasting, leading to prolonged

Figure 3.4: Dynamic Responses to an Immigration Shock: Benchmark (5 Years)



declines in consumption and output, while labor market variables such as employment and unemployment go back to the steady states as immigration shocks die away 10 years after the shock. In contrast with the results of VARs, labor share return to its steady-state value quickly.

3.3.3 The Effects of Immigration Shocks with Bargaining Power Shock

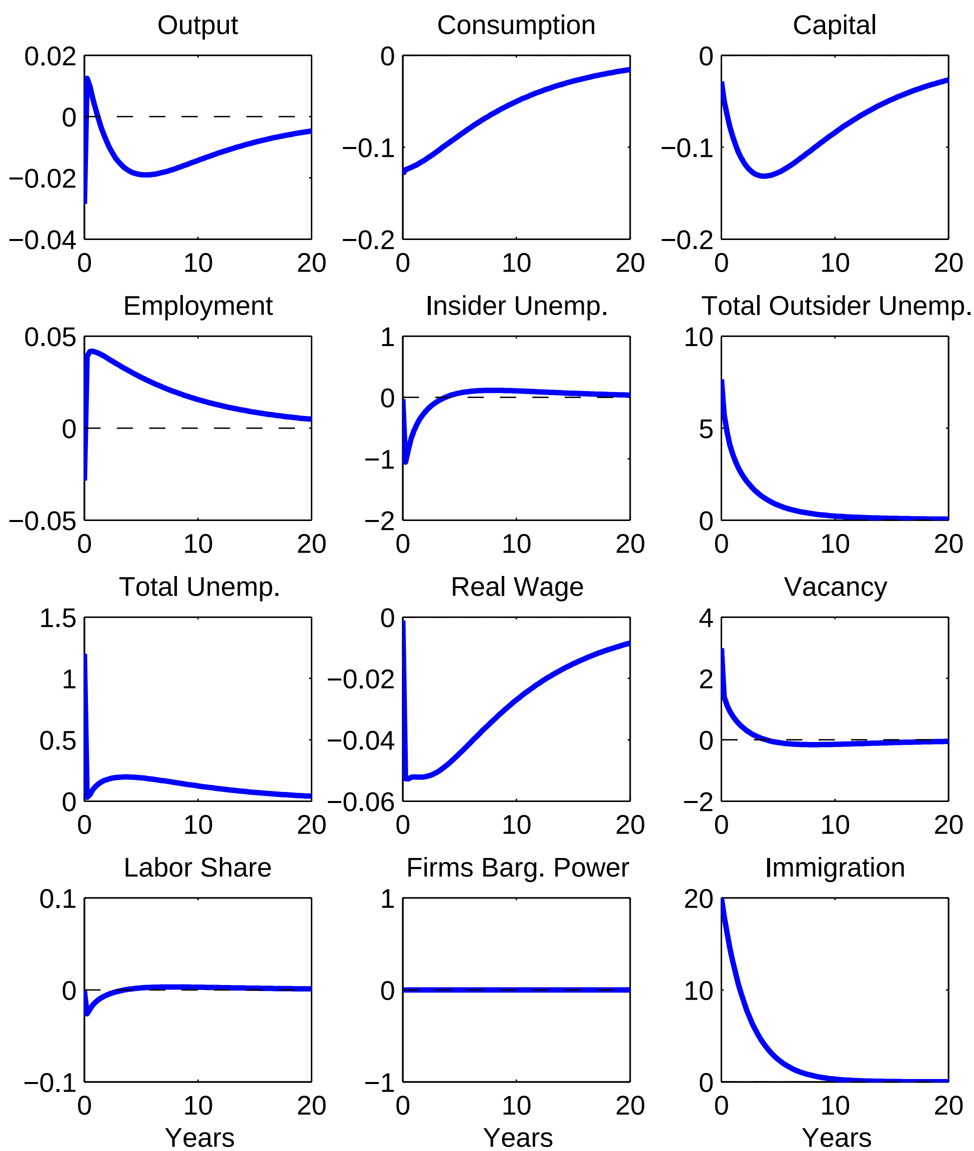
Next, we assume that an increase in immigration will reduce the bargaining power of workers. In reality, the direction of causality between immigration and bargaining power could go both ways. For example, immigration could cause a fall in the bargaining power of workers since immigrant workers are harder to organize into a union. However, the direction of causality could be reversed. This is because a fall in workers' bargaining power could worsen the working conditions of jobs which native workers abandon. Immigrant labor might be increased to fill the vacancies. We do not argue the causality between them and just assume these shocks are correlated with each other. In this setting, the bargaining power for firms, ϑ, is no longer constant, and the log-linearized Nash wage equation is now replaced by:

$$\begin{aligned} w\widehat{w}_t =& (1-\vartheta)(1-\varphi)\frac{y}{e}(\widehat{x}_t + \widehat{y}_t - \widehat{e}_t) - \vartheta\left[(1-\varphi)x\frac{y}{e} - c^{\eta}(\Phi l^{-\zeta} - \sigma\lambda_u)\right]\widehat{\vartheta}_t \\ & - \zeta\vartheta c^{\eta}\Phi l^{-\zeta}\widehat{l}_t - \vartheta c^{\eta}\sigma\lambda_u\widehat{\lambda}_{ut} + \vartheta c^{\eta}\eta(\Phi l^{-\zeta} - \sigma\lambda_u)\widehat{c}_t. \end{aligned} \tag{3.22}$$

A smaller bargaining power of workers, i.e., a larger value of ϑ, means that the workers receive a smaller share of the surplus, and hence a smaller wage.

The green lines in Figure 3.6 plot the responses in the case where a bargaining shock of 1% occurs simultaneously with the same immigration shock as before. For reference, the

Figure 3.5: Dynamic Responses to an Immigration Shock: Benchmark (20 Years)



responses following an immigration shock only (the baseline) are also shown. As is clear from the figure, the decline in the real wage is amplified in the presence of a bargaining shock, and more vacancies are created. As a consequence, insider unemployment declines and employment increases by a larger amount, leading to an increase in output. Note also that changes in bargaining power and immigration generate a larger decline in labor share.

We next study how the degree of persistence of bargaining power shocks affects our results. The darker green lines in Figure 3.7 displays the responses to an AR(2) bargaining shock with an immigration shock. That persistent bargaining shock causes a delayed response for real wage, and hence labor share, which is close to the empirical response.

3.3.4 The Effects of an Immigration Shock with Rigid Wages

The literature emphasizes the need for the wage rigidity to explain the cyclical behavior of unemployment and vacancies in the search and matching model. For example, Hall (2005) showed how wage stickiness allows labor market frictions model to explain unemployment variability. Uhlig (2007) showed how real wage rigidity can allow macro models to explain asset market behavior. Blanchard and Galí (2007) have shown how real wage rigidity can allow the New Keynesian model to account for inflation and unemployment dynamics. The wage rigidity may also play an important role for explaining the recent decline in labor share, coupled with immigration shock. Here, we introduce a simple wage rigidity rule to the benchmark model following Shimer (2010). The wage is expressed as a weighted average of the wage in the previous period and Nash bargained wage in this period:

$$w_t = \chi^w w_{t-1} + (1 - \chi^w) w_t^{Nash} \tag{3.23}$$

Figure 3.6: Dynamic Responses to an Immigration Shock with a Bargaining Shock

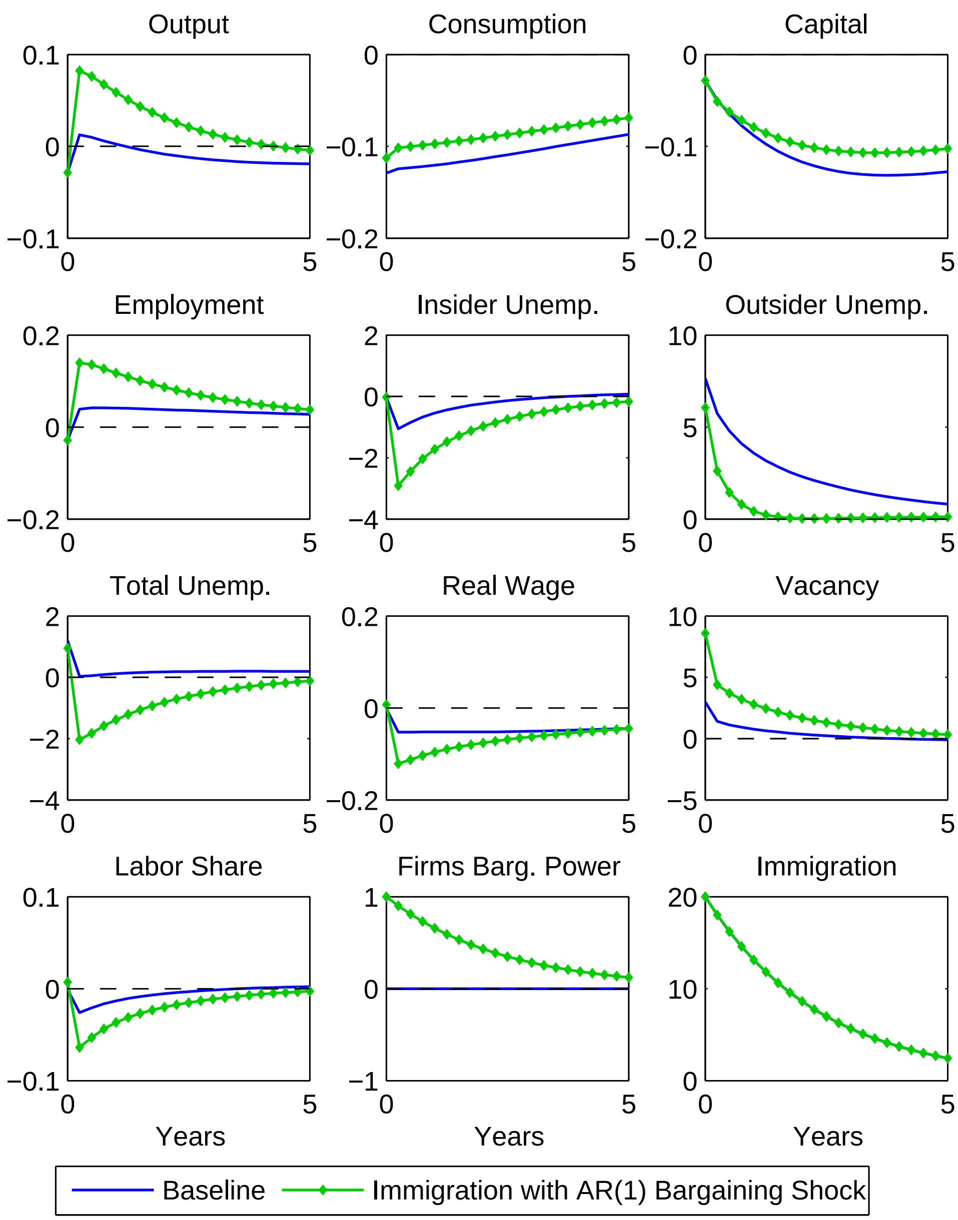
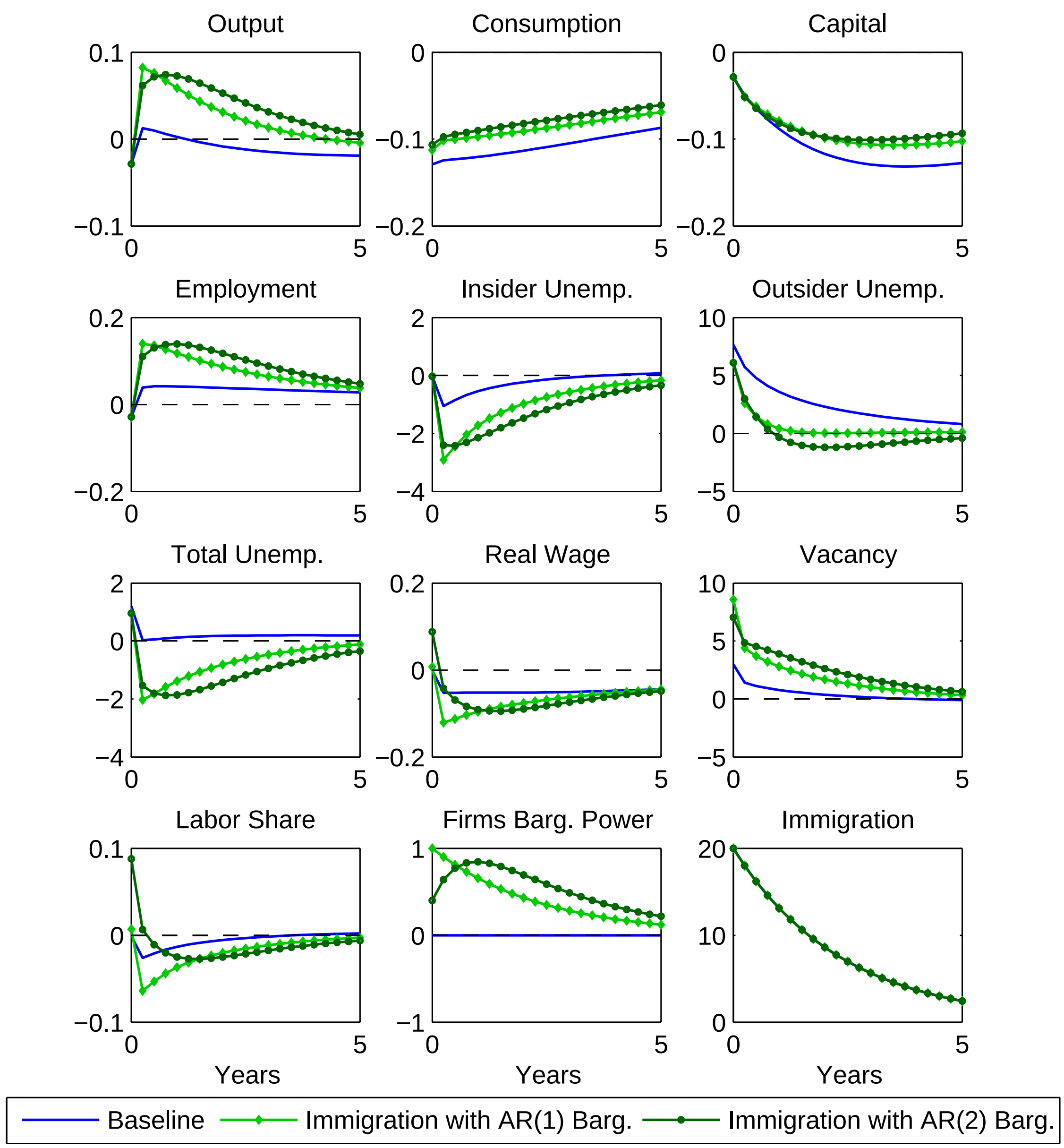


Figure 3.7: Dynamic Responses to Immigration Shocks with an AR(2) Bargaining Shock



where $\chi^w \in [0, 1]$ denotes the degree of wage stickiness and w_t^{Nash} is a Nash bargained wage given by (3.21). When $\chi^w = 0$, wages are flexible, and therefore, the model corresponds to the benchmark one.

Figure 3.8 plots the impulse responses following a shock to immigration in the presence of the wage rigidity for alternative parameter for wage rigidity, $\chi^w = 0$ (baseline), 0.5, or 0.9. No bargaining shock occurred here. Figure 3.9 shows the responses to an immigration and a bargaining power shock with wage rigidity.

According to Figure 3.8, the presence of wage rigidity leads to more gradual declines in real wages, and in turn, less increases in job vacancies. Consequently, unemployment of insiders is higher, and employment and output are lower than those in the baseline model.

As shown in Figure 3.9, the difference in responses of real wages in the case with the mild wage rigidity ($\chi^w = 0.5$) and an AR(2) bargaining shock, and the case without them lies in the impact effects. After that they show similar responses.

3.4 Extension

In this section, we extend the baseline model in two ways by changing the assumption of treating immigrants as outsiders. First, we assume that immigrants enter a host economy with jobs. Next, we assume that immigrants enter as insiders.

Figure 3.8: Dynamic Responses to an Immigration Shock with Rigid Wages

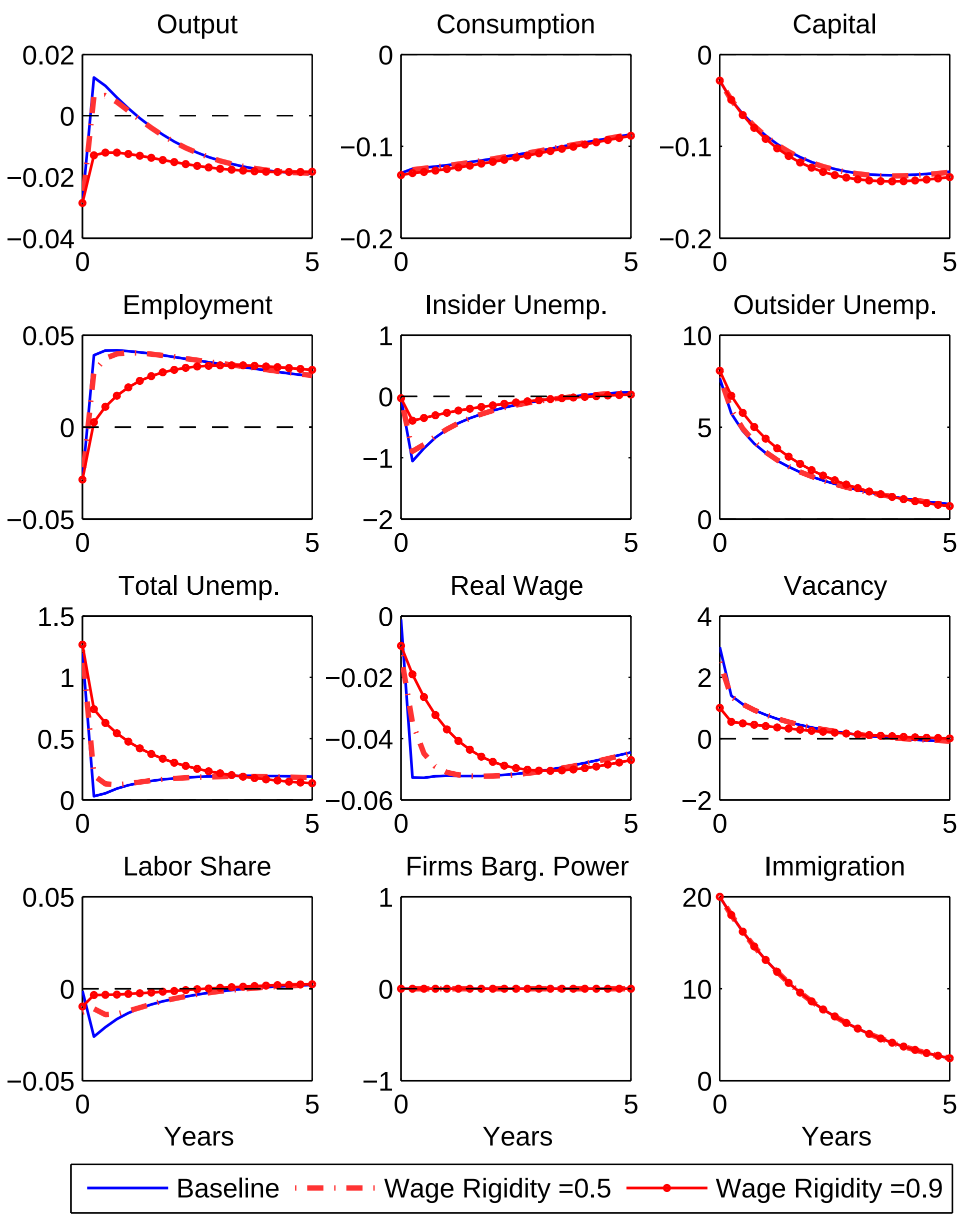
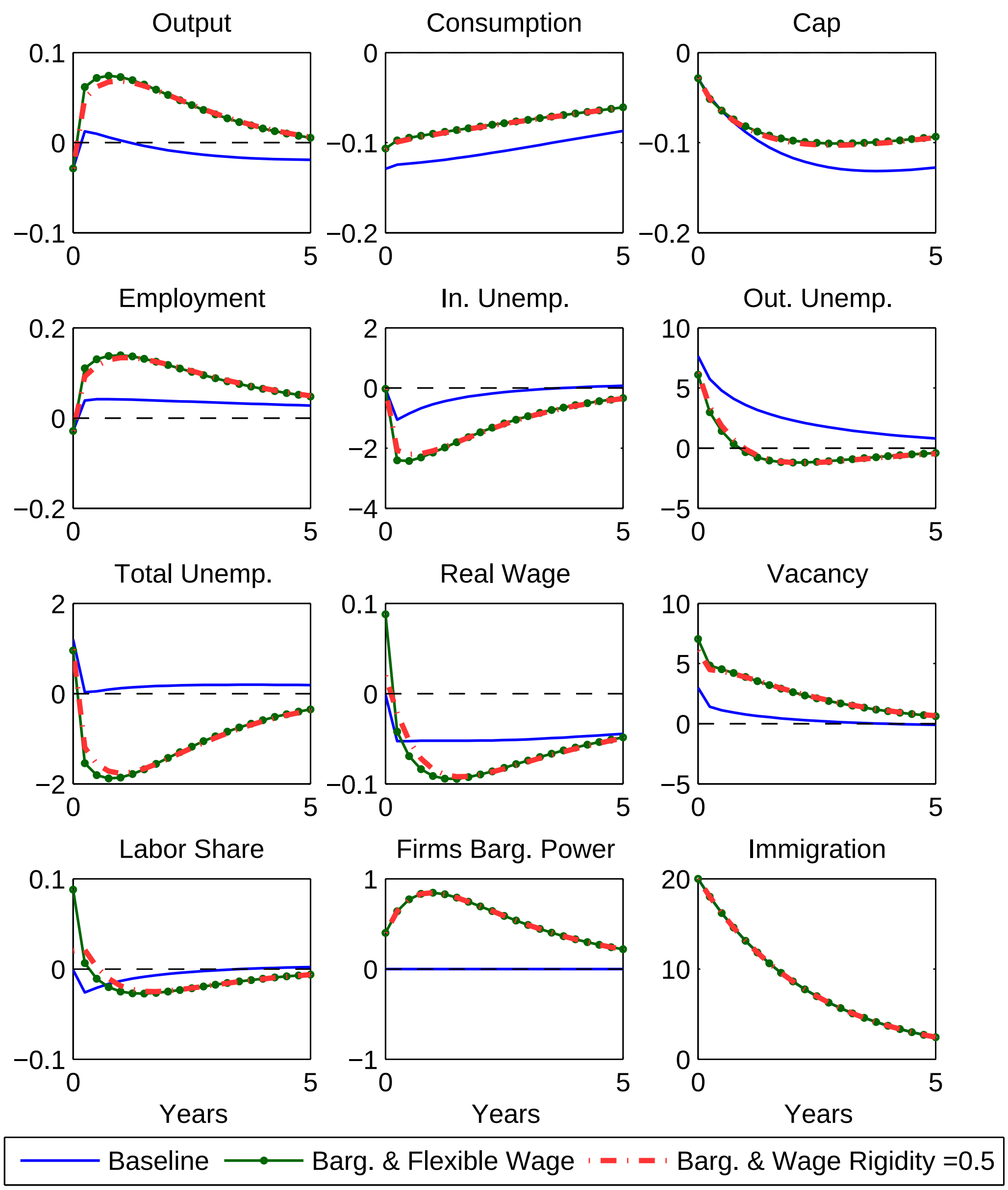


Figure 3.9: Dynamic Responses to Immigration Shock with Rigid Wages and Bargaining Shock



3.4.1 Immigration with Jobs

If we consider an increase in immigration as an exogenous shock to the number of employed workers, then the transition of equation for employed workers is rewritten as

$$\begin{aligned} E_{t+1} &= (1-\sigma)E_t + (1-\mu_1)Mig_t + M_t^I + M_t^O \\ \Leftrightarrow \widetilde{e}_{t+1} &= (1-\sigma)e_t + (1-\mu_1)mig_t + m_t^I + m_t^O \end{aligned} \tag{3.24}$$

where $\mu_1 \in (0,1)$ is the exogenous job separation rate. We assume that immigrants become outsiders when their jobs are terminated. The first-order conditions for household's problem are unaffected to this change. The Cobb-Douglas production function is now given by

$$Y_t = K_t^{\varphi}(E_t + Mig_t)^{1-\varphi}. \tag{3.25}$$

Thus, the marginal product of labor now becomes;

$$MPL_t := \frac{\partial Y_t}{\partial E_t} = (1-\varphi)\frac{Y_t}{E_t + Mig_t} = (1-\varphi)\frac{y_t}{e_t + mig_t} \tag{3.26}$$

Note that an increase in immigration leads to a fall in the marginal product of labor if other things are equal. The resulting optimal vacancy posting condition is

$$\frac{\kappa}{\gamma_t^f} = \beta \mathbb{E}_t \left(\frac{c_t}{c_{t+1}}\right)^{\eta} \left[(1-\varphi)\frac{y_{t+1}}{e_{t+1} + mig_{t+1}} - w_{t+1} + (1-\sigma)\frac{\kappa}{\gamma_{t+1}^f}\right], \tag{3.27}$$

and Nash bargained wage is

$$w_t = (1-\vartheta)\left[(1-\varphi)x_t\frac{y_t}{e_t + mig_t} + \frac{\kappa\gamma_t^{Oh}}{\gamma_t^f}\right] + \vartheta ben - \vartheta c_t^\eta \sigma\beta\mathbb{E}_t\mathbb{V}_t^{UI}. \tag{3.28}$$

An increase in employed immigration has two counteracting effects on job vacancies. On one hand, it creates more incentives for firms to post vacancies by reducing the marginal product of labor, and hence, wage. On the other hand, the decrease in the marginal product of labor caused by employed immigration creates less incentive to open vacancies by reducing the marginal benefit of posting them.

