# Non-invasive Longitudinal Beam Profile Diagnostic Exploiting Coherent Cherenkov Diffraction Radiation 

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A thesis submitted to the University of London for the degree of Doctor of Philosophy

## Declaration of Authorship

I Kirill Fedorov hereby declare that this thesis and the work presented in it is entirely my own. Where information has been derived from other sources, I confirm that this has been indicated in the document.

Signed:

Date:

To my parents, Albina and Valeriy, for your constant support.

## Abstract

In this thesis, the possibility of noninvasive longitudinal beam profile diagnostic exploiting Coherent Cherenkov Diffraction Radiation (ChDR) was considered in details. To achieve this a number of practical and theoretical problems have been solved.

First, a system for Coherent Cherenkov Diffraction Radiation generation has been developed as well as system for signal detection and spectral characteristic analysis. Experimental studies has been organized at the CLARA facilities based at Daresbury (United Kingdom) laboratory. With sub-ps long electron bunches, the measurements of the emitted coherent radiation spectra extend up to the THz frequency range was enough to provide its spectral analysis using Martin-Pupplet interferometer.

Second, investigation of the theoretical properties of ChDR taking into account the dielectric properties of the radiator (Teflon target) has been provided. The Polarization Current Approach (PCA) has been used for the computation of the ChDR single electron spectrum generated by a prismatic shape target. The chosen approach allowed us to account for all essential experimental parameters, such as electron beam properties, the distance between electron beam and radiator, the radiator dimensions, and the radiator dielectric properties.

Finally, by using obtained experimental data and theoretical evaluations, set of the longitudinal bunch profiles have been reconstructed. To reconstruct the bunch profile, the Kramers-Kronig analysis has been used

As a result, it was the first demonstration of CChDR being used for noninvasive (not cutting the beam) longitudinal beam profile diagnostic, and the reliability of the results obtained are confirmed by the well-studied method based on Coherent Transition Radiation (CTR).

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## List of abbreviations

| BA1 | CLARA (Beam Area 1) |
| :--- | :--- |
| BPM | Beam Position Monitor |
| BTR | Backward Transition Radiation |
| CCD | Charge-Coupled Device |
| ChDR | Cherenkov Diffraction Radiation |
| CChDR | Coherent Cherenkov Diffraction Radiation |
| CLARA | Compact Linear Accleerator for Research and Applications |
| CSR | Coherent Synchrotron Radiation |
| CTR | Coherent Transition Radiation |
| CW | Constant Wave |
| DR | Diffraction Radiation |
| EOTD | Electro-Optical Time Domain method |
| EOSD | Electro-Optical Spectral Decoding |
| FEL | Free-Electron LASer |
| FFT | Fast Fourier Transform |
| IP | Interaction Point |
| MPI | Martin-Pupplet Interferometer |
| LCLS | Linac Coherent Light Source |
| SASE | Self Amplified Spontaneous Emission |
| SPR | Smith Purcell Radiation |
| TR | Transition Radiation |

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## Introduction

In modern accelerators of various types we coordinate operations of precise electromagnetic devices, such as dipoles and focusing quadrupole magnets, electrostatic deflecting devices, accelerating high-frequency cavities, pulse kickers, etc. to group, focus, accelerate, cool and transport charged particle beams. Imperfections of such complex systems and external environment influence lead to the fact that the motion of the particle beam in real machine is different from the theoretical estimations, and the efficient operation of an accelerator is almost impossible without accurate and reliable beam diagnostics systems that provide capability to adjust accelerator parameters based on the measurement results, and effeciently control beam dynamics.

Nowadays, one of the forefront issues in beam diagnostic is monitoring and control of short picosecond and subpicosecond relativistic bunches parameters. The best examples of machines that require such short bunches are linac-based fourth-generation light sources (free-electron lasers) and next-generation linear colliders.

### 1.1 Challenges in longitudinal beam profile diagnostic

At the forefront of modern accelerator physics and, in particular of linear accelerators, is the concept of the Compact Linear Collider (CLIC) - future sub-TeV energies for future electron-positron collider. The main purpose of such colliders is to extend our understanding beyond the standard model which can be done using "clean" collisions
of fundamental particles. Luminosity of linear colliders can be defined as:

$$
\begin{equation*}
\mathcal{L}=H_{D} \frac{N^{2}}{\sigma_{x}^{*} \sigma_{y}^{*}} n_{b} f_{r} \tag{1.1}
\end{equation*}
$$

