

1 Investigating the Occurrence of Hierarchies of Cyclicity in Platform Carbonates

2 D.A. Pollitt^{1*}, P.M. Burgess² and V.P. Wright³

3 ¹ Chevron Corporation, 1500 Louisiana St., Houston, TX. 77018, USA.

4 ² Department of Earth Sciences, Royal Holloway University of London, Egham, Surrey,
5 TW20 0EX, UK.

6 ³ Department of Geology, National Museum of Wales, Cardiff, Cathays Park, Cardiff, CF10
7 3NP, UK.

8 *Corresponding author (e-mail: david.pollitt@chevron.com)

9 7379 words, 19 Figures, 1 Table

10 Running header: Hierarchies of Cyclicity in Carbonates

11 Keywords: Carbonates, cyclicity, hierarchy, ice-house

12 **Abstract**

13 Hierarchies of cyclicity have been described from a wide variety of carbonate platform strata, and
14 are assumed to be a consequence of Milankovitch-forced variations in accommodation, although
15 descriptions of hierarchical strata, including 'cycles' and what they constitute, are typically
16 qualitative, subjective, and in some cases difficult to reproduce. One reason for this is the lack of any
17 detailed definition of what constitutes a hierarchy, as well as the implicit and largely untested
18 nature of the assumptions underpinning most interpretations of hierarchical strata.

19 In this study we aim to investigate the response of depositional systems if they were to behave in
20 the way implied by sequence stratigraphic (hierarchical) models, to clearly state the assumptions of
21 these models and illustrate the consequences of these assumptions when they are employed in a
22 simple, internally-consistent forward model with plausible parameters.

23 We define hierarchies, in both the time-domain (chronostratigraphic) and thickness-domain
24 (stratigraphic), as two or more high-frequency sequences in which there exists a repeated trend of

25 decreasing high-frequency sequence (HFS) thickness such that within a single low-frequency
26 sequence (LFS) each high-frequency sequence is thinner than the previous sequence.

27 Based on this definition, results from 110,000 numerical model runs suggest that ordered forcing
28 via cyclical eustatic sea-level oscillations rarely results in an easily identifiable hierarchy of stacked
29 cycles. Hierarchies measured in the chronostratigraphic time-domain occur in only 9% of model
30 run cases, and in 15% of cases when measured in the thickness-domain, suggesting that vertical
31 thickness trends are probably not a useful way to identify products of ordered periodic external
32 forcing. Variability in relative forcing periodicity results in significant variation in both HFS and LFS
33 thickness trends making accurate identification of hierarchy and any forcing controls from
34 thickness data alone very difficult. Comparison between model results and outcrop sections
35 suggests that hierarchies are often assumed to be present despite a lack of adequate supporting
36 evidence and quantitative analysis of these sections suggests that they are not hierarchical in any
37 meaningful sense.

38 **Introduction**

39 Platform carbonates are important recorders of climatic and tectonic history and form hydrocarbon
40 reservoirs in many basins (e.g. Saller *et al.* 1994). During ice-house periods of global climate
41 platform interior strata are typically characterised by stacked high-frequency sequences (HFSs)
42 that often show clear evidence for high-frequency high-amplitude relative sea-level oscillations
43 (Goldhammer *et al.* 1990). HFSs deposited during ice-house periods are typically defined as a
44 shallowing-upward sequence of sub-tidal strata capped by sub-aerial exposure (Rankey 2004).
45 Individual HFSs are interpreted as 'stacking' into thicker low-frequency sequences (LFSs). LFSs are
46 themselves therefore unconformity bounded packages of strata, following the standard definition of
47 sequence, and are often identified by a vertical trend of decreasing HFS thickness (e.g. Lehrmann
48 and Goldhammer 1999; Kenter *et al.* 2006). These trends, based on variations of facies and
49 thickness, form the basis for identification of a hierarchy of stacked cycles or sequences (Figure 1).

50 Sedimentary hierarchies are potentially important because, if present, they allow systematic
51 subdivision of strata and also because implicit in hierarchy interpretation is an assumption that it
52 was generated by an ordered forcing-mechanism. The control is typically assumed to be periodic
53 variations in accommodation usually attributed to climate variations resulting from Milankovitch-
54 scale orbital variations (e.g. Cozzi *et al.* 2005; Schwarzacher 2005; Algeo and Hinnov 2006), which

55 vary global sea-level primarily by dictating the amount of water stored as continental ice. The
56 accommodation changes are therefore inferred to be periodic creating a sedimentary hierarchy via
57 interaction of various wavelengths of Milankovitch oscillations (for a review see de Boer and Smith
58 1994). Hierarchies also have additional implications related to order and completeness of the
59 stratigraphic record, the assumption being that since an ordered forcing mechanism is present in
60 the climate system the stratigraphic record preserves this signal with sufficient fidelity and without
61 significant loss so as to be recognizable. This signal is demonstrably recorded in deep marine
62 settings which undergo almost continuous pelagic sedimentation (e.g. Zachos *et al.* 2001) but less
63 obvious in shallow marine settings which contain presumably significant periods hiatus (as much
64 as 80% of the total depositional period; Barnett *et al.*, 2002).

65 Despite being such a potentially useful concept and approach, a critical problem with interpretation
66 of sedimentary hierarchies to date is the lack of an agreed, detailed definition for the term. A lack of
67 a rigorous definition allows interpretation of hierarchical strata without sufficient evidence to
68 support the resulting conclusions about controlling factors (e.g. Kerans *et al.* 1994), makes
69 objective comparison of proposed examples more difficult, and limits the degree to which
70 hierarchies can be understood.

71 In this study we aim to investigate the response of depositional systems if they were to behave in
72 the way implied by sequence stratigraphic (hierarchical) models, to clearly state the assumptions of
73 these models and illustrate the consequences of these assumptions when they are employed in an
74 internally-consistent forward model with plausible parameters. We also critically examine the
75 conditions necessary to generate a sedimentary hierarchy and propose a definition for the term as
76 well as a new objective method for quantifying the degree of hierarchy displayed in a sedimentary
77 section. We apply this method to both numerical models of carbonate accumulation and outcrop
78 data in order to determine the likely frequency of occurrence of hierarchical strata in the ancient
79 record, and use this analysis to comment on fidelity of carbonate platform strata as a recorder of
80 external forcing.

81 ***Previous definitions of sedimentary hierarchies***

82 Strata can be interpreted to be hierarchical despite the lack of a clear definition, although only in a
83 subjective and qualitative way. Different qualitative definitions implied by various authors prevent
84 consensus, decrease reproducibility and inhibit testing for presence of hierarchical strata (*cf.*
85 Goldhammer *et al.* 1991; Drummond and Wilkinson 1993a; Lerat *et al.* 2000). For example, two
86 distinct sedimentary hierarchies were described from the same sedimentary sections (in the

87 Paradox Basin, southeastern Utah) by Goldhammer *et al.* (1991) and Lerat *et al.* (2000) but differ
88 significantly. In order to establish a “cycle hierarchy”, Lerat *et al.* state that the criteria rest not on
89 thickness but on the “extent of changes in the depositional environments recorded within a cycle”
90 and “the importance of the cycle bounding surfaces in regional correlations” (Lerat *et al.* 2000;
91 p78). Following from this they state that a “5th-order cycle” or “genetic unit” is defined by a “cyclic
92 but minor change in bathymetry or accommodation as deduced from facies”, and is “the expression
93 of a short term cyclic variation of relative sea-level”. “4th-order sequences”, in contrast, are said to
94 represent “major changes in bathymetry or accommodation”. There are numerous problems with
95 these interpretations given the possibility of complex and incomplete strata (Burgess and Wright
96 2003; Rankey 2004; Burgess 2006), the lack of strong evidence for ordered strata in many cases
97 (Drummond and Wilkinson 1996; Wilkinson *et al.* 1996; Wilkinson *et al.* 1997; Wilkinson *et al.*
98 1998), and the evidence against simple lithology and depth relations for shallow-water facies
99 (Rankey 2004). The uncertainty in using “minor bathymetric changes” to define cycle boundaries is
100 therefore clearly problematic and boundaries of a HFS should therefore only be interpreted where
101 there is unambiguous evidence of change in relative sea-level. In shallow-marine settings, due to
102 non-unique facies-depth relations this dictates the use of sub-aerial exposure as bounding events
103 for cycles.

104 Deriving information on sea-level history from exposure surfaces is, however, difficult because the
105 nature and degree of development of subaerial exposure surfaces varies greatly (e.g. Davies 1991;
106 Vanstone 1998; Sattler *et al.* 2005) and limits the ability to quantify the nature of exposure periods
107 using sedimentary features (Budd *et al.* 2002). Goldhammer *et al.* (1991; 1994) used HFS thickness
108 trends to define a sedimentary hierarchy, providing a clear example of what a hierarchy is
109 interpreted to constitute. Based on observations of variations in cycle thickness throughout a
110 sequence, a hierarchy was interpreted from the relative position of thicker and thinner “fifth-order”
111 high-frequency sequences within a “lower-order” sequence (Goldhammer *et al.* 1991). Fifth-order
112 cycles were interpreted, measured and were found to thin upwards within a succession. When a
113 fifth-order cycle that was thicker than the underlying cycle was observed, it was considered to be
114 the end of that particular sequence and the start of a new lower-frequency sequence (“fourth-
115 order”). From this kind of analysis, “bundles” of cycles in the form of HFS are said to occur within
116 each lower-frequency sequence, with a bundling ratio defined based on how many HFSs occur
117 within the LFS. This ‘cycle-bundling’ concept is now commonly advocated by workers as evidence
118 for the operation of Milankovitch-forced glacio-eustasy. In the case of the Paradox Basin strata,
119 cycle-bundling is manifest at a maximum ratio of 9:1 (with a minimum of 3:1), contrasting with the

120 usual 5:1 ratio quoted for many successions (e.g. Goldhammer *et al.* 1987). A shortfall in the
121 number of high-frequency sequences per sequence is usually accounted for by citing “missed beats”
122 as a cause (*sensu* Goldhammer *et al.* 1994; p262), although it is notable that authors rarely explain
123 how more beats than the usual 5:1 (e.g. the 9:1 described above) are accounted for via this method.