The corresponding log-linearized equations are now given by

$$\begin{aligned}
\widehat{\widehat{e}}_{t+1} =& \left(\frac{1-\sigma}{\zeta^N}\right)\widehat{e}_t + \left(\frac{mig}{e\,\zeta^N}\right)\widehat{mig}_t + \left(\frac{m^I}{e\,\zeta^N}\right)\widehat{m}_t^I + \left(\frac{m^O}{e\,\zeta^N}\right)\widehat{m}_t^O,\\
\widehat{y}_t =& \varphi\widehat{k}_t + (1-\varphi)\left[\frac{e}{e+mig}\;\widehat{e}_t + \frac{mig}{e+mig}\;\widehat{mig}_t\right],\\
\frac{\kappa}{\beta\gamma^f}\widehat{\gamma}_t^f + \frac{\kappa\eta}{\beta\gamma^f}\widehat{c}_t =& \frac{\kappa\eta}{\beta\gamma^f}\mathbb{E}_t\widehat{c}_{t+1} + (1-\varphi)\frac{y}{e+mig}\mathbb{E}_t\left[\left(\frac{e}{e+mig}\;\widehat{e}_t + \frac{mig}{e+mig}\;\widehat{mig}_t\right) - \widehat{x}_{t+1} - \widehat{y}_{t+1}\right]\\
&+ w\mathbb{E}_t\widehat{w}_{t+1} + (1-\sigma)\frac{\kappa}{\gamma^f}\mathbb{E}_t\widehat{\gamma}_{t+1}^f,\\
w\widehat{w}_t =& (1-\vartheta)(1-\varphi)x\frac{y}{e+mig}\left[\widehat{x}_t + \widehat{y}_t - \left(\frac{e}{e+mig}\;\widehat{e}_t + \frac{mig}{e+mig}\;\widehat{mig}_t\right)\right)\\
&- \zeta\vartheta c^\eta\Phi l^{-\zeta}\widehat{l}_t - \vartheta c^\eta\sigma\lambda_u\widehat{\lambda}_{ut} + \vartheta c^\eta\eta(\Phi l^{-\zeta} - \sigma\lambda_u)\widehat{c}_t.
\end{aligned}$$

Figure 3.10 displays impulse responses to an immigration shock of the same magnitude as the baseline, but now immigrants are assumed to enter as employed workers. This employed immigration shock leads to a fall in real wages, which is similar to the prediction of the baseline model of outsider-immigrants. However, the model with employed immigrants predicts a decrease in job vacancies, while the model with outsider-immigrants predicts the opposite. In the baseline model, as explained in Subsection 3.3.2, the main reason why an immigration shock increases job vacancies is that it causes the job finding rate for outsiders to fall, which leads to negative pressure on real wage and a positive pressure on vacancies. In the model with employed immigration, the effect of a reduction in the marginal benefit of posting a vacancy, due to a fall in the marginal product of labor, dominates the effect of an increase in the marginal benefit due to a fall in real wages, and as a result of that, job vacancies decrease. This reduction of vacancies makes it harder for insiders to find jobs, and hence, unemployment of insiders increases. Unemployment of outsiders also increases on the impact. This is because non-participants reduces leisure and enter the labor market in order to increase consumption. However, it becomes slightly below the steady state by reflecting the fact they leave the labor market and become non-participants. As a consequence, the model with employed immigration generates slightly larger impacts on total employment and output. The responses of consumption and capital are similar between the two models. Labor share decreases more on the impact.

3.4.2 Immigration as Insiders

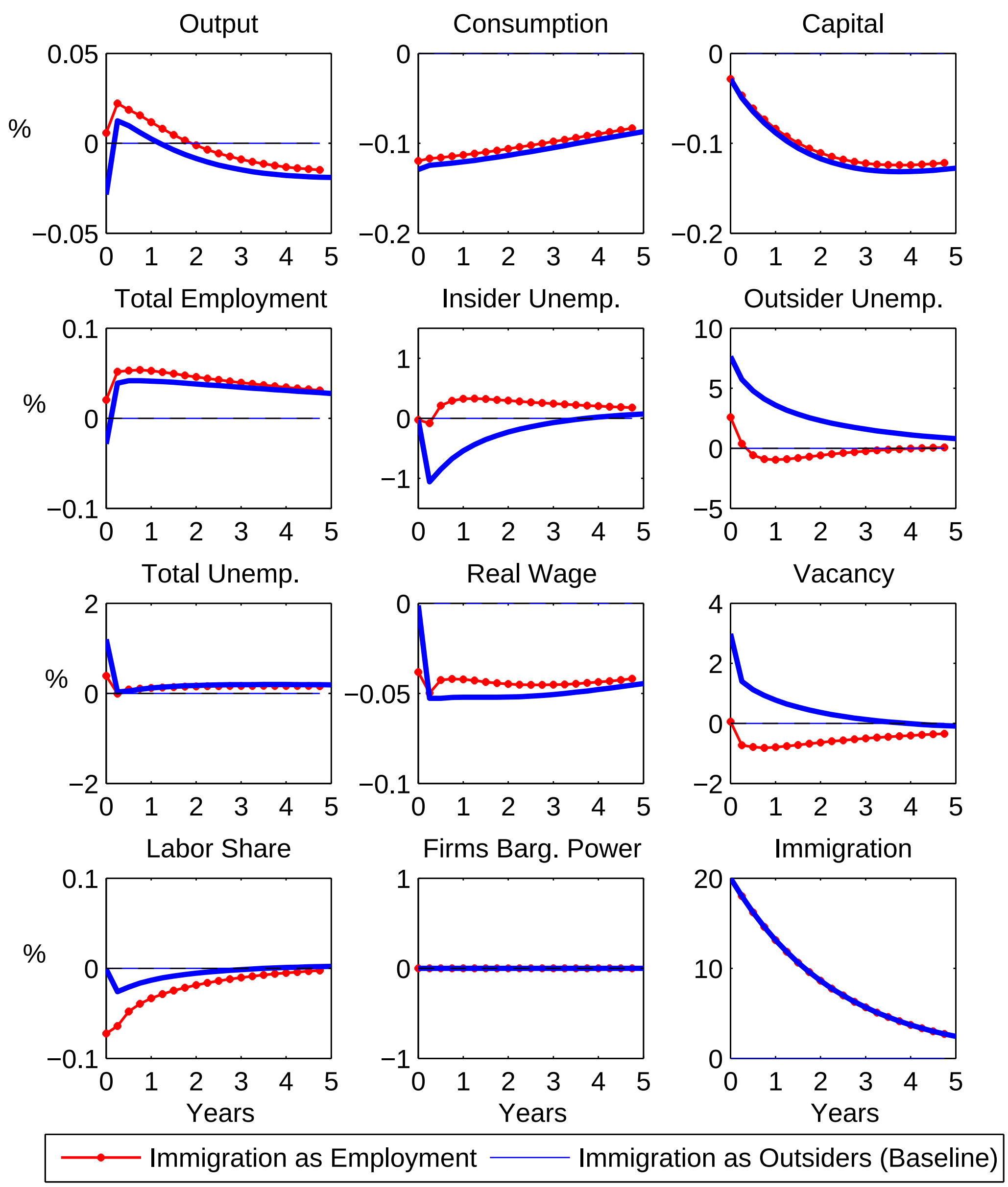
Next, we turn to the case where immigrants enter a host economy as insiders. In this case, the matching function for insiders is now given by

$$M_t^I = \rho_m^I V_t^\alpha [(u_t^I + mig_t)N_t]^{1-\alpha}, \tag{3.29}$$

and hence, the job finding rate for insiders is

$$\gamma^{Ih} = \frac{M_t^I}{(u_t^I + mig_t)N_t} = \frac{m_t^I}{u_t^I + mig_t}. \tag{3.30}$$

Figure 3.10: Immigration with Employment



The law of motion for insiders is replaced by

$$U^I_{t+1} = (1-\mu)U^I_t + \sigma E_t - M^I_t + (1-\mu_2)Mig_t$$
$$\Leftrightarrow \widetilde{u}^I_{t+1} = (1-\mu)u^I_t + \sigma e_t - \gamma^{Ih}_t u^I_t + (1-\mu_2)mig_t \tag{3.31}$$

where $\mu_2 \in (0,1)$ is the probability that immigrants become outsiders.

As a result, the corresponding log-linearized equations are now replaced by

$$\widehat{m}^I_t = \alpha\, \widehat{v}_t + (1-\alpha)\left[\frac{u^I}{u^I+mig}\,\widehat{u}^I_t + \frac{mig}{u^I+mig}\,\widehat{mig}_t\right],$$
$$\widehat{\gamma}^{Ih}_t = \widehat{m}^I_t - \frac{u^I}{u^I+mig}\,\widehat{u}^I_t - \frac{mig}{u^I+mig}\,\widehat{mig}_t,$$
$$\widehat{\widetilde{u}}^I_{t+1} = \left(\frac{1-\mu}{\zeta^N}\right)\widehat{u}^I_t + \sigma\left(\frac{e}{u^I\zeta^N}\right)\widehat{e}_t - \left(\frac{m^I}{u^I\zeta^N}\right)\widehat{m}^I_t + (1-\mu_2)\left(\frac{mig}{u^I\zeta^N}\right)\widehat{mig}_t.$$

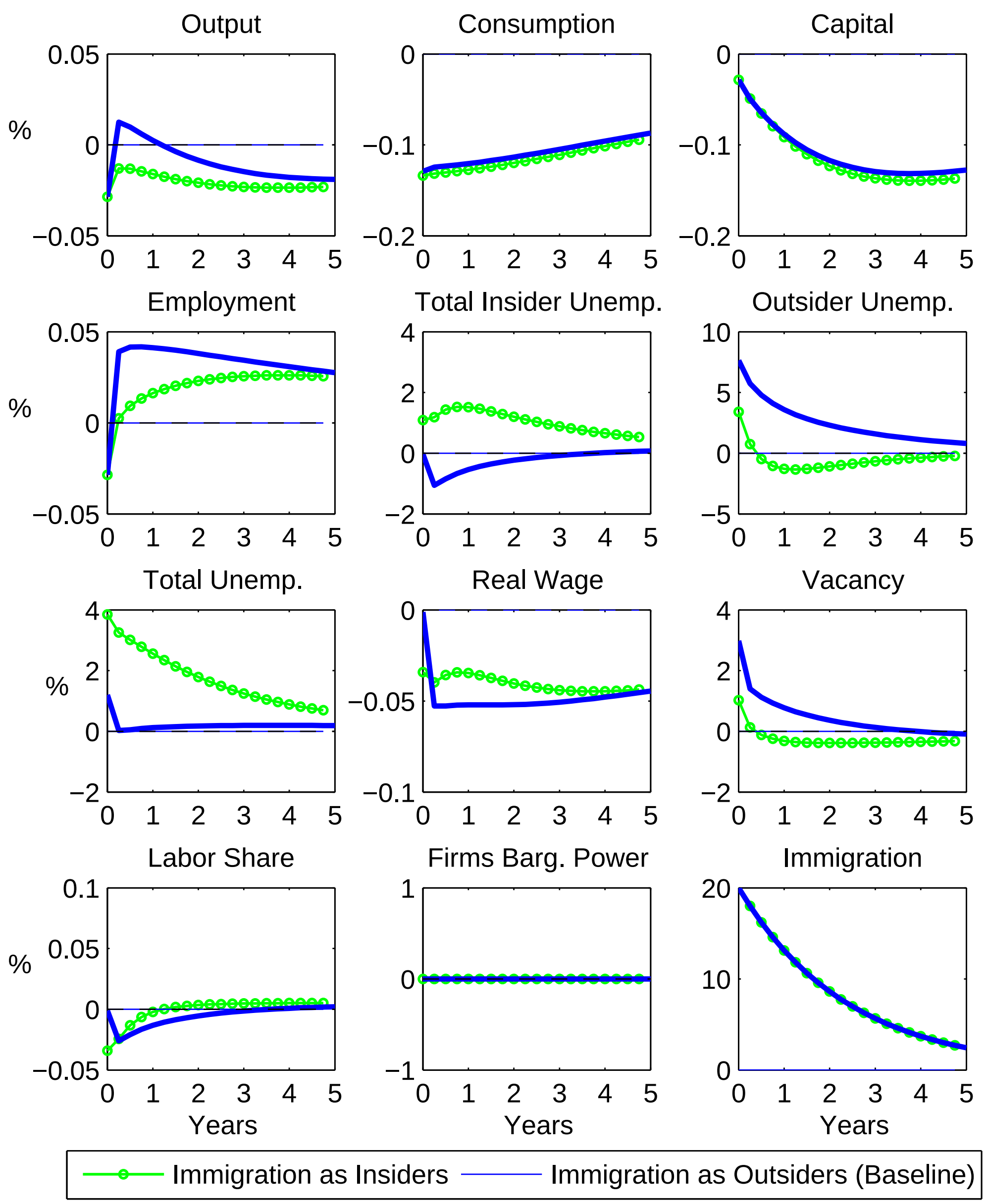
Figure 3.11 shows the dynamic responses to an immigration shock with the assumption that immigrants enter a host economy as insiders. The magnitude of the shock is the same as the baseline model. For comparison, the responses of the baseline model are also displayed. An increase in immigration generates an increase in unemployment of insiders as we have assumed. On impact, real wages fall due to a fall in the marginal value of being an insider, leading to an instantaneous rise in vacancies. After that, however, vacancies show a gradual decrease and become slightly below the steady state level since the marginal cost of a vacancy increases gradually due to a fall in the vacancy filling rate (not shown). Unemployment of outsiders also declines gradually just after an initial rise, reflecting the fact that the job finding rate for outsiders declines since some of them leave the labor market. Less increase in vacancies leads to less increase in employment, and hence, less output. Again, the responses of consumption and capital to the shock are unaffected by changing the assumption on how immigrants enter an economy. Labor share decreases on impact and soon goes back to the steady state.

3.5 Conclusion

This paper has analyzed the macroeconomic effects of immigration on labor market dynamics using a New Keynesian model with labor market search and heterogeneous unemployed workers. The structure of the model enables us to study the different hypotheses on how immigrations enter a host country. We find that, when they enter as outsiders and reduce the bargaining power of workers, the labor share of national income shows a hump-shaped decline, which is in line with empirical evidence produced by a VAR analysis. This suggests the importance of the role of the worker's bargaining power in investigating the dynamic macroeconomic impacts of immigration, to which no role is given in the standard New Keynesian model.

We also find that the reduction in wages caused by an outsider-immigration creates incentive for firms to post more vacancies, so the unemployment of insiders declines, i.e., an outsider-immigration shock is bad for outsiders and beneficial to insiders in terms of unemployment. When immigrants enter as employed workers, it generates a fall in the marginal product of labor, which has a negative effect on wage and vacancies, and hence, workers lose out in terms of wages and insiders lose out in terms of unemployment. Unemployment of outsiders falls gradually since they leave the labor market and enjoy leisure. Lastly, when immigrants enter as insiders, they are bad for insiders in terms of unemployment and workers in terms of wages, while they are good for outsiders in terms of unemployment. To sum up, immigration adversely effects, or directly competes with, the sector immigrants enter, but benefits other sector(s).

Figure 3.11: Immigration as Insiders



There are some significant issues for future research. One issue to be pursued in future work is to take into account the difference in wages between matched insiders and outsiders. In the model we have used above, the wages paid to matched insiders are the same as the wages paid to matched outsiders. Considering the equilibrium wage gap between inside and outside workers makes our model more realistic and may have important implications for the design of immigration policies. Another issue to be considered is to incorporate a constant elasticity of the substitution (CES) production function where migrant labor is a substitute for capital but a complement to domestic labor, which allows for an analysis of the impacts of immigration when capital-skill complementarity is present. These would be fruitful for future research.

Appendix B

Appendices to Chapter 3

B.1 Log-linearizing around the Steady State

This appendix presents the details of how we derived the first-order conditions. We then gives the details of the derivations of log-linearized equations of the model as well as the steady state conditions in terms of per-capita variables.

B.2 Derivations of the First-Order Conditions

B.2.1 Household's Problem

The household's problem is expressed as

$$\mathcal{J}(k_t, e_t, u_t^I, b_t) = \max_{c_t, k_{t+1}, b_{t+1}, e_{t+1}, u_{t+1}^I, u_t^O} \left(\frac{c_t^{1-\eta}}{1-\eta} + \Phi \frac{l_t^{1-\zeta}}{1-\zeta} \right) N_t$$
$$+ \beta \mathbb{E}_t \mathcal{J}(k_{t+1}, e_{t+1}, u_{t+1}^I, b_{t+1})$$

subject to

$$
\begin{aligned}
c_tN_t + i_tN_t + \frac{b_{t+1}N_{t+1}}{p_tR_t} &\leq r_tk_tN_t + w_te_tN_t + ben(u_t^I + u_t^O + mig_t)N_t + \frac{b_tN_t}{p_t} + pro_tN_t - t_tN_t, \\
k_{t+1}N_{t+1} &= (1-\delta)k_tN_t + i_tN_t - \frac{\omega}{2}\left(\frac{k_{t+1}N_{t+1}}{k_tN_t} - \zeta^N\right)^2 k_tN_t, \\
1 &= e_t + u_t^I + u_t^O + mig_t + l_t \\
e_{t+1}N_{t+1} &= (1-\sigma)e_tN_t + M_t^I + M_t^O, \\
u_{t+1}^I N_{t+1} &= (1-\mu)u_t^I N_t + \sigma e_tN_t - M_t^I, \\
M_t^I &= \gamma_t^{Ih} u_t^I N_t, \\
M_t^O &= \gamma_t^{Oh}(u_t^O + mig_t)N_t.
\end{aligned}
$$

The household's problem can be rewritten as

$$
\begin{aligned}
\mathcal{J}(k_t, e_t, u_t^I, b_t) = \max_{c_t, k_{t+1}, b_{t+1}, e_{t+1}, u_{t+1}^I, u_t^O} & \left(\frac{c_t^{1-\eta}}{1-\eta} + \Phi\frac{(1 - e_t - u_t^I - u_t^O - mig_t)^{1-\zeta}}{1-\zeta}\right) N_t \\
& + \beta\mathbb{E}_t\mathcal{J}(k_{t+1}, e_{t+1}, u_{t+1}^I, b_{t+1}) \\
& - \lambda_{ct}\Big(c_tN_t + k_{t+1}N_{t+1} - (1-\delta)k_tN_t + \frac{\omega}{2}\left(\frac{k_{t+1}N_{t+1}}{k_tN_t} - \zeta^N\right)^2 k_tN_t + \frac{b_{t+1}N_{t+1}}{p_tR_t} \\
& - r_tk_tN_t - w_te_tN_t - ben(u_t^I + u_t^O + mig_t)N_t - \frac{b_tN_t}{p_t} - pro_tN_t + t_tN_t\Big) \\
& + \lambda_{et}((1-\sigma)e_tN_t + \gamma_t^{Ih}u_t^IN_t + \gamma_t^{Oh}(u_t^O + mig_t)N_t - e_{t+1}N_{t+1}) \\
& + \lambda_{ut}((1-\mu)u_t^IN_t + \sigma e_tN_t - \gamma_t^{Ih}u_t^IN_t - u_{t+1}^IN_{t+1}).
\end{aligned}
\tag{B.1}
$$

The first-order conditions are:

$$
\begin{aligned}
[c_t] &: c_t^{-\eta} N_t - \lambda_{ct} N_t = 0 \\
[k_{t+1}] &: -\lambda_{ct} N_{t+1} \left[1 + \omega \left(\frac{k_{t+1} N_{t+1}}{k_t N_t} - \zeta^N \right) \right] + \beta \mathbb{E}_t \mathcal{J}_{k,t+1} = 0 \\
[b_{t+1}] &: -\lambda_{ct} \frac{N_{t+1}}{p_t R_t} + \beta \mathbb{E}_t \mathcal{J}_{b,t+1} = 0 \\
[e_{t+1}] &: \beta \mathbb{E}_t \mathcal{J}_{e,t+1} - \lambda_{et} N_{t+1} = 0 \qquad \text{(B.2)} \\
[u^I_{t+1}] &: \beta \mathbb{E}_t \mathcal{J}_{u^I,t+1} - \lambda_{ut} N_{t+1} = 0 \qquad \text{(B.3)} \\
[u^O_t] &: -\Phi l_t^{-\zeta} N_t + \lambda_{ct} ben N_t + \lambda_{nt} \gamma_t^{Oh} N_t = 0.
\end{aligned}
$$

