Breadth vs. Depth

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Basic Question

The Breadth / Depth Question

When choosing among multiple unknown alternatives, is it better to learn a little about all of them or a lot about only one of them?

- Breadth Strategy: A little about all options.
- Depth Strategy: A lot about a single option.

Model

A risk-neutral agent faces the following choice problem:

- There are N objects and N attributes.
- Each object has a value drawn i.i.d from a mean-zero distribution F for each attribute.
- The payoff from choosing an object is the sum of its values.
- The agent knows F, but not the realizations.

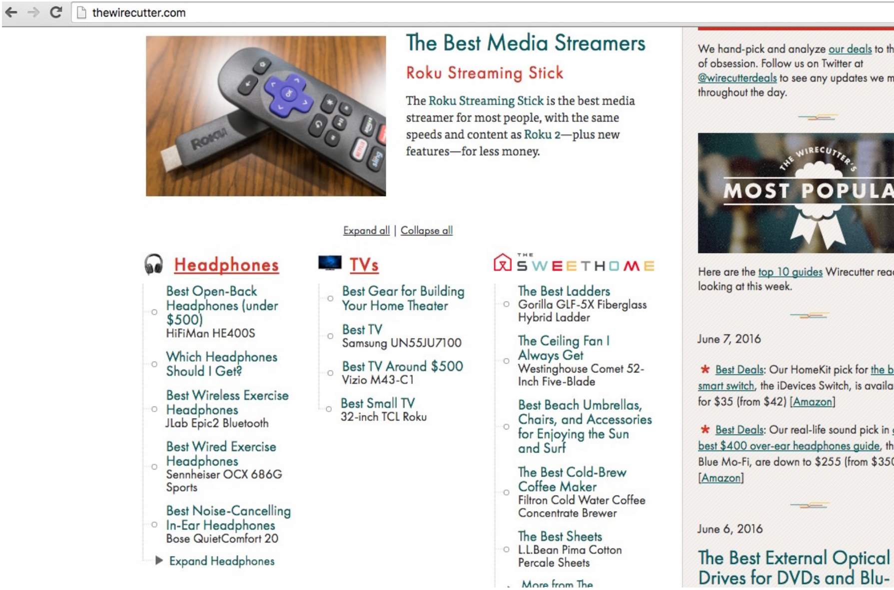
	A_1	A_2
O_1	F	F
O_2	F	F

Search Strategies

	A_1	A_2
O_1	F	F
O_2	F	F

- "Breadth" is learning all of the values for a single attribute
- "Depth" is learning all of the values for a single good
- If I want to learn about a particular phone, I can go to the store, borrow a friend's, ask questions, etc...
- If I want to learn about an attribute (photo quality), I can learn about megapixels, focus lengths, shutter speed, etc...

Depth Example - Wirecutter



Depth Example - Wirecutter

Our pick

The Roku Streaming Stick offers the widest selection of content, the best search, the best interface, and the best user experience. Unlike prior versions, the current Roku Streaming Stick offers all the speed and performance of the more expensive Roku 2, drops some extra features that many people probably don't need, and adds other benefits.

Our pick

$50 from Roku

The best media streamer for most people

Roku Streaming Stick

The Roku Streaming Stick is the best media streamer for most people because it offers the largest selection of streaming content, a clean and responsive user interface, and a useful search function.

Roku has a larger selection of content than anyone else, and it continues to grow. Amazon, Google Play Movies & TV, HBO Go and Now, Hulu, Netflix, Pandora, Showtime, Sling TV, Spotify, Vudu, and more all have support. Finding something that Roku doesn't support is the hard part. The only major service missing is iTunes, but Apple doesn't open that up to anyone. When new services launch, Roku is typically among the first—if not the first—to offer support.

Roku's search displays results in a specific order: First, results from channels you have installed, sorted by price (lowest first). After this, you get results from channels you don't have installed, which are also ordered by price. Not only does this approach help you find content more easily, but it also lets you choose content from the least expensive source. If a movie or TV show is available for free from Netflix but for purchase from Amazon and Vudu, for example, Roku's search function shows Netflix first. For people who subscribe to multiple streaming services, where content changes monthly, Roku's search function makes finding what you want, for the lowest price, easier than the search tools on competing boxes.

For example, contrast all of that to Amazon's Fire TV and Google Chromecast. The Fire TV's search is currently limited to Amazon, Crackle, Hulu Plus, Showtime, and Vevo. Because of how Google's Chromecast works, it offers no search across different platforms. Amazon was the first to offer voice search on its streaming box, but the search on the Roku Streaming Stick is better implemented and looks across more content than Amazon's feature does. After all, a search feature that is easier to use because of voice control but is unable to find what you're searching for really isn't useful.

Roku also lets users create their own "channels," which can provide access to content even if official Roku support doesn't exist. Lifehacker has some helpful tips regarding great, free streaming channels available on Roku and how to find them.