124 Further examples of more rigorous definitions of hierarchies come from studies critical of
125 interpretations of order in carbonate successions (Drummond and Wilkinson 1993a; 1993b;
126 Wilkinson *et al.* 1997). Drummond and Wilkinson (1993b; p688) state that “...many cyclic
127 sequences exhibit a distinct stacking hierarchy wherein a pattern of thickness is repeated
128 throughout an individual sequence”. Drummond and Wilkinson (1993a; p369) expand upon this:
129 “Explicit in this argument is that each meter-scale cycle represents a single excursion of sea-level
130 and that repeated patterns in cycle thickness faithfully represent the constructive interference of
131 forcing functions of different frequency”. These statements further reiterate the concept that
132 thickness of cycles bears direct relation to the order of forcing, and that trends in thickness are
133 related to interference of multiple orders of forcing.

134 ***An objective definition of a sedimentary hierarchy***

135 These common themes in previous descriptions of sedimentary hierarchies can be used to
136 formulate a more rigorous definition. Since cycle development can be complex (Burgess 2006) and
137 lithofacies are not uniquely diagnostic over shallow-water depth ranges (Rankey 2004), any
138 cyclicity and hierarchies defined using facies transitions must include significant uncertainty. In
139 this study we therefore focus on hierarchies defined in terms of thickness. Hierarchies described in
140 other studies tend to include the following:

- 141 (a) The assumption that an ordered forcing-mechanism causes an ordered pattern to be
142 recorded in sedimentary strata by influencing accommodation;
- 143 (b) The observation that two or more smaller HFSs “stack” or bundle into a larger LFS;
- 144 (c) That ostensibly ordered variations in thickness are used to define the larger-scale LFS;
- 145 (d) That a new LFS begins when the thickness of a given HFS exceeds that of the underlying
146 HFS.

147 The concept of a sedimentary hierarchy in ice-house platform carbonates as proposed by earlier
148 workers can be formalized as the following definition:

149 “a sedimentary hierarchy consists of two or more high-frequency sequences, each bounded
150 by unambiguous evidence of sub-aerial exposure, in which there exists a repeated trend of

151 decreasing high-frequency sequence thickness such that within a single low-frequency
152 sequence each high-frequency sequence is thinner than the previous sequence .”

153 In this definition subaerial exposure surfaces define the thickness of an individual HFS. Stacking of
154 these HFSs and trends of decreasing vertical thickness define any sedimentary hierarchy present.

155 In ice-house environments sub-aerial unconformities generally provide unambiguous evidence of a
156 sea-level fall and can therefore be reasonably used as boundaries to HFSs. Greenhouse
157 environments may also display evidence for sub-aerial exposure (e.g. Bover-Arnal *et al.* 2013,
158 Cariou *et al.* 2013) although well-developed surfaces may be lacking (Haas, 2004), and so the
159 selection of appropriate bounding surfaces is more difficult. For greenhouse carbonate successions,
160 bounding surfaces could be reasonably placed at facies transitions that show an unequivocal fall of
161 relative sea-level. An example of such a transition may be from inter-tidal to sub-tidal strata
162 (Wright 1996). However, similar transitions can also occur due to autocyclic processes (Burgess
163 2001) and from tectonic-forcing overriding any eustatic controls (Bosence *et al.* 2009). A good
164 understanding of three-dimensional stratal geometries and careful consideration of allo versus
165 autocyclic processes (Burgess 2006) is therefore needed to define HFS boundaries in greenhouse
166 strata. To avoid these issues, this study focuses on the definition of a HFS as it applies to ice-house
167 cycles; i.e. bound by sub-aerial exposure surfaces.

168 Careful application of the above definition of hierarchy should decrease uncertainty in description
169 and increase reproducibility from outcrop description. Further to this, numerical modelling can be
170 used to give some indication as to the likelihood of creating hierarchical strata, as defined, under
171 varying conditions arising from the combined interactions of eustatic controls of varying
172 periodicity and amplitude. Absolute and even most relative age-dating of ancient successions is not
173 able to differentiate individual HFSs at the scale of the interpreted cyclicity (<20-100ka), nor is it
174 able to detect significant changes in sedimentation rate that may distort any original signal.
175 Forward modeling therefore represents a method to determine how an ordered forcing mechanism
176 is recorded as strata and allows us to examine the accuracy of the recording mechanism (Figure 2).

177 **Model formulation**

178 The model presented here is a one-dimensional numerical process-response stratigraphic forward
179 model of carbonate accumulation (Pollitt 2008; Burgess and Pollitt, 2012). The model records
180 accumulation on a simulated carbonate platform at a single point in space. Dominant processes

181 affecting icehouse accommodation creation are glacio-eustatic sea-level change and subsidence
 182 (Burgess 2001; Barnett *et al.* 2002) which operate as independent variables. One-dimensional
 183 modelling lends itself well to the evaluation of stacking-patterns in platform top carbonates
 184 because aggradational stratal geometries are common in isolated platform interiors during ice-
 185 house periods (e.g. Goldhammer *et al.* 1994, Della Porta *et al.* 2002) and it is often assumed that
 186 high-frequency high-amplitude eustasy is a dominant control on stacking. Previous studies
 187 incorporating one-dimensional modelling of carbonate cyclicity have also attempted to address the
 188 conditions which would lead to Milankovitch-type 5:1 bundling of cycles (e.g. Walkden and
 189 Walkden, 1990). The benefit of using a simple one-dimensional model is that many thousands of
 190 long-duration simulations can be run allowing criteria such as thickness to be systematically
 191 evaluated against controlling parameters like eustatic period and amplitude, and production and
 192 subsidence rates.

193 Proponents of Milankovitch forcing of cyclic sequences have based their arguments primarily on
 194 the assumption that the periodicity of individual cycles (as calculated from numbers of cycles and
 195 sequence duration) commonly falls within the same range as that of Milankovitch-band parameters
 196 (20-400ka; Vail *et al.*, 1977, Berger 1978). Intermediate-frequency oscillations, with periodicity of
 197 100ka to 400ka, are commonly interpreted to oscillate with amplitude of 45-75m (Crowley and
 198 Baum 1991) although amplitudes of up to 95m have also been suggested (Heckel 1986, Read *et al.*
 199 1986, Wright and Vanstone 2001). High-frequency oscillations are typically envisaged to have
 200 amplitudes up to 35m, with a periodicity of ~20ka to 40ka (e.g. Read *et al.* 1986, Paterson *et al.*
 201 2006).

202 Allocyclic eustatic fluctuations forced by glacial build-up and melting are modelled using an
 203 asymmetrically-modified sinusoid, according to the function

$$204 \quad f(x) = \begin{cases} \sin\left[\frac{\pi x}{\alpha}\right] & \text{for } -\alpha/2 < x < \alpha/2 \text{ and} \\ \sin\left[\frac{\pi}{2} + \frac{\pi}{\beta} \times \left(x - \alpha/2\right)\right] = \cos\left[\frac{\pi}{\beta} \times \left(x - \alpha/2\right)\right] & \text{for } \alpha/2 < x \leq \alpha/2 + \beta \end{cases}$$

205 where α is the relative proportion of the period represented by the positive gradient limb, and β is
 206 the relative proportion of the period represented by the negative gradient limb. Outside this range
 207 $f(x)$ is defined to be periodic with period $\alpha + \beta$. Since this function is odd (i.e. $f(-x) = f(+x)$) its
 208 Fourier series consists of sines, therefore

209

$$f(x) = \sum b_n \times \sin\left[\frac{2\pi mx}{\alpha + \beta}\right]$$

210 where

211

$$b_n = \left[\frac{2}{\pi}\right] \times \left[\frac{1}{1 + \frac{\beta}{\alpha} - 2n} - \frac{1}{1 + \frac{\beta}{\alpha} + 2n} + \frac{1}{1 + \frac{\alpha}{\beta} + 2n} - \frac{1}{1 + \frac{\alpha}{\beta} - 2n} \right] \times \cos\left[\frac{\pi n}{1 + \frac{\beta}{\alpha}}\right]$$

212 ***Carbonate accumulation***

213 Carbonate accumulation is calculated iteratively according to operation of several simple processes
214 summarised as

215

$$c_{(t)} = e_{(z)} + o_{(z)} + a_{(z)} \vee d_{(z)}$$

216 where t is time, z is platform surface elevation, and c , e , o , a and d are rates of carbonate
217 accumulation, euphotic production, oligophotic production, aphotic production and surface
218 lowering respectively. All rates are expressed in metres per million-years. Carbonate producers
219 within the model are categorised as euphotic, oligophotic and aphotic (after Pomar 2001; Figure 3).
220 Utilising multiple curves for carbonate production provides a way to define discrete lithofacies as
221 simulation outputs, however this was not included in this study due to the aforementioned
222 evidence against simple facies-depth relationships.

223 Euphotic biota are autotrophic and autoheterotrophic organisms requiring well-lit water and thus
224 inhabiting shallow depths in the euphotic zone, which extends typically to 20-30m (Milliman 1974,
225 Hallock and Schlager 1986). Estimates of euphotic zone sedimentation rates vary widely (e.g.
226 Demicco and Hardie 2002; Strasser and Samankassou 2003) although they are well documented in
227 modern environments for framework building organisms (e.g. Bosscher and Schlager 1993). The
228 rates used in the model take into account the usually limited geographic extent of framework-
229 building organisms in inner-platforms (*cf.* Smith and Kinsey 1976). The euphotic production
230 component of the model is based on the work of Bosscher and Schlager (1993) and is applied in the
231 model as

$$e_{(t)} = e_{(m)} \times \tanh[k \times \exp(d \times w_{(t)})]$$

232 where w is water depth, t is the current timestep, e is carbonate accumulation, d is a decay constant
233 and k is a rate constant.

234 Oligophotic organisms (autotrophic and autoheterotrophic) inhabit the oligophotic zone,
235 characterised by lower light levels and sometimes lower temperature (Milliman 1974; Pomar
236 2001). Rates of production for deeper-water oligophotic carbonate factories are uncertain, but
237 estimates suggest between 30-60% of euphotic factory rates (Pomar 2001; Schlager 2003). The
238 oligophotic production component is represented by

$$o_{(t)} = o_{(m)} \times w_{(t)} < o_{(a)} \\ \Rightarrow \tanh\{o_{(k)} \times \exp[o_{(d)} \times (o_{(a)} - w_{(t)})]\} \vee \tanh\{o_{(a)} \times \exp[o_{(d)} \times (w_{(t)} - o_{(a)})]\}$$

239 where w is water depth, t is the current timestep, o is carbonate accumulation, a is a turn-around
240 depth constant, d is a decay constant and k is a rate constant.