Next we derive the first derivatives of $\mathcal{J}$ defined above. Differentiating (B.1) with respect to k_t, b_t, e_t, and u^I_t:

$$
\begin{aligned}
\mathcal{J}_{k,t} &= \lambda_{ct} \left((1-\delta) + r_t + \left[\omega \left(\frac{k_{t+1} N_{t+1}}{k_t N_t} - \zeta^N \right) \left(\frac{k_{t+1} N_{t+1}}{k_t N_t} \right) - \frac{\omega}{2} \left(\frac{k_{t+1} N_{t+1}}{k_t N_t} - \zeta^N \right)^2 \right] \right) N_t \\
&= \lambda_{ct} \left((1-\delta) + r_t + \frac{\omega}{2} \left[\left(\frac{k_{t+1} N_{t+1}}{k_t N_t} \right)^2 - (\zeta^N)^2 \right] \right) N_t \\
\mathcal{J}_{b,t} &= \lambda_{ct} \frac{N_t}{p_t} \\
\mathcal{J}_{e,t} &= (-\Phi l_t^{-\zeta} + \lambda_{ct} w_t + (1-\sigma)\lambda_{et} + \sigma \lambda_{ut}) N_t \\
\mathcal{J}_{u^I,t} &= (-\Phi l_t^{-\zeta} + \lambda_{ct} ben + \gamma_t^{Ih} \lambda_{et} + ((1-\mu) - \gamma_t^{Ih}) \lambda_{ut}) N_t
\end{aligned}
$$

Forwarding these expressions one period:

$$
\begin{aligned}
\mathcal{J}_{k,t+1} &= \lambda_{ct+1}\left((1-\delta)+r_{t+1}+\frac{\omega}{2}\left[\left(\frac{k_{t+2}N_{t+2}}{k_{t+1}N_{t+1}}\right)^2-(\zeta^N)^2\right]\right)N_{t+1} \\
\mathcal{J}_{b,t+1} &= \lambda_{ct+1}\frac{N_{t+1}}{p_{t+1}} \\
\mathcal{J}_{e,t+1} &= (-\Phi l_{t+1}^{-\zeta}+\lambda_{ct+1}w_{t+1}+(1-\sigma)\lambda_{et+1}+\sigma\lambda_{ut+1})N_{t+1} \\
\mathcal{J}_{u^I,t+1} &= (-\Phi l_{t+1}^{-\zeta}+\lambda_{ct+1}ben+\gamma_{t+1}^{Ih}\lambda_{et+1}+((1-\mu)-\gamma_{t+1}^{Ih})\lambda_{ut+1})N_{t+1}.
\end{aligned}
$$

Thus, substituting these expressions into the first-order conditions, we get:

$$
\begin{aligned}
[c_t] &: c_t^{-\eta} = \lambda_{ct} \\
[k_{t+1}] &: \lambda_{ct}\left(1+\omega\left[\frac{k_{t+1}N_{t+1}}{k_tN_t}-\zeta^N\right]\right) = \beta\mathbb{E}_t\lambda_{ct+1}\left((1-\delta)+r_{t+1}+\frac{\omega}{2}\left[\left(\frac{k_{t+2}N_{t+2}}{k_{t+1}N_{t+1}}\right)^2-(\zeta^N)^2\right]\right) \\
[b_{t+1}] &: \lambda_{ct}\frac{N_{t+1}}{p_tR_t} = \beta\mathbb{E}_t\lambda_{ct+1}\frac{N_{t+1}}{p_{t+1}} \Leftrightarrow 1 = \beta\mathbb{E}_t\left(\frac{\lambda_{ct+1}}{\lambda_{ct}}\right)\left(\frac{p_t}{p_{t+1}}\right)R_t \\
[e_{t+1}] &: \lambda_{et} = \beta\mathbb{E}_t[\lambda_{ct+1}w_{t+1}+(1-\sigma)\lambda_{et+1}+\sigma\lambda_{ut+1}-\Phi l_{t+1}^{-\zeta}] \\
[u_{t+1}^I] &: \lambda_{ut} = \beta\mathbb{E}_t[\lambda_{ct+1}ben+\gamma_{t+1}^{Ih}\ +((1-\mu)-\gamma_{t+1}^{Ih})\lambda_{ut+1}-\Phi l_{t+1}^{-\zeta}] \\
[u_t^O] &: \lambda_{et} = \frac{\Phi l_t^{-\zeta}-\lambda_{ct}ben}{\gamma_t^{Oh}}.
\end{aligned}
\tag{B.4}
$$

B.2.2 Value Definitions

We define the marginal value to the household of having one member employed as follows:

$$
\mathbb{V}_t^E := \mathcal{J}_{e,t} = (-\Phi l_t^{-\zeta}+\lambda_{ct}w_t+(1-\sigma)\lambda_{et}+\sigma\lambda_{ut})N_t \tag{B.5}
$$

$$
\mathbb{V}_t^{UI} := \mathcal{J}_{u^I,t} = (-\Phi l_t^{-\zeta}+\lambda_{ct}ben+\gamma_t^{Ih}\lambda_{et}+((1-\mu)-\gamma_t^{Ih})\lambda_{ut})N_t \tag{B.6}
$$

By defining next-period value of employment and insider-unemployment as

$$\mathbb{V}^E_{t+1} := \mathcal{J}_{e,t+1}$$

$$\mathbb{V}^{UI}_{t+1} := \mathcal{J}_{u^I,t+1}$$

and using the first-order conditions for e_{t+1} (B.2) , u^O_t (B.4), and u^I_{t+1} (B.3), we have

$$\begin{aligned} \lambda_{et} N_{t+1} =& \beta \mathbb{E}_t \mathbb{V}^E_{t+1} \\ \Leftrightarrow \lambda_{et} =& \beta \mathbb{E}_t \frac{\mathbb{V}^E_{t+1}}{N_{t+1}} = \frac{\Phi l_t^{-\zeta} - \lambda_{ct} ben}{\gamma^O_t} \end{aligned} \tag{B.7}$$

and

$$\lambda_{ut} N_{t+1} = \beta \mathbb{E}_t \mathbb{V}^{UI}_{t+1} \tag{B.8}$$

Thus, eliminating λ_{et} and λ_{ut} from (B.5) and (B.6),

$$\mathbb{V}^E_t = \left[-\Phi l_t^{-\zeta} + c_t^{-\eta} w_t + (1-\sigma)\beta \mathbb{E}_t \frac{\mathbb{V}^E_{t+1}}{N_{t+1}} + \sigma \beta \mathbb{E}_t \frac{\mathbb{V}^{UI}_{t+1}}{N_{t+1}} \right] N_t \tag{B.9}$$

$$\mathbb{V}^{UI}_t = \left[-\Phi l_t^{-\zeta} + c_t^{-\eta} ben + \gamma^{Ih}_t \beta \mathbb{E}_t \frac{\mathbb{V}^E_{t+1}}{N_{t+1}} + ((1-\mu) - \gamma^{Ih}_t)\beta \mathbb{E}_t \frac{\mathbb{V}^{UI}_{t+1}}{N_{t+1}} \right] N_t. \tag{B.10}$$

B.2.3 Firm's Problem

The value function of a firm with E_t currently employed workers is:

$$\mathcal{V}(E_t) = \max_{K_t, V_t} \; x_t F(K_t, E_t) - w_t E_t - r_t K_t - \kappa V_t + \mathbb{E}_t \Lambda_{t+1} \mathcal{V}((1-\sigma)E_t + \gamma^f_t V_t). \tag{B.11}$$

The first-order conditions are:

$$
\begin{aligned}
[K_t] &: x_t F_{Kt} - r_t = 0 \Leftrightarrow \varphi x_t \frac{Y_t}{K_t} = r_t \\
[V_t] &: -\kappa + \mathbb{E}_t \Lambda_{t+1} \mathcal{V}_{E,t+1} \gamma_t^f = 0 \\
&\Leftrightarrow \frac{\kappa}{\gamma_t^f} = \mathbb{E}_t \Lambda_{t+1} \mathcal{V}_{E,t+1}. \qquad \text{(B.12)}
\end{aligned}
$$

Next, we derive the first derivative of $\mathcal{V}$ with respect to E_{t+1}, i.e., $\mathcal{V}_{E,t+1}$. Substituting the optimal value of vacancy, which is denoted by $V_t^*(E_t)$, into (B.11) gives

$$
\mathcal{V}(E_t) = x_t F(K_t, E_t) - w_t E_t - r_t K_t - \kappa V_t^*(E_t) + \mathbb{E}_t \Lambda_{t+1} \mathcal{V}((1-\sigma)E_t + \gamma_t^f V_t^*(E_t))
$$

Differentiate with respect to E_t gives

$$
\begin{aligned}
\mathcal{V}_{E,t} &= x_t F_{Et} - w_t - \kappa V_t^{*'}(E_t) + \underbrace{\mathbb{E}_t \Lambda_{t+1} \mathcal{V}_{E,t+1}}_{=\kappa/\gamma_t^f \text{ from FOC}} ((1-\sigma) + \gamma_t^f V_t^{*'}(E_t)) \\
&= x_t F_{Et} - w_t - \kappa V_t^{*'}(E_t) + (1-\sigma)\frac{\kappa}{\gamma_t^f} + \kappa V_t^{*'}(E_t) \\
&= x_t F_{Et} - w_t + (1-\sigma)\frac{\kappa}{\gamma_t^f} \qquad \text{(B.13)}
\end{aligned}
$$

Forwarding the last expression one period

$$
\mathcal{V}_{E,t+1} = x_{t+1} F_{E,t+1} - w_{t+1} + (1-\sigma)\frac{\kappa}{\gamma_{t+1}^f} \qquad \text{(B.14)}
$$

By substituting (B.14) into (B.12), the first-order condition for v_t is now given by

$$
\frac{\kappa}{\gamma_t^f} = \beta \mathbb{E}_t \left(\frac{c_t}{c_{t+1}}\right)^\eta \left[x_{t+1} F_{Et+1} - w_{t+1} + (1-\sigma)\frac{\kappa}{\gamma_{t+1}^f}\right],
$$

where we use $\Lambda_{t+1} = \beta u_{ct+1}/u_{ct} = \beta(c_t/c_{t+1})^\eta$.

B.2.4 Value Definitions

From (B.13), the marginal value to the firm of filling a vacancy is

$$\mathbb{V}_t^F := \underbrace{\frac{\partial \mathcal{V}(E_t)}{\partial E_t}}_{=\mathcal{V}_{E,t}} \underbrace{\frac{\partial E_t}{\partial e_t}}_{=N_t} = \left[(1-\varphi)x_t\frac{Y_t}{E_t} - w_t + (1-\sigma)\frac{\kappa}{\gamma_t^f}\right]N_t, \tag{B.15}$$

where we use $E_t = e_t N_t$ and $F_{Et} = (1-\varphi)x_t\frac{Y_t}{E_t}$. Thus, the first-order condition for vacancy, (B.12), is now given by

$$\frac{\kappa}{\gamma_t^f} = \beta\mathbb{E}_t\left(\frac{c_t}{c_{t+1}}\right)^\eta \frac{\mathbb{V}_{t+1}^F}{N_{t+1}}. \tag{B.16}$$

B.2.5 Bargaining over Wages

The Nash bargaining problem is to maximize the weighted sum of log surpluses:

$$\max_{w_t} \;\; (1-\vartheta)\ln\mathbb{V}_t^E + \vartheta\ln\mathbb{V}_t^F$$

where $\mathbb{V}_t^E$ and $\mathbb{V}_t^F$ are defined in (B.9) and (B.15) respectively. The first-order condition with respect to w_t is

$$\begin{aligned} 0 =& (1-\vartheta)\frac{c_t^{-\eta}N_t}{\mathbb{V}_t^E} + \vartheta\frac{-N_t}{\mathbb{V}_t^F} \\ \Leftrightarrow \vartheta\mathbb{V}_t^E =& (1-\vartheta)c_t^{-\eta}\mathbb{V}_t^F \qquad \text{(B.17)} \end{aligned}$$

Forwarding (B.17) one period and taking expectations at period t gives

$$\vartheta \mathbb{E}_t \mathbb{V}^E_{t+1} = (1-\vartheta)\mathbb{E}_t c_{t+1}^{-\eta} \mathbb{V}^F_{t+1}$$

Multiplying both sides by $\beta c_t^\eta / N_{t+1}$

$$\begin{aligned}
\vartheta \beta c_t^\eta \mathbb{E}_t \frac{\mathbb{V}^E_{t+1}}{N_{t+1}} =& (1-\vartheta) \underbrace{\beta \mathbb{E}_t \left(\frac{c_t}{c_{t+1}}\right)^\eta \frac{\mathbb{V}^F_{t+1}}{N_{t+1}}}_{=\kappa/\gamma_t^f \text{ by } (B.16)} \\
\vartheta c_t^\eta \underbrace{\beta \mathbb{E}_t \frac{\mathbb{V}^E_{t+1}}{N_{t+1}}}_{(B.7)} =& (1-\vartheta)\frac{\kappa}{\gamma_t^f} && \text{(B.18)} \\
\vartheta c_t^\eta \left(\frac{\Phi l_t^{-\zeta} - c_t^{-\eta} ben}{\gamma_t^{Oh}}\right) =& (1-\vartheta)\frac{\kappa}{\gamma_t^f} \\
\vartheta c_t^\eta \Phi l_t^{-\zeta} =& \vartheta ben + (1-\vartheta)\frac{\kappa \gamma_t^{Oh}}{\gamma_t^f}. && \text{(B.19)}
\end{aligned}$$

Multiplying the definition of the marginal value of employment, (B.9), by $\vartheta_t c_t^\eta$;

$$\begin{aligned}
\vartheta c_t^\eta \mathbb{V}^E_t &= \vartheta c_t^\eta \left[c_t^{-\eta} w_t - \Phi l_t^{-\zeta} + +(1-\sigma)\beta \mathbb{E}_t \frac{\mathbb{V}^E_{t+1}}{N_{t+1}} + \sigma \beta \mathbb{E}_t \frac{\mathbb{V}^{UI}_{t+1}}{N_{t+1}} \right] N_t \\
&= \left[\vartheta w_t - \underbrace{\vartheta c_t^\eta \Phi l_t^{-\zeta}}_{(B.19)} + (1-\sigma) \underbrace{\vartheta c_t^\eta \beta \mathbb{E}_t \frac{\mathbb{V}^E_{t+1}}{N_{t+1}}}_{(B.18)} + \vartheta c_t^\eta \sigma \beta \mathbb{E}_t \frac{\mathbb{V}^{UI}_{t+1}}{N_{t+1}} \right] N_t \\
&= \left[\vartheta w_t - \left[\vartheta ben + (1-\vartheta)\frac{\kappa \gamma_t^{Oh}}{\gamma_t^f} \right] + (1-\sigma)(1-\vartheta)\frac{\kappa}{\gamma_t^f} + \vartheta c_t^\eta \sigma \beta \mathbb{E}_t \frac{\mathbb{V}^{UI}_{t+1}}{N_{t+1}} \right] N_t \\
&= \left[\vartheta w_t - \vartheta_t ben + (1-\vartheta_t)\frac{\kappa}{\gamma_t^f}\left(-\gamma_t^{Oh} + 1 - \sigma\right) + \vartheta_t c_t^\eta \sigma \beta \mathbb{E}_t \frac{\mathbb{V}^{UI}_{t+1}}{N_{t+1}} \right] N_t. && \text{(B.20)}
\end{aligned}$$

Therefore, by rewiring (B.17) using (B.20) and (B.15), we can derive the following optimal wage:

$$
\begin{aligned}
&\vartheta c_t^{\eta}\mathbb{V}_t^{E} = (1-\vartheta)\mathbb{V}_t^{F} \\
\Leftrightarrow &\left[\vartheta w_t - \vartheta ben + (1-\vartheta)\frac{\kappa}{\gamma_t^f}\left(-\gamma_t^{Oh} + 1 - \sigma\right) + \vartheta c_t^{\eta}\sigma\beta\mathbb{E}_t\frac{\mathbb{V}_{t+1}^{UI}}{N_{t+1}}\right]N_t \\
&= (1-\vartheta)\left[(1-\varphi)\frac{Y_t}{E_t} - w_t + (1-\sigma)\frac{\kappa}{\gamma_t^f}\right]N_t \\
\Leftrightarrow w_t &= (1-\vartheta)\left[(1-\varphi)\frac{y_t}{e_t} + [\gamma_t^{Oh} - (1-\sigma) + (1-\sigma)]\frac{\kappa}{\gamma_t^f}\right] + \vartheta ben N_t - \vartheta c_t^{\eta}\sigma\beta\mathbb{E}_t\frac{\mathbb{V}_{t+1}^{UI}}{N_{t+1}} \\
\Leftrightarrow w_t &= (1-\vartheta)\left[(1-\varphi)\frac{y_t}{e_t} + \frac{\kappa\gamma_t^{Oh}}{\gamma_t^f}\right] + \vartheta ben - \vartheta c_t^{\eta}\sigma\beta\mathbb{E}_t\frac{\mathbb{V}_{t+1}^{UI}}{N_{t+1}}
\end{aligned}
$$

B.3 Derivations of Log-linearized Equations

We log-linearize the equations around the steady state. Hats over variables denote log-deviations from the steady-state values. The per-capita variables are defined so that $c_t = C_t/N_t$. For notational ease, we also define three variables;

$$\widetilde{k}_t = \frac{K_t}{N_{t-1}},\ \widetilde{e}_t = \frac{E_t}{N_{t-1}},\ \widetilde{u}_t^I = \frac{U_t^I}{N_{t-1}}.$$

Thus, $\widetilde{k}_t$, $\widetilde{e}_t$ and $\widetilde{u}_t^I$ are not adjusted for the increase in population. Note that

$$\widetilde{k}_t = \frac{K_t}{N_t}\frac{N_t}{N_{t-1}} = k_t\zeta_t^N$$

or

$$\widetilde{k}_{t+1}N_t = \frac{K_{t+1}}{N_t}N_t = \frac{K_{t+1}}{N_{t+1}}N_{t+1} = k_{t+1}N_{t+1}$$

B.3.1 Law of Motion for Employment

Dividing $\widetilde{e}_{t+1}N_t = (1-\sigma)e_tN_t + M_t^I + M_t^O$ by N_t gives:

$$\begin{aligned}
\widetilde{e}_{t+1} &= (1-\sigma)e_t + m_t^I + m_t^O \\
\widetilde{e}\ (1+\widehat{\widetilde{e}}_{t+1}) &= (1-\sigma)e\ (1+\widehat{e}_t) + m^I(1+\widehat{m}_t^I) + m^O(1+\widehat{m}_t^O) \\
\widetilde{e}\ \widehat{\widetilde{e}}_{t+1} &= (1-\sigma)e\ \widehat{e}_t + m^I\widehat{m}_t^I + m^O\widehat{m}_t^O \\
\widehat{\widetilde{e}}_{t+1} &= (1-\sigma)\left(\frac{e}{\widetilde{e}}\right)\ \widehat{e}_t + \left(\frac{m^I}{\widetilde{e}}\right)\widehat{m}_t^I + \left(\frac{m^O}{\widetilde{e}}\right)\widehat{m}_t^O \\
&= \left(\frac{1-\sigma}{\zeta^N}\right)\widehat{e}_t + \left(\frac{m^I}{e\ \zeta^N}\right)\widehat{m}_t^I + \left(\frac{m^O}{e\ \zeta^N}\right)\widehat{m}_t^O
\end{aligned}$$

since

$$\frac{e}{\widetilde{e}} = \frac{E/N}{E/N_{-1}} = \frac{N_{-1}}{N} = \frac{1}{\zeta^N}$$

and

$$\frac{m^I}{\widetilde{e}} = \frac{M^I/N}{E/N_{-1}} = \frac{M^I}{E}\frac{N_{-1}}{N} = \frac{m^I}{e}\frac{1}{\zeta^N}.$$

At the steady state:

$$\widetilde{e} = (1-\sigma)e + m^I + m^O$$

B.3.2 Adjusted Employment per Person

$$e_t = \frac{E_t}{N_t} = \frac{E_t}{N_{t-1}}\frac{N_{t-1}}{N_t} = \frac{\widetilde{e}_t}{\zeta_t^N} \Leftrightarrow \widehat{e}_t = \widehat{\widetilde{e}}_t - \widehat{\zeta}_t^N$$

B.3.3 Matching Function for Insiders

$$M_t^I = \rho_m^I V_t^\alpha (u_t^I N_t)^{1-\alpha} \Leftrightarrow m_t^I = \rho_m^I v_t^\alpha (u_t^I)^{1-\alpha} \Leftrightarrow \widehat{m}_t^I = \alpha \widehat{v}_t + (1-\alpha)\widehat{u}_t^I$$

B.3.4 Matching Function for Outsiders

$$\begin{aligned}
M_t^O &= \rho_m^O V_t^\alpha ((u_t^O + mig_t)N_t)^{1-\alpha} \\
\Leftrightarrow m_t^O &= \rho_m^O v_t^\alpha (u_t^O + mig_t)^{1-\alpha} \\
\Leftrightarrow \widehat{m}_t^O &= \alpha\ \widehat{v}_t + (1-\alpha)\left[\frac{u^O}{u^O + mig}\ \widehat{u}_t^O + \frac{mig}{u^O + mig}\ \widehat{mig}_t\right]
\end{aligned}$$

B.3.5 Job Finding Rate for Insiders

$$\gamma_t^{Ih} = \frac{M_t^I}{u_t^I N_t} = \frac{m_t^I}{u_t^I} \Leftrightarrow \widehat{\gamma}_t^{Ih} = \widehat{m}_t^I - \widehat{u}_t^I$$

B.3.6 Job Finding Rate for Outsiders

$$\begin{aligned}
\gamma_t^{Oh} =& \frac{M_t^O}{(u_t^O + mig_t)N_t} = \frac{m_t^O}{u_t^O + mig_t} \\
\Leftrightarrow \widehat{\gamma}_t^{Oh} =& \widehat{m}_t^O - \frac{u^O}{u^O + mig}\ \widehat{u}_t^O - \frac{mig}{u^O + mig}\ \widehat{mig}_t
\end{aligned}$$

B.3.7 Private Capital Accumulation

$$\widetilde{k}_{t+1} = (1-\delta)k_t + i_t - \frac{\omega}{2}\left(\frac{\widetilde{k}_{t+1}}{k_t} - \zeta^N\right)^2 k_t$$

At the steady state:

$$k\zeta^N = (1-\delta)k + i \Leftrightarrow \frac{i}{k\zeta^N} = 1 - \frac{1-\delta}{\zeta^N}$$

Log-linearizing gives

$$\widehat{\widetilde{k}}_{t+1} = \left(\frac{1-\delta}{\zeta^N}\right)\widehat{k}_t + \left(1 - \frac{1-\delta}{\zeta^N}\right)\widehat{i}_t$$

since the adjustment cost is equal to zero when log-linearizing.