Examples

Primary Examples -

- Phones
  - Resolution, Reception Quality, Battery Life, Camera Quality
- Restaurants
  - Yelp Rating, Spiciness, Distance
- Politicians
  - Domestic and Foreign Policy Issues

Alternate (and Mathematically Equivalent) Example -

- Investments
  - There are N possible states of the world which may be realized tomorrow
  - Each state is equally likely
  - Θ_1 = positive jobs report, Θ_2 = negative jobs report
  - Payoffs = expected value
  - A search reveals state-dependent payoffs

	$\Theta_1 = \uparrow$	$\Theta_2 = \downarrow$
I_1	F	F
I_2	F	F

  - Event-driven trading strategies

2x2 Example

	A_1	A_2
O_1	x_{11}	x_{12}
O_2	x_{21}	x_{22}

- Ex-ante: $U_i = \mathbb{E}\left[\sum_{j=1}^{2} x_{ij}\right] = 0$

2x2 Example

	A_1	A_2
O_1	x_{11}	x_{12}
O_2	x_{21}	x_{22}

- Ex-ante: $U_i = \mathbb{E}\left[\sum_{j=1}^{2} x_{ij}\right] = 0$
- Breadth search:

$$U_1 = x_{11} + \mathbb{E}[x_{12}] = x_{11} \qquad U_2 = x_{21} + \mathbb{E}[x_{22}] = x_{21}$$

2x2 Example

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- Breadth search:

  $$U_1 = x_{11} + \mathbb{E}[x_{12}] = x_{11} \qquad U_2 = x_{21} + \mathbb{E}[x_{22}] = x_{21}$$

- Choose the maximizer:

  $$\text{Payoff} = \mathbb{E}\left[\max(x_{11}, x_{21})\right]$$

2x2 Example

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2x2 Example

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- Ex-ante: $U_i = \mathbb{E}\left[\sum_{j=1}^{2} x_{ij}\right] = 0$
- Depth search:

  $U_1 = x_{11} + x_{12}, \quad U_2 = 0$

2x2 Example

	A_1	A_2
O_1	x_{11}	x_{12}
O_2	x_{21}	x_{22}

- Ex-ante: $U_i = \mathbb{E}\left[\sum_{j=1}^{2} x_{ij}\right] = 0$
- Depth search:

  $U_1 = x_{11} + x_{12}, \quad U_2 = 0$
- Choose 1 if above-average. Otherwise, choose 2.

  Payoff $= \mathbb{E}\left[\max(x_{11} + x_{12}, 0)\right]$

The 2x2 Case - Bernoulli

F Coin Flip, $Prob(1) = Prob(-1) = 1/2$

	A_1	A_2
O_1	F	F
O_2	F	F

Search an Object

	A_1	A_2
O_1	1	1
O_2	F	F

$\downarrow$

2

	A_1	A_2
O_1	1	-1
O_2	F	F

$\downarrow$

0

	A_1	A_2
O_1	-1	1
O_2	F	F

$\downarrow$

0

	A_1	A_2
O_1	-1	-1
O_2	F	F

$\downarrow$

0

Depth Payoff $= (1/4) * 2 = 1/2$

Search an Attribute

	A_1	A_2
O_1	1	F
O_2	1	F

$\downarrow$

1

	A_1	A_2
O_1	1	F
O_2	-1	F

$\downarrow$

1

	A_1	A_2
O_1	-1	F
O_2	1	F

$\downarrow$

1

	A_1	A_2
O_1	-1	F
O_2	-1	F

$\downarrow$

-1

$$\text{Breadth Payoff} = (3/4) * 1 + (1/4) * -1 = 1/2$$

Search an Attribute

	A_1	A_2
O_1	1	F
O_2	1	F

$\downarrow$

1

	A_1	A_2
O_1	1	F
O_2	-1	F

$\downarrow$

1

	A_1	A_2
O_1	-1	F
O_2	1	F

$\downarrow$

1

	A_1	A_2
O_1	-1	F
O_2	-1	F

$\downarrow$

-1

$$\text{Breadth Payoff} = (3/4) * 1 + (1/4) * -1 = 1/2$$
$$= \text{Depth Payoff}$$