where $\sigma_{x}^{*}$ and $\sigma_{y}^{*}$ are the transverce bunch dimensions; $f_{r}$ is the machine operation frequency; $H_{D}$ is the "hour-glass" parameter (describes the change of a bunch transverse size in longitudinal direction); $n_{b}$ and $N$ are the number of bunches in the train and number of particles in one bunch. To reduce the "hour-glass" effect and increase the luminosity we need to reduce longitudinal bunch duration. In Fig 1.1 schematic of the future CLIC is shown. BC1 and BC2 blocks represent bunch compressors which should provide the value of the longitudinal bunch length of 140 fs . Thus, the resolution of $\approx 10 \mathrm{fs}$ is needed and it is necessary to provide monitoring of the bunch shape and length in many point along the beam line.


Figure 1.1: CLIC complex. Figure taken from [1]

Another field where the use of short and ultra-short charged particle beams is required is the generation of X-radiation. This radiation, which was first observed as a byproduct of circular accelerators, nowadays attracts a huge community of researchers.

It founds applications in different fields from medicine and safety to geology and archeology, which is a factor of the ever-growing demand for high power X-rays sources. One of the possible X-ray sources are Free Electron Lasers (FELs) based on linear accelerators. This machines will be able to provide the best solutions for researches, such as radiation power, coherence level, and brilliance. The key to achieving these parameters is the use of the effect of Self-Amplified Spontaneous Emission (SASE) - when the electron bunch density is periodically modulated (microbunched) by the radiation inside undulator. If we will use long undulator, the number of electrons radiating in phase will increase and thus enables a high-gain FEL process. In [2] the relationship between the gain length and bunch density was derived as:

$$
\begin{equation*}
L_{G}=C \gamma\left(\frac{\sigma_{t}^{2}}{I_{0}}\right)^{\frac{1}{3}} \tag{1.2}
\end{equation*}
$$

where $\gamma$ is the Lorenz factor, $\sigma_{t}$ is the RMS transverse size of the beam, $I_{0}$ is the bunch current, C - is the constant defined by the undulator parameters. It can be seen that to have a reasonable short gain length $L_{G}$ it is needed to achieve high charge densities in the electron bunch, which is emphasized the need for longitudinal bunch compression up to tens of fs. Usually, the system of 2 or 3 bunch compressions between linacs is used in modern FEL machines, each of them requires precise control and bunch diagnostic system. Examples of modern X-ray FELs operating with SASE principle are Linear Coherent Light Source (LCLS) in USA [3], EUROPEAN X-FEL [4], Freeelectron LASer (FLASH) in Germany [5], SPring-8 Compact SASE Source (SCSS) in Japan [6], Free-electron laser (SwissFEL) in Switzerland [7]. Main beam parameters of these machines are shown in Table 1.1.

Already existing diagnostic techniques have a number of limitations that make them difficult to use in modern machines. In the next section, all actual diagnostic techniques will be considered in details, and their advantages and disadvantages will be highlighted.

Table 1.1: Typical parameters for SASE FEL's

| SASE FELs | LCLS | EU X-FEL | FLASH | SCSS |
| :---: | :---: | :---: | :---: | :---: |
| Beam energy E [GeV] | 14 | 17.5 | 1.5 | 8 |
| Peak current I [kA] | 3.4 | 5 | 2.5 | 3 |
| Normalized emittance | 1.2 | $<1$ | $<1.4$ | 1 |
| $[\mathrm{~mm}$ mrad] | 100 | 100 | $30-200$ | 80 |

### 1.2 Concept of the bunch length

To start a discussion about longitudinal profile diagnostic we should briefly review how bunch length is actually formed within the accelerator. Basic bunched beam formation occurs in the Radio Frequency (RF) cavity and the length of the bunch depends on its operating frequency. Usually, the basic bunch length in RMS is about 1 degree of RF period, which, for example, for 3 GHz accelerating structure ( 1 period $=333 \mathrm{ps}$ ) will be about 925 fs RMS. Any further manipulations with the electron beam (in order to enhance its peak current, or reduce a bunch length for THz generation) is called bunch compression.

The basic method for relativistic beam compression is called Magnetic bunch compression. Magnetic bunch compression operates in the following way: when particles are accelerated on the rising edge of the RF phase (parameter known as off-crest phase) of the last accelerating structure, an energy spread is introducing that grows from the bunch tail to its head. This process is also called bunch "chirping". Then, in the dispersive area (a chicane of magnets) the higher energy particles have longer path length then particles with lower energies. Thus, the bunch tail gets closer to the head, leading to longitudinal compression. Described principle of longitudinal bunch compression shown in Figure 1.2.