B.3.8 Adjusted Capital per Person

$$k_t = \frac{K_t}{N_t} = \frac{K_t}{N_{t-1}}\frac{N_{t-1}}{N_t} = \frac{\widetilde{k}_t}{\zeta_t^N}$$
$$\Leftrightarrow \widehat{k}_t = \widehat{\widetilde{k}}_t - \widehat{\zeta}_t^N$$

B.3.9 Law of Motion for Insider Unemployment

$$\widetilde{u}_{t+1}^I = (1-\mu)\ u_t^I + \sigma\ e_t - m_t^I$$

At the steady state:

$$\widetilde{u}^I = (1-\mu)\ u^I + \sigma\ e - \gamma^{Ih}\ u^I$$
$$\Leftrightarrow \widetilde{u}^I - (1-\mu)\ u^I = \sigma\ e - \gamma^{Ih}\ u^I$$
$$\Leftrightarrow (\zeta^N - 1 + \mu)\ \widetilde{u}^I = \sigma\ e - \gamma^{Ih}\ u^I$$

Log-linearizing

$$
\begin{aligned}
\widetilde{u}^I(1+\widehat{\widetilde{u}}^I_{t+1}) &= (1-\mu)\ u^I(1+\widehat{u}^I_t) + \sigma e\ (1+\widehat{e}_t) - m^I(1+\widehat{m}^I_t) \\
\widehat{\widetilde{u}}^I_{t+1} &= (1-\mu)\left(\frac{u^I}{\widetilde{u}^I}\right)\widehat{u}^I_t + \sigma\left(\frac{e}{\widetilde{u}^I}\right)\widehat{e}_t - \left(\frac{m^I}{\widetilde{u}^I}\right)\widehat{m}^I_t \\
&= \left(\frac{1-\mu}{\zeta^N}\right)\widehat{u}^I_t + \sigma\left(\frac{e}{u^I\zeta^N}\right)\widehat{e}_t - \left(\frac{m^I}{u^I\zeta^N}\right)\widehat{m}^I_t
\end{aligned}
$$

where we use the fact that $u^I/\widetilde{u}^I = 1/\zeta^N$

B.3.10 Adjusted Insider Unemployment per Person

$$
u^I_t = \frac{U^I_t}{N_t} = \frac{U^I_t}{N_{t-1}}\frac{N_{t-1}}{N_t} = \frac{\widetilde{u}^I_t}{\zeta^N_t} \Leftrightarrow \widehat{u}^I_t = \widehat{\widetilde{u}}^I_t - \widehat{\zeta}^N_t
$$

B.3.11 Total Population

$$
1 = e_t + u^I_t + u^O_t + mig_t + l_t
$$

$$
\Leftrightarrow 0 = e\widehat{e}_t + u^I\widehat{u}^I_t + u^O\ \widehat{u}^O_t + mig\ \widehat{mig}_t + l\widehat{l}_t
$$

B.3.12 FOC for Capital Holding

$$
\begin{aligned}
c_t^{-\eta}\left(1+\omega\left[\frac{k_{t+1}N_{t+1}}{k_tN_t} - \zeta^N\right]\right) &= \beta\mathbb{E}_t c_{t+1}^{-\eta}\left(1-\delta+r_{t+1}+\frac{\omega}{2}\left[\left(\frac{k_{t+2}N_{t+2}}{k_{t+1}N_{t+1}}\right)^2 - (\zeta^N)^2\right]\right) \\
\Leftrightarrow c_t^{-\eta}\left(1+\omega\left[\frac{\widetilde{k}_{t+1}}{k_t} - \zeta^N\right]\right) &= \beta\mathbb{E}_t c_{t+1}^{-\eta}\left(1-\delta+r_{t+1}+\frac{\omega}{2}\left[\left(\frac{\widetilde{k}_{t+2}}{k_{t+1}}\right)^2 - (\zeta^N)^2\right]\right)
\end{aligned}
$$

At the steady state

$$c^{-\eta} = \beta c^{-\eta}(1-\delta+r) \Leftrightarrow \frac{1}{\beta} = 1-\delta+r$$

Log-linearizing

$$\begin{aligned}
\text{LHS} =& c^{-\eta} e^{-\eta\widehat{c}_t}\left(1+\omega\left[\zeta^N \exp(\widehat{\widehat{k}}_{t+1}-\widehat{k}_t)-\zeta^N\right]\right)\\
=& c^{-\eta}(1-\eta\widehat{c}_t)\left(1+\omega\zeta^N\left[\widehat{\widehat{k}}_{t+1}-\widehat{k}_t\right]\right)\\
=& c^{-\eta}\left(1+\omega\zeta^N\left[\widehat{\widehat{k}}_{t+1}-\widehat{k}_t\right]-\eta\widehat{c}_t\right)
\end{aligned}$$

$$\begin{aligned}
\text{RHS} =& \beta\mathbb{E}_t c^{-\eta}\exp(-\eta\widehat{c}_{t+1})\left(1-\delta+re^{\widehat{r}_{t+1}}+\frac{\omega}{2}(\zeta^N)^2\left[\exp(2\widehat{\widehat{k}}_{t+2}-2\widehat{k}_{t+1})-1\right]\right)\\
=& \beta c^{-\eta}E_t(1-\eta\widehat{c}_{t+1})\left(\underbrace{(1-\delta+r)}_{=1/\beta}+r\widehat{r}_{t+1}+\frac{\omega}{2}(\zeta^N)^2[2\widehat{\widehat{k}}_{t+2}-2\widehat{k}_{t+1}]\right)\\
=& \beta c^{-\eta}\mathbb{E}_t\left[\frac{1}{\beta}+r\widehat{r}_{t+1}+\omega(\zeta^N)^2(\widehat{\widehat{k}}_{t+2}-\widehat{k}_{t+1})-\frac{\eta}{\beta}\widehat{c}_{t+1}\right]\\
=& c^{-\eta}\mathbb{E}_t\left[1+\beta r\widehat{r}_{t+1}+\beta\omega(\zeta^N)^2(\widehat{\widehat{k}}_{t+2}-\widehat{k}_{t+1})-\eta\widehat{c}_{t+1}\right]
\end{aligned}$$

Combining gives

$$\begin{aligned}
c^{-\eta}(1+\omega\zeta^N[\widehat{\widehat{k}}_{t+1}-\widehat{k}_t]-\eta\widehat{c}_t) &= c^{-\eta}\mathbb{E}_t\left[1+\beta r\widehat{r}_{t+1}+\beta\omega(\zeta^N)^2(\widehat{\widehat{k}}_{t+2}-\widehat{k}_{t+1})-\eta\widehat{c}_{t+1}\right]\\
\omega\zeta^N(\widehat{\widehat{k}}_{t+1}-\widehat{k}_t)-\eta\widehat{c}_t &= \mathbb{E}_t\left[\beta r\widehat{r}_{t+1}+\beta\omega(\zeta^N)^2(\widehat{\widehat{k}}_{t+2}-\widehat{k}_{t+1})-\eta\widehat{c}_{t+1}\right]\\
-\eta\widehat{c}_t-\omega\zeta^N\widehat{k}_t &= \mathbb{E}_t[-\eta\widehat{c}_{t+1}+\beta r\widehat{r}_{t+1}+\beta\omega(\zeta^N)^2\widehat{\widehat{k}}_{t+2}-w\zeta^N\widehat{\widehat{k}}_{t+1}-\beta\omega(\zeta^N)^2\widehat{k}_{t+1}]\\
\frac{\eta}{\beta}\widehat{c}_t+\frac{\omega\zeta^N}{\beta}\widehat{k}_t &= \mathbb{E}_t\left[\frac{\eta}{\beta}\widehat{c}_{t+1}-r\widehat{r}_{t+1}-\omega(\zeta^N)^2\widehat{\widehat{k}}_{t+2}+\frac{w\zeta^N}{\beta}\widehat{\widehat{k}}_{t+1}+\omega(\zeta^N)^2\widehat{k}_{t+1}\right]
\end{aligned}$$

B.3.13 FOC for Outsider Unemployment

$$
\begin{aligned}
\lambda_{et} &= \frac{\Phi l_t^{-\zeta} - c_t^{-\eta} ben}{\gamma_t^{Oh}} \\
\Leftrightarrow \gamma^{Oh}\lambda_e(1 + \widehat{\gamma}_t^{Oh} + \widehat{\lambda}_{et}) &= \Phi l^{-\zeta}(1 - \zeta \widehat{l}_t) - c^{-\eta} ben(1 - \eta \widehat{c}_t) \\
\Leftrightarrow \gamma^{Oh}\lambda_e(\widehat{\gamma}_t^{Oh} + \widehat{\lambda}_{et}) &= -\Phi l^{-\zeta}\zeta \widehat{l}_t + c^{-\eta} ben \eta \widehat{c}_t
\end{aligned}
$$

or using the steady-state relationship $\Phi l^{-\zeta} = \gamma^{Oh}\lambda_e + c^{-\eta} ben$

$$
\frac{\gamma^{Oh}\lambda_e}{\gamma^{Oh}\lambda_e + c^{-\eta} ben}(\widehat{\gamma}_t^{Oh} + \widehat{\lambda}_{et}) - \frac{c^{-\eta} ben \eta}{\gamma^{Oh}\lambda_e + c^{-\eta} ben}\widehat{c}_t = -\zeta \widehat{l}_t
$$

B.3.14 Value of Unemployment

$$
\lambda_{ut} = \beta \mathbb{E}_t[c_{t+1}^{-\eta} ben + \gamma_{t+1}^{Ih}\lambda_{nt+1} + ((1-\mu) - \gamma_{t+1}^{Ih})\lambda_{ut+1} - \Phi l_{t+1}^{-\zeta}]
$$

At the steady state

$$
\lambda_u = \beta[c^{-\eta} ben + \gamma^{Ih}\lambda_e + ((1-\mu) - \gamma^{Ih})\lambda_u - \Phi l^{-\zeta}]
$$

Log-linearizing

$$
\begin{aligned}
\lambda_u(1 + \widehat{\lambda}_{ut}) = \beta \mathbb{E}_t[& c^{-\eta} ben(1 - \eta \widehat{c}_{t+1}) + \gamma^{Ih}\lambda_e(1 + \widehat{\gamma}_{t+1}^{Ih} + \widehat{\lambda}_{et+1}) \\
&+ (1-\mu)\lambda_u(1 + \widehat{\lambda}_{ut+1}) - \gamma^{Ih}\lambda_u(1 + \widehat{\gamma}_{t+1}^{Ih} + \widehat{\lambda}_{ut+1}) - \Phi l^{-\zeta}(1 - \zeta \widehat{l}_{t+1})] \\
\lambda_u \widehat{\lambda}_{ut} = \beta \mathbb{E}_t[& -c^{-\eta} ben \eta \widehat{c}_{t+1} + \gamma^{Ih}\lambda_e \widehat{\lambda}_{et+1} + \lambda_u[(1-\mu) - \gamma^{Ih}]\widehat{\lambda}_{ut+1} \\
&+ \gamma^{Ih}[\lambda_e - \lambda_u]\widehat{\gamma}_{t+1}^{Ih} + \Phi l^{-\zeta}\zeta \widehat{l}_{t+1}]
\end{aligned}
$$

B.3.15 Value of Employment

$$\lambda_{et} = \beta \mathbb{E}_t[c_{t+1}^{-\eta} w_{t+1} + (1-\sigma)\lambda_{et+1} + \sigma \lambda_{ut+1} - \Phi l_{t+1}^{-\zeta}]$$

Log-linearizing

$$\begin{aligned} \lambda_e(1+\widehat{\lambda}_{et}) =& \beta \mathbb{E}_t[c^{-\eta} w(1-\eta \widehat{c}_{t+1} + \widehat{w}_{t+1}) + (1-\sigma)\lambda_e(1+\widehat{\lambda}_{et+1}) + \sigma \lambda_u(1+\widehat{\lambda}_{ut+1}) \\ & - \Phi l^{-\zeta}(1-\zeta \widehat{l}_{t+1})] \\ \Leftrightarrow \lambda_e \widehat{\lambda}_{et} =& \beta \mathbb{E}_t[-\eta c^{-\eta} w \widehat{c}_{t+1} + c^{-\eta} w \widehat{w}_{t+1} + (1-\sigma)\lambda_e \widehat{\lambda}_{et+1} + \sigma \lambda_u \widehat{\lambda}_{ut+1} + \Phi l^{-\zeta} \zeta \widehat{l}_{t+1}] \end{aligned}$$

B.3.16 Return on Bond

$$1 = \mathbb{E}_t \left[\beta \left(\frac{c_t}{c_{t+1}} \right)^{\eta} \left(\frac{p_t}{p_{t+1}} \right) R_t \right]$$

At the steady state:

$$1 = \beta R \Leftrightarrow R = 1/\beta$$

Log-linearizing

$$\widehat{c}_t = \mathbb{E}_t \widehat{c}_{t+1} - \frac{1}{\eta}(\widehat{R}_t - \mathbb{E}_t \widehat{\pi}_{t+1})$$

B.3.17 Production Function

$$Y_t = K_t^{\varphi} E_t^{1-\varphi} \Leftrightarrow y_t = k_t^{\varphi} e_t^{1-\varphi} \Leftrightarrow \widehat{y}_t = \varphi \widehat{k}_t + (1-\varphi)\widehat{e}_t$$

B.3.18 Vacancy Filling Rate for Insiders

$$\gamma_t^{If} = \frac{M_t^I}{V_t} = \frac{m_t^I}{v_t} \Leftrightarrow \widehat{\gamma}_t^{If} = \widehat{m}_t^I - \widehat{v}_t$$

B.3.19 Vacancy Filling Rate for Outsiders

$$\gamma_t^{Of} = \frac{M_t^O}{V_t} = \frac{m_t^O}{v_t} \Leftrightarrow \widehat{\gamma}_t^{Of} = \widehat{m}_t^O - \widehat{v}_t$$

B.3.20 Job Creation

$$\frac{\kappa}{\gamma_t^f} = \beta \mathbb{E}_t \left(\frac{c_t}{c_{t+1}} \right)^\eta \left[(1-\varphi) x_t \frac{y_t}{e_t} - w_{t+1} + (1-\sigma) \frac{\kappa}{\gamma_{t+1}^f} \right]$$

At the steady state:

$$\begin{aligned} \frac{\kappa}{\gamma_t^f} &= \beta \left[(1-\varphi) \frac{y}{e} - w + (1-\sigma) \frac{\kappa}{\gamma^f} \right] \\ \Leftrightarrow \frac{\kappa}{\gamma_t^f} (1 - \beta(1-\sigma)) &= \beta \left[(1-\varphi) \frac{y}{e} - w \right] \end{aligned}$$

Log-linearizing

$$
\begin{aligned}
\frac{\kappa}{\gamma^f}(1-\widehat{\gamma}_t^f) =&\beta\mathbb{E}_t(1+\eta(\widehat{c}_t-\widehat{c}_{t+1}))\times \\
&\left[(1-\varphi)x\frac{y}{e}(1+\widehat{x}_{t+1}+\widehat{y}_{t+1}-\widehat{e}_{t+1})-w(1+\widehat{w}_{t+1})+(1-\sigma)\frac{\kappa}{\gamma^f}(1-\widehat{\gamma}_{t+1}^f)\right] \\
=&\beta\mathbb{E}_t(1+\eta(\widehat{c}_t-\widehat{c}_{t+1}))\times \\
&\left[\frac{\kappa}{\beta\gamma^f}+(1-\varphi)x\frac{y}{e}(\widehat{x}_{t+1}+\widehat{y}_{t+1}-\widehat{e}_{t+1})-w\widehat{w}_{t+1}-(1-\sigma)\frac{\kappa}{\gamma^f}\widehat{\gamma}_{t+1}^f\right] \\
=&\beta\mathbb{E}_t\left[\left(\frac{\kappa}{\beta\gamma^f}+(1-\varphi)x\frac{y}{e}(\widehat{x}_{t+1}+\widehat{y}_{t+1}-\widehat{e}_{t+1})-w\widehat{w}_{t+1}-(1-\sigma)\frac{\kappa}{\gamma^f}\widehat{\gamma}_{t+1}^f\right)\right. \\
&\left.+\frac{\kappa\eta}{\beta\gamma^f}(\widehat{c}_t-\widehat{c}_{t+1})\right] \\
\frac{\kappa}{\beta\gamma^f}\widehat{\gamma}_t^f+\frac{\kappa\eta}{\beta\gamma^f}\widehat{c}_t =&\frac{\kappa\eta}{\beta\gamma^f}\mathbb{E}_t\widehat{c}_{t+1}+(1-\varphi)x\frac{y}{e}\mathbb{E}_t(\widehat{e}_{t+1}-\widehat{x}_{t+1}-\widehat{y}_{t+1})+w\mathbb{E}_t\widehat{w}_{t+1}+(1-\sigma)\frac{\kappa}{\gamma^f}\mathbb{E}_t\widehat{\gamma}_{t+1}^f.
\end{aligned}
$$

B.3.21 Nash Wage Equation

$$
w_t=(1-\vartheta)\left[(1-\varphi)x_t\frac{y_t}{e_t}+\frac{\kappa\gamma_t^{Oh}}{\gamma_t^f}\right]+\vartheta ben-\vartheta c_t^\eta\sigma\beta\mathbb{E}_t\frac{\mathbb{V}_{t+1}^{UI}}{N_{t+1}}. \tag{B.21}
$$

At the steady state:

$$
w=(1-\vartheta)\left[(1-\varphi)x\frac{y}{e}+\frac{\kappa\gamma^{Oh}}{\gamma^f}\right]+\vartheta ben-\vartheta c^\eta\sigma\lambda_{ut},
$$

where we use $\lambda_{ut}N_{t+1}=\beta E_t\mathbb{V}_{t+1}^{UI}$ from (B.8). Substituting (B.19) into (B.21) and log-linearizing gives

$$\begin{aligned}
w_t =&(1-\vartheta_t)(1-\varphi)x_t\frac{y_t}{e_t} + \underbrace{(1-\vartheta_t)\frac{\kappa\gamma_t^{Oh}}{\gamma_t^f} + \vartheta_t b}_{=\vartheta_t c_t^\eta \Phi l_t^{-\zeta}} - \vartheta_t c_t^\eta \sigma\lambda_{ut} \\
w(1+\widehat{w}_t) =&(1-\vartheta)(1-\varphi)\frac{y}{e}(1+\widehat{x}_t+\widehat{y}_t-\widehat{e}_t) + \vartheta c^\eta \Phi l^{-\zeta}(1+\widehat{\vartheta}_t+\eta\widehat{c}_t-\zeta\widehat{l}_t) \\
&-\vartheta c^\eta \sigma\lambda_u(1+\widehat{\vartheta}_t+\eta\widehat{c}_t+\widehat{\lambda}_{ut}) \\
w\widehat{w}_t =&(1-\vartheta)(1-\varphi)x\frac{y}{e}(\widehat{x}_t+\widehat{y}_t-\widehat{e}_t) \\
&-\zeta\vartheta c^\eta \Phi l^{-\zeta}\widehat{l}_t - \vartheta c^\eta \sigma\lambda_u\widehat{\lambda}_{ut} + \vartheta c^\eta \eta(\Phi l^{-\zeta}-\sigma\lambda_u)\widehat{c}_t.
\end{aligned}$$

B.3.22 Capital Rental Rate

$$r_t = \varphi x_t \frac{Y_t}{K_t} = \varphi x_t \frac{y_t}{k_t} \Leftrightarrow \widehat{r}_t = \widehat{x}_t + \widehat{y}_t - \widehat{k}_t$$

B.3.23 Resource Constraint

$$\begin{aligned}
Y_t &= C_t + I_t + \kappa V_t + G_t \\
y_t &= c_t + i_t + \kappa v_t + g_t \\
\widehat{y}_t &= \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \kappa\frac{v}{y}\widehat{v}_t + \frac{g}{y}\widehat{g}_t
\end{aligned}$$

B.3.24 Aggregate Vacancy Filling Rate

$$\gamma_t^f = \gamma_t^{If} + \gamma_t^{Of} \Leftrightarrow \gamma^f\widehat{\gamma}_t^f = \gamma^{If}\widehat{\gamma}_t^{If} + \gamma^{Of}\widehat{\gamma}_t^{Of}$$

B.3.25 Monetary Policy

$$R_t = R\exp(\zeta_\pi \pi_t)$$

$$\Leftrightarrow \log R_t = \log R + \zeta_\pi \pi_t \Leftrightarrow \widehat{R}_t = \zeta_\pi \pi_t$$

B.2.26 Growth of Population

$$\frac{N_{t+1}}{N_t} = \zeta^N_{t+1} = \frac{1}{1 - mig_{t+1}}.$$

Log-linearizing this expression, it is easy to define a new variable $f_t := 1 - mig_{t+1}$. Thus, $\zeta^N_{t+1} = \zeta^N \exp\big(\widehat{\zeta}^N_{t+1}\big) = f^{-1}\exp\big(-\widehat{f}_t\big)$, hence $\widehat{\zeta}^N_{t+1} = -\widehat{f}_t$. Now log-linearize f_t gives:

$$\begin{aligned}
f(1+\widehat{f}_{t+1}) &= 1 - mig(1 + \widehat{mig}_{t+1}) \\
\Leftrightarrow \widehat{f}_{t+1} &= -\frac{mig}{f}\widehat{mig}_{t+1} = -\frac{mig}{1-mig}\widehat{mig}_{t+1} \\
\Leftrightarrow \widehat{\zeta}^N_{t+1} &= \frac{mig}{1-mig}\widehat{mig}_{t+1}.
\end{aligned}$$

Chapter 4

Mitigating Fiscal Crisis through Population Growth

4.1 Introduction

The recent increases in the debt to GDP ratio in many developed countries, especially after the financial crisis in 2008, have led to a great deal of debate about fiscal consolidation polices, i.e., how best to reduce the level of debt in an economy. The accumulated public debt is considered to be problematic since it begins to hamper economic growth when it exceeds some threshold level– see, e.g., Reinhart and Rogoff (2010) and Checherita-Westphal and Rother (2012). Thus, much public and academic attention has been paid to the effects of fiscal policy and the size of fiscal multipliers, coupled with the fact that monetary policy is constrained by the zero lower bound on nominal interest rates. See Ramey (2011b) for an overview of the literature on fiscal policy.

In this paper we analyze one policy whose effects have not received a great deal of analysis: raising the rate of population growth. It is intuitive that raising the rate of population growth will reduce the rate of debt per capita and the debt to GDP ratio, *ceteris paribus*, since the number of taxpayers increases. However, an increase in the rate of population will also reduce the domestic private capitals per person if other things are equal. This is what is called a "capital dilution effect". This capital dilution effect may cause a slowdown in the growth of GDP per capita, which in turn potentially may lead to a rise in the level of debt.

This paper analyzes the effects of such a debt reduction policy using the dynamic stochastic general equilibrium (DSGE) framework of Uhlig (2010) and Trabandt and Uhlig (2011). We show that if the tax revenue from increased population growth is spent effectively on productive public capital and at the correct time, then population growth can mitigate the capital dilution without increasing the present value of budget deficits.

The analysis in this paper treats population growth simply as a policy parameter which is open to interpretation. A straightforward interpretation would be that the rate of population growth in a developed economy can be controlled by the rate of immigration, which is in the power of government. In Section 2 below we show that the rate of immigration has been inversely related to the growth rate of the debt to GDP ratio in the U.S. economy since 1950, and we detail briefly the U.S. immigration legislation in the periods.