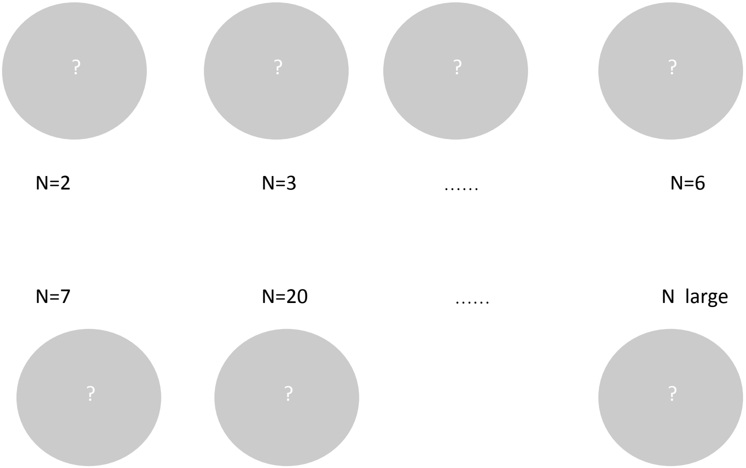
Related Literature

- Weitzman (1979) - Pandora's box search
- Bordalo, Gennaioli, Shleifer (2013) - agents pay weighted attention to attributes
- Speigler (2006) - an IO framework where agents sample one price attribute of each object
- Klabjan, Olszewski, Wolinsky (2014) - optimal attribute search selection for a single good
- Gabaix, Laibson, Moloche, Weinberg (2006) - experiment on searching through an unknown matrix with F normal
- Sanjuro (2017) - simulations and establishes some rules for searching from above

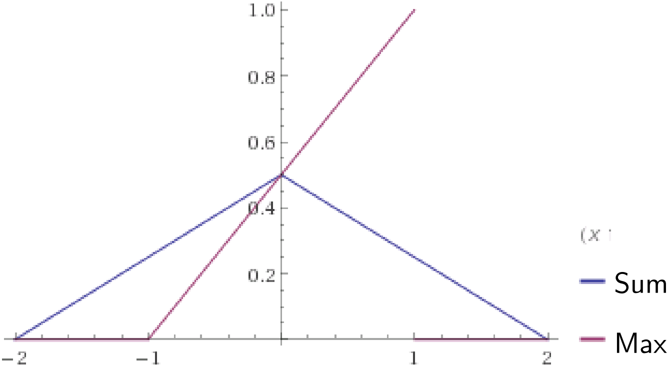
Outline

1. $N = 2$
2. $3 \leq N \leq 6$
3. N Large, Thin Tails
4. N Large, Fat Tails
5. Political Competition
6. Strategic Settings

Goal

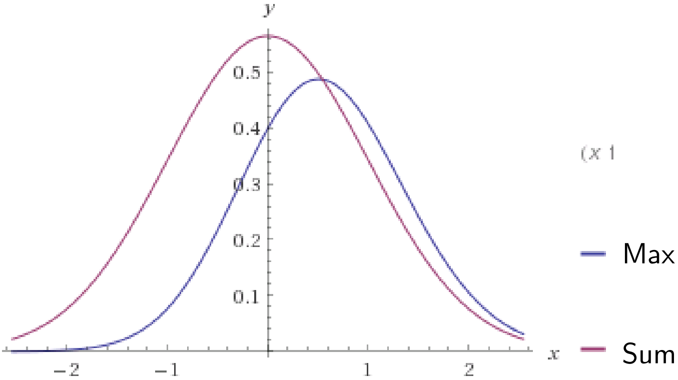


N=2, Uniform



- Expectation of Max of 2 Uniforms = $1/3$
- Expectation of Sum of 2 Uniforms = $1/3$

N=2, Normal



- Expectation of Max of 2 Normals $= 1/\sqrt{\pi}$
- Expectation of Sum of 2 Normals $= 1/\sqrt{\pi}$

Symmetric Distributions

For the Bernoulli, Uniform, and Normal Distributions, Breadth $=$ Depth.

Symmetric Distributions

For the Bernoulli, Uniform, and Normal Distributions, Breadth $=$ Depth.

Theorem

For $N = 2$ and F symmetric, breadth$=$depth.

That is, the payoffs of searching an object or searching an attribute are the same.

Partition

- Fix $x \geq y \geq 0$ s.t. $x, y \in \operatorname{supp}(F)$.
- The realizations $(x, y), (x, -y), (-x, y), (-x, -y)$ are equally likely by symmetry.
- This partitions the possible realizations.
- It suffices to demonstrate that Breadth $=$ Depth for each cell of the partition.

Object Search

	A_1	A_2
O_1	x	y
O_2	F	F

$\downarrow$

$x + y$

	A_1	A_2
O_1	x	$-y$
O_2	F	F

$\downarrow$

$x - y$

	A_1	A_2
O_1	$-x$	y
O_2	F	F

$\downarrow$

0

	A_1	A_2
O_1	$-x$	$-y$
O_2	F	F

$\downarrow$

0

Conditional Depth Payoff

$$= (1/4) * (x + y) + (1/4) * (x - y) = x/2$$

Attribute Search

	A_1	A_2
O_1	x	F
O_2	y	F

$$\downarrow$$
$$x$$

	A_1	A_2
O_1	x	F
O_2	$-y$	F

$$\downarrow$$
$$x$$

	A_1	A_2
O_1	$-x$	F
O_2	y	F

$$\downarrow$$
$$y$$

	A_1	A_2
O_1	$-x$	F
O_2	$-y$	F

$$\downarrow$$
$$-y$$

$$\begin{aligned}&\text{Conditional Breadth Search Payoff}\\ &\quad = (2/4) * x + (1/4) * y + (1/4) * (-y)\\ &\quad = x/2 = \text{Conditional Depth Search Payoff}\end{aligned}$$

Breadth with an Outside Option

With an outside option of 0, Breadth is strictly better.