241 Production rates for aphotic sedimentation (from heterotrophic organisms) at shallow water
242 depths are poorly constrained and are often categorised along with euphotic sedimentation rates
243 (Pomar 2001). Aphotic sedimentation into deeper water is more constrained, with evidence from
244 Pleistocene and Holocene data suggesting pelagic sedimentation to occur at a rate of 52-66m Ma⁻¹
245 (Vollbrecht and Kudrass 1990). Aphotic production is modelled using the function

$$a_{(t)} = a_{(m)} \times w_{(t)} < a_{(w)} \Rightarrow w_{(t)} \times a_{(w)} \vee w_{(t)} < a_{(p)} \Rightarrow 1 - \left[\frac{(a_{(w)} - w_{(x)})}{(a_{(w)} - a_{(p)})} \right] \times a_{(r)} \vee a_{(r)}$$

246 where w is water depth, t is the current timestep, a is carbonate accumulation, p is a turn-around
247 depth constant, r is a rate constant.

248 ***Model parameters and model runs***

249 Each individual simulation in this study has a runtime of 3Ma. This was selected as an appropriately
250 long duration to allow a significant number of LFSs to be generated. It also allows the simulated
251 carbonate platform to come to a state of equilibrium relative to longer-term sea-level behaviour. A
252 state of equilibrium in this model either means the platform always aggrades at or near sea-level or
253 the platform drowns. Since the longest period of forcing simulated was 112ka, 3Ma allows for 26
254 such oscillations and therefore enough time to reach either state and determine if hierarchical
255 strata form. Periodicities of oscillation were fixed at 23ka for high-frequency, 112ka for
256 intermediate-frequency and 1Ma for low-frequency (Berger, 1978). Asymmetric sea-level

257 oscillations are assumed to represent the faster melting of continental ice-sheets than their
258 accumulation and typify ice-house sea-level behavior (de Boer and Smith 1994). High and
259 intermediate-frequency oscillations were therefore modelled with 95% asymmetry while low-
260 frequency oscillations were modelled as symmetrical (refer to Goldhammer 1991 for discussion of
261 asymmetry in modelling sea-level behaviour). Although most of the other parameters and variables
262 within the model are appropriate for both ice-house and greenhouse regimes, both the asymmetry
263 and amplitude of the eustatic components are focused on characterizing the most likely sea-level
264 behavior in ice-house periods.

265 Sensitivities to the iterative value (time-step) were evaluated by running a single simulation many
266 times with different time-steps until a measured criterion became stable (i.e. did not vary between
267 simulations). Through this sensitivity analysis a time-step of 0.000025Ma (25a) was selected. This
268 time-step was chosen as it is at the upper limit of numerical stability (Figure 4).

269 Parameters varied in the simulations are given in Table 1. The range of sea-level amplitudes was
270 chosen in order to bracket the range of oscillation amplitudes documented in published literature
271 (e.g. Heckel 1986; Crowley and Baum 1991; Wright and Vanstone 2001). Subsidence was modelled
272 at a constant rate, varied within a range considered to represent a reasonable spectrum of likely
273 scenarios. A minimum rate of 10m Ma⁻¹ was modelled in order to represent intra-cratonic basins
274 while a maximum rate of 900m Ma⁻¹ was considered to represent a rapidly subsiding basin on an
275 active plate margin.

276 The simulations presented in this study did not include any sub-aerial denudation during exposure
277 since rates of carbonate dissolution from ancient platforms are poorly constrained. Although some
278 empirical estimates of carbonate dissolution are available (e.g. Plan, 2005) the high degree of
279 uncertainty surrounding these values as applied to ancient platform interior sediments introduces
280 significant complexity into evaluating the response of stratigraphy to external forcing. Additionally,
281 including erosion makes comparison with previous studies that did not invoke erosion (e.g.
282 Goldhammer *et al.* 1994) more troublesome. Future work will be needed to quantify the effect of
283 sub-aerial denudation on the likelihood of generating a sedimentary hierarchy.

284 In each simulation a parameter is modified within the range according to a predefined stepping
285 value, thus each simulation represents the unique combination of the five variables. 110,000
286 simulations were run, spanning the complete range of all five variables.

287 ***Model Output***

288 Although the model outputs simulated thickness and chronostratigraphic sections (Figure 2) that
289 are visually similar to outcrop measured sections a more quantitative measure of hierarchy is
290 required to objectively compare both simulated and outcrop examples. This measure is used to
291 provide a single value per simulation that could be evaluated statistically across several thousand
292 model runs.

293 This metric is presented here as a ratio which represents the number of HFSs per LFS and is
294 referred to as the “*h*-value”. The calculation of *h* is made with complete information in both a time
295 and thickness-domain since within the model all parameters and responses are known. The *h*-value
296 can be expressed as $1/n$, where *n* is the number of HFSs (Figure 5). This provides a convenient way
297 of comparing the degree of hierarchy development from multiple simulations. In a strongly non-
298 hierarchical system, for instance, there may only be one HFS per LFS, which would result in a ratio
299 of 1:1 (or $h=1$). Over the course of a 3Ma simulation there will be occasions when more than one
300 HFS occurs per LFS, and so the average ratio for the entire simulation would be $h<1$. If conditions
301 consistently allowed two HFSs per LFS, the ratio would be $h=0.5$, and if there were three then
302 $h=0.33$. By this measurement and the definition of hierarchy employed here, a sedimentary section,
303 simulated or otherwise, can be considered to display weak evidence of a sedimentary hierarchy if
304 there are on average ≥ 2 HFSs per LFS ($h<0.5$), and strong evidence of a sedimentary hierarchy if
305 there are on average ≥ 3 HFSs per LFS ($h<0.33$).

306 **Results**

307 Figure 6 shows the results of four simulations where the amplitude of the high-frequency eustatic
308 component has been varied to result in *h*-values reflecting a range of hierarchical and non-
309 hierarchical outcomes. All parameters except high-frequency amplitude were constant in these
310 simulations, including intermediate-frequency amplitude which was fixed at 40m. As high-
311 frequency amplitude increases, more oscillations of sea-level are recorded as discrete units of
312 sedimentation separated by sub-aerial exposure surfaces (i.e. a HFS). This is a direct consequence
313 of amplitude, since higher amplitude oscillations have a greater likelihood of exposing the platform.

314 Figure 7 displays the corresponding stratigraphic diagrams to the chronostratigraphic results
315 shown in Figure 6. All simulations produce stratigraphic sections that resemble metre-scale ice-
316 house HFSs described from outcrop. Variation in the amplitude of high-frequency oscillation results
317 in markedly different cycle thicknesses although all simulations generate a similar total sediment

318 thickness (204-219m). Non-hierarchical sections ($h > 0.5$) have cycle thicknesses that are roughly
319 similar to those described from outcrops of ice-house platform carbonates (10-15m). Weakly to
320 strongly hierarchical sections ($h < 0.5$) display thinner cycles the higher the h -value. The most
321 hierarchical section has an average cycle thickness of 6.06m. This is thinner than usually described
322 from ice-house successions and is a consequence of having rapid glacio-eustatic oscillations; many
323 oscillations are recorded as discrete periods of sedimentation, but since the oscillations are of
324 short-duration the resulting cycles are thin.

325 The likelihood of generating a sedimentary hierarchy for 110,000 parameter combinations across
326 the investigated range is shown in Figure 8. These results are displayed in the chronostratigraphic
327 time-domain and show that only 9% of possible parameter values lead to strata displaying at least
328 weak evidence of a sedimentary hierarchy (i.e. having on average two HFSs per LFS or $h < 0.5$).
329 Furthermore, only 4% of simulations display strong evidence of a sedimentary hierarchy (having
330 on average three HFSs per LFS or $h < 0.33$).

331 The cumulative probability distribution for these results shows that the majority of simulations
332 (78%) have $h > 0.75$ meaning that these simulations average between 1 and 2 HFSs per LFS (Figure
333 8). These 78% of models run do not exhibit consistent trends of vertically decreasing thickness
334 within a LFS and so do not generate hierarchical strata. This suggests that hierarchies are created
335 only under a specific and limited set of allocyclic forcing conditions.

336 The cumulative probability distribution for the same set of simulations but for a thickness-domain
337 hierarchy shows that based on analysis of thickness alone (Figure 9), without additional
338 information about, for example, duration of deposition, only 15% of simulations exhibit weak
339 evidence of a sedimentary hierarchy. Furthermore less than 1% of simulations display strong
340 evidence of a sedimentary hierarchy. In contrast to results measured in the chronostratigraphic
341 time-domain, where a majority of simulations showed strong evidence against a sedimentary
342 hierarchy ($h > 0.75$), thickness-domain results show relatively few simulations with either very
343 strong evidence for or against a hierarchy. Fewer than 1% display strong evidence for a hierarchy
344 ($h < 0.33$) and only 6% display strong evidence against a hierarchy ($h > 0.75$). The majority of
345 simulations therefore have between 1.5 and 2 HFSs per LFS demonstrating that when measured in
346 the thickness-domain alone, most modelled strata display evidence against a sedimentary
347 hierarchy.

348 Using the parameter-space distribution of time-domain hierarchical and non-hierarchical
349 simulations we can investigate further why evidence for a thickness hierarchy is weaker than
350 combined time-thickness evidence (Figure 10). Under relatively simple conditions with only two
351 eustatic variables there is a clear relationship between amplitude of low-frequency oscillation
352 relative to that of intermediate-frequency oscillation and the ability to generate a hierarchy. It is
353 only when the amplitude of high-frequency oscillation exceeds ~70% of the intermediate-
354 frequency amplitude that hierarchies are consistently generated throughout a simulated section
355 (i.e. returning an average of $h < 0.5$). This relationship between relative amplitudes of oscillation also
356 provides an insight into why simulations with the most extreme h -values occur. The least
357 hierarchical strata occur when intermediate-frequency amplitude is greatest, while the most
358 strongly hierarchical strata form when high-frequency amplitude is largest and so end-member h -
359 values are limited to extremes of oscillation amplitude.

