Comment on “Unified Formalism of Andreev Reflection at a Ferromagnet/Superconductor Interface”

Recently, a “unified” Andreev reflection (AR) formalism, claimed to be “the most general to date,” was suggested [1]. Here, we show that while there are numerous works [2–7] correctly solving the problem formulated in Ref. [1] for arbitrary spin polarization \( P \), Ref. [1] fails to correctly incorporate \( P \neq 0 \) effects and is incompatible with basic physical laws.

At \( P = 0 \), the approach [1] is identical to the nonmagnetic 1D Blonder-Tinkham-Klapwijk (BTK) model [8]. For \( P \neq 0 \), they postulate an AR wave function with an additional evanescent wave at \( x < 0 \): 
\[
\psi_{\text{AR}} = a_{1\parallel}^{(0)} \exp[(\alpha + i)q^+ x] \quad \text{and} \quad \alpha = 2\sqrt{P/(1-P)}.
\]
This is completely unsubstantiated, and the proposed wave function violates charge conservation [9]: its current density is 
\[
J_{\text{AR}}(x) \propto \text{Im}[\psi_{\text{AR}}^\dagger \nabla \psi_{\text{AR}}] = q^+ (\alpha^2 \exp[2aq^+ x]).
\]

Even for a half-metal (HM, \( P = 1 \)), where this expression correctly gives \( j_{\text{AR}} = 0 \), it remains false. Reference [1] postulates an infinitesimally small decay length of the AR electron into a HM, while, in fact, it decays with a finite length, depending on the electronic structure, primarily the gap in the nonmetallic spin channel [10].

The rationale for introducing a new \( \psi_{\text{AR}} \) [1] was that (a) allegedly, the result in Ref. [3] only applies to HM with \( \tilde{\psi}_{\text{AR}} = a_{1\parallel}^{(0)} \exp(\kappa x) \) and a nonmagnetic metal with \( \tilde{\psi}_{\text{AR}} = a_{1\parallel}^{(0)} \exp(ikx) \), while the current for an instantaneous \( P \) is assumed to be a linear combination of these cases, (b) no prior work had treated the \( 0 < P < 1 \) case, and (c) one can define \( P \) for an individual electron. Regarding (a) and (b), the derivation in Ref. [3] and in other works [4–7] is rigorous for any \( P \). Regarding (c), one cannot define a single channel BTK model with an arbitrary \( P \) [11]: any given electron in a metal Andreev reflects into either a propagating or evanescent wave. Finite \( P \) means that while some current-carrying electrons propagate, others evanesce; one can only define spin polarization in a multielectron system, where the numbers of states at the Fermi surface (FS) (conductivity channels) for the two spin directions differ. If the 2D wave vector \( k_\parallel \), parallel to the interface, is conserved [12], then after quantization of \( k_\parallel \), the total number of conductivity channels in the \( x \) direction for a given spin is proportional to the area of the FS projection on the interface \( n = <N(E_F)\upsilon_{Fx}>_\text{FS} \) [3]. After summation over all states, the total current \( i \) is a linear combination (with the weights defined by \( P \)) of the solutions of the BTK model with \( P = 0 \) and \( P = 1 \). This was derived in numerous papers (see Ref. [7]).

In contrast to the “universal” \( P \) in Ref. [1], independent of electronic mass, \( k_F \)’s, or any band structure at all, the real transport spin polarization in AR depends on the overall FS properties. Moreover, there is no unique spin polarization; it depends on an experimental probe. In fact, the definition used in Ref. [1] (neglecting the velocities) does not correspond to the AR but rather to spin-polarized photoemission [13].

Reference [1] has overlooked the previous works where the posed problem has been correctly solved for an arbitrary \( P \) and has replaced this solution with an incorrect formula, postulating an unphysical wave function that does not conserve charge and has an incorrect HM limit. They calculated the current due to \( \psi_{\text{AR}} \) at \( x = 0 \), but the actual current is measured far away from the interface (where they would predict zero \( j_{\text{AR}} \) for any \( P \)). They erroneously assumed that the decay length for an electron inside the band gap is uniquely defined by the spin polarization (these two quantities are unrelated). While the formula postulated in Ref. [1] provides a fit for their experimental data, which is essentially identical to using Ref. [3], this does not constitute an argument for the validity of this approach, particularly considering unphysical predictions of this formalism in the finite \( Z \) case (kinks at zero bias and notches near the gap; see Fig. 2(c) of Ref. [1]). The inclusion of inelastic scattering by adding broadening, another claimed novelty, was done previously [6,14].

M. Eschrig, 1 A. A. Golubov, 2 I. I. Mazin, 3 B. Nadgorny, 4 Y. Tanaka, 5 O. T. Valls, 6 and Igor Žutić 7
1Department of Physics, Royal Holloway, University of London, Egham, Surrey TW20 0EX
2Faculty of Science and Technology and MESA+ Institute of Nanotechnology, University of Twente, 7500 AE Enschede, Netherlands
3Naval Research Laboratory, Washington, D.C., 20375, USA
4Department of Physics and Astronomy, Wayne State University, Detroit, Michigan 48201, USA
5Department of Applied Physics, Nagoya University, Aichi 464-8603, Japan
6School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA
7Department of Physics, University at Buffalo, SUNY, Buffalo, New York 14260, USA

Received 15 January 2013; published 24 September 2013
DOI: 10.1103/PhysRevLett.111.139703
PACS numbers: 72.25.–b, 74.25.F-, 75.47.−m, 85.75.−d


[9] The quasiparticle current conservation is also violated.

[10] Earlier works often used the limit of infinitely short decay because the final result hardly depends on this parameter, but it is important to use in the derivation a physically meaningful wave function that allows for a finite decay.


[12] This holds for a ballistic flat interface.