	A_1	A_2
O_1	x	F
O_2	y	F

$\downarrow$

x

	A_1	A_2
O_1	x	F
O_2	$-y$	F

$\downarrow$

x

	A_1	A_2
O_1	$-x$	F
O_2	y	F

$\downarrow$

y

	A_1	A_2
O_1	$-x$	F
O_2	$-y$	F

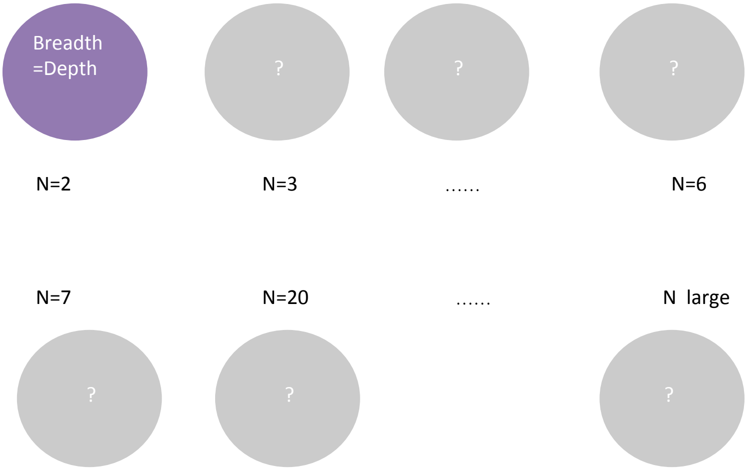
$\downarrow$

0

Conditional Breadth Search Payoff

$$= (2/4) * x + (1/4) * y$$
$$= x/2 + y/4 \geq \text{Conditional Depth Search Payoff}$$

N=2



$N = 3$

$F = \pm 1$ with 50% probability.

- Depth Payoff $= 3/4$
- Breadth Payoff $= 3/4$

$N = 3$

$F = \pm 1$ with 50% probability.

- Depth Payoff $= 3/4$
- Breadth Payoff $= 3/4$

$F \sim N(0,1)$

- Depth Payoff $= \frac{\sqrt{3}}{\sqrt{2\pi}} \approx 0.69$
- Breadth Payoff $= \frac{3}{2\sqrt{\pi}} \approx 0.85$

Partition

- Fix $x_1 \geq x_2 \geq x_3 \geq 0$ s.t. $x_1, x_2, x_3 \in \text{supp}(F)$.
- The realizations $(\pm x_1, \pm x_2, \pm x_3)$ are equally likely.
  - With probability 1/2, $(x_1, \ldots)$ $\rightarrow x_1$
  - With probability 1/4, $(-x_1, x_2, \ldots)$ $\rightarrow x_2$
  - With probability 1/8, $(-x_1, -x_2, x_3)$ $\rightarrow x_3$
  - With probability 1/8, $(-x_1, -x_2, -x_3)$ $\rightarrow -x_3$
- Therefore, Breadth Search Payoff $= 1/2x_1 + 1/4x_2$
- Depth Search Payoff is more complicated.
- It depends upon how x_1 relates to $x_2 + x_3$

Small N

- Breadth Payoff $= 1/2x_1 + 1/4x_2$
- Depth Payoff is either $1/2x_1$ or $1/4x_1 + 1/4x_2 + 1/4x_3$
- Either way Breadth $\geq$ Depth
- In general, for $N = 3, \ldots, 6$, there could be many cases
  - But, an inductive argument suffices
  - Until $N = 7$

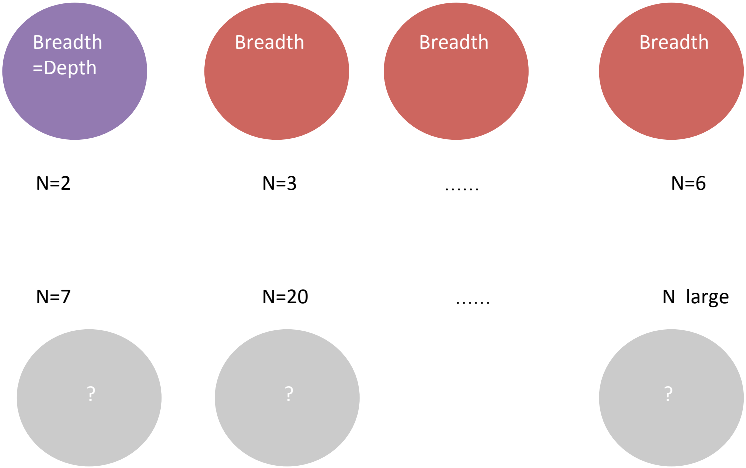
Small N Theorem

Theorem

For F symmetric, $N = 3, 4, 5, 6$, Breadth $\geq$ Depth.