360 The distribution of h -values for this group of simulations is without discrete steps or tipping points
361 (Figure 11). As the relative amplitude of high- to intermediate-frequency oscillations increases, the
362 mean h -value of the simulated strata also increases. For the range of parameter values tested here,
363 a hierarchy is increasingly likely with increasing higher-frequency eustatic oscillation relative to
364 the amplitude of the lower frequency oscillation. Given these results, if the amplitude of high-
365 frequency eustatic oscillations is known for given strata (in the Pleistocene for instance), it may be
366 possible to estimate the h -value range and likely intermediate-frequency amplitude.

367 Examination of a single hierarchical and a single non-hierarchical example from this parameter-
368 space distribution provides insight into why this relationship exists (Figure 12). In the non-
369 hierarchical case fourteen HFSs are generated. Nine HFSs are clearly forced by the intermediate-
370 frequency sea-level oscillation and five HFSs are forced by a high-frequency oscillation of sea-level
371 during the falling-stage of an intermediate-order oscillation. The small amplitude high-frequency
372 oscillation (20m) during the relatively large amplitude intermediate-frequency oscillation (90m)
373 provides only a short interval during the falling-stage of the intermediate-frequency oscillation
374 when a HFS can be generated. In contrast, in the hierarchical example during the same period, 33
375 HFSs are created because the high-frequency higher-amplitude oscillations have a much greater
376 impact on the geometry of the relative sea-level curve. Instead of only triggering deposition during
377 the falling stage of an intermediate-frequency oscillation, higher amplitude high-frequency
378 oscillations are sufficient to regularly trigger and then truncate sedimentation regardless of the

379 position on the intermediate-frequency curve. This creates hierarchical strata with many HFSs per
380 LFS.

381 Stratigraphic completeness in these two simple examples is surprisingly similar given the variation
382 in regularity of exposure events (Figure 12). In both cases, mean stratigraphic completeness is
383 calculated by determining the percentage of time during each HFS where sedimentation occurs. The
384 mean value represents the average for all HFSs over the entire simulation. These results do not
385 suggest that hierarchical strata will tend to be more incomplete than non-hierarchical strata. In this
386 example, the non-hierarchical example has average stratigraphic completeness of 49%, while the
387 hierarchical example is 47% complete. In both of these cases the minimum and maximum values for
388 stratigraphic completeness are similar, suggesting that in each simulation there are end-members
389 of similarly long and short periods of sedimentation. Although short duration sub-aerial exposure
390 occurs more regularly in the hierarchical simulation, the non-hierarchical simulation has long
391 periods of less frequent exposure at intermediate-frequency lowstands of sea-level. The net result
392 is similar overall periods of non-deposition.

393 Although the relationship between hierarchy and stratigraphic completeness is straightforward
394 under these simple forcing conditions, as the complexity of multiple nested eustatic curves is
395 introduced the incidence of hierarchical strata decreases. This is demonstrated in Figure 13 with
396 the addition of a third forcing component; a eustatic curve with a 1Ma symmetric oscillation of
397 varying amplitude. Generally, as the amplitude of this eustatic component increases, fewer cases of
398 hierarchical strata occur. More rapid accommodation change results in a greater likelihood of
399 drowning during the transgressive stage.

400 Burgess and Pollitt (2012) used the same forward model to study controls on lithofacies thickness
401 distributions. They showed that increasing the complexity of the forcing eustatic curve with
402 additional frequencies of oscillation created exponential thickness distributions of the kind
403 observed in outcrop. Simpler curves created non-exponential thickness distributions. Exponential
404 thickness distributions are significant because statistical theory suggests that they arise from
405 random processes. Burgess and Pollitt (2012) showed that they can also arise from complex forcing
406 functions in a deterministic model when rapid changes in water depth generate strata with many
407 thin lithofacies units and few thick lithofacies units. Aside from raising interesting issues about the
408 nature of randomness versus complexity, this result is significant here because more complex
409 forcing also tends to decrease the likelihood of hierarchical strata. These results suggest that

410 hierarchical strata represent the effects of simple external forcing and not complex interactions of
411 multiple forcing components, as previous workers have suggested (e.g. Paterson *et al.* 2006).

412 Only 9% of the 10,000 simulations shown in Figure 13 show at least weak evidence for a time-
413 domain hierarchy and these are restricted to a relatively small region of parameter-space. This
414 small region represents a 'Goldilocks' zone of suitable parameters for time-domain hierarchy
415 development. In the case of the model runs represented by Figure 13 this zone represents the
416 following conditions:

- 417 1. Subsidence rate is sufficient for development of stacked cycles rather than a continuously
418 exposed platform ($\geq 100\text{m Ma}^{-1}$).
- 419 2. Subsidence rate is not so high that accommodation significantly outpaces sediment supply
420 and drowns the carbonate factory ($< 500\text{m Ma}^{-1}$).
- 421 3. The low-frequency forcing component is of low amplitude ($< 120\text{m}$). Higher amplitudes
422 increase the likelihood of drowning and decrease the potential for generation of
423 hierarchical strata.
- 424 4. The high-frequency forcing component is of sufficient amplitude to regularly cause HFS
425 development ($> 70\%$ that of intermediate-frequency amplitude).

426 These criteria are true for this particular rate of maximum carbonate productivity (2000m Ma^{-1}).
427 Different productivity rates would change the zone of preferential hierarchy development. These
428 results support the view that hierarchical strata require quite specific conditions to develop and so
429 are likely to be relatively rare.

430 The results depicted in Figure 13 demonstrate the range of modelled parameter values in which
431 hierarchies defined in the chronostratigraphic domain can occur. Figure 14 shows thickness-
432 domain (stratigraphic) results from the same set of simulations with h -value calculated from
433 thickness information alone. Stratigraphic hierarchies observed in model results may be used to
434 infer how likely it is that a hierarchy can be reasonably interpreted from outcrop. In these cases, the
435 h -value represents the degree of hierarchy development indicated by stacking of HFS thickness,
436 where a new LFS is started if a given HFS is thicker than the last. Figure 14 shows that using a
437 thickness-only definition there is a broader parameter-space range in which hierarchical sections
438 may be identified. However, fewer model runs can be categorized as strongly hierarchical. There is
439 less of a dependency on low-frequency amplitude, with apparently hierarchical sections occurring
440 at large low-frequency amplitudes. Similarly, high h -values are recorded at low subsidence rates,

441 although drowning of the platform still terminates hierarchy development above rates of 400 m Ma-
442 1.

443 These two different methods of defining hierarchies give different results because many examples
444 of hierarchies defined using thickness information do not constitute hierarchies when considered
445 using full chronostratigraphic information (e.g. Figure 2). Indications of hierarchy from thickness
446 data alone are often inaccurate. This effect is particularly acute if high-frequency oscillations are of
447 large amplitude. In these cases high-frequency oscillations regularly force HFS deposition on the
448 transgressive part of the intermediate-frequency curve. Although of short-duration, these HFSs are
449 often thicker than the underlying HFS and would be identified, using thickness information alone,
450 as a new LFS. An example of this behavior is shown in Figure 2. When analysed using thickness
451 information alone, these strata appear hierarchical when in a true sense they are not. A significant
452 number of the hierarchical sections shown in Figure 14 can be considered false-positives. In these
453 situations the hierarchy present in the stacking patterns of HFS thickness is not truly representative
454 of the forcing mechanism since the LFSs in these sections will be composed of HFSs forced by
455 different oscillations of sea-level (as is the case in Figure 2). In effect, HFSs are bundled incorrectly
456 and the stratigraphic record – in the process of converting a time-domain sedimentation rate to the
457 thickness-domain – is an imperfect record. This result suggests that determining which sections
458 truly display stacking representative of the forcing mechanism is not possible using thickness
459 information alone. Comparison between the number of strongly-hierarchical simulations in the
460 time-domain versus the much greater number of weakly-hierarchical simulations in the thickness-
461 domain suggests that the wide distribution of hierarchical sections indicated by the thickness-only
462 analysis is not an accurate representation of the forcing mechanism and over-estimates the number
463 of hierarchical simulations.

464 **Comparison to outcrops**

465 Results from the model runs discussed above suggest that truly hierarchical strata are relatively
466 rare and restricted to a small area of the modelled parameter-space. However, hierarchies defined
467 based on thickness data have been defined many times, both explicitly and implicitly, in the
468 interpretation of carbonate outcrops. Lehrmann and Goldhammer (1999) contained interpretations
469 of data from 93 outcrops, five of which contained explicit interpretations of sedimentary
470 hierarchies composed of stacked “4th-order parasequences” (LFSs) and “5th-order parasequences”
471 (HFSs) and based on thickness trends. For instance, the Hermosa Group logged by Goldhammer *et*

472 *al.* (1994) is defined as “composite stratigraphic cyclicity, in which small depositional cycles build
473 into larger sequences according to vertical stacking patterns” (Goldhammer *et al.* 1994; p267) and
474 is clearly defined by diagrams in that study. Two of these five outcrops, the Pennsylvanian Hermosa
475 Group and Gobbler Formation in the south-western USA were also logged independently by Pollitt
476 (2008; Figure 15). Figure 16 shows the hierarchy *h*-value as per the original workers’
477 interpretation of HFSs and LFSs as well as the *h*-value using the HFS definition made in this study.

478 *h*-values resulting from the original worker’s interpretation of HFSs and LFSs range from 0.08 to
479 0.30. This is aligned with the interpretation that these sections are hierarchical (e.g. Goldhammer *et*
480 *al.* 1994). However using the hierarchy definition made in this study (i.e. consistently starting a new
481 LFS when a given HFS is thicker than the last) results in higher *h*-values ranging from 0.57 to 0.75,
482 suggesting that in fact the strata are not hierarchical in any meaningful sense. In these cases a
483 rigorous application of stacking according to vertical thickness does not result in a hierarchy, but
484 instead results in approximately 1.6 HFSs per LFS which is not representative of Milankovitch
485 bundling ratios. Similarly, the Holder and Gobbler Formations, described as cyclic by previous
486 workers (Wilson 1972; Algeo *et al.* 1991), respectively have *h*-values of 0.65 and 0.76 which
487 equates to an average of 1.5 and 1.3 HFSs per LFS.