Bunch compression for the CLARA machine, where experiments were carried out for this work, is based on the same principle. Simulated variation of electron bunch length and electron energy spread with linac off-crest phase shown in Fig. 1.3. The maximum compression of about 300 fs RMS is expected to be at $\approx 8^{\circ}$ off-crest phase.


Figure 1.2: Principle of longitudinal bunch compression in a dispersive area. Accelerator structure operates off-crest and the tale of a beam gains more energy than the head. After that, inside dispersive area (Magnetic Chicane) higher energy particles follow a longer pass, which allows the tail to catch the head of the bunch


Figure 1.3: Simulated variation of RMS electron bunch length ( $\sigma_{t}$, red) and RMS energy spread ( $\sigma_{E}$, black) with linac off-crest phase at CLARA/VELA facility

### 1.3 Techniques of longitudinal diagnostics for short electron beam bunches

### 1.3.1 Streak Cameras

The operation principle of the streak camera is to convert the temporal distribution of the light pulses into a spatial one (Fig. 1.4). A short radiation pulse induced by the beam (synchrotron radiation from bending magnets or transition radiation from special targets) passes through the optical system and enters the photocathode. The photocathode converts the incident light into an electron beam, which with some accuracy repeats the temporal distribution of the light pulse. Then the electron beam passes through a system of deflecting electrodes, to which a sawtooth sweep pulse is applied. This leads to a vertical deflection of the beam particles, proportional to the longitudinal coordinate. Thus, the longitudinal distribution of particles is transformed into the transverse. This bunch then fall into fluorescent screen, and its shape registered by a CCD camera. The resolution is defined by by the sawtooth sweep speeds and CCD


Figure 1.4: Operation principle of a streak camera (Figure taken from [8])
camera pixel sizes. Best resolution of $0.2 p s$ was achieved by Hamamatsu company [9]. The main advantages of streak cameras are high scanning frequency (sweeps with a repetition rate of up to tens of kHz ) and fairly high sensitivity [10]. It is possible to
use synchrotron radiation for incident light pulse production, thus, diagnostic can be non-invasive. In the linear accelerators we need to use TR screens and diagnostic is always invasive.

### 1.3.2 Transverse Deflecting Cavities

Transverse Deflecting Cavities apply the longitudinal to transverse rotation to the bunch itself. During measurements, the RF cavity and deflecting voltage are phased: beam arrives with zero degree off-crest phase. In this case, bunch will be vertically deflected (see Figure 1.5). After it, we can measure transverse bunch profile and conclude about longitudinal distribution [11, 12].


Figure 1.5: Operation principle of RF deflecting cavity (Figure taken from [12])

The best Deflecting Cavity resolution, of about 1 fs , was demonstrated at LCLS FEL [13]. However, the deflecting cavities, are expensive, because require additionally RF system.

### 1.3.3 Electro-Optic Techniques

A novel method for longitudinal bunch profiling in the time domain is an electro-optic (EO) technique. The very best explanation of how this method works is given in [14], and here I will highlight some main features. In EO method, polarization of short (femptosecond) laser pulse is modulated by an electric field of the bunch inside GaP or ZnTe crystal. This effect is also knows as the Pockels effect. Laser probe pulse can then be demodulated with two different methods: electro-optic spectral decoding
(EOSD)[15] and electro-optic temporal decoding (EOTD) [16].
In EOSD (see Fig. 1.6 top), laser pulse is linearly chirped - it's frequency depends on the longitudinal position. It is then passes GaP or ZnTe crystal inside a vacuum chamber simultaneously with a bunch of charged particles. Laser polarization now encoded with a bunch length. Then, using an analyzer (A) we can convert polarization modulation to intensity modulation. Then we can use grating spectrometer and CCD camera to visualize longitudinal bunch profile. EOSD is easier to implement than EOTD method, however the result is affected by the mixing of the bunch electron field and laser pulse field. [16, 17].


Figure 1.6: Schematic of EOSD (a) and EOTD (b) methods. Figure taken from [12]

To avoid this mixing we can use EOTD method. In this case we also have intensity modulated probe pulse after analyzer, but also using a second short laser pulse with a fixed delay relative to the probe pulse. When this two pulses hit the $\beta$-barium borate crystal (BBO - nonlinear optical crystal) it emits visible light, which intensity is a function of two pulses delay within a crystal. Thus, this light can be detected and analysed.