The above is generally strict. Only equalities are $N = 2$ or $N = 3, 5$ and F Bernoulli.

$N \leq 6$



Breadth =Depth
Breadth
Breadth
Breadth
N=2
N=3
......
N=6
N=7
N=20
......
N large
?
?
?

Small N Theorem: Tight

Theorem

For F symmetric, $N = 3, 4, 5, 6$, *Breadth* $\geq$ *Depth.*

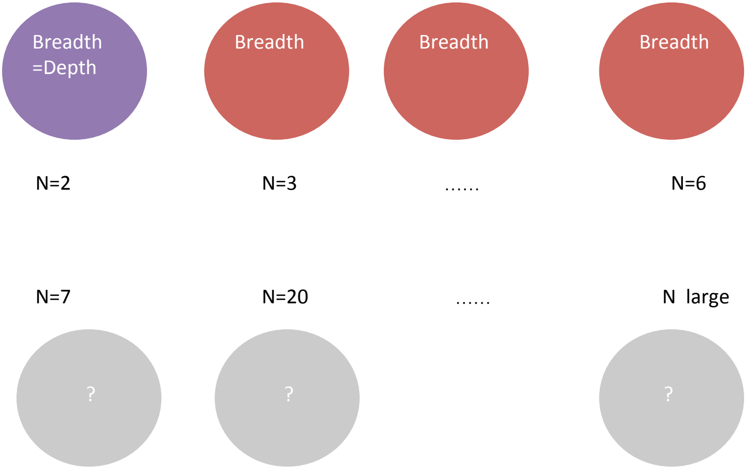
Tightness

For any $N \geq 7$, $\exists F_N, G_N$ symmetric s.t.

$\text{Breadth}(F_N) > \text{Depth}(F_N)$
$\text{Breadth}(G_N) < \text{Depth}(G_N)$.

Zero-Inflated Distributions, $F_N = p * 0 + (1 - p) * Binom(-1, 1)$

N$\leq$6


Breadth
=Depth
Breadth
Breadth
Breadth
N=2
N=3
......
N=6
N=7
N=20
......
N large
?
?
?

Why $N \times N$?

- In a $N \times N$ problem, breadth and depth both reveal N out of N^2 squares
- Interpretation: Searching a $N_O \times N_A$ matrix where $N_O, N_A \geq N$.
- Results on $N \times N$ have implications for other sized matrices

Large N

Theorem

For any given F with finite variance, for all large enough N, Depth $>$ Breadth.

Bounded Intuition

- $F \sim \pm 1$ coin flip
- Breadth Payoff ≤ 1
- Central Limit Theorem: $\sum_{j=1}^{N} \frac{X_{ij}}{\sqrt{N}} \to Normal(0, 1)$
- $\sum_{j=1}^{N} X_{ij} \sim \sqrt{N} * Normal(0, 1)$
- Above-average normal draws: $\int_0^\infty x \frac{e^{-x^2/2\sigma^2}}{\sigma\sqrt{2\pi}} dx = \frac{\sigma}{\sqrt{2\pi}}$
- Depth Payoff $\sim \frac{\sigma}{\sqrt{2\pi}} \sqrt{N}$

Normal Intuition

- $F \sim Normal(0, \sigma)$
- Breadth Payoff $= \mathbb{E}\left[\max(X_1, \ldots, X_N)\right] \leq \sigma\sqrt{2 \log N}$
- Depth Payoff $\sim \frac{\sigma}{\sqrt{2\pi}}\sqrt{N}$