488 The fact that all seven cases shown in Figure 16 were interpreted as hierarchical based on thickness
489 and stacking patterns but this was not reproduced with a more rigorous definition suggests that
490 workers have an inherent bias towards inferring patterns of order in sedimentary sections.

491 Analysis of the remaining 88 sections in the data published by Lehrmann and Goldhammer (1999)
492 suggests that *h*-values are relatively consistent and usually within the range $h=0.6-0.7$ (Figure 17).
493 This is surprising given that the data comes from a wide variety of depositional settings and ages
494 from the Paleozoic to Cenozoic. The outlying datapoints (Figure 17) are likely to be a result of
495 undersampling in terms of number of HFSs. Sequences with less than 20 HFSs show greater scatter
496 than those with $n>20$ (Figure 18). This data suggests that in the majority of these measured
497 sections, when a rigorous definition of a sedimentary hierarchy is applied, the number of HFSs per
498 LFS is between 1.5 and 2. This is not suggestive of any type of bundling according to climatic
499 forcing, and suggests that consistent vertical trends in HFS thickness are not present in any of the
500 studied outcrops. Analysis of the number of “runs” of decreasing or increasing lithofacies unit
501 thickness in the Lehrmann and Goldhammer (1999) dataset shows that there are typically
502 approximately two lithofacies units in a given run (Figure 19). This data suggests that long term
503 consistent trends in lithofacies unit thickness are rare and most are indistinguishable from random,

504 a finding which supports that of earlier workers (Wilkinson *et al.* 1996). It also suggests the
505 possibly of bias inherent to stratigraphic interpretation, where workers tend to ‘even out’ thickness
506 of lithofacies units and avoid extremes in thickness (Burgess 2008); where thin units are rarely
507 interpreted and are commonly lumped as interbeds and thick units are broken into smaller units
508 using a variety of criteria such as sorting or grainsize.

509 **Discussion**

510 “All models are wrong, but some are useful” (Box 1987) is an important statement to consider when
511 interpreting the results from any type of forward model. The one-dimensional forward modelling in
512 this study, while certainly simplistic and “wrong” in many respects, is useful in the sense that it
513 forces us to objectify and quantify our concept and definition of sedimentary hierarchies in
514 carbonate successions. Key assumptions in the model have a strong bearing on its output. In the
515 case of generating a sedimentary hierarchy, key assumptions are the parameters relating to the
516 ordered forcing of sea-level behavior and the definition of a sedimentary hierarchy.

517 Clearly the results presented here are very dependent on the definition of hierarchy. Different
518 results would be obtained with a different definition. Other definitions of a hierarchy are certainly
519 possible and have been employed elsewhere. For instance, a hierarchy could be defined in terms of
520 facies partitioning whereby proportions of facies are altered relative to depositional position within
521 a systems tract. This may be particularly important in greenhouse environments and to evaluate
522 hierarchies in greenhouse strata it is likely that further model runs are required with greenhouse-
523 specific parameters (e.g. low-amplitude oscillations, autocyclic component). A comparison of the
524 occurrence of hierarchies by different definitions in a controlled model environment could be
525 important future work.

526 With these limitations in mind, these results from numerical modelling suggest that forcing by
527 ordered cyclical sea-level oscillations rarely results in an easily identifiable hierarchy of stacked
528 cycles, defined either with total chronostratigraphic information (9% of cases) or with just
529 thickness data (15% of cases). The fact that only 9% of sections result in a hierarchical
530 chronostratigraphic section is particularly illuminating, since it suggests that vertically decreasing
531 trends in thickness are not an appropriate way to identify ordered patterns in sea-level behavior.
532 Were this not true, then clearer trends in duration of HFS deposition would result from the trends
533 in accommodation caused by oscillation of sea-level (as depicted conceptually in Figure 1).

534 One key difference between the conceptual depiction in Figure 1 and the model simulations
535 conducted in this study is the periodicity of orbital oscillation. In the conceptual example, and in
536 many other numerical studies of nested cyclicity (e.g. Goldhammer 1994) the period of the high-
537 frequency oscillation is evenly divisible from the intermediate-frequency, such that five high-
538 frequency cycles fit exactly within one intermediate-frequency oscillation. High-frequency cycles
539 therefore occur in the exact same position relative to the intermediate-frequency oscillation on a
540 consistent basis. There is no reason to expect Milankovitch parameters to be evenly divisible in this
541 way. Behavior of individual Milankovitch-scale orbital variations is known to vary through time
542 quasi-chaotically, suggesting that such a state is unlikely to occur and be maintained over a
543 significant period (Laskar *et al.* 2011). Given this, it seems more reasonable in this simple model to
544 simulate sedimentation with periodicities that vary relative to one another through time, meaning
545 that high-frequency oscillations occur in different relative positions to each intermediate-frequency
546 oscillation (for an example see Figure 6).

547 The net effect of variation in the relative position of high-frequency oscillations is to change the
548 duration and thickness of HFSs in each successive intermediate-frequency oscillation. Thus the
549 hierarchy depicted in Figure 1 cannot occur, and this has a fundamental effect on generation of the
550 vertical thickness trends used to identify hierarchies. For example, it means that the thickest HFS is
551 not always at the start of an intermediate-frequency oscillation and the thinnest HFS is not always
552 at the end. This leads to thickness trends that do not always bear an obvious relationship to the
553 sequence stratigraphic position and vary significantly from the simple and convenient conceptual
554 models put forward by earlier workers. It also means that vertical trends in cycle thickness alone
555 should not be used to identify hierarchies of stratigraphic cyclicity. Given this, are existing
556 qualitative models of carbonate sequence stratigraphy useful? Where the nature of bundling of
557 higher-frequency sequences in lower-frequency sequences changes through time, it is probably not
558 possible to describe cycle stacking with a single simple conceptual model. Given these important
559 implications, further research into the ability to generate sedimentary hierarchies under different
560 combined periodicities of orbital oscillation may increase our understanding of where hierarchies
561 are likely to occur.

562 Results presented here demonstrate that even in a simple 1D model, strata may not accurately
563 represent the signal from external forcing factors. For example, in Figure 2, the LFSs defined in the
564 thickness domain do not correspond to those defined in the time-domain. In this case, thickness of
565 successive HFSs, although decreasing vertically, span multiple oscillations of intermediate-

566 frequency eustatic sea-level and so cannot be said to accurately record the forcing mechanism. In
567 this sense, even a much-simplified representation of the stratigraphic record is demonstrated to be
568 an imperfect record of hierarchies on the basis of vertical thickness trends. Comparison of model
569 results to outcrop data supports this conclusion and suggests that even for the best documented
570 examples of hierarchies in carbonate strata the degree of vertical trends in cycle stacking is likely to
571 be overstated when a strict definition of a hierarchy is applied.

572 In these model results, hierarchies occur in only a small region of parameter space. This raises the
573 question what happens in the remaining >80% of the parameter space where hierarchies do not
574 occur? Are there other stratal patterns that reflect order arising from allocyclic forcing that have
575 not yet been described? It is also interesting to consider how introduction of additional complexity
576 such as sediment erosion, transport and diagenesis in 3D would affect occurrence of hierarchical
577 strata. Could some parts of a carbonate platform preserve a stratal hierarchy while others do not?
578 Further experimental work combined with careful outcrop examination is required to investigate
579 all of these questions.

580 **Conclusions**

- 581 1. The numerical forward modeling experiments presented here investigate the response of
582 depositional systems if they were to behave in the way implied by sequence stratigraphic
583 (hierarchical) models. This model is internally-consistent and uses plausible parameters.
- 584 2. These experiments suggest that in the simplest cases, with two superimposed orders of
585 allocyclic forcing, the higher-frequency forcing needs to have an amplitude of 70% that of
586 the lower-frequency oscillation in order to consistently effect cyclicity and generate a
587 sedimentary hierarchy. Additional frequencies of allocyclic forcing attenuate the eustatic
588 curve further and decrease the likelihood of any hierarchy being preserved in the
589 stratigraphic record.
- 590 3. Results from a wide range of allocyclic forcing amplitude, subsidence and carbonate
591 production rates suggest that ordered forcing via cyclical eustatic sea-level change rarely
592 results in an easily identifiable hierarchy of stacked cycles. Hierarchies defined with full
593 chronostratigraphic information occur in 9% of model run cases, and in 15% of cases when
594 defined purely in terms of thickness information.

- 595 4. The lack of a significant number of strongly and weakly hierarchical sections suggests
596 vertical thickness trends in strata are unlikely to be hierarchical, even in situations where
597 external forcing is present.
- 598 5. Hierarchical sections do not intrinsically contain more missing time than non-hierarchical
599 sections. Hierarchical sections necessarily miss fewer “beats” than non-hierarchical
600 sections, but simulations from the two groups can show a similar amount of missing time;
601 non-hierarchical sections can be just as incomplete as hierarchical sections.
- 602 6. Strata that appear weakly hierarchical in the thickness-domain are unlikely to be
603 representative of the true ‘bundling’ of orbital forcing parameters. If frequencies of
604 oscillation vary through time, relative to one another, the resultant variability in attenuation
605 of the relative sea-level curve results in significant variation in sediment thickness per
606 oscillation. This has a tendency to disrupt trends of vertical thickness and leads to false-
607 positive identification of hierarchical sections. This also suggests that vertical thickness
608 trends are not an appropriate proxy for identify eustatic forcing events in shallow-water
609 carbonate platforms.
- 610 7. Comparison to studied outcrops suggests that hierarchies defined in terms of stratigraphic
611 thickness are often assumed to be present despite a lack of adequate supporting evidence.
612 Quantitative analysis of many of these sections suggests that they are not in fact hierarchical
613 in any meaningful sense.

614

615 **Acknowledgements**

616 The authors are grateful to Dan Lehrmann for providing the data published in Lehrmann and
617 Goldhammer (1999). Thanks also to Dan Bosence and Bruce Wilkinson for their thorough critique
618 of the original manuscript which prompted numerous and significant revisions.

619 **References**

620 ALGEO, T. J. AND HINNOV, L. A. 2006. Milankovitch cyclicity in the Ohio and Sunbury shales: astronomical calibration of
621 the Late Devonian-Early Carboniferous Timescale (Abstract). *Geological Society of America Abstracts with*
622 *Programs*, **38**, 268.