In the analysis we treat increased population as identical to all other representative agents and so increased population growth, e.g., immigrants, makes no extra demands on government finances. We regard this as a reasonable assumption since the evidence shows that, if anything, immigrants are net contributors to a government's budget. For example, Storesletten (2000) uses a calibrated general equilibrium overlapping generations model to compute the net discounted gain to the U.S. government of one additional average immigrant and shows that it is positive. More recent results for the U.K. government have been obtained by Dustman, Frattini and Halls (2010) and Dustman and Frattini (2014), and these show that immigrants that arrived since 2000, especially those from the European Economic Area, have made a significant net contribution to public finances using the static approach. This paper focuses on the dynamic approach in the sense that all future taxes and expenditures are considered in a forward-looking manner using the concept of the net present value, not focusing on a particular year.

Our paper is most similar to the work by Ben-Gad (2012), which has shown that immigration creates an incentive, or a bias, for a current native population to support higher deficits because the cost of financing them can be partially shifted to future immigrants. We reverse his argument; government deficits create an incentive, a rationale, to admit more immigrants. The main difference between our approach and his is that we analyze adjustments of labor income using a DSGE model where labor supply is endogenized, while he examines changes in capital income taxes using an overlapping dynasties model with inelastic labor.

This paper is not meant to explore what caused the crisis, but rather to investigate whether population growth, or immigration, policies are effective given high level of debts. To do so, we keep our model simple by introducing an exogenous *debt shock*, which can cause a sharp rise in government debt. We assume that a debt shock is exogenous in the context of our theoretical analysis. One can think that an unexpected rise in government debt, which can be calculated as the actual rise in debt above its forecast, is an exogenous debt shock. Another interpretation for a debt shock is that it represents an increase in debt resulting from revisions of debt statistics as the Greek government experienced during the recent financial crisis. Thus, we abstract from financial sectors and shocks that directly affect banks' balance sheets as a source of a crisis. Analyses of such financial shock can be found in e.g., Gertler and Karadi (2011), Gertler and Kiyotaki (2009), Jermann and Quadrini (2011), and Christiano, Motto, and Rostagno (2014).

This paper is organized as follows. In Section 2 we briefly describe the recent evolution of immigration and debt to GDP ratios in the U.S. economy. Section 3 outlines the model, emphasizing the departures from Uhlig (2010). Section 4 discusses the results. Section 5 concludes.

4.2 Empirical Background

This section presents the broad trends in the growth of the debt to GDP and immigration in the U.S. Figure 4.1 shows that until around the end of the 1970's the debt to GDP ratio fell as the U.S. economy boomed and the level of immigration was low with the share of foreign born in the U.S. population falling from 5.4% in 1960 to 4.7% in 1970, which is shown on the right scale in Figure 4.2. Since then the immigration share has kept rising as a result of generous immigration policies in the U.S. government. For example, the Immigration Reform and Control Act of 1986, which is also known as the 1986 Immigration Amnesty, legalized three million illegal immigrants. Subsequently, the Immigration Act of 1990 placed more emphasis on employment considerations and increased the number of highly educated and skilled immigrations. In 2010, immigrants account for 12.9% of the U.S. population.

This rise in immigration was accompanied by a fall in the growth rate of the debt to GDP ratio in most years since 1970. There were some exceptions in the 1980's, when military spending increased in response to the Soviet invasion of Afghanistan, and in the 2000's, which experienced the wars in Afghanistan and Iraq and the Great Recession

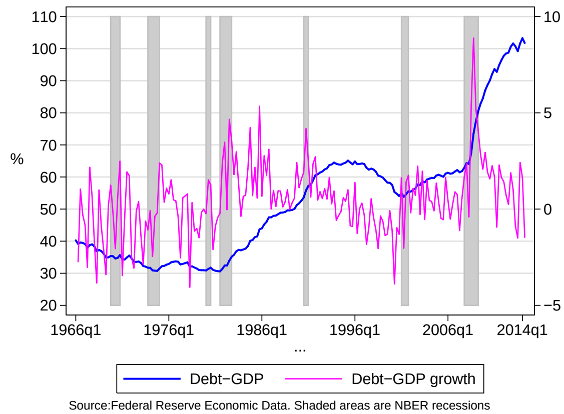


Figure 4.1: Debt to GDP in the U.S.

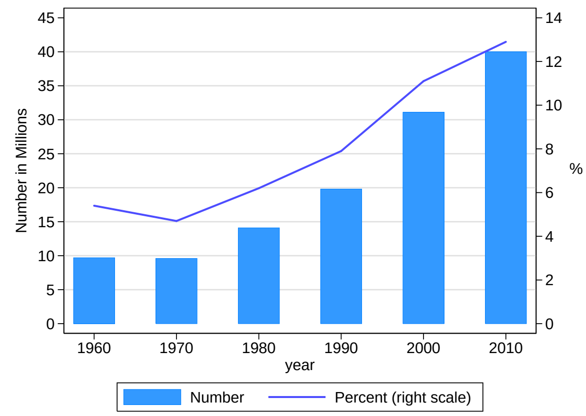


Figure 4.2: Immigrants in the U.S., Number and Percent

of 2008-2009. Of course this evidence is not proof that the inverse relationship between growth in the debt to GDP ratio and immigration levels is causal. However the evidence in Section 3 shows how population growth can help mitigate fiscal problems and so a political economy model of immigration quota level setting would be an intuitive extension of this analysis, although it is beyond the scope of this paper.

4.3 Elements of the Model

We use the model of Uhlig (2010) as a starting point, extending it in three directions. Firstly and the most importantly, population growth is introduced in the economy. Secondly, public capital is introduced as a factor of production as in Baxter and King (1993), Kamps (2004) and Leeper, Walker and Yang (2010). Third, we introduce a debt shock. An unanticipated shock in debt provides the source of a crisis.

4.3.1 Population Growth

Our first departure from Uhlig (2010) is the introduction of population growth. The total population of the economy, which is denoted by N_t, grows exogenously at a gross rate ζ_{t+1}^N,

$$N_{t+1} = \zeta_{t+1}^N N_t, \tag{4.1}$$

where ζ_{t+1}^N follows the stationary AR(1) stochastic process:

$$\zeta_{t+1}^N = \rho_N \zeta_t^N + \epsilon_{t+1}^N. \tag{4.2}$$

4.3.2 Representative Household

The infinitely-lived representative household maximizes its lifetime utility (4.3) subject to the budget constraint (4.4) and the capital accumulation equation (4.5). We will use lower-case letters to denote per capita terms. By the introduction of population growth, the household's lifetime utility is expressed by multiplying the instantaneous utility of representative household by the total population of economy. Formally, it is given by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t N_t \left(\frac{(c_t \Phi(l_t))^{1-\eta} - 1}{1-\eta} \right) \right] \tag{4.3}$$

where c_t and l_t denote consumption and labor per capita respectively, and η is the coefficient of relative risk aversion. If we interpret that population growth is caused by immigration, we regard the immigrants as homogeneous in age and skills and as identical to natives, and that they start working on entering the economy. Since this is an infinitely-lived agent model, life cycle aspects are abstracted altogether.

The budget constraint takes the form

$$
\begin{aligned}
(1+\tau^c)c_tN_t + x_tN_t + b_tN_t =& R_t^b b_{t-1}N_{t-1} + s_tN_t + m_tN_t \\
&+ (1-\tau_t^l)w_tl_tN_t + (r_t - \tau^k(r_t-\delta))k_tN_t
\end{aligned}
\tag{4.4}
$$

where x_t is private investment, b_t denote government bonds, R_t^b represents the interests of government bonds purchased in period $t-1$, s_t denote transfers from the government, m_t denote transfers from the rest of the world, w_t represents wage, r_t is capital rental rate, and k_t is private capital. The tax which are levied on consumption, labor income, and capital income are denoted by τ^c, τ_t^l, and τ^k respectively. Consumption taxes and capital income taxes are held fixed, and thus there are no time subscripts. Capital income are taxed on net of depreciation $\delta \in [0,1]$. The private capital accumulation equation is given by

$$
k_{t+1}N_{t+1} = [(1-\delta)k_t + x_t]N_t. \tag{4.5}
$$

4.3.3 Representative Firm

The representative firm produces output in period t, which is denoted by Y_t, by combining aggregate labor L_t, aggregate private capital K_t, and aggregate public capital K_t^G. This introduction of public capital as a factor of production is the second departure from Uhlig (2010). The production function takes the form:

$$
Y_t = A_t^{\theta} K_t^{1-\theta} L_t^{\theta} (K_t^G)^{\theta_G}, \tag{4.6}
$$

where A_t is a technology parameter and $\zeta_t^A = A_t/A_{t-1}$ follows a stationary exogenous stochastic process. A key parameter in this model is the productiveness of public capital, θ^G. If it is strictly positive, public capital is productive since it raises marginal product of private capital and labor. If it is equal to zero, public capital is unproductive. The firm maximizes profits

$$Y_t - r_t K_t - w_t L_t \tag{4.7}$$

subject to (4.6) taking r_t and w_t as given.

4.3.4 Government

The government remaining debt before levying labor tax and issuing new bond, which is denoted by D_t, is written as[1]

$$\begin{aligned} D_t =& G_t + S_t + R_t^b B_{t-1} - \tau^c C_t - \tau^k (r_t - \delta) K_t && (4.8) \\ =& B_t + \tau_t^l w_t L_t, && (4.9) \end{aligned}$$

where G_t is aggregate government purchases, and S_t, C_t and B_t denote aggregate variables of corresponding per capita variables. Excess debts above steady-state values are paid back at speed $\psi \in (0, 1]$ per

$$\begin{aligned} \psi(D_t - A_t N_t D) &= \tau_t^l w_t L_t - A_t N_t \tau^l \bar{w} L, && (4.10) \\ (1 - \psi)(D_t - A_t N_t D) &= B_t - A_t N_t B, \end{aligned}$$

[1]We call D_t as "remaining *debt*" considering it is the stock variable although Uhlig (2010) refers it as "remaining *deficit*". The budget deficit in period t can be defined as the change in government debt in period t, i.e., $B_t - B_{t-1}$.

where variables without time subscripts denote steady state values of the corresponding variables. Note that $\bar{w}$ is the steady state value of $w_t = A_t \widetilde{w}_t$. The choice of labor income tax, τ_t^l, will also determine the level of debt. That is we only have one degree of freedom to determine the debt and the tax rate. Thus the adjustment speed, ψ, is effectively choosing both τ_t^l and the level of debt. A higher speed adjustment will mean higher labor taxes and lower debt. Following Kamps (2004), we assume that aggregate government purchases comprises basic government purchases G_t^B and government investment I_t^G:

$$G_t = G_t^B + I_t^G, \tag{4.11}$$

where both G_t^B and I_t^G follow stationary AR(1) stochastic processes. Public capital accumulates as private capital according to

$$K_{t+1}^G = (1 - \delta_G) K_t^G + I_t^G \tag{4.12}$$

where $\delta_G \in [0, 1]$ is the depreciation rate of public capital.

4.3.5 Equilibrium

Market clearing condition in the goods market requires

$$C_t + X_t + G_t = Y_t + M_t,$$

for all t. Furthermore, in equilibrium aggregate variables are equal to their per-capita counterparts multiplied by the total population of the economy.

$$
\begin{aligned}
Y_t &= y_t N_t, \\
C_t &= c_t N_t, \\
X_t &= x_t N_t, \\
K_t &= k_t N_t, \\
&\vdots
\end{aligned}
$$

and so on.

4.3.6 Equilibrium Dynamics

In order to be able to log-linearize around the steady state, we need to detrend variables on the balanced growth path. Following Uhlig (2010), we will denote log-deviations by hats so that $\widehat{c}_t = \log(\widetilde{c}_t) - \log(c) \approx (\widetilde{c}_t - c)/c$ where $\widetilde{c}_t := c_t/A_t$ and c is the steady state of $\widetilde{c}_t$. The exceptions are the labor tax rates, τ_t^l, which is expressed as percentage point deviations, i.e., $\widehat{\tau}_t^l = \tau_t^l - \tau^l$, and the debt, d_t, government spending and consumption, g_t and g_t^B, government bond, b_t, government transfer, s_t, are expressed relative to steady state output, e.g., $\widehat{d}_t^* = (\widetilde{d}_t - d)/y$, where asterisk is used to emphasize that it is expressed relative to output.

The log-linearized dynamics of the model is shown in Table 4.1.[2] Equations (1) to (16) determine 16 endogenous variables: $y_t, c_t, x_t, k_t, K_t^G, r_t, w_t, l_t, \tau_t^n, R_t^k, R_t^b, b_t, d_t, g_t, A_t$ and λ_t. Equations (a) to (e) represent exogenous processes. Two things are worth noting. First, equation (6) shows that there is a capital dilution effect. An increase in population

[2]See Section C.3 in Appendix C for the details of derivations.

growth rate reduces per capita private capital. Next, equation (11) shows that there is a tax reduction effect. An increase in population growth rate reduce the remaining budget deficits per capita.

4.4 Calibration and Simulation

4.4.1 Parameter Values

In the baseline calibration, we take the period in the model to correspond to a quarter and use the same parameter values as in Uhlig (2010). The new parameter introduced here is the productiveness of public capital, θ^G, which is assumed to be 0.2 following Kamps (2004). The benchmark parameters are displayed in Table 4.2.

4.4.2 Policy Experiments

An Increase in Debt and/or Population Growth

First, we analyze the effects of an unanticipated increase in debt shock and assume that there is no population growth. Figure 4.3 displays the dynamics of nine macroeconomic variables (output per capita, labor, capital per capita, consumption per capita, investment per capita, return on capital, labor tax rate, population growth, and government debt per capita) in response to a 10 percent increase in debt shock, which is assumed to persist as an AR(1) process with 0.9 coefficient. According to the blue lines in Figure 4.3, government debts per capita begin to rise quickly following an unanticipated debt shock, leading to nearly 60 percent higher than the steady-state GDP per capita about five years after the shock. These responses of government debts are mimicked by those of labor taxes since a fixed ψ fraction of higher government debts must be financed by higher labor taxes as

Table 4.1: The Log-Linearized Equations of the Model

Uhlig (2010) model with population growth and public capital	
Household	
(1)	$\widehat{w}_t = \frac{1}{\omega_S}\widehat{l}_t + \frac{1}{1-\tau^l}\widehat{\tau}^l_t + \widehat{c}_t$
(2)	$\widehat{\lambda}_t = -\eta\ \widehat{c}_t - (1-\eta)\kappa\ \widehat{l}_t$
(3)	$0 = \mathbb{E}_t[\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{R}^k_{t+1}]$
(4)	$0 = \mathbb{E}_t[\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{R}^b_{t+1}]$
(5)	$\widehat{R}^k_t = \left(1 - (1-(1-\tau^k)\delta)\frac{\tilde{\beta}}{\zeta^A}\right)\widehat{r}_t$
(6)	$\widehat{k}_{t+1} = \left(1 - \frac{\tilde{\delta}}{\zeta^A\zeta^N}\right)\widehat{k}_t + \frac{\tilde{\delta}}{\zeta^A\zeta^N}\widehat{x}_t - \widehat{\zeta}^A_{t+1} - \widehat{\zeta}^N_{t+1}$
Firm	
(7)	$\widehat{y}_t = \theta\ \widehat{l}_t + (1-\theta)\ \widehat{k}_t + \theta_G\ (\widehat{k}^G_t + \widehat{A}_t)$
(8)	$\widehat{w}_t = (1-\theta)(\widehat{k}_t - \widehat{l}_t) + \theta_G\ (\widehat{k}^G_t + \widehat{A}_t)$
(9)	$\widehat{r}_t = \widehat{y}_t - \widehat{k}_t$
(10)	$\widehat{A}_{t+1} = \widehat{A}_t + \widehat{\zeta}^A_{t+1}$
Government	
(11)	$\widehat{d}^*_t = \widehat{g}^*_t + \widehat{s}^*_t + \frac{1}{\bar{\beta}\zeta^N}\ \widehat{b}^*_{t-1} + \frac{\bar{b}}{\bar{\beta}y\zeta^N}\ (\widehat{R}^b_t - \widehat{\zeta}^A_t - \widehat{\zeta}^N_t) - \tau^c\frac{c}{y}\ \widehat{c}_t$ $-\tau^k\left(1 - \theta - \frac{\delta}{\tilde{\delta}}\frac{x}{y}\right)\ \widehat{k}_t - \tau^k(1-\theta)\ \widehat{r}_t + \widehat{\varepsilon}^d_t$
(12)	$\psi\widehat{d}^*_t = \theta\ \widehat{\tau}^l_t + \theta\bar{\tau}^l\ (\widehat{w}_t + \widehat{l}_t)$
(13)	$(1-\psi)\ \widehat{d}^*_t = \widehat{b}^*_t$
(14)	$\widehat{g}^*_t = \widehat{g}^{*B}_t + (I^G/y)\ \widehat{I}^G_t$
(15)	$\widehat{K}^G_{t+1} = \left(1 - \frac{\tilde{\delta}_G}{\zeta^A}\right)\widehat{K}^G_t + \frac{\tilde{\delta}_G}{\zeta^A}\widehat{I}^G_t - \widehat{\zeta}^A_{t+1}$
Resource Constraint	
(16)	$\widehat{y}_t = \frac{\bar{c}}{\bar{y}}\ \widehat{c}_t + \frac{\bar{x}}{\bar{y}}\ \widehat{x}_t + \widehat{g}_t$
Exogenous Processes	
(a)	$\widehat{\zeta}_{t+1} = \rho_\zeta\ \widehat{\zeta}_t + \epsilon^\zeta_{t+1}$
(b)	$\widehat{g}^B_{t+1} = \rho_B\ \widehat{g}^B_t + \epsilon^B_{t+1}$
(c)	$\widehat{I}^G_{t+1} = \rho_I\ \widehat{I}^G_t + \epsilon^I_{t+1}$
(d)	$\widehat{h}_{t+1} = \rho_\zeta\ \widehat{h}_t + \epsilon^h_{t+1}$
(e)	$\widehat{\varepsilon}^d_{t+1} = \rho_d\ \widehat{\varepsilon}^d_t + \epsilon^d_{t+1}$

This table shows the equations of the log-linearized version of the model. The asterisk denotes that the variable is expressed relative to steady state output.

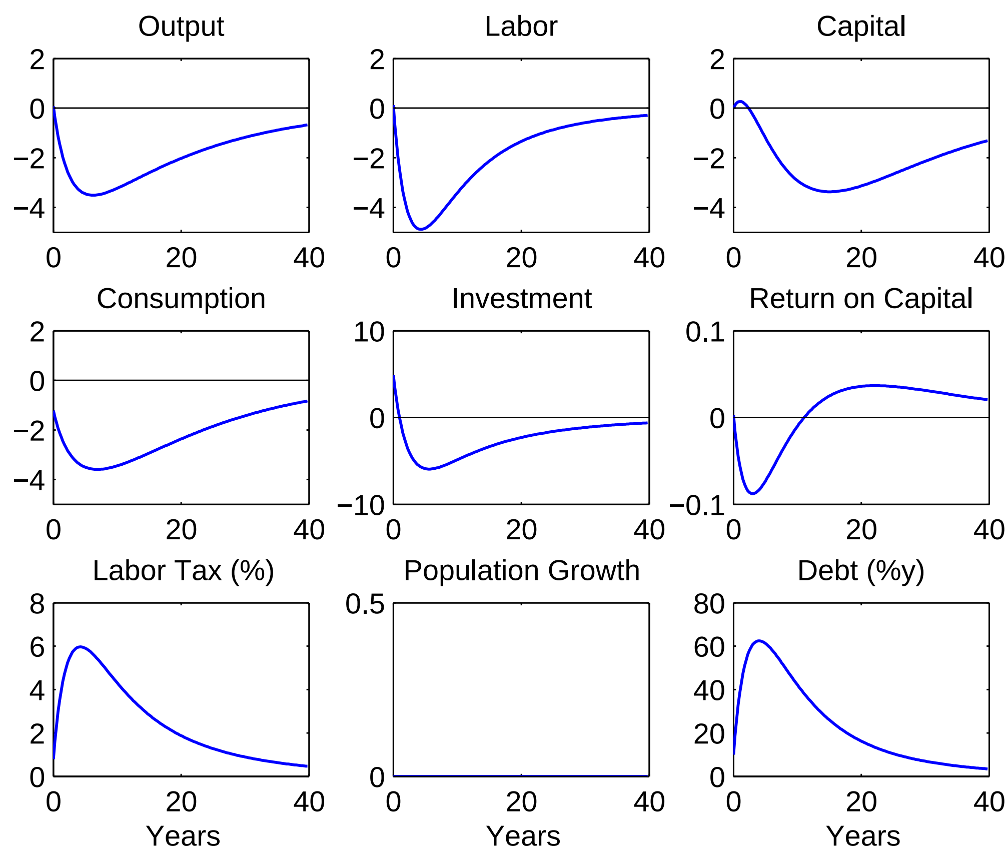
Table 4.2: Common Parameter Values

Household			
Discount rate	$\beta = 0.99$	Tech. growth rate	$\zeta^A = 1.005$
Depreciation	$\delta = 0.02$	Risk Aversion	$\eta = 2$
Pop. growth	$\zeta^N = 1$		
Firm			
Labor share	$\theta = 0.62$	Productiveness of Pub.Cap	$\theta_G = 0.2$
Government			
Consumption tax	$\tau^c = 0.05$	Capital Tax	$\tau^k = 0.36$
Labor tax	$\tau^l = 0.28$	Depreciation	$\delta_G = 0.02$
Financing speed	$\psi = 0.05$		
Steady State Properties			
Consumption-GDP	$c/y = 0.59$	Investment-GDP	$x/y = 0.27$
Gov.spending-GDP	$g/y = 0.18$	Gov.inv.-GDP	$I^G/y = 0.05$
Debt-GDP	$b/y = 0.63$	Net imports-GDP	$m/y = 0.04$
Gov.transfer-GDP	$s/y = 0.07$		

the debt payment rule (4.10), or (12) in Table 4.1 when log-linearized, shows (we assume $\psi = 0.05$ in the baseline). This sharp increase in labor taxes creates a strong disincentive to work, which leads the household to reduce labor. Per capita values in output, private consumption, and private capital also show persistent hump-shaped declines.