General Intuition

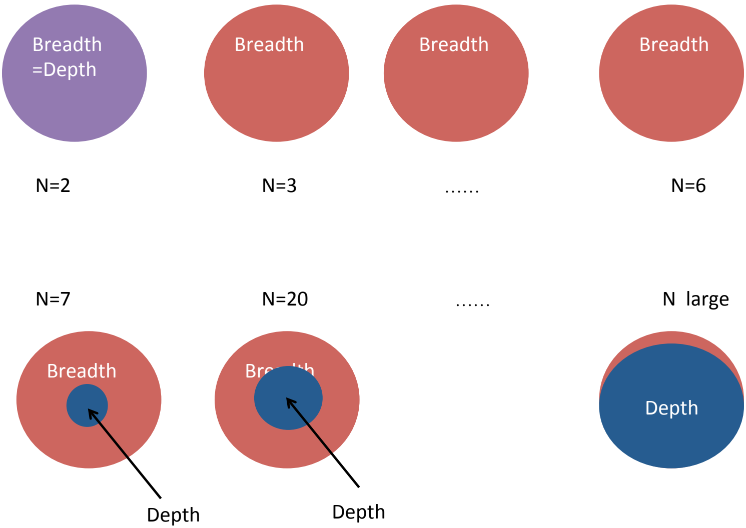
- Central Limit Theorem: $\sum_{j=1}^{N} \frac{X_{ij}}{\sqrt{N}} \to Normal(0, \sigma)$
- Depth Payoff $\sim \frac{\sigma}{\sqrt{2\pi}}\sqrt{N}$
- Gumbel (1954), shows for any. F with mean μ and std. dev σ, that $\mathbb{E}\left[\max_{i \leq N} X_i\right] \leq \mu + \sigma\frac{N-1}{\sqrt{2N-1}} \sim \mu + \sigma\sqrt{N}$
- Not good enough

General Intuition Continued

Depth Payoff $\sim \frac{\sigma}{\sqrt{2\pi}}\sqrt{N}$

- Truncating $X^{|c} = \max(c, X)$, increases μ, decreases σ
- $\mathbb{E}\left[\max_{i \leq N} X_i\right] \leq \mathbb{E}\left[\max_{i \leq N} X_i^{|c}\right] \leq \hat{\mu} + \hat{\sigma}\sqrt{N} < \frac{\sigma}{\sqrt{2\pi}}\sqrt{N}$
- The Gumbel bound for $X^{|c}$ is sufficient. $\blacksquare$

Illustration



Observability with Noise

Corollary

All previous results hold if a signal $s_{i,j} = x_{i,j} + \epsilon_{i,j}$ is observed where $\epsilon_{i,j} \sim G$ for a symmetric G.

IO Model

- N firms selling K attribute goods
- Each firm chooses F_i s.t. $\mu_i = 1$ and $F_i(x) = 0, \ \forall x < 0.$
- Agents choose to search by object or attribute and which object or attribute to search.
- They then select an object and receive its expected payoff according to their own search.
- Firms' payoffs are the probability of being selected.
- We restrict attention to symmetric equilibria.

Object Equilibrium

- Firm i's payoff $= \Pr(i \text{ chosen})$
  $= 1/N \cdot \Pr(i \text{ chosen} \mid i \text{ searched})$
  $+(N-1)/N \cdot \Pr(i \text{ chosen} \mid i \text{ not searched})$
- Firm i only controls the first term.
- If F Bernoulli between ϵ and $1+\epsilon$, then $\lim_{\epsilon \to 0} \Pr(i \text{ chosen} \mid i \text{ searched}) = 1$.
- Therefore, in equilibrium $\Pr(i \text{ chosen} \mid i \text{ searched}) = 1 \Rightarrow$
- F_i is a unit mass at 1.
- Agents randomize searching between all objects
- If the realized object is weakly above average, they choose it, otherwise, they randomly choose an unsearched object.

Breadth Equilibrium

Firm i's Payoff: $\Pr_{F_i}(i \text{ chosen} \mid \text{breadth search}) =$

$$\Pr(x_i > \max_{k \neq i} x_k) + \frac{\Pr(x_i = \max_{k \neq i} x_k)}{\#\{x_k | x_k = \max_{k'} x_{k'}\}}$$

Theorem

In the unique attribute equilibrium, each firm employs the same distribution $F(x) = (x/N)^{1/N-1}$ *on* $[0, N]$.

Attribute Equilibrium

Claim

In a symmetric equilibrium, there can be no positive masses.

- If there were at $x > 0$, then the firm can shift $1 - \epsilon$ this weight to $x + \frac{\epsilon^2}{1-\epsilon}$ weight above x and ϵ of the mass to $x - \epsilon$ via a mean preserving spread.
- The firm's probability of winning is only affected when his value was x and the maximum value of all other firms is x.
- In those situations, the firm's probability of winning increases goes from at most $1/2$ to $1 - \epsilon$.
- A mass at 0 can similarly be profitably shifted.