623 ALGEO, T. J., WILSON, J. L. AND LOHMANN, K. C. 1991. Eustatic and tectonic controls on cyclic sediment accumulation
624 patterns in Lower-Middle Pennsylvanian strata of the Orogrande Basin, New Mexico. In *Geology of the Sierra*
625 *Blanca, Sacramento and Capitan Ranges, New Mexico: New Mexico Geological Society Guidebook. 42nd Field*
626 *Conference*, (ed. BARKER, J. M., KUES, B. S., AUSTIN, G. S. AND LUCAS, S. G.), New Mexico Geological Society, 1991
627 Field Conference, 203-12.

628 BARNETT, A.J., BURGESS, P.M. AND WRIGHT, V.P. 2002. Icehouse world sea-level behaviour and stratal patterns in late
629 Visean (Mississippian) carbonate platforms; integration of numerical forward modelling and outcrop studies.
630 *Basin Research*, **14**, 417-439.

631 BARRELL, J. 1917. Rhythms and the Measurements of Geologic Time: Bulletin of the Geological Society of America,
632 **28**, 745-904.

633 BERGER, A. L. 1978. Long-term variations of caloric insolation resulting from the Earth's orbital elements. *Quaternary*
634 *Research*, **9**, 139-67.

635 BOSENCE, D., PROCTER, E., AURELL, M., BEL KAHLA, A., BOUDAGHER-FADEL, M. K., CASAGLIA, F., CIRILLI, S., MEHDIE, M., NIETO, L.,
636 REY, J., SCHERREIKS, R., SOUSSI, M. AND WALTHAM., D. 2009. A dominant tectonic signal in high-frequency, peritidal
637 carbonate cycles? A regional analysis of Liassic platforms from western Tethys. *Journal of Sedimentary*
638 *Research*, **79**, 389-415.

639 BOSSCHER, H. AND SCHLAGER, W. 1993. Accumulation rates of carbonate platforms. *Journal of Geology*, **101**, 345-355.

640 BOVER-ARNAL, T. AND STRASSER, A. 2013. Relative sea-level change, climate, and sequence boundaries: insights from
641 the Kimmeridgian to Berriasian platform carbonates of Mount Salève (E France), *International Journal of Earth*
642 *Sciences*, **102**, 493-515.

643 BOX, G.E.P. AND DRAPER, N.R. 1987. *Empirical Model-Building and Response Surfaces*. Wiley, New York, 424 pp.

644 BUDD, D. A., GASWIRTH, S. B. AND OLIVER, W. L. 2002. Quantification of macroscopic subaerial exposure features in
645 carbonate rocks. *Journal of Sedimentary Research*, **72**, 917-28.

646 BURGESS, P. M. 2001. Modelling carbonate sequence development without relative sea-level oscillations. *Geology*,
647 **29**, 1127-30.

648 BURGESS, P. M. 2006. The signal and the noise: forward modeling of allocyclic and autocyclic processes influencing
649 peritidal carbonate stacking patterns. *Journal of Sedimentary Research*, **76**, 962-77.

650 BURGESS, P.M. 2008. The nature of shallow-water carbonate lithofacies thickness distributions. *Geology*, **36**, 235-
651 238.

- 652 BURGESS P. M. AND POLLITT, D. A. 2012. The origins of shallow-water carbonate lithofacies thickness distributions:
653 one-dimensional forward modelling of relative sea-level and production rate control. *Sedimentology*, **59**, 57–
654 80.
- 655 CARIOU, E., OLIVIER, N., PITTET, B., MAZIN, J.-M. AND HANTZPERGUE, P. 2013. Dinosaur track record on a shallow
656 carbonate-dominated ramp (Loule section, Late Jurassic, French Jura), *Facies*, *In Press*.
- 657 COZZI, A., HINNOV, L. A. AND HARDIE, L. A. 2005. Orbitally forced Lofer cycles in the Dachstein Limestone of the Julian
658 Alps (northeastern Italy). *Geology*, **33**, 789-92.
- 659 CROWLEY, T.J. AND BAUM, S.K. 1991. Estimating Carboniferous sea-level fluctuations from Gondwanan ice-extent.
660 *Geology*, **19**, 975-977.
- 661 DAVIES, J. R. 1991. Karstification and pedogenesis on a late Dinantian carbonate platform, Anglesey, North Wales.
662 *Proceedings of the Yorkshire Geological Society*, **48**, 297-321.
- 663 DE BOER, P.L. AND SMITH, D.G. 1994. Orbital forcing and cyclic sequences. In: *Orbital Forcing and Cyclic Sequences* (Eds
664 P.L. de Boer and D.G. Smith), *IAS Special Publication*, **19**, pp. 1-14. IAS, Oxford.
- 665 DELLA PORTA, G., KENTER, J. A. M., IMMENHAUSER, A. AND BAHAMONDE, J. R. 2002. Lithofacies character and architecture
666 across a Pennsylvanian inner-platform transect (Sierra de Cuera, Asturias, Spain). *Journal of Sedimentary*
667 *Research*, **72**, 898-916.
- 668 DEMICCO, R. V. AND HARDIE, L. A. 2002. The "carbonate factory" revisited: a re-examination of sediment production
669 functions used to model deposition on carbonate platforms. *Journal of Sedimentary Research*, **72**, 849-57.
- 670 DRUMMOND, C. N. AND WILKINSON, B. H. 1993a. Carbonate cycle stacking patterns and hierarchies of orbitally forced
671 eustatic sea-level change. *Journal of Sedimentary Petrology*, **63**, 369-77.
- 672 DRUMMOND, C. N. AND WILKINSON, B. H. 1993b. On the Use of Cycle Thickness Diagrams as Records of Long-Term
673 Sealevel Change during Accumulation of Carbonate Sequences. *The Journal of Geology*, **101**, 687-702.
- 674 DRUMMOND, C. N. AND WILKINSON, B. H. 1996. Stratal thickness frequencies of and the prevalence of orderedness in
675 stratigraphic sequences. *Journal of Geology*, **104**, 1-18.
- 676 GOLDSHAMMER, R. K., DUNN, P. A. AND HARDIE, L. A. 1987. High frequency glacio-eustatic sealevel oscillations with
677 Milankovitch characteristics recorded in Middle Triassic platform carbonates in northern Italy. *American*
678 *Journal of Science*, **287**, 853-92.

- 679 GOLDHAMMER, R. K., DUNN, P. A. AND HARDIE, L. A. 1990. Depositional cycles, composite sea level changes, cycle
680 stacking patterns, and the hierarchy of stratigraphic forcing: example from platform carbonates of the Alpine
681 Triassic. *Geological Society of America Bulletin*, **102**, 535-62.
- 682 GOLDHAMMER, R.K., OSWALD, E.J. AND DUNN, P.A. 1991. Hierarchy of stratigraphic forcing: example from Middle
683 Pennsylvanian shelf carbonates of the Paradox Basin. In: *Sedimentary Modeling: Computer Simulations and*
684 *Methods for Improved Parameter Definition* (Eds E.K. Franseen, W.L. Watney, C.G.S.C. Kendall and W. Ross),
685 *Kansas Geological Survey Bulletin*, **233**, 361-414. Kansas Geological Survey, Lawrence, KS.
- 686 GOLDHAMMER, R. K., OSWALD, E. J. AND DUNN, P. A. 1994. High-frequency, glacio-eustatic cyclicity in the Middle
687 Pennsylvanian of the Paradox Basin: an evaluation of Milankovitch forcing. In *Orbital Forcing and Cyclic*
688 *Sequences*, (ed. DE BOER, P. L. AND SMITH, D. G.), **19**. IAS Special Publication, Oxford, 243-83.
- 689 HAAS, J. 2004. Characteristics of peritidal facies and evidences for subaerial exposures in Dachstein-type cyclic
690 platform carbonates in the Transdanubian Range, Hungary. *Facies*, **50**, 263-86.
- 691 HALLOCK, P. AND SCHLAGER, W. 1986. Nutrient excess and the demise of coral reefs and carbonate platforms. *Palaios*,
692 **1**, 389-398.
- 693 HECKEL, P.H. 1986. Sea-level curve for Pennsylvanian eustatic marine transgressive-regressive depositional cycles
694 along midcontinent outcrop belt, North America. *Geology*, **14**, 330-334.
- 695 KERANS, C., LUCIA, F. J. AND SENGER, R. K. 1994. Integrated characterization of carbonate ramp reservoirs using Permian
696 San Andres Formation outcrop analogs. *American Association of Petroleum Geologists Bulletin*, **78**, 181-216.
- 697 Kenter, J. A. M., Harris, P. M., Collins, J. F., Weber, L. J., Kuanysheva, G. and Fischer D. J., 2006. Late Visean to
698 Bashkirian Platform Cyclicity in the Central Tengiz Buildup, Precaspian Basin, Kazakhstan: Depositional
699 Evolution and Reservoir Development. In *Giant Hydrocarbon Reservoirs of the World: From Rocks to Reservoir*
700 *Characterization and Modeling*, (ed. Harris, P. M. and Weber, L. J.), AAPG Memoir, **88**, 55-95.
- 701 LASKAR, J., FIENGA, A., GASTINEAU, M. AND MANCHE, H. 2011. La2010: a new orbital solution for the long-term motion of
702 the Earth. *Astronomy and Astrophysics*, **532**, A89
- 703 LEHRMANN, D. J. AND GOLDHAMMER, R. K. 1999. Secular variation in parasequence and facies stacking patterns of
704 platform carbonates: a guide to application of stacking patterns analysis in strata of diverse ages and settings.
705 In *Advances in Carbonate Sequence Stratigraphy: Applications to Reservoirs, Outcrops and Models*, (ed. SALLER,
706 A. H., HARRIS, P. M. AND SIMO, J. A.), SEPM Special Publication, **63**. SEPM, Tulsa, Oklahoma, 187-225.
- 707 LERAT, O., VAN BUCHEM, F. S. P., ESCHARD, R., GRAMMER, G. M. AND HOMEWOOD, P. W. 2000. Facies distribution and
708 control by accommodation within high-frequency cycles of the Upper Ismay interval (Pennsylvanian, Paradox

709 Basin, Utah). In *Genetic Stratigraphy at the Exploration and Production Scales*, (ed. HOMEWOOD, P. W. AND
710 EBERLI, G. P.), Elf EP, Pau, France, 71-91.