Time-domain method has a better resolution than spectral-domain, since it does
not depend on the CCD pixels dimensions and does not have frequency mixing issue. But setup for EOTD is much more complicated, taking into account the use of second laser beam and delay unit. Nowadays EO techniques are implemented in many modern facilities. The best resolution ( 60 fs ) was achieved at FLASH facility [14]. The limit set by the accuracy of the synchronization of the electron bunch and probe pulse arrival time.

Summing up, at this moment EO method is of the most potential technologies with the possibility to provide non-invasive and single-shot diagnostic. However, applicability of this technique has many limitations: physical limit of EO material properties, stability of lasers with very short pulse duration, and of the most importance - bulky optical setups and total cost.

### 1.3.4 Frequency domain method

The frequency-domain method (also can be known as the coherent radiation (CR) method) is based on the measurements of coherent radiation spectrum and the reconstruction of longitudinal distribution. Among the physical effects which allow us to generate coherent radiation from charged particles are: Synchrotron Radiation (SR), Transition Radiation (TR) [18], Diffraction Radiation (DR) [19], Smith-Purcell Radiation (SPR) [20], Vavilov-Cherenkov [21] and Cherenkov Diffraction Radiation [22, 23] (VCR and ChDR). Synchrotron Radiation stay aside since it can be generated naturally by a particle moving in magnetic field. For others we need to use specific radiators.

The spectral information of coherent radiation can be obtained, for example, by using Fourier spectroscopy. In the heart of spectroscopy devices are usually MartinPupplet [24] or Michelson interferometer [25]. Since such devices measure only intensity spectrum - all information about the phase shift of the emitted radiation is lost. The phase can be recovered using so called phase-retrieval techniques such Kramers-Kronig [26] analysis, Blaschke phase retrieval [27-29], or more novel approaches such as phaseconstrained iterative method [30]. Using a CR method in pair with a Fourier spectroscopy we only limited by the quality of interferometer positioning system, optics, detectors frequency limits and detector signal-to-noise ratio. For modern particle accelerators with sub-ps bunches we need to properly observe sub-THz $(100 \mathrm{GHz}+)$ region,
as this is region where coherency decay is happening. This technique and complete reconstruction process will be researched in Chapter 2.3.

### 1.4 Mechanisms of radiation generation for a frequency domain method

As it can be seen from an analysis of modern diagnostic techniques - the majority of them use electromagnetic radiation emitted by a beam of charged particles. In Streak Cameras this is usually synchrotron radiation or transition radiation. For EO method no induced radiation is used, however, we can use THz radiation induced in radiator (CTR, CChDR) to avoid beam penetration inside the crystal ( THz radiation also induces The Pockels effect, and this effect is using for EO THz spectroscopy). The frequency-domain method is based on different types of coherent radiation such as CSR, CDR, CTR, CVCR, or CChDR effects. Since the frequency domain method is the basis of this work - these phenomena will be reviewed in this section to highlight the main features, advantages, and disadvantages.

### 1.4.1 Synchrotron Radiation

Synchrotron radiation can be induced by a high-energy electron beam inside the field of bending magnets or inside special devices: wigglers and undulators.

Figure 1.7 demonstrates the principles of Synchrotron radiation generation in bending magnets, wigglers, and undulators. In bending magnets, electrons move along a circular path and produce a smooth spectrum. The wiggler is a sequence of $N$ bending magnets with alternate polarities. The outgoing radiation is not different from that of a bending magnet, aside from that the intensity will be proportional to the product $N * N_{e}$, where $N_{e}$ is a number of electrons. The undulator is a periodical structure with N alternating poles, where the electrons perform small transverse periodic oscillations. Radiation emitted at any point of the undulator interferes constructively and the result is that it is squeezed into a discrete spectrum. In an undulator, emission from the different bends is coherent but the individual electrons are uncoherent, therefore overall Synchrotron radiation intensity is proportional to $\propto N^{2} * N_{e}$. FEL's operate by the
same principle as undulators, but there is also coherence between the emission from the different electrons due to short electron bunch (the bunch length is smaller than the emitted wavelength). Thus, FEL's intensity is proportional to $\propto N^{2} * N_{e}^{2}$.


Figure 1.7: Different Synchrotron source operation modes. Figure adapted from [31]

CSR was first observed in 1971 [32] as emission of Bremsstrahlung in a periodic magnetic field and then in 1989 [33] in Synchrotron machine. However, the fundamental relationship between CSR and bunch length was demonstrated by Nodvick and Saxon in 1954 [34], where authors theoretically demonstrated that SR becomes coherent at the wavelengths larger than the bunch length. Later it was concluded, that by knowing the bunch coherency function (bunch form factor) we can estimate the longitudinal distribution itself.