Attribute Equilibrium

- Because there are no mass points, the firm's objective function is: $Pr(x_i \geq \max_{k \neq i} x_k)$.
- Holding other firm's strategies fixed as F, a firm solves:

$$\max_g \int_{x=0}^{\infty} F^{N-1}(x) g(x) dx \text{ s.t.}$$

$$\int_{x=0}^{\infty} x g(x) dx = 1 \tag{1}$$

$$\int_{x=0}^{\infty} g(x) dx = 1 \tag{2}$$

$$g(x) \geq 0 \tag{3}$$

- Calculus of variations $\Rightarrow F(x) = (x/N)^{1/N-1}$ on $[0, N]$

Comparative Statics

- For every N, there is both a breadth-search and depth-search equilibrium
- Both are observed in everyday life
- The breadth-search equilibrium is payoff-dominant
- $U^{att} \approx N/2$ and $U^{obj} = 0$
- Social planner:
  - Choose search method and F on $[0, N]$ to maximize agents' utility
  - Optimal Dist. is $\Pr(0) = (N-1)/N$ and $\Pr(N) = 1/N$.
  - Optimal search method is breadth search.
  - This yields utility $\to (1 - 1/e)N \approx 0.63N$.
  - Breadth search is 79.1% of social optimum

- For games against nature, the marginal benefit from either depth or breadth search was at most $O(\sqrt{N})$.
- But, here an agent's benefit is much larger.
- Two benefits from competition
  1. As N increases, an agent gets more draws
  2. The firms' equilibrium distributions change in a fashion which benefits agents.

Conclusion

- Exogenous Distributions
  - Small $N \to$ Breadth
  - Large $N \to$ Depth
- "If you can search only a little, search different objects."
- "If you can search a lot, search the same object"

Conclusion

- Endogenous Distributions $\rightarrow$ Breadth
- Fat Tails $\rightarrow$ Breadth
- Correlation $\rightarrow$ Breadth
- Future Work:
  - Cell-by-cell Attention Allocation
  - Sequential firm/agent choice
  - Tournament Incentives

Political Competition (context suggested by Bhattarchaya)

- In political competition, voters tend to learn exclusively about their favorite candidate
- Behavioral Justification: I can't stand to hear about my dispreferred candidate
- Two Candidates - A, B
- Two Attributes - I, II
- $F = \pm 1$ with 50% probability.
- A voter has a small bias b for Candidate A.
- $U(A) = A_I + A_{II} + b$
- $U(B) = B_I + B_{II}$

Political Competition

Proposition

In the 2x2x2 model with a bias b,
Breadth $=$ Depth$_A$ $=$ Depth$_B$.

Political Competition

Expected Utility with

- $U(A) = u(A_I + A_{II} + b)$ where
- $u(x) = \begin{cases} x & \text{if } x \geq 0 \\ \lambda x & \text{if } x < 0 \end{cases}$
- where $\lambda > 1$

Political Competition

Proposition

If $\lambda < 9$, then searching A, the preferred candidate is optimal.
If $\lambda > 9$, then searching B, the dispreferred candidate is optimal.
Searching an attribute is not optimal.

Loss Aversion - Empirical

Proposition

If $\lambda < 9$, then searching A, the preferred candidate is optimal.

Abdellaoi, Bleichrodt, Paraschiv (2007) present the following:

Study	Definition	Domain	Estimates
Fishburn and Kochenberger (1979)	$\frac{U'(-x)}{U'(x)}$	Money	4.8
Tversky and Kahneman (1992)	$\frac{-U(-1)}{U(1)}$	Money	2.25
Bleichrodt et al. (2001)	$\frac{-U(-x)}{U(x)}$	Health	2.17 3.06
Schmidt and Traub (2002)	$\frac{U'(-x)}{U'(x)}$	Money	1.43
Pennings and Smidts (2003)	$\frac{U'(-x)}{U'(x)}$	Money	1.81
Booij and van de Kuilen (2006)	$\frac{U'_{\uparrow}(0)}{U'_{\downarrow}(0)}$	Money	1.79 1.74

Consequence: If u concave and $\frac{u'(-2)}{u'(2+b)} < 9$, then searching preferred candidate is optimal.

Fat Tails

- The case of infinite variance is more complicated.
- For large N, such cases can be studied via
- Generalized Central Limit Theorem $\sum_{k=1}^{N} \frac{X_{ik}-a_N}{b_N} \to$ Stable Laws
- Extreme Value Theory $\frac{\max(X_1,\ldots,X_N)-a_N}{b_N} \to$ Extreme Value distributions.
- In the case of finite variances, generally the sum grows at a higher rate than the maxima
- For infinite variances, the rates of growth are generally the same, so the constants drive the relationships

Fat Tails

- For a mean zero distribution, infinite variance $\rightarrow \int x^2 f(x)dx = \infty$