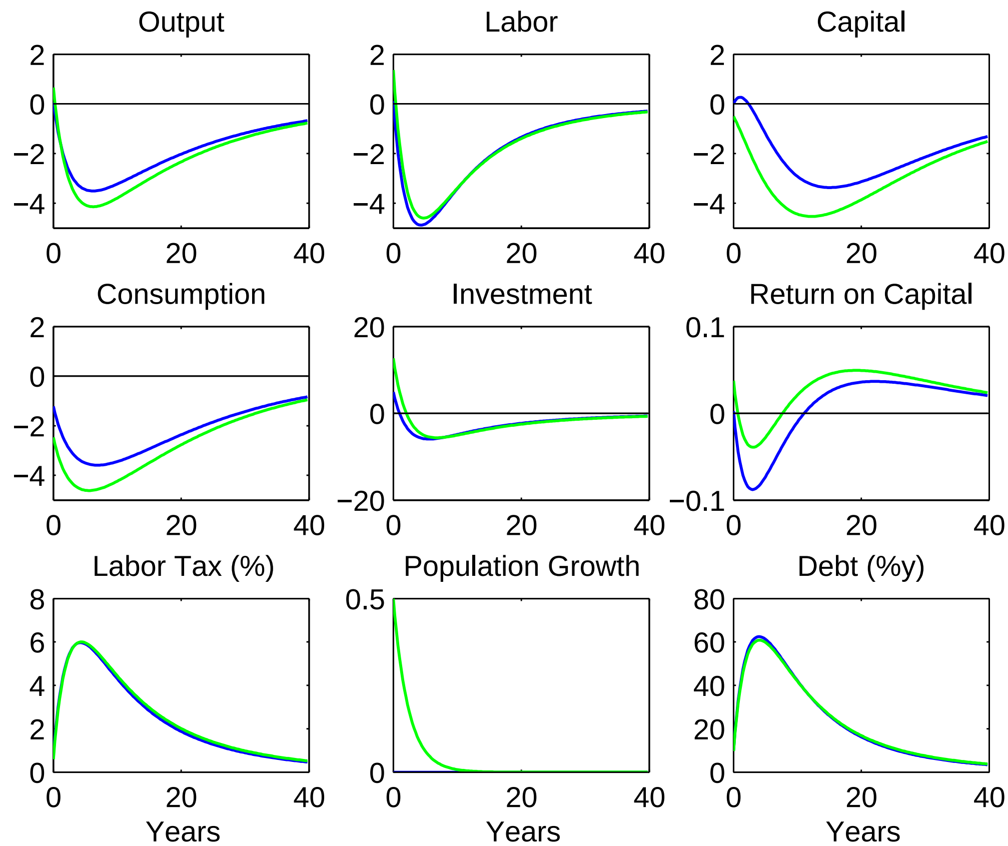
Next, we make different assumptions on population growth. The green lines in Figure 4.4 assume population growth, which also follows an AR(1) process, while the blue lines assume no population growth as before. The green lines show that if a debt shock is followed by a simultaneous increase in population, it causes stronger capital dilution effects. The top right panel shows that population growth leads to a reduction in private capital per capita more than in the absence of population growth. In contrast, the dynamic responses of government debts and, hence, those of labor taxes, are nearly identical under these two cases. That is because tax reduction effects brought about by population growth are offset by a reduction in labor tax revenues due to a decrease in wages brought about by a rise in capital-labor ratio. As a consequence, in the case of population growth,

Figure 4.3: Dynamic Responses to a Debt Shock. Blue lines: no population growth



stronger capital dilution and smaller tax reduction effects generate larger drops in output and consumption per capita.

Figure 4.4: Dynamic Responses to Debt Shock. Blue lines: no population growth; green lines: population growth.

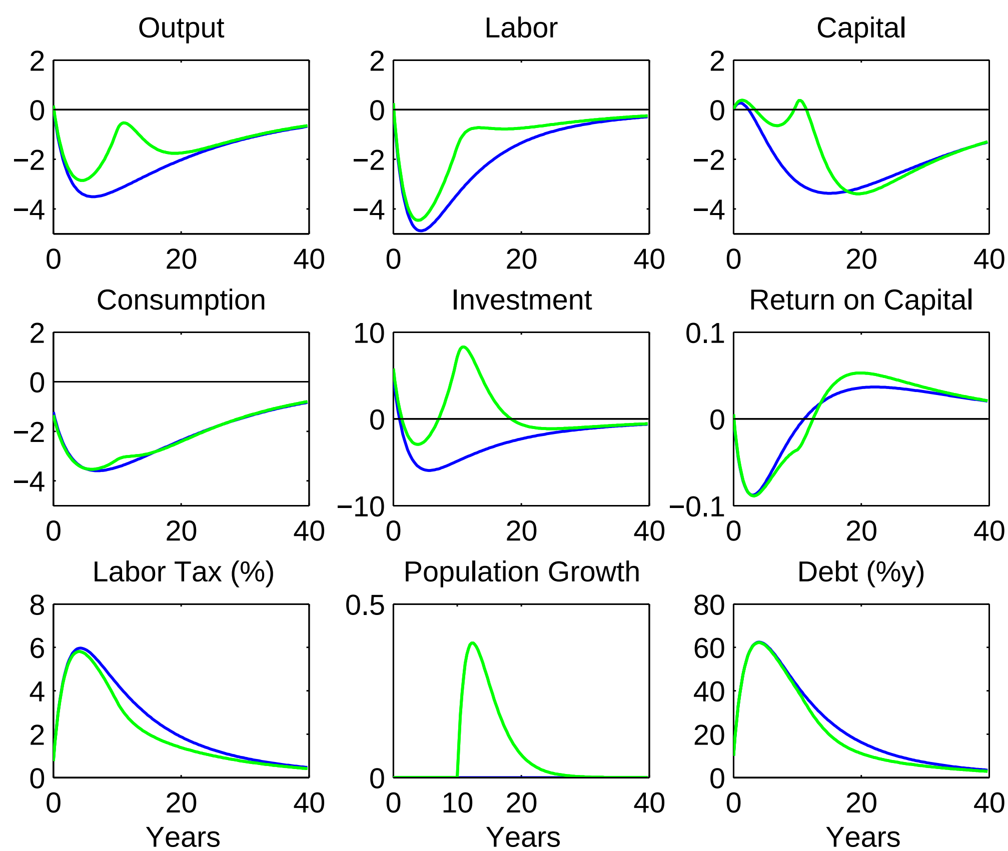


Even if we change the sizes of the debt shock and the population growth shock, the result, that population growth does not mitigate capital dilution effects, is unchanged. Furthermore, even if we change the responses of population growth from AR(1) processes to AR(2) processes, the result does not differ markedly though there is a slightly larger tax reduction effect (Figures are not shown here).

An Increase in Population Growth 10 Years after Shock

We have so far found that a simultaneous increase in population following a debt shock leads to strong capital dilution. Next, in order to mitigate this dilution effect we change the timing of the occurrence of population growth. The green lines in Figure 4.5 show the dynamic responses caused by an increase in population ten years after the same size of the debt shock as in Figure 4.4.

Figure 4.5: Dynamic Responses to Debt Shock. Blue lines: no population growth; green lines: population growth 10 years later



As shown in Figure 4.5, changing the timing of population growth affects the dynamics greatly, especially for private investment per capita, and hence, private capital per capita.

If the supply of labor increases with population growth, then wages are expected to drop due to a fall in the capital-labor ratio and, in contrast, return on capital is expected to rise. Therefore, private investment begins to rise before the actual population growth and shows larger positive responses. As a result, private capital falls less during the initial years and even becomes positive until the dilution effects occur. The timing of population growth also matters for the responses of labor taxes and, hence, labor. When population growth occurs ten years after the shock, labor taxes begin to fall faster and labor falls less.

Tax Revenues Spent on Public Capital

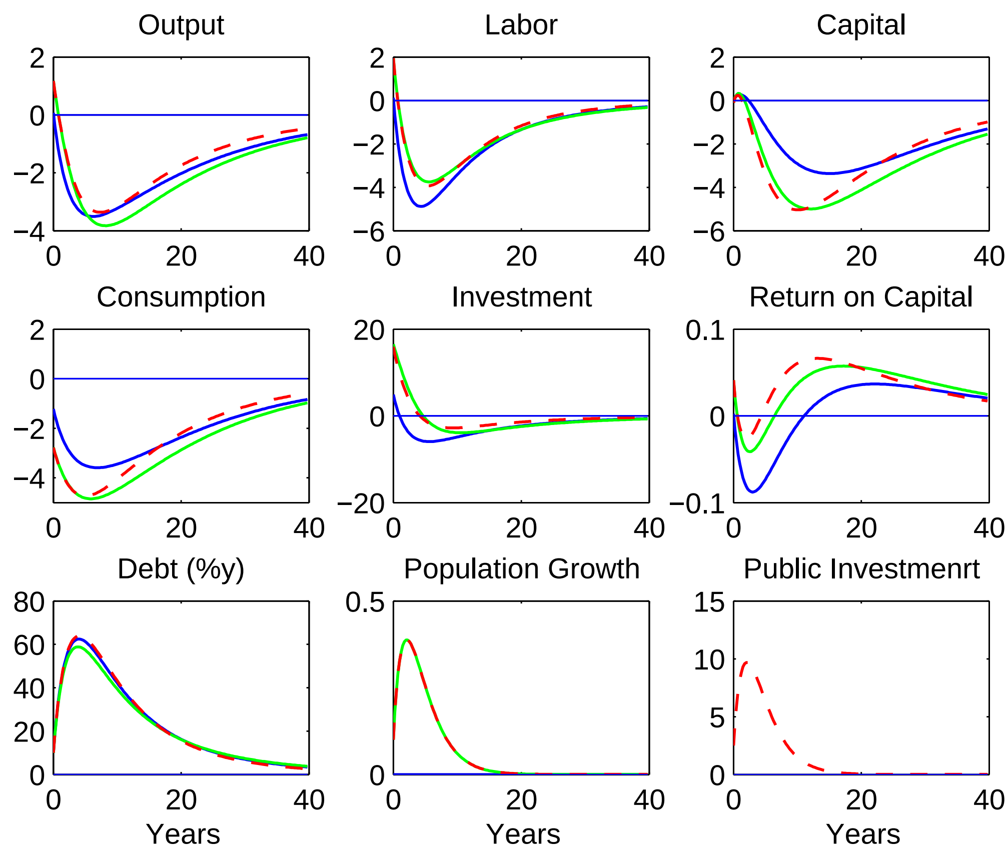
Next, we compare the following two scenarios; the first one is the same as before. That is, the growth path of the economy with a high debt shock. The other one is the growth path of an economy with a high debt shock and with population growth when the extra tax revenue is spent productively on public capital stock. In doing this comparison, the present value of debts,

$$PVD_t := \sum_{s=0}^{t} R^{-s}\widehat{d}_s, \tag{4.13}$$

is kept the same between the two scenarios.

The blue line in Figure 4.6 is the model with no population growth (the same as Figure 4.4), and the red dashed line is the model with population growth and government investment in public capital using the extra tax revenue. For comparison, the green lines report the responses under the model with population growth and no government investment.

Figure 4.6: Dynamic Responses to Debt Shock. Blue lines: debt shock only; green lines: population growth; red dashed lines: population growth with government investment



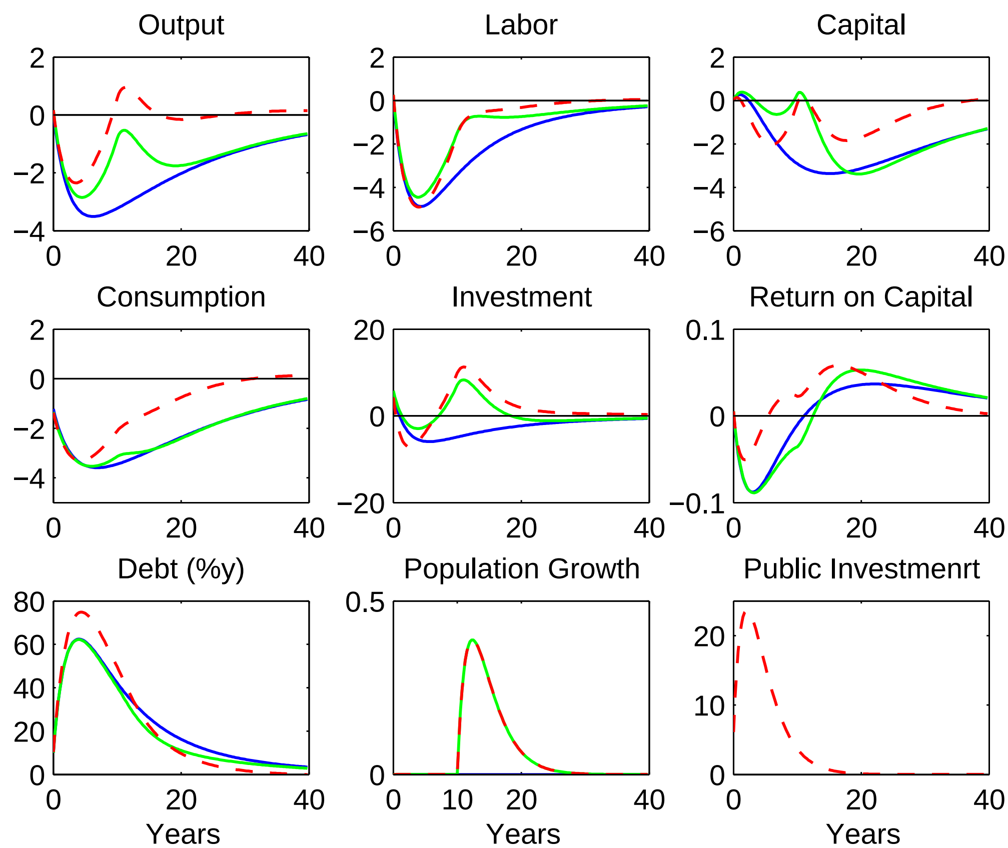
In the case where there is the presence of government investment, private capital per capita quickly begins to increase about ten years after the shock and returns to the steady state. This is because an increase in the marginal product of capital induced by the accumulation of productive public capital generates an expansion in private investment and, therefore, capital dilution effects are mitigated. Output and consumption per capita also converge faster to the steady state due to increases in private and public capital per capita.

Expected Tax Revenues Spent on Public Capital

We next turn to the case where population growth occurs ten years after the initiating shock in Figure 4.7. If government invests in public capital, government debts rise more during the first ten years due to a rise in aggregate government purchases. After that, however, the debts begin to decrease more rapidly due to an increase in population, i.e., the number of taxpayers. Although the resulting dynamic responses of government debts look different from those with no population growth, their present values, which are calculated using (4.13), are nearly identical.

In this case, capital dilution effects that are supposed to occur ten years later due to population growth are mitigated by a much larger increase in private investment induced by the accumulation of productive capital. The increase in public capital and the lower decrease in private capital are responsible for a much larger increase in output and, hence, a smoother consumption path. That is, if the expected tax revenues from future population growth are spent on productive capital, capital dilution effects are mitigated greatly.

Figure 4.7: Dynamic Responses to Debt Shock. Blue lines: debt shock only; green lines: population growth; red dashed lines: population growth with government investment



Tax Revenues Spent on Private Capital

Next we consider the case where government can invest directly in building up the private capital stock. Specifically, we assume that log-linearized capital accumulation equation is now given by

$$\widehat{k}_{t+1} = \left(1 - \frac{\tilde{\delta}}{\zeta^A \zeta^N}\right) \widehat{k}_t + \frac{\tilde{\delta}}{\zeta^A \zeta^N} \widehat{x}_t + a \left(\frac{\tilde{\delta}}{\zeta^A \zeta^N}\right) \widehat{I}_t^G - \widehat{\zeta}_{t+1}^A - \widehat{\zeta}_{t+1}^N,$$

where the third term in the right hand side is newly introduced. We assume that government is less efficient in building up private capital than the private sector, and the parameter $a \in (0, 1)$ captures this inefficiency. That is, the parameter governs efficiency of transformation of government investment into private capital stock.

We set the parameter a equal to 0.9 in the baseline. The red dashed lines in Figure 4.8 reports the dynamic responses for this case. The top right panel in Figure 4.8 shows that capital dilution effects are not mitigated even ten years after the shock, as opposed to the case with government investment in public capital in Figure 4.6.

Expected Tax Revenues Spent on Private Capital

Next we change the timing of population growth as before. Figure 4.9 reports for this case. Unlike the public capital investment case, even if expected tax revenue is spend on building up private capital, capital dilution effects induced by population growth are not mitigated.

4.4.3 Sensitivity Analysis

The results above depend on the parameters we set in Tables 4.2. We therefore conduct sensitivity tests to examine how different parameter values affect the shape of impulse

Figure 4.8: Dynamic Responses to Debt Shock. Blue lines: debt shock only; green lines: population growth; red dashed lines: population growth with direct government investment in private capital

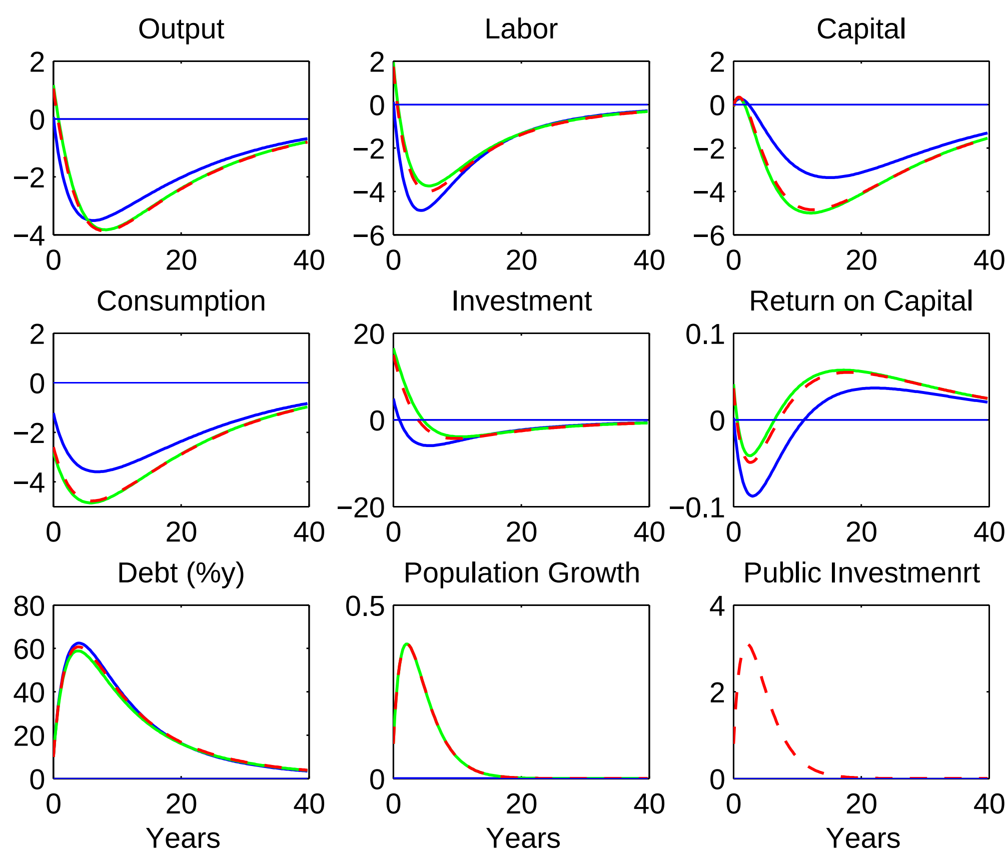
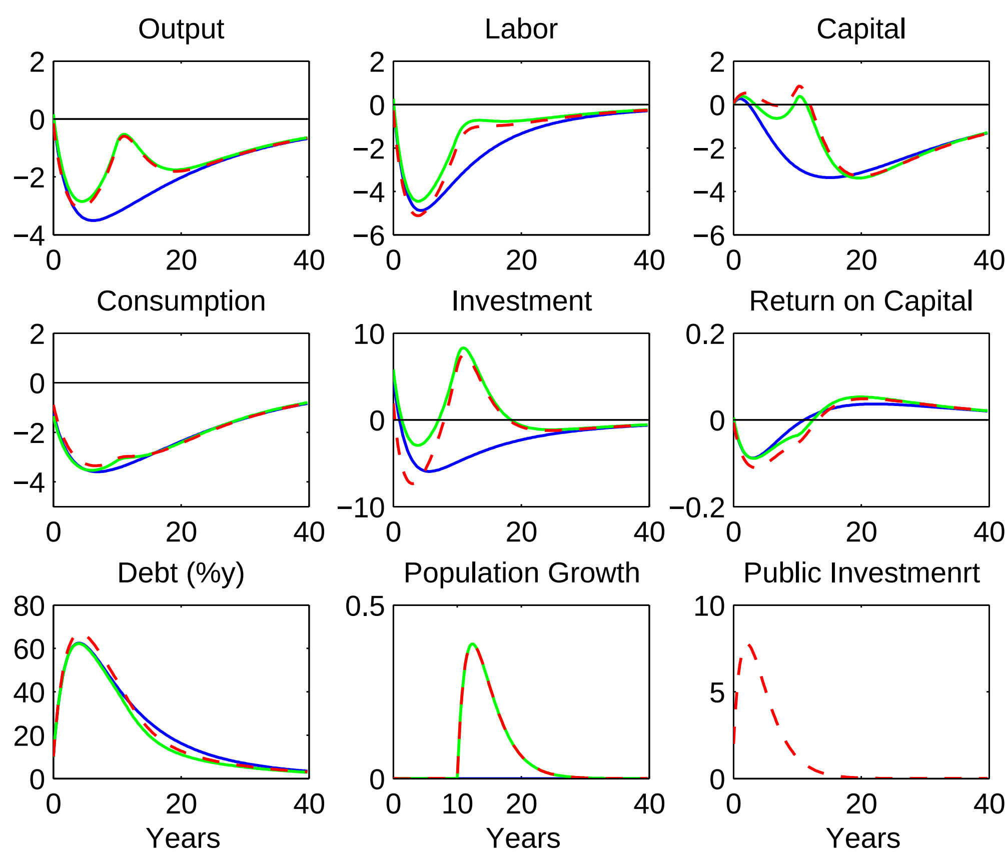


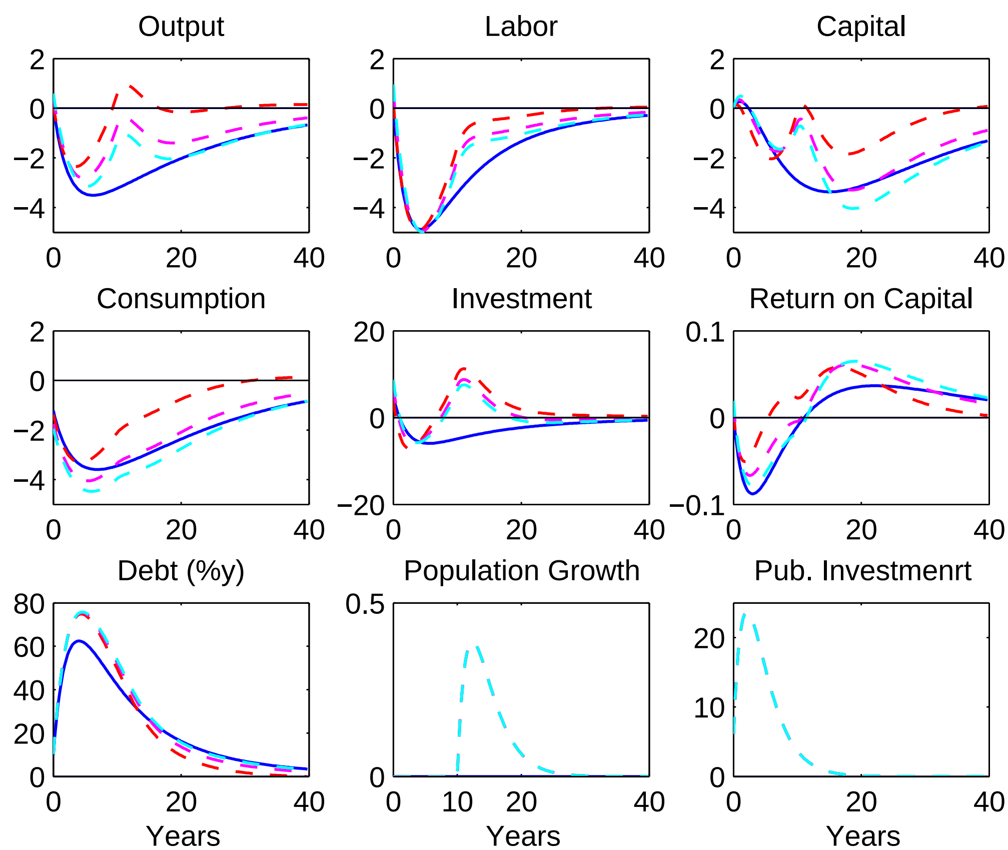
Figure 4.9: Dynamic Responses to Debt Shock. Blue lines: debt shock only; green lines: population growth; red dashed lines: delayed population growth with direct government investment in private capital



response functions to a debt shock. In particular, we experiment with variations in the productiveness of public capital θ^G in the case of delayed population growth with public capital investment, which has led to mitigate capital dilution greatly as we have seen in Figure 4.7.