711 MILLIMAN, J. D. 1974. *Marine Carbonates. Part 1, Recent Sedimentary Carbonates*. Springer-Verlag, New York, 375
712 pp.

713 PATERSON, R. J., WHITAKER, F. F., JONES, G. D., SMART, P. L., WALTHAM, D. A. AND FELCE, G. 2006. Accommodation and
714 sedimentary architecture of isolated icehouse carbonate platforms: insights from forward modeling with
715 CARB3D+. *Journal of Sedimentary Research*, **76**, 1162-82.

716 PLAN, L., 2005. Factors controlling carbonate dissolution rates quantified in a field test in the Austrian alps.
717 *Geomorphology*, **68**, 201-12.

718 POLLITT, D.A. 2008. *Outcrop and forward modelling analysis of ice-house cyclicity and reservoir lithologies*. PhD
719 thesis, Cardiff University, UK.

720 POMAR, L. 2001. Types of carbonate platforms: a genetic approach. *Basin Research*, **13**, 313-34.

721 RANKEY, E. C. 2004. On the Interpretation of Shallow Shelf Carbonate Facies and Habitats: How Much Does Water
722 Depth Matter? *Journal of Sedimentary Research*, **74**, 2-6.

723 READ, J. F., GROTZINGER, J. P., BOVA, J. A. AND KOERSCHNER, W. F. 1986. Models for generation of carbonate cycles.
724 *Geology*, **14**, 107-10.

725 SALLER, A. H., DICKSON, J. A. D. AND BOYD, S. A. 1994. Cycle stratigraphy and porosity in Pennsylvanian and Lower
726 Permian shelf limestone, Eastern Central Basin Platform, Texas. *American Association of Petroleum Geologists*
727 *Bulletin*, **83**, 1835-54.

728 SATTLER, U., IMMENHAUSER, A., HILLGÄRTNER, H. AND ESTEBAN, M. 2005. Characterization, lateral variability and lateral
729 extent of discontinuity surfaces on a Carbonate Platform (Barremian to Lower Aptian, Oman). *Sedimentology*,
730 **52**, 339-61.

731 SCHLAGER, W. 2003. Benthic carbonate factories of the Phanerozoic. *International Journal of Earth Sciences*, **92**, 445-
732 64.

733 SCHWARZACHER, W. 2005. The stratification and cyclicity of the Dachstein Limestone in Lofer, Leogang and Steinernes
734 Meer (Northern Calcareous Alps, Austria). *Sedimentary Geology*, **181**, 93-106.

735 SMITH, S. V. AND KINSEY, D. W. 1976. Calcium carbonate budget production, coral reef growth and sea-level changes.
736 *Science*, **194**, 937-39.

- 737 STRASSER, A. AND SAMANKASSOU, E. 2003. Carbonate Sedimentation Rates Today and in the Past: Holocene of Florida
738 Bay, Bahamas, and Bermuda vs. Upper Jurassic and Lower Cretaceous of the Jura Mountains (Switzerland and
739 France). *Geologia Croatica*, **56**, 1-18.
- 740 VAIL, P. R., MITCHUM, R. M. J., TODD, R. G., WIDMIER, J. M., THOMPSON, S. I., SANGREE, J. B., BUBB, J. N. AND HATLELID, W. G.
741 1977. Seismic stratigraphy and global changes of sea level. In *Seismic Stratigraphy: Applications to*
742 *Hydrocarbon Exploration*, (ed. PAYTON, C. E.), AAPG Memoir, **26**. AAPG, Houston, 49-212.
- 743 VANSTONE, S. D. 1998. Late Dinantian palaeokarst of England and Wales: implications for exposure surface
744 development. *Sedimentology*, **45**, 19-37.
- 745 VOLLBRECHT, R. AND KUDRASS, H. R. 1990. Geological results of a pre-site survey for ODP drill sites in the SE Sulu Basin.
746 In *Proceedings of the Ocean Drilling Program, Initial Reports*, (eds. RANGIN, C., SILVER, E. AND VON BREYMAN, M.
747 T.), **124**, 105-11.
- 748 WALKDEN, G. M. AND WALKDEN, G. D., 1990. Cyclic sedimentation in carbonate and mixed carbonate clastic
749 environments: four simulation programs for a desktop computer. In *Carbonate Platforms (Facies, Sequences*
750 *and Evolution)*, (ed. TUCKER, M. E., WILSON, J. L., CREVELLO, P. D., SARG, J. F. AND READ, J. F.), IAS Special Publication,
751 **9**, 55-78.
- 752 WILKINSON, B. H., DIEDRICH, N. W. AND DRUMMOND, C. N. 1996. Facies successions in peritidal carbonate sequences.
753 *Journal of Sedimentary Research*, **66**, 1065-78.
- 754 WILKINSON, B. H., DIEDRICH, N. W., DRUMMOND, C. N. AND ROTHMAN, E. D. 1998. Michigan hockey, meteoric
755 precipitation, and rhythmicity of accumulation on peritidal carbonate platforms. *Geological Society of America*
756 *Bulletin*, **110**, 1075-93.
- 757 WILKINSON, B. H., DRUMMOND, C. N., ROTHMAN, E. D. AND DIEDRICH, N. W. 1997. Stratal order in peritidal carbonate
758 sequences. *Journal of Sedimentary Research*, **67**, 1068-82.
- 759 WILSON, J. L. 1972. Influence of local structure in sedimentary cycles of Beeman and Holder Formations,
760 Sacramento Mountains, Otero County, New Mexico. In *Cyclic Sedimentation in the Permian Basin, A*
761 *Symposium*, (ed. ELAM, J. C. AND CHUBER, S.), West Texas Geological Society, Midland, TX, 100-14.
- 762 WRIGHT, V. P. 1996. Use of palaeosols in sequence stratigraphy of peritidal carbonates. In *Sequence Stratigraphy in*
763 *British Geology*, (ed. HESSELBO, S. P. AND PARKINSON, D. N.), Geological Society Special Publication, **103**. The
764 Geological Society, London, 63-74.
- 765 WRIGHT, V. P. AND VANSTONE, S. D. 2001. Onset of Late Palaeozoic glacio-eustasy and the evolving climates of low
766 latitude areas: a synthesis of current understanding. *Journal of the Geological Society*, **158**, 579-82.

767 **Figure Captions**

768 **Figure 1:** Diagrammatic example of how cyclical variation in accommodation, in this case the combined effects of
769 two sea-level curves, results in ostensibly cyclical patterns of sedimentation. Modified after Barrell (1917) and
770 Goldhammer *et al.* (1994).

771 **Figure 2:** (a) Chronostratigraphic and (b) stratigraphic diagrams showing carbonate accumulation. The carbonate
772 platform starts from 0m elevation and accumulates sediment according to the carbonate accumulation functions
773 while water depth is greater than lag depth (2m). Submarine hardgrounds are created if water depth is greater
774 than 0m but less than lag depth. If the carbonate platform is exposed (water depth less than 0) no sedimentation
775 occurs. Sedimentation is plotted on a secondary axis with a maximum rate of 2000m Ma^{-1} . Hardgrounds and sub-
776 aerial exposure surfaces are displayed in the appropriate chronostratigraphic position with an arbitrary y-axis
777 value. This output from a single model run illustrates that the definition of a hierarchy is domain-dependent and
778 that the specific characteristics of the hierarchy depend on whether it is measured in time or thickness. Both plots
779 show the result of a single simulation run over a period of 0.4Ma producing 130m of strata.

780 (a) the results in time. All model time is accounted for, including time present in the thickness domain as
781 unconformities, and so a new HFS begins when the underlying HFS ends (i.e. when the platform becomes
782 exposed). HFSs are stacked into LFSs according to the definition presented in this study. In the time-domain, this
783 means that a new LFS is started when a given HFS is of shorter duration than the previous HFS.

784 (b) the results plotted in thickness. HFSs are defined between sub-aerial exposure surfaces. LFSs are defined as
785 stacked HFSs, beginning when a given HFS is thicker than the underlying HFS. Carbonate production rate is shown
786 along the x-axis as a proxy for lithofacies typically depicted on stratigraphic columns from output. Large
787 thicknesses of hardgrounds depict sedimentation occurring slightly below lag depth and outpacing sea-level rise
788 (thus creating thin beds separated by hardgrounds).

789 Four LFSs are defined in the time-domain while five LFSs are defined in the thickness-domain. The difference arises
790 during the fourth HFS which incorporates a long period of exposure and a short period of sedimentation. In the
791 time-domain this HFS is assigned to the second LFS since it was precipitated by the second intermediate-frequency
792 rise in sea-level in the simulation. In the thickness-domain, this third HFS is thick enough to start a new LFS, but the
793 next HFS is also thicker than the underlying HFS, and so forms a LFS comprising a single HFS. The thickness-domain
794 therefore gives a false impression of the actual behaviour of sea-level since it suggests five major oscillations of
795 sea-level, when in fact there were only four.

796 **Figure 3:** (a) An example of a depth-dependent carbonate production curve used in the model.

797 **Figure 4:** Water depth history from the model depicting a 50ka interval from a single simulation run with a range of
798 different time steps. The calculated water depths change substantially for time steps greater than 0.00005Ma (50
799 years) due to approximation (aliasing) error. A time-step of 0.000025Ma was chosen for the simulations as it is
800 within the accurate range.

801 **Figure 5:** Example output of the model showing how a ratio is recorded to represent the proportion of HFSs per
802 LFS.

803 (a) Chronostratigraphic diagram showing carbonate platform growth through time. For a description of the
804 diagram refer to Figure 2.

805 (b) Chronostratigraphic diagram showing carbonate production and water-depth relative to a water-depth of 0m.

806 In this example there are three LFSs, where each successive LFS contains two, four and four HFSs. The ratio is
807 simply the number of HFSs per LFS ($1/x$ where x =number of HFSs) averaged over the entire simulation. In this case
808 $h=0.33$ and would be strong evidence of the existence of a hierarchy (an average of three HFSs per LFS over the
809 entire simulation).