- An intuitive candidate for f(x) is $k * \frac{1}{x^{\alpha-1}}$, for which

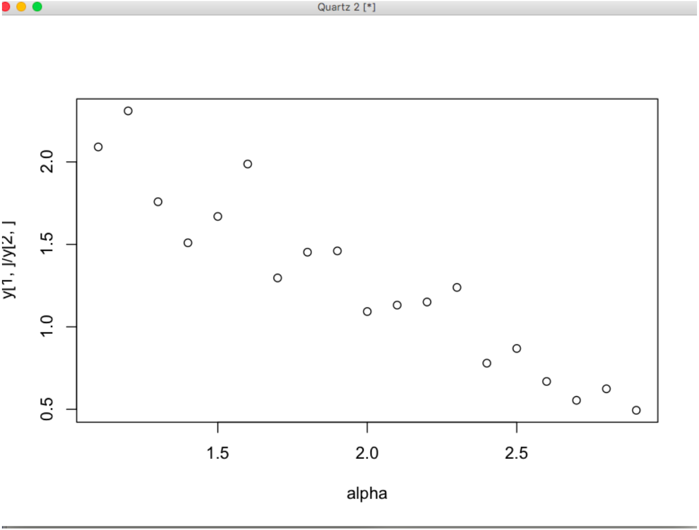
$$\int x^2 f(x)dx = k \int x^{1-\alpha}dx = \infty \text{ for } \alpha \leq 2$$

- These natural laws, with distribution $F(x) = 1 - \left(\frac{k}{x}\right)^{\alpha}$ are known as Pareto (or Power) laws

- Pareto laws have been widely studied in economics (see Mandelbrot (1963), Gabaix (2009)).

Simulation

Breadth Benefit / Depth Benefit



$N = 100000$, trials=100, Total Draws = 210 million

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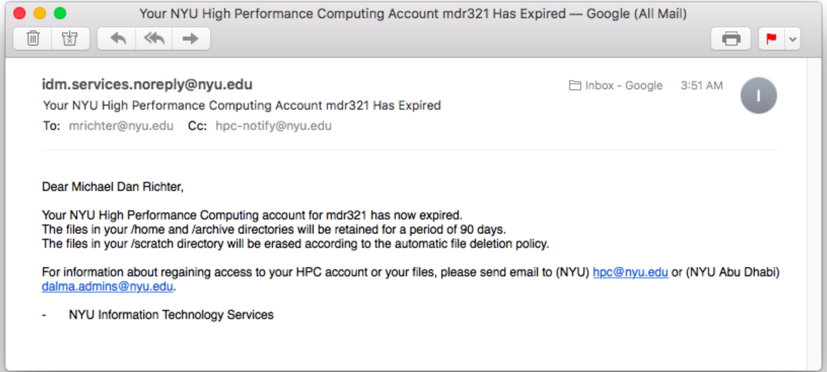

Your NYU High Performance Computing Account mdr321 Has Expired — Google (All Mail)

idm.services.noreply@nyu.edu

Inbox - Google 3:51 AM

Your NYU High Performance Computing Account mdr321 Has Expired

To: mrichter@nyu.edu Cc: hpc-notify@nyu.edu

Dear Michael Dan Richter,

Your NYU High Performance Computing account for mdr321 has now expired.
The files in your /home and /archive directories will be retained for a period of 90 days.
The files in your /scratch directory will be erased according to the automatic file deletion policy.

For information about regaining access to your HPC account or your files, please send email to (NYU) hpc@nyu.edu or (NYU Abu Dhabi) dalma.admins@nyu.edu.

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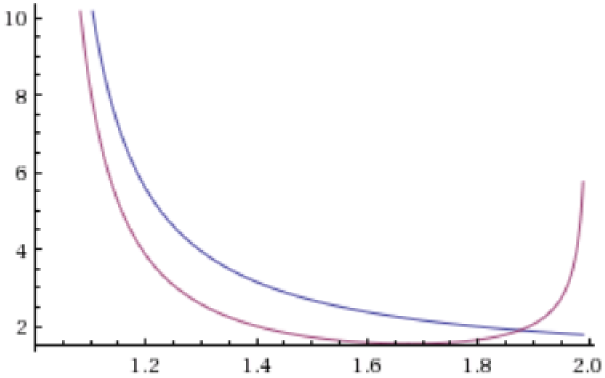
Fat Tails

- $\frac{1}{C_\alpha}\sum_{n=1}^{N}\frac{X_{in}-n\mu}{\sqrt[\alpha]{nk}} \to S(\alpha,1)$ (a stable distribution)
- $\frac{\max(X_1,\ldots,X_N)-n\mu}{\sqrt[\alpha]{nk}} \to \Phi_\alpha$ where $\Phi_\alpha(x) = e^{-x^{-\alpha}}$ (Frechet).
- To compare the search methods requires calculating $\mathbb{E}[\Phi_\alpha]$, $\mathbb{E}[\max(S(\alpha,1),0]$ and C_α



Breadth: always decreasing in α

Depth: growing when $\alpha \to 2$ as convergence constant blows up

Fat Tails Takeaway

- There is a tail-index threshold $\hat{\alpha}$ s.t.
  - For distributions with thicker Pareto tails, breadth is better
  - For distributions with thinner Pareto tails, depth is better
- In a fatter-tailed world, not only do the alternatives become riskier, but there is a second heretofore hidden effect:
  - Agents optimal search procedure leads to the choice of mostly unknown alternatives.