Figure 4.10 displays the dynamics for three alternative parameter choices ($\theta^G = 0.2, 0.1$, or 0.05) in the case with population growth and government investment in public capital. It is very intuitive that the lower productiveness will lead to higher capital dilution effects and, therefore, smaller outputs.

Figure 4.10: Alternative choices of θ^G



4.5 Conclusion

This paper has extended the Uhlig (2010) model to allow for population growth and public capital in order to analyze the effects of a policy that increases an economy's rate of population growth in response to an unexpected increase in its debt to GDP ratio.

We find that there is indeed a potential for such a policy to boost economic activity without increasing the present value of government debts if the expected tax revenue from future increased population growth is spent effectively on productive public capital at the correct time. However, there is also scope for depressing the economy further if the dilution of the domestic capital stock by increased population is not properly managed. For example, the direct government investment in private capital or less productive public capital is not useful in mitigating capital dilution.

Future research would need a quantitative criterion to assess the performances of immigration policies considered above. Introducing heterogeneity between the natives and immigrants and calculating immigration surplus accruing to the native populations on the lines of Ben-Gad (2008) would be an important contribution. We also require assessment against empirical evidence obtained, for example, from simulating on the lines of Trabandt and Uhlig (2011) as well as estimating on the lines of An and Schorfheide (2007).

Appendix C

Appendices to Chapter 4

C.1 Log-linearizing around the Balanced Growth Path

This appendix provides the details of how we derived the first-order conditions and detrended equations. Then we presents the derivations of the log-linearized equations of the model as well as the steady state relationships.

C.2 Derivations of the First-Order Conditions and Detrended Equations

C.2.1 Household's Problem

The Lagrangian for household problem is given by

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg[N_t \frac{(c_t \Phi(l_t))^{1-\eta} - 1}{1-\eta}$$
$$-\lambda_t((1+\tau^c)c_t N_t + \overbrace{(k_{t+1}N_{t+1} - (1-\delta)k_t N_t)}^{x_t N_t} + b_t N_t - R_t^b b_{t-1} N_{t-1} - s_t N_t - m_t N_t$$
$$- (1-\tau_t^l) w_t n_t N_t - (r_t - \tau^k(r_t - \delta))k_t N_t) \Bigg].$$

Then, the first-order conditions for c_t, l_t, k_{t+1} and b_t are;

$$
\begin{aligned}
[c_t] &: N_t c_t^{-\eta}(\Phi(l_t))^{1-\eta} - \lambda_t(1+\tau^c)N_t = 0 \\
&\Leftrightarrow \lambda_t = \frac{c_t^{-\eta}(\Phi(l_t))^{1-\eta}}{1+\tau^c} && \text{(C.1)} \\
[l_t] &: N_t c_t^{1-\eta}(\Phi(l_t))^{-\eta}\Phi'(l_t) + \lambda_t(1-\tau_t^l)w_t N_t = 0 \\
&\Leftrightarrow c_t^{1-\eta}(\Phi(l_t))^{-\eta}\Phi'(l_t) + \lambda_t(1-\tau_t^l)w_t = 0 && \text{(C.2)} \\
[k_{t+1}] &: -\lambda_t N_{t+1} + \beta\mathbb{E}_t[\lambda_{t+1}((1-\delta) + (r_{t+1} - \tau^k(r_{t+1}-\delta)))N_{t+1}] = 0 \\
&\Leftrightarrow 1 = \mathbb{E}_t\left[\beta\left(\frac{\lambda_{t+1}}{\lambda_t}\right)R_{t+1}^k\right] && \text{(C.3)} \\
[b_t] &: -\lambda_t N_t + \beta\mathbb{E}_t\left[\lambda_{t+1}R_{t+1}^b N_t\right] = 0 \\
&\Leftrightarrow 1 = \mathbb{E}_t\left[\beta\left(\frac{\lambda_{t+1}}{\lambda_t}\right)R_{t+1}^b\right], && \text{(C.4)}
\end{aligned}
$$

where $R_t^k := (1-\delta)+(r_t-\tau^k(r_t-\delta))$. These expressions are unchanged from the original Uhlig (2010) model. In order to detrend the variables, we divide by A_t. Then, equation (C.1) now becomes:

$$
\begin{aligned}
\lambda_t &= \frac{(c_t/A_t)^{-\eta}A_t^{-\eta}(\Phi(l_t))^{1-\eta}}{1+\tau^c} \\
\Leftrightarrow \widetilde{\lambda}_t &= \frac{(\widetilde{c}_t)^{-\eta}(\Phi(l_t))^{1-\eta}}{1+\tau^c} && \text{(C.5)}
\end{aligned}
$$

where $\lambda_t := \lambda_t A_t^{\eta}$. Equation (C.2) is now given by:

$$
\begin{aligned}
(c_t/A_t)^{1-\eta}(\Phi(l_t))^{-\eta}\Phi'(l_t) + \lambda_t A_t^{\eta}(1-\tau_t^l)(w_t/A_t) = 0 \\
\Leftrightarrow (\widetilde{c}_t)^{1-\eta}(\Phi(l_t))^{-\eta}\Phi'(l_t) + \widetilde{\lambda}_t(1-\tau_t^l)\widetilde{w}_t = 0. && \text{(C.6)}
\end{aligned}
$$

Similarly, equation (C.3) and (C.4) are rewritten as:

$$
\begin{aligned}
1 =& \mathbb{E}_t \left[\beta \left(\frac{\lambda_{t+1} A_{t+1}^{\eta}}{\lambda_t A_t^{\eta}} \right) \left(\frac{A_t^{\eta}}{A_{t+1}^{\eta}} \right) R_{t+1} \right] \\
=& \mathbb{E}_t \left[\beta \left(\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t} \right) \left(\frac{A_t}{A_{t+1}} \right)^{\eta} R_{t+1} \right]
\end{aligned} \tag{C.7}
$$

where $R_{t+1} = R_{t+1}^b = R_{t+1}^k$.

C.2.2 The Balanced Growth Path

Along the balanced growth path of growing consumption ($c_{+1} = \zeta^A c$) and constant labor ($l_{+1} = l$), the asset price equation (C.7) becomes:

$$
\begin{aligned}
1 =& \beta \left(\frac{1}{\zeta^A} \right)^{\eta} R \\
\Leftrightarrow R =& \frac{1}{\beta (\zeta^A)^{-\eta}} = \frac{\zeta^A}{\widetilde{\beta}},
\end{aligned} \tag{C.8}
$$

where $\widetilde{\beta} := \beta (\zeta^A)^{1-\eta}$ and $R = R^b = R^k$.

C.2.3 Production

The production function is expressed in per capita terms as follows:

$$
\begin{aligned}
y_t := \frac{Y_t}{N_t} &= A_t^{\theta} \frac{K_t^{1-\theta} L_t^{\theta}}{N_t^{1-\theta} N_t^{\theta}} \left(K_t^G \right)^{\theta_G} \\
&= A_t^{\theta} k_t^{1-\theta} l_t^{\theta} (K_t^G)^{\theta_G}.
\end{aligned} \tag{C.9}
$$

Note that K_t^G is not expressed in per capita terms since we assume that K_t^G is a pure public good, and therefore, it is not affected by the number of people. The first-order condition delivers capital rental rates and wages,

$$[k_t] : (1-\theta)A_t^{\theta}k_t^{-\theta}l_t^{\theta}(K_t^G)^{\theta_G} = r_t \tag{4.14}$$

$$[l_t] : \quad \theta A_t^{\theta}k_t^{1-\theta}l_t^{\theta-1}(K_t^G)^{\theta_G} = w_t \tag{C.10}$$

C.2.4 Government

The government budget constraint is written as

$$\begin{aligned} D_t =& G_t + S_t + R_t^b B_{t-1} - \tau^c C_t - \tau^k(r_t - \delta)K_t \\ =& B_t + \tau_t^l w_t L_t \end{aligned}$$

Dividing both side by N_t gives

$$\begin{aligned} d_t =& g_t + s_t + R_t^b b_{t-1}\left(\frac{1}{h_t}\right) - \tau^c c_t - \tau^k(r_t - \delta)k_t \\ =& b_t + \tau_t^l w_t l_t \end{aligned}$$

Debt and tax dynamics are

$$\begin{aligned} \psi(D_t - A_tN_tD) &= \tau_t^l w_t L_t - A_tN_t\tau^l wL, \\ (1-\psi)(D_t - A_tN_tD) &= B_t - A_tN_tB. \end{aligned}$$

Dividing both sides of these two equations by N_t gives

$$\psi(d_t - A_t D) = \tau_t^l w_t l_t - A_t \tau^l w L$$

$$(1-\psi)(d_t - A_t D) = b_t - A_t B.$$

Aggregate government purchases G_t consists of basic government purchases G_t^B and government investment I_t^G. That is,

$$G_t = G_t^B + I_t^G$$

$$\Leftrightarrow g_t = g_t^B + I_t^G.$$

since we assume that I_t^G is not affected by the number of people.

C.2.5 Resource Constraint

$$C_t + X_t + G_t = Y_t + M_t$$

$$\Leftrightarrow c_t + x_t + g_t = y_t + m_t$$

C.3 Derivations of Log-linearized Equations

C.3.1 Labor Supply

Since the first-oeder conditions for c_t and l_t are unchanged, the labor supply equation is also unchanged from the original Uhlig model. In fact, substituting (C.5) into the (C.6),

$$\begin{aligned}(\widetilde{c}_t)^{1-\eta}(\Phi(l_t))^{-\eta}\Phi'(l_t) + \frac{\overbrace{(\widetilde{c}_t\Phi(l_t))^{-\eta}\Phi(l_t)}^{=\widetilde{\lambda}_t}}{1+\tau^c}(1-\tau_t^l)\widetilde{w}_t = 0 \\ \Leftrightarrow \widetilde{c}_t\Phi'(l_t) + \frac{1}{1+\tau^c}\Phi(l_t)(1-\tau_t^l)\widetilde{w}_t = 0\end{aligned} \tag{C.11}$$

In the steady state,

$$\begin{aligned}c\ \Phi'(l) + \frac{1}{1+\tau^c}\Phi(l)(1-\tau^l)w = 0 \\ \Leftrightarrow -\frac{\Phi'(l)l}{\Phi(l)} = \frac{1-\tau^l}{1+\tau^c}\theta\frac{y}{c} \equiv \kappa,\end{aligned}$$

where we use the steady state relationship of $\theta = wl/y$. Taking the first-order Taylor expansion of (C.11) around the steady state,

$$\begin{aligned}0 =& \Phi'(l)c\frac{(\widetilde{c}_t - c)}{c} + \left(c\ \Phi''(l) + \frac{1-\tau^l}{1+\tau^c}\Phi'(l)w\right)l\frac{(l_t-l)}{l} \\ &+ \frac{1-\tau^l}{1+\tau^c}\Phi(l)w\frac{(\widetilde{w}_t - w)}{w} - \frac{w}{1+\tau^c}\Phi(l)\underbrace{(\tau_t^l - \tau)}_{=\widehat{\tau}_t^l}\end{aligned}$$

Dividing both sides by $\Phi'(l)c$,

$$
\begin{aligned}
0 =& \widehat{c}_t + \left(\frac{\Phi''(l)l}{\Phi'(l)} + \underbrace{\frac{1-\tau^l}{1+\tau^c}\frac{wl}{c}}_{=\kappa}\right)\widehat{l}_t + \frac{1-\tau^l}{1+\tau^c}\frac{\Phi(l)}{\Phi'(l)}\frac{w}{c}\widehat{w}_t - \frac{1}{1+\tau^c}\frac{\Phi(l)}{\Phi'(l)}\frac{w}{c}\widehat{\tau}^l_t \\
\Leftrightarrow 0 =& \widehat{c}_t + \underbrace{\left(\frac{\Phi''(l)l}{\Phi'(l)} + \kappa\right)}_{=1/\omega_S}\widehat{l}_t + \frac{1-\tau^l}{1+\tau^c}\left(-\overbrace{\frac{1+\tau^c}{1-\tau^l}\frac{c}{w}}^{\frac{\Phi(l)}{\Phi'(l)}}\right)\frac{w}{c}\widehat{w}_t - \frac{1}{1+\tau^c}\left(-\overbrace{\frac{1+\tau^c}{1-\tau^l}\frac{c}{w}}^{\frac{\Phi(l)}{\Phi'(l)}}\right)\frac{w}{c}\widehat{\tau}^l_t \\
\Leftrightarrow \widehat{w}_t =& \widehat{c}_t + \frac{1}{\omega_S}\widehat{l}_t + \frac{1}{1-\tau^l}\widehat{\tau}^l_t
\end{aligned}
$$

C.3.2 Lagrange Multiplier

Taking the first-order Taylor expansion of (C.5),

$$
\begin{aligned}
\lambda \exp(\widehat{\lambda}_t) =& \lambda - \eta\frac{c^{-\eta}\Phi(l)^{1-\eta}}{1+\tau^c}\frac{(c_t - c)}{c} + (1-\eta)\frac{c^{-\eta}\Phi(l)^{-\eta}\Phi'(l)l}{1+\tau^c}\frac{(l_t - l)}{l} \\
\Leftrightarrow \lambda\widehat{\lambda}_t =& -\eta\lambda\widehat{c}_t + (1-\eta)\underbrace{\frac{c^{-\eta}\Phi(l)^{1-\eta}}{1+\tau^c}}_{=\lambda}\underbrace{\frac{\Phi'(l)l}{\Phi(n)}}_{=-\kappa}\widehat{l}_t \\
\Leftrightarrow \widehat{\lambda}_t =& -\eta\widehat{c}_t - (1-\eta)\kappa\widehat{l}_t,
\end{aligned}
$$

where we use the steady state relationship of $\lambda = c^{-\eta}\Phi(l)^{1-\eta}/(1+\tau^c)$.

C.3.3 Asset Price Equations for the Bond Return and Capital Return

Log-linearizing (C.7) gives:

$$
\begin{aligned}
1 &= \mathbb{E}_t\left[\beta\left(\frac{1}{\zeta^A}\right)^{\eta} R\exp(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \eta\widehat{\zeta}^A_{t+1} + \widehat{R}_{t+1})\right] \\
1 &= \mathbb{E}_t\left[1 + \widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \eta\widehat{\zeta}^A_{t+1} + \widehat{R}_{t+1}\right] \\
0 &= \mathbb{E}_t\left[\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \eta\widehat{\zeta}^A_{t+1} + \widehat{R}_{t+1}\right].
\end{aligned}
$$

C.3.4 Capital Return

$$R_t^k = (1-\delta) + (r_t - \tau^k(r_t - \delta))$$
$$= (1-\delta) + (1-\tau^k)r_t + \tau^k\delta \tag{C.12}$$

In the steady state,

$$R^k = (1-\delta) + (1-\tau^k)r + \tau^k\delta, \quad \text{or}$$
$$r = \frac{R^k - 1 + (1-\tau^k)\delta}{(1-\tau^k)} \tag{C.13}$$

Log-linearizing (C.12) gives

$$R\exp(\widehat{R}_t^k) = (1-\delta) + (1-\tau^k)r\exp(\widehat{r}_t) + \tau^k\delta$$
$$\Leftrightarrow \widehat{R}_t^k = (1-\tau^k)\frac{r}{R}\widehat{r}_t$$

Substituting (C.13),

$$\widehat{R}_t^k = (1-\tau^k)\left(\frac{R - 1 + (1-\tau^k)\delta}{(1-\tau^k)}\right)\frac{1}{R}\widehat{r}_t$$
$$= \left(1 - (1-(1-\tau^k)\delta)\frac{1}{R}\right)\widehat{r}_t$$

Substituting (C.8),

$$\widehat{R}_t^k = \left(1 - (1-(1-\tau^k)\delta)\frac{\tilde{\beta}}{\bar{\zeta}}\right)\widehat{r}_t$$

C.3.5 Private Capital Accumulation

The derivation of the log-linearized equation for private capital accumulation is the same as that of Subsection A.2.5 in Appendix A. Therefore,

$$\widehat{k}_{t+1} = \left(1 - \frac{\tilde{\delta}}{\zeta^A \zeta^N}\right) \widehat{k}_t + \frac{\tilde{\delta}}{\zeta^A \zeta^N} \widehat{x}_t - \widehat{\zeta}^A_{t+1} - \widehat{\zeta}^N_{t+1},$$

where

$$\tilde{\delta} := \zeta^A \zeta^N - 1 + \delta = x/k. \tag{C.14}$$

C.3.6 Production Function

Divide both sides of (C.9) by A_t to denote detrended equations,

$$\begin{aligned} \frac{y_t}{A_t} &= \frac{A_t^\theta k_t^{1-\theta}}{A_t^\theta A_t^{1-\theta}}\ l_t^\theta \left(\frac{K_t^G}{A_t}\right)^{\theta_G} A_t^{\theta_G} \\ \Leftrightarrow \widetilde{y}_t &= \widetilde{k}_t^{1-\theta} l_t^\theta (\widetilde{K}_t^G)^{\theta_G} A_t^{\theta_G} \\ \Leftrightarrow \widehat{y}_t &= (1-\theta)\widehat{k}_t + \theta\ \widehat{l}_t + \theta^G(\widehat{K}_t^G + \widehat{A}_t). \end{aligned}$$

C.3.7 Wage

Divide both sides of (C.10) by A_t to denote detrended equations,

$$\begin{aligned} \frac{w_t}{A_t} &= \theta \frac{A_t^\theta k_t^{1-\theta}}{A_t^\theta A_t^{1-\theta}}\ l_t^{\theta-1} \left(\frac{K_t^G}{A_t}\right)^{\theta_G} A_t^{\theta_G} \\ \Leftrightarrow \widetilde{w}_t &= \theta \widetilde{k}_t^{1-\theta} l_t^{\theta-1} (\widetilde{K}_t^G)^{\theta_G} A_t^{\theta_G} \\ \Leftrightarrow \widehat{w}_t &= (1-\theta)(\widehat{k}_t - \widehat{l}_t) + \theta_G(\widehat{K}_t^G + \widehat{A}_t). \end{aligned}$$

C.3.8 Capital Rental Rates

To derive $\widehat{r}_t$, we make use of the fact that the sum of factor earnings exhaust the total output.

$$\begin{aligned}
1 =& \frac{r_t k_t}{y_t} + \underbrace{\frac{w_t l_t}{y_t}}_{=\theta} \\
\Leftrightarrow 1 - \theta =& r_t \frac{k_t/A_t}{y_t/A_t} = \frac{\widetilde{k}_t}{\widetilde{y}_t} \\
\Leftrightarrow r_t =& (1-\theta)\frac{\widetilde{y}_t}{\widetilde{k}_t} \\
\Leftrightarrow \widehat{r}_t =& \widehat{y}_t - \widehat{k}_t
\end{aligned}$$

C.3.9 Technology Growth

$$\zeta_t^A = \frac{A_t}{A_{t-1}} \Leftrightarrow \widehat{\zeta}_t^A = \widehat{A}_t - \widehat{A}_{t-1}$$

C.3.10 Public Capital Accumulation

Public capital accumulation is expressed as follows:

$$K_{t+1}^G = (1-\delta)K_t^G + I_t^G.$$

Divide both sides by A_t to denote detrended equations,

$$\begin{aligned}
\frac{K_{t+1}^G}{A_{t+1}} \frac{A_{t+1}}{A_t} &= (1-\delta)\frac{K_t^G}{A_t} + \frac{I_t^G}{A_t} \\
\Leftrightarrow \widetilde{K}_{t+1}^G \zeta_{t+1}^A &= (1-\delta)\widetilde{K}_t^G + \widetilde{I}_t^G.
\end{aligned}$$

Log-linearizing gives

$$
\begin{aligned}
K^G\zeta^A \exp(\widehat{K}^G_{t+1} + \widehat{\zeta}^A_{t+1}) &= (1-\delta)K^G \exp(\widehat{K}^G_t) + I^G \exp(\widehat{I}^G_t) \\
\Leftrightarrow \zeta^A(\widehat{K}^G_{t+1} + \widehat{\zeta}^A_{t+1}) &= (1-\delta)\widehat{K}^G_t + \frac{I^G}{K^G}\widehat{I}^G_t \\
\Leftrightarrow \widehat{K}^G_{t+1} &= \frac{1-\delta}{\zeta^A}\widehat{K}^G_t + \frac{\tilde{\delta}_G}{\zeta^A} I^G_t - \widehat{\zeta}^A_{t+1} \\
\Leftrightarrow \quad &= \left(1 - \frac{\tilde{\delta}_G}{\zeta^A}\right)\widehat{K}^G_t + \frac{\tilde{\delta}_G}{\zeta^A} I^G_t - \widehat{\zeta}^A_{t+1},
\end{aligned}
$$

where we use the steady state relationship of $I^G/K^G = \zeta^A - 1 + \delta =: \tilde{\delta}_G$.