810 **Figure 6:** Chronostratigraphic diagrams of simulations that resulted in a range of h -values. Amplitude of high-
811 frequency eustatic oscillations is the only variable modified between simulations. Intermediate-frequency
812 amplitude is fixed at 40m, run-time in all cases is 1Ma. As high-frequency oscillation amplitude increases, more
813 HFSs are recorded and the h -value decreases. Above a high-frequency amplitude of 30m, the simulations could be
814 said to be strongly hierarchical. Strongly hierarchical simulations do not show a strong correlation with the mean
815 amount of total simulation time recorded as sedimentation (referred to here as preservation). The simulation with
816 the most strongly non-hierarchical results preserves the least amount of time as sedimentation. Other simulations
817 preserve a similar amount of time as sedimentation.

818 **Figure 7:** Stratigraphic columns of the same set of simulations shown as chronostratigraphic diagrams in Figure 6.
819 See text for description and discussion.

820 **Figure 8:** Cumulative probability plot showing the results of 110000 model runs to evaluate the likelihood of
821 generating a hierarchy defined in the time-domain. This plot shows that there is a 9% probability of generating a
822 mean ratio of $h<0.5$. $h<0.5$ is taken here as weak evidence for the existence of a sedimentary hierarchy (an average
823 of two HFSs per LFS). $h<0.33$ is taken as strong evidence of a sedimentary hierarchy (an average of three HFSs per
824 LFS).

825 Using the $h<0.5$ criterion it can be said that approximately one in ten simulations (9%) display some evidence for a
826 hierarchy in the time-domain. Using the $h<0.33$ criterion it can be said that only approximately one in twenty
827 simulations (4%) display strong evidence for a hierarchy in the time-domain.

828 **Figure 9:** Cumulative probability plot showing the results of 110000 simulations which evaluates the likelihood of
829 generating a hierarchy in the thickness-domain. This plot shows that there is a 15% probability of generating a
830 mean ratio of $h < 0.5$. $h < 0.5$ is taken here as weak evidence for the existence of a sedimentary hierarchy (an average
831 of two HFSs per LFS). $h < 0.33$ is taken as strong evidence of a sedimentary hierarchy (an average of three HFSs per
832 LFS).

833 Using the $h < 0.5$ hurdle it can be said that approximately one in seven simulations (15%) display some evidence for
834 a hierarchy in the thickness-domain. Using the $h < 0.33$ hurdle it can be said that only approximately one in one
835 hundred simulations (1%) display strong evidence for a hierarchy in the thickness-domain.

836 **Figure 10:** Scatter diagram showing the distribution of time-domain hierarchical conditions under a range of sea-
837 level parameters. For this group of 100 simulations oscillation of low-frequency sea-level amplitude was fixed at
838 0m, subsidence rate was fixed at 200m Ma^{-1} and maximum production rate fixed at 2000m Ma^{-1} . Axes represent
839 varying conditions of intermediate- and high-frequency sea-level amplitude. h -value (degree of hierarchy) is
840 represented by a gradient colour-scale as depicted. (a) and (b) represent the parameter-space positions of the
841 simulations depicted in Figure 11, where (a) is an example of a non-hierarchical simulation and (b) is an example of
842 a hierarchical simulation. (a) and (b) have h -values of 0.84 and 0.43 respectively.

843 For a given simulation hierarchies are seen to be generated at large amplitudes of high-frequency oscillation
844 relative to the amplitude of intermediate-frequency oscillation. This trend can generally be said to be linear. It can
845 be said that for this set of simulations a hierarchy will be generated as long as the amplitude of high-frequency
846 oscillation is greater than 70% of the amplitude of intermediate-oscillation.

847 **Figure 11:** Line diagram showing the distribution of time-domain mean ratio h -values for the simulations depicted
848 in Figure 9. The probability of generating a sedimentary hierarchy increases with larger high-frequency amplitude.
849 h -values returned from these simulations have a minimum of 0.2, which corresponds to the maximum possible
850 number of high-frequency oscillations occurring within an intermediate-frequency oscillation. Consequently a LFS
851 can contain a maximum of five HFSs.

852 **Figure 12:** Chronostratigraphic and stratigraphic diagrams showing (a) an example of a non-hierarchical simulation
853 and (b) a hierarchical simulation (distinguished with a grey background). The parameter-space location of these
854 simulations is shown in Figure 9 (a and b). Sedimentation occurs when the relative sea-level curve (RSL) has a
855 positive elevation relative to the platform surface (represented by a 0-line) and is represented by a horizontal
856 chronostratigraphic column showing a schematic representation of lithofacies. Per cycle, the amount of time
857 recorded as deposition, as opposed to non-deposition during emergence, is recorded and presented as a
858 percentage of the total duration of that cycle. This is averaged over the entire simulation and those values are
859 presented here. It is notable that both simulations record, on average, a similar proportion of the total simulation

860 period. This suggests that highly hierarchical sections, even though they have more periods of sub-aerial exposure,
861 do not record less time.

862 In the hierarchical example the high-frequency sea-level curve impacts much more upon the overall sea-level curve
863 and consequently generates several HFSs per intermediate-frequency oscillation. In the non-hierarchical example,
864 the high-frequency curve impacts less on overall sea-level behaviour due to its smaller amplitude, and therefore
865 fewer HFSs are generated. This leads to an h -value of 0.43 in the hierarchical case and 0.84 in the non-hierarchical
866 case.

867 **Figure 13:** Parameter-space plot showing the time-domain hierarchy mean ratio recorded by 10000 individual
868 simulations. Each point represents the mean ratio for a given simulation. These points are arranged into 10x10
869 rectangles, the axes of which are (x) high-frequency amplitude and (y) intermediate-frequency amplitude. The
870 squares are arranged into a trellis such that each cell of the trellis has a common low-frequency amplitude and
871 subsidence rate. Carbonate production rate was not varied in these simulations (2000m Ma^{-1}). Platforms which
872 drowned, and therefore never attained equilibrium relative to sea-level variation, are indicated with a different
873 symbol. Drowning is defined by a simulation having no production other than aphotic during a single intermediate-
874 frequency oscillation (i.e. water-depth is always greater than 55m). No results are recorded after a platform
875 becomes drowned.

876 Generally, it can be said that hierarchical simulations are generated at a range of high- and intermediate-frequency
877 amplitudes, but only at small low-frequency amplitudes ($<100\text{m}$) and low subsidence rates ($<400\text{m Ma}^{-1}$).

878 **Figure 14:** Parameter-space plot showing the thickness-domain hierarchy mean ratio recorded by 10000 individual
879 simulations. Each point represents the mean ratio for a given simulation. These points are arranged into 10x10
880 rectangles, the axes of which are (x) high-frequency amplitude and (y) intermediate-frequency amplitude. The
881 squares are arranged into a trellis such that each cell of the trellis has a common low-frequency amplitude and
882 subsidence rate. Carbonate production rate was not varied in these simulations (2000m Ma^{-1}).

883 Generally, it can be said that hierarchical simulations are generated at a range of high-, intermediate- and low-
884 frequency amplitudes, but only at low subsidence rates ($<500\text{m Ma}^{-1}$).

885 **Figure 15:** Representative photographs and measured sections from the two outcrop localities documented in
886 Pollitt (2008). Outcrops are ostensibly cyclic but do not exhibit a sedimentary hierarchy when evaluated
887 objectively. Position of measured sections on photographs is depicted with black lines. List of abbreviations: SS =
888 sandstone, GS = grainstone, PS = packstone, WS = wackestone, BS = boundstone, MS = mudstone.

889 **Figure 16:** Line diagram showing a comparison between the mean ratio of a series of sedimentary hierarchies
890 described by various authors and the mean ratio of the same section with the hierarchy evaluated under the strict

891 definition from this study. Sections indicated by a * are from Pollitt (2008). All data except those indicated by a * is
892 from Lehrmann and Goldhammer (1999). Of the 93 measured outcrop sections published in Lehrmann and
893 Goldhammer (1999), only five had an explicitly defined hierarchy (with both 5th- and 4th-order cycles defined).

894 Circles indicate the mean and mode *h-value* using the HFS and LFS definitions of the original workers. Diamonds
895 indicate the mean and mode ratio resulting from the application of a “strict” definition of a hierarchy presented in
896 this study (i.e. a new LFS is started when a given HFS is thicker than the last).

897 Using the author-defined 4th-5th order cycles results in a much higher ratio: <0.3, or more than three 5th order
898 cycles per 4th. Using a strict definition of a hierarchy by thickness results in strongly non-hierarchical sections:
899 average ratio of >0.6, meaning on average, less than two 5th order cycles make up a 4th order cycle.

900 **Figure 17:** Scatter diagram showing the mean *h-value* and number of HFSs for 88 measured sedimentary sections.
901 Data is from Lehrmann and Goldhammer (1999). Only three measured sections have a *h-value* of <0.5 and can
902 therefore be considered hierarchical. Furthermore, a greater degree of scatter is seen in measured sections with
903 few HFSs. With more samples ($n > 40$), *h-value* is seen to occur in a range between 0.6 and 0.7. Short sedimentary
904 sections with few interpreted HFSs may therefore give a false impression of a sedimentary hierarchy.

905 **Figure 18:** Scatter diagram showing the relationship between the number of HFSs in an outcrop section and the
906 mean *h-value* of that section. Data is from Lehrmann and Goldhammer (1999). The amount of scatter in *h-value*
907 decreases as the number of recorded HFSs increases. The amount of scatter associated with fewer HFSs suggests
908 these sections are under-sampled. This plot suggests that to accurately determine the degree of hierarchy inherent
909 to a stratigraphic section that section should have >~20 HFSs.

910 **Figure 19:** Line chart showing the mean and mode “run” of bed thickness per outcrop from data in Lehrmann and
911 Goldhammer (1999). For example, a series of thinning upwards beds with the consecutive thickness; 4m, 3m, 2m,
912 1m, 4m, would constitute a run of four. Error bars represent the standard distribution of the mean number of beds
913 per HFS for that outcrop.

914 **Table 1:** Model parameters used. In each simulation parameters were varied within a range according to a
915 stepping value. All simulations had a run time of 3Ma. Maximum oligophotic and aphotic production rates were
916 modelled as a proportion of the maximum euphotic rate (20% and 5% respectively). Lag depth was fixed at 2m. In
917 order to evaluate the interaction of all possible parameter permutations, 110000 simulations were conducted.