C.3.11 Government Budget Constraint

The government budget constraint per capita in terms of detrended variables is:

$$
\begin{aligned}
\frac{d_t}{A_t} &= \frac{g_t}{A_t} + \frac{s_t}{A_t} + R^b_t \frac{b_{t-1}}{A_{t-1}} \frac{A_{t-1}}{A_t}\left(\frac{1}{\zeta^N_t}\right) - \tau^c \frac{c_t}{A_t} - \tau^k(r_t - \delta)\frac{k_t}{A_t} \\
\Leftrightarrow \widetilde{d}_t &= \widetilde{g}_t + \widetilde{s}_t + R^b_t \widetilde{b}_{t-1}\left(\frac{1}{\zeta^A_t \zeta^N_t}\right) - \tau^c \widetilde{c}_t - \tau^k(r_t - \delta)\widetilde{k}_t
\end{aligned}
$$

Log-linearizing gives:

$$
d\widehat{d}_t = g\widehat{g}_t + s\widehat{s}_t + \frac{R^b b}{\zeta^A \zeta^N}(\widehat{R}^b_t + \widehat{b}_{t-1} - \widehat{\zeta}^A_t - \widehat{\zeta}^N_t) - \tau^c c\widehat{c}_t - \tau^k(r\widehat{r}_t + (r-\delta)k\widehat{k}_t).
$$

To denote $\widehat{d}_t^* := (\widetilde{d}_t - d)/y$, $\widehat{g}_t^* := (\widetilde{g}_t - g)/y$, $\widehat{s}_t^* := (\widetilde{s}_t - s)/y$, and $\widehat{b}_t^* := (\widetilde{b}_t - b)/y$, we divide both sides by y,

$$\begin{aligned}\widehat{d}_t^* &= \widehat{g}_t^* + \widehat{s}_t^* + \frac{R^b b}{y\zeta^A\zeta^N}(\widehat{R}_t^b - \widehat{\zeta}_t^A - \widehat{\zeta}_t^N) + \frac{R^b}{\zeta^A\zeta^N}\widehat{b}_{t-1}^* - \tau^c\frac{c}{y}\widehat{c}_t - \tau^k\frac{rk}{y}r_t - \tau^k\left(\frac{rk}{y} - \frac{k}{y}\delta\right)\widehat{k}_t \\ &= \widehat{g}_t^* + \widehat{s}_t^* + \frac{b}{\widetilde{\beta}y\zeta^N}(\widehat{R}_t^b - \widehat{\zeta}_t^A - \widehat{\zeta}_t^N) + \frac{1}{\widetilde{\beta}\zeta^N}\widehat{b}_{t-1}^* - \tau^c\frac{c}{y}\widehat{c}_t - \tau^k(1-\theta)\widehat{r}_t - \tau^k\left(1 - \theta - \frac{\delta}{\tilde{\delta}}\frac{x}{y}\right)\widehat{k}_t\end{aligned}$$

since $R^b = \zeta^A/\widetilde{\beta}$, $rk/y = 1-\theta$ by Cobb-Douglas production function, and $k/y = (1/\tilde{\delta})(x/y)$ by (C.14).

C.3.12 Low of Motion of Labor Tax

In terms of detrended variables,

$$\begin{aligned}\psi(d_t - A_t D) &= \tau_t^l w_t l_t - A_t\tau^l\bar{w}L \\ \Leftrightarrow \psi(\widetilde{d}_t - D) &= \tau_t^l\widetilde{w}_t l_t - \tau^l\bar{w}L\end{aligned}$$

At the steady state,

$$\begin{aligned}\psi(d - D) &= \tau^l wl - \tau^l\bar{w}L \\ \Leftrightarrow \psi d(1 - AN) &= \tau^l wl(1 - AN) \\ \Leftrightarrow \psi d &= \tau^l wl,\end{aligned}$$

where we use the steady state relationship of $D = ANd$, $\bar{w} = Aw$, and $L = lN$ with A and N denoting the steady state values of A_t and N_t respectively. Log-linearizing gives

$$\psi(d(1+\widehat{d}_t) - D) = \tau^l wl(1 + \widehat{\tau}^l_t + \widehat{w}_t + \widehat{l}_t) - \tau^l wL$$

$$\Leftrightarrow \psi d\widehat{d}_t = \tau^l wl(\widehat{\tau}^l_t + \widehat{w}_t + \widehat{l}_t)$$

$$\Leftrightarrow \psi\widehat{d}^*_t = \tau^l \frac{wl}{y}(\widehat{\tau}^l_t + \widehat{w}_t + \widehat{l}_t)$$

$$\Leftrightarrow \quad = \theta\widehat{\tau}^l_t + \theta\tau^l(\widehat{w}_t + \widehat{l}_t),$$

where we use $wl/y = \theta$

C.3.13 Low of Motion of Debt

In terms of detrended variables,

$$(1-\psi)(d_t - A_t D) = b_t - A_t B$$

$$\Leftrightarrow (1-\psi)(\widetilde{d}_t - D) = \widetilde{b}_t - B$$

At the steady state,

$$(1-\psi)(d - D) = b - B$$

$$\Leftrightarrow (1-\psi)d(1 - AN) = b(1 - AN)$$

$$\Leftrightarrow (1-\psi)d = b,$$

where we use the steady state relationship of $B = ANb$. Log-linearizing gives

$$(1-\psi)(d(1+\widehat{d}_t) - D) = b(1+\widehat{b}_t) - B.$$

$$\Leftrightarrow (1-\psi)d\widehat{d}_t = b\widehat{b}_t$$

$$\Leftrightarrow (1-\psi)\widehat{d}_t^* = \widehat{b}_t^*$$

C.3.14 Aggregate Government Purchases

$$\widetilde{g}_t = \widetilde{g}_t^B + \widetilde{I}_G$$

$$\Leftrightarrow g\exp(\widehat{g}_t) = g^B\exp(\widehat{g}_t^B) + I^G\exp(\widehat{I}_t^G)$$

Dividing by y

$$\frac{g}{y}\widehat{g}_t = \frac{g^B}{y}\widehat{g}_t^B + \frac{I^G}{y}\widehat{I}_t^G$$

$$\Leftrightarrow \widehat{g}_t^* = (\widehat{g}_t^{*B}) + \frac{I^G}{y}\widehat{I}_t^G,$$

Note that $g_t^{*B} = (\widetilde{g}_t^B - g^B)/y$.

C.3.15 Resource Constraint

$$\widetilde{y}_t = \widetilde{c}_t + \widetilde{x}_t + \widetilde{g}_t$$

$$\Leftrightarrow y\exp(\widehat{y}_t) = y\exp(\widehat{c}_t) + y\exp(\widehat{x}_t) + y\exp(\widehat{g}_t)$$

$$\Leftrightarrow \widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{x}{y}\widehat{x}_t + \widehat{g}_t^*.$$

Note that $g_t^* = (\widetilde{g}_t - g)/y$.

C.4 Derivation of a Second-Order Approximation to a Household's Welfare per Capita

The period utility per capita is given by

$$u(c_t, l_t) = \frac{(c_t \Phi(l_t))^{1-\eta} - 1}{1-\eta}.$$

The first, the second, and the cross derivatives evaluated at a steady state (c, l) are:

$$\begin{aligned}
u_c &= c^{-\eta}\Phi(l)^{1-\eta} \\
u_{cc} &= -\eta c^{-\eta-1}\Phi(l)^{1-\eta} \\
u_l &= c^{1-\eta}\Phi(l)^{-\eta}\Phi'(l) \\
u_{ll} &= c^{1-\eta}[-\eta\Phi(l)^{-\eta-1}(\Phi'(l))^2 + \Phi(l)^{-\eta}\Phi''(l)] \\
u_{cl} &= (1-\eta)c^{-\eta}\Phi(l)^{-\eta}\Phi'(l).
\end{aligned}$$

For later use, we calculate the following using derivatives above:

$$
\begin{aligned}
\frac{u_l l}{u_c c} &= \frac{c^{1-\eta}\Phi(l)^{-\eta}\Phi'(l)l}{c^{1-\eta}\Phi(l)^{1-\eta}} = \frac{\Phi'(l)l}{\Phi(l)} = -\kappa \\
\frac{(u_{cc}c + u_c)c}{u_c c} &= \frac{u_{cc}c^2}{u_c c} + 1 = \frac{-\eta c^{1-\eta}\Phi(l)^{1-\eta}}{c^{1-\eta}\Phi(l)^{1-\eta}} + 1 = 1 - \eta \\
\frac{(u_{ll}l + u_l)l}{u_c c} &= \frac{u_{ll}l^2}{u_c c} + \frac{u_l l}{u_c c} = \frac{c^{1-\eta}[-\eta\Phi(l)^{-\eta-1}(\Phi'(l))^2 + \Phi(l_t)^{-\eta}\Phi''(l)]l^2}{c^{1-\eta}\Phi(l)^{1-\eta}} - \kappa \\
\Leftrightarrow \qquad &= -\eta\frac{(\Phi'(l))^2 l^2}{\Phi(l)^2} + \frac{\Phi''(l)l^2}{\Phi(l)} - \kappa \\
\Leftrightarrow \qquad &= -\eta\kappa^2 + \underbrace{\frac{\Phi''(l)l}{\Phi'(l)}}_{=1/\omega_S - \kappa} \underbrace{\frac{\Phi'(l)l}{\Phi(l)}}_{=-\kappa} - \kappa \\
\Leftrightarrow \qquad &= -\eta\kappa^2 + \kappa^2 - \frac{\kappa}{\omega_S} - \kappa \\
\Leftrightarrow \qquad &= (1-\eta)\kappa^2 - \left(1 + \frac{1}{\omega_S}\right)\kappa \\
\frac{u_{cl}cl}{u_c c} &= \frac{(1-\eta)c^{1-\eta}\Phi(l)^{-\eta}\Phi'(l)l}{c^{1-\eta}\Phi(l)^{1-\eta}} = (1-\eta)\frac{\Phi'(l)l}{\Phi(l)} = -(1-\eta)\kappa
\end{aligned}
$$

The second-order Taylor expansion of u_t around a steady state (c, l) yields

$$
\begin{aligned}
u_t &\simeq u + u_c c\ \hat{c}_t + u_l l\ \hat{l}_t + \frac{1}{2}(u_{cc}c + u_c)c\ \hat{c}_t^2 + \frac{1}{2}(u_{ll}l + u_l)l\ \hat{l}_t^2 + u_{cl}cn\ \hat{c}_t\hat{l}_t \\
\Leftrightarrow \frac{u_t - u}{u_c c} &= \hat{c}_t + \left(\frac{u_l l}{u_c c}\right)\ \hat{l}_t + \frac{1}{2}\left[\frac{(u_{cc}c + u_c)c}{u_c c}\right]\ \hat{c}_t^2 + \frac{1}{2}\left[\frac{(u_{ll}n + u_l)n}{u_c c}\right]\ \hat{l}_t^2 + \left(\frac{u_{cl}cn}{u_c c}\right)\ \hat{c}_t\hat{l}_t \\
&= \hat{c}_t - \kappa\ \hat{l}_t + \frac{1-\eta}{2}\ \hat{c}_t^2 + \frac{1}{2}\left[(1-\eta)\kappa^2 - \left(1 + \frac{1}{\omega_S}\right)\kappa\right]\ \hat{l}_t^2 - (1-\eta)\kappa\ \hat{c}_t\hat{l}_t \\
&= \left(\hat{c}_t + \frac{1-\eta}{2}\ \hat{c}_t^2\right) - \kappa\left(\hat{l}_t - \frac{1}{2}\left[(1-\eta)\kappa - \left(1 + \frac{1}{\omega_S}\right)\right]\ \hat{l}_t^2\right) - (1-\eta)\kappa\ \hat{c}_t\hat{l}_t \\
&= \left(\hat{c}_t + \frac{1-\eta}{2}\ \hat{c}_t^2\right) - \kappa\left(\hat{l}_t + \frac{1}{2}\left[(\eta-1)\kappa + \left(1 + \frac{1}{\omega_S}\right)\right]\ \hat{l}_t^2\right) - (1-\eta)\kappa\ \hat{c}_t\hat{l}_t
\end{aligned}
$$

Hence, a second-order approximation to the household's welfare losses can be expressed as a fraction of steady state consumption:

$$
\begin{aligned}
\mathbb{W} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{u_t - u}{u_c c} \right) \\
&= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\hat{c}_t + \frac{1-\eta}{2} \, \hat{c}_t^2 \right) - \kappa \left(\hat{l}_t + \frac{\varphi}{2} \, \hat{l}_t^2 \right) - (1-\eta)\kappa \; \hat{c}_t \hat{l}_t \right],
\end{aligned}
$$

where $\varphi := (\eta - 1)\kappa + \left(1 + \frac{1}{\omega_S}\right)$. We can calculate this value since we know the path of $\hat{c}_t$ and $\hat{l}_t$.

Bibliography

[1] An, S., and F. Schorfheide (2007), "Bayesian Analysis of DSGE Models," *Econometric Reviews*, Vol. 26(2-4), 113-172.

[2] Andolfatto, D. (1996). "Business cycles and labor-market search," *The American Economic Review*, Vol. 86(1), 112-132.

[3] Atkinson A. B., T. Piketty, and E. Saez (2011). "Top incomes in the long run of history," *Journal of Economic Literature*, Vol. 49(1), 3-71.

[4] Auerbach, A., and Y. Gorodnichenko (2012). "Fiscal Multipliers in Recession and Expansion," NBER Chapters, in: Fiscal Policy after the Financial Crisis National Bureau of Economic Research, Inc.

[5] Baxter, M., and R. King (1993), "Fiscal Policy in General Equilibrium," *The American Economic Review*, Vol. 83(3), 315-334.

[6] Ben-Gad, M. (2004). "The economic effects of immigration-a dynamic analysis," *Journal of Economic Dynamics and Control*, Vol. 28(9), 1825-1845.

[7] Ben-Gad, M. (2008). "Capital-skill complementarity and the immigration surplus," *Review of Economic Dynamics*, Vol. 11(2), 335-365.

[8] Ben-Gad, M. (2012). "On deficit bias and immigration," Working Papers 12/09, Department of Economics, City University London.

[9] Blanchard, O. (1997). "The Medium Run," Brookings Papers on Economic Activity, Economic Studies Program, The Brookings Institution, Vol. 28(2), 89-158.

[10] Blanchard, O., and J. Galí (2007). "Real wage rigidities and the New Keynesian model," *Journal of Money, Credit and Banking*, Vol. 39(s1), 35-65

[11] Blanchard, O., and J. Galí (2010). "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment," *American Economic Journal: Macroeconomics*, Vol. 2(2), 1-30.

[12] Brückner, M., and E. Pappa (2012). "Fiscal Expansions, Unemployment and Labor Force Participation," *International Economic Review*, Vol. 53(4), 1205-1228.

[13] Cahuc, P., and A. Zylberberg, (2004). *Labor Economics*, MIT Press.

[14] Cantore, C., F. Ferroni, and M. León-Ledesma (2012). "The dynamics of hours worked and technology," Banco de España Working Papers 1238, Banco de España.

[15] Canova, F. (2007). *Methods for Applied Macroeconomic Research*, Princeton University Press.

[16] Calvo, G. (1983). "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, Vol. 12(3), 383-398.

[17] Chassamboulli, A., and T. Palivos (2014). "A Search - Equilibrium Approach to the Effects of Immigration on Labor Market Outcomes," *International Economic Review*, Vol. 55(1), 111-129.

[18] Checherita-Westphal, C., and P. Rother (2012). “The impact of high government debt on economic growth and its channels: An empirical investigation for the euro area,” *European Economic Review*, Vol. 56(7), 1392-1405.

[19] Christiano, L., M. Eichenbaum, and C. Evans (2005). “Nominal rigidities and the dynamic effects of a shock to monetary policy,” *Journal of Political Economy*, Vol. 113(1), 1-45.

[20] Christiano, L., R. Motto, and M. Rostagno (2014), “Risk Shocks,” *The American Economic Review*, Vol. 104(1), 27-65.

[21] Cociuba, S., E. Prescott, and A. Ueberfeldt (2009). “U.S. Hours and Productivity Behavior Using CPS Hours worked Data: 1947-III to 2009-III,” Mimeo.

[22] Diamond, P. (1982). “Wage determination and efficiency in search equilibrium,” *The Review of Economic Studies*, Vol. 49(2), 217-227.

[23] Del Negro, M., S. Eusepi, M. Giannoni, A. Sbordone, A. Tambalotti, M. Cocci, R. Hasegawa, H. Linder (2013). “The FRBNY DSGE Model,” No. 647. Staff Report, Federal Reserve Bank of New York.

[24] Doepke, M., M. Hazan, and Y. Maoz (2012). “The Baby Boom and World War II: A Macroeconomic Analysis. Working paper Northwestern University.

[25] Duenhaupt P. (2011). “The Impact of Financialization on Income Distribution in the USA and Germany: A Proposal for a New Adjusted Wage Share,” IMK working Paper, Hans Böckler Foundation.

[26] Durand, J., D. Massey, and E. Parrado (1999). “The New Era of Mexican Migration to the United States. *Journal of American History*, Vol. 86(2), 518-536.

[27] Dustman, C., T. Frattini, and C. Halls (2010). "Assessing the Fiscal Costs and Benefits of A8 Migration to the UK," *Fiscal Studies*, Vol. 31(1), 1-41.

[28] Dustmann, C., T. Frattini, and I. Preston (2008). "The Effect of Immigration along the Distribution of Wages," CReAM Discussion Paper No.03/08.

[29] Dustmann, C., and T. Frattini (2014). "The Fiscal Effects of Immigration to the UK," *The Economic Journal*, Vol.124(580), F593-F643

[30] Elsby, M., B. Hobijn, and A. Şahin (2013). "The decline of the US labor share," *Brookings Papers on Economic Activity*, Vol. 2, 1-63.

[31] Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle*, Princeton University Press.

[32] Galí, J. (2011). *Unemployment Fluctuations and Stabilization Policies: A New Keynesian Perspective*, Princeton University Press.

[33] Gertler, M., and P. Karadi (2011). "A model of unconventional monetary policy," *Journal of monetary Economics*, Vol. 58(1), 17-34.

[34] Gertler, M., and N. Kiyotaki (2010). "Financial intermediation and credit policy in business cycle analysis," *Handbook of Monetary Economics*, Vol. 3(11), 547-599.

[35] Hall, R. (2005). "Employment fluctuations with equilibrium wage stickiness," *The American Economic Review*, Vol. 95(1), 50-65.

[36] Heathcote, J., F. Perri, and G. L. Violante (2010). "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States: 1967-2006," *Review of Economic Dynamics*, Vol. 13(1), 15-51.

[37] Jermann, U., and V. Quadrini (2012), "Macroeconomic Effects of Financial Shocks," *American Economic Review*, Vol. 102(1), 238-271.

[38] Kaldor, N. (1957). "A Model of Economic Growth," *The Economic Journal*, Vol. 67 (268), 591—624

[39] Kamps, C. (2004). *The Dynamic Macroeconomic Effects of Public Capital: Theory and Evidence for OECD Countries*, Springer.

[40] Karabarbounis, L., and B. Neiman (2013). "The global decline of the labor share," No. w19136, National Bureau of Economic Research.

[41] Krusell, P., L. Ohanian, V. Ros - Rull, and G. Violante (2000). "Capital - skill complementarity and inequality: A macroeconomic analysis," *Econometrica*, Vol. 68(5), 1029-1053.

[42] Kumhof, M., D. Muir, S. Mursula, and D. Laxton (2010). *The Global Integrated Monetary and Fiscal Model (GIMF)—Theoretical Structure*, International Monetary Fund.

[43] Leeper, E. M., T. B. Walker, and S. S. Yang (2010), "Government Investment and Fiscal Stimulus," *Journal of Monetary Economics*, Vol. 57(8), 1000-1012.

[44] Lütkepohl, H. (2005). *New Introduction to Multiple Time Series Analysis*, Springer.

[45] Manacorda, M., A. Manning, and J. Wadsworth (2012). "The Impact of Immigration on the Structure of Wages: Theory and Evidence from Britain," *Journal of the European Economic Association*, Vol. 10(1), 120-151.

[46] Merz, M. (1995). "Search in the labor market and the real business cycle," *Journal of Monetary Economics*, Vol. 36(2), 269-300.

[47] Monacelli, T., R. Perotti, and A. Trigari (2010). "Unemployment fiscal multipliers," *Journal of Monetary Economics*, Vol. 57(5), 531-553.

[48] Mortensen, D., and C. Pissarides (1994). "Job creation and job destruction in the theory of unemployment," *The Review of Economic Studies*, Vol. 61(3), 397-415.

[49] Mountford, A., and H. Uhlig (2009). "What are the effects of fiscal policy shocks?," *Journal of Applied Econometrics*, Vol. 24(6), 960-992.

[50] Ortega, J. (2000). "Pareto - improving Immigration in an Economy with Equilibrium Unemployment," *The Economic Journal*, Vol. 110(460), 92-112.

[51] Ortega, J., and G. Verdugo (2014). "The impact of immigration on the French labor market: Why so different?," *Labour Economics*, Vol. 29, 14-27.

[52] Ottaviano G. I. P., and G. Peri (2012). "Rethinking the Effect of Immigration on Wages," *Journal of the European Economic Association*, Vol. 10(1), 152-197.

[53] Piketty, T., and E. Saez (2003). "Income Inequality in the United States, 1913-1998," *Quarterly Journal of Economics*, Vol. 118(1), 1-39.

[54] Piketty, T., and E. Saez (2006). "The evolution of top incomes: A historical and international perspective," *The American Economic Review, Papers and Proceedings*, Vol.96(2), 200-205.

[55] Ramey, V. (2011a). "Identifying Government Spending Shocks: It's all in the Timing," *Quarterly Journal of Economics*, Vol. 126(1), 1-50.

[56] Ramey, V. (2011b). "Can government purchases stimulate the economy?," *Journal of Economic Literature*, Vol. 49(3), 673-685.

[57] Ravn, M. (2008). "The Consumption-Tightness puzzle," NBER International Seminar on Macroeconomics 2006, 9-63.

[58] Reinhart, C., and K. Rogoff (2010). "Growth in a Time of Debt," *The American Economic Review*, Vol. 100(2), 573-78.

[59] Shimer, R. (2010). *Labor Markets and Business Cycles*, Princeton University Press.

[60] Smets, F., and R. Wouters (2003) "An estimated dynamic stochastic general equilibrium model of the euro area," *Journal of the European Economic Association*, Vol.1(5), 1123-1175.

[61] Smets, F., and R. Wouters (2007) "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *The American Economic Review*, Vol.97(3), 586-606.

[62] Storesletten, K. (2000): "Sustaining Fiscal Policy through Immigration," *Journal of Political Economy*, Vol. 108(2), 300-323.

[63] Trabandt, M., and H. Uhlig (2011). "The Laffer Curve Revisited," *Journal of Monetary Economics*, Vol. 58(4), 305-327.

[64] Trigari, A. (2006). "The Role of Search Frictions and Bargaining for Inflation Dynamics," IGIER Working Paper No. 304

[65] Uhlig, H. (1999). "A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily," In R. Marimon and A. Scott, editors *Computational Methods for the Study of Dynamic Economies*, 30-61.

[66] Uhlig, H. (2005). "What are the effects of monetary policy on output? Results from an agnostic identification procedure," *Journal of Monetary Economics*, Vol. 52(2), 381-419.

[67] Uhlig, H. (2007). "Explaining asset prices with external habits and wage rigidities in a DSGE model," *The American economic review*, Vol. 97(2), 239-243.

[68] Uhlig, H. (2010). "Some Fiscal Calculus," *The American Economic Review, Papers and Proceedings*, American Economic Association, Vol. 100(2), 30-34.

[69] Van Vliet, O., and K. Caminada (2012). "Unemployment Replacement Rates Dataset Among 34 Welfare States, 1971-2009: An Update, Extension and Modification of the Scruggs' Welfare State Entitlements Data Set," NEUJOBS Special Report NO. 2.

[70] Wadsworth, J. (2010). "The UK Labour Market and Immigration," *National Institute Economic Review*, Vol. 213(1), 35-42.

[71] Walsh, C. (2005). "Labor market search, sticky prices, and interest rate policies," *Review of Economic Dynamics*, Vol. 8(4), 829-849.

[72] Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy* , Princeton University Press.