The most detailed work on the bunch length diagnostic with CSR is given in [35], where experiments conducted at the Vacuum Ultraviolet Free Electron Laser (VUVFEL) at DESY are described. The resolution of this technique was identified to be 130 fs. The use of CSR has a great advantage for circular machines as no extra devices to generate it is needed. However, synchrotron spectrum is difficult to predict, because it crucially affected by coherent backgrounds generated from other components of the machine.

### 1.4.2 Transition radiation

Transition Radiation (TR) was first considered by Frank and Ginzburg in 1945 [18]. It is induced when a charged particle crosses an interface between two media with different dielectric permittivities $\epsilon_{1}$ and $\epsilon_{2}$ (the role of one of the media may be played by vacuum). It is believed that the physical nature of the TR phenomenon is that under the influence of the charged particle Coulomb field, the electron shells of target material atoms are displaced relative to the nucleus, which leads to the formation of an elementary dipole (polarization), the appearance of polarization currents, and the emission of Transition Radiation. This radiation occurs both in forward direction (FTR), along with the particle's momentum, and in backward (BTR), at an angle of specular reflection to the interface.


Figure 1.8: Schematic drawing of TR generation process. a - normal incidence of charged particle beam, b - oblique incidence usually used for beam diagnostic.

For a particle with charge $q$ passing with normal incidence (Fig. 1.8 left) through the interface between a perfect conductor of infinite dimensions and vacuum, the spectralangular distribution will be [18]:

$$
\begin{equation*}
\frac{d^{2} W_{T R}}{d \omega d \Omega}=\frac{\beta^{2} q^{2}}{\pi^{2} c} \frac{\sin \theta^{2}}{\left(1-\beta^{2} \cos \theta^{2}\right)^{2}} \tag{1.3}
\end{equation*}
$$

where $\omega$ is the angular frequency; $\beta=v / c$ is the speed parameter, $\theta$ is the angle between light radiation and the particle direction, $c$ is the speed of light. For $\beta \sim 1$, the maximum of angular distribution is located at $\theta \sim 1 / \gamma$. By reasons related to the convenience of radiation extraction from a beam line, a scheme with $45^{\circ}$ oblique angle and BTR geometry is usually used for diagnostic of charged particle bunches (Fig. 1.8
right).
Transition Radiation was first detected in 1956 [36]. Coherent Transition Radiation was first observed in in 1991 [37]. The first measurements of the electron longitudinal distribution exploiting CTR were made in 1994 [38, 39]. The most recent and significant work in terms of CTR for longitudinal diagnostic has been done at the FLASH facility in 2018, where a sub-picosecond electron bunch length and shape were measured using a grating spectrometer isolated in a vacuum chamber [40]. The main advantages of diagnostics using CTR are relatively high intensity and the possibility to extract light with a narrow angular cone of the order of $\theta \sim 1 / \gamma$ at higher frequencies. On the other hand, TR is an invasive process. In the next chapter, Transition Radiation will be considered in details, since it was used in this experimental work as a gauge method.

### 1.4.3 Diffraction Radiation

Transition and Diffraction radiation mechanisms have the same nature. In case of DR, the field of the charged particle polarises atoms at the radiator surface, and creates changing electric field. Diffraction radiation appears when a particle does not cross the medium but moves in the vicinity of it. This effect was predicted by I.M. Frank in the early 1940s [41]. Then it was directly observed in 2003 [19]. Diffraction Radiation also emits in two directions: Forward Diffraction Radiation (FDR) and Backward Diffraction Radiation (BDR). The last is usually used for beam diagnostic (see Fig. 1.9). DR spectral-angular distribution for different incidence angles of the charged particle and the different distance between particle and radiator (slit) can be calculated as [42, 43]:

$$
\begin{array}{r}
\frac{d^{2} W_{D R}^{\text {slit }}}{d \omega d \Omega}=\frac{e^{2} \gamma^{2}}{2 \pi^{2}} \frac{\exp \left(-\frac{2 \pi a \sin \theta_{0}}{\gamma \lambda} \sqrt{1+t_{x}^{2}}\right)}{\left(1+t_{x}^{2}+t_{y}^{2}\right)\left(1+t_{x}^{2}\right)} \\
\times\left[\left(1+2 t_{x}^{2}\right) \cosh \left(\frac{4 \pi a_{x}}{\gamma \lambda} \sqrt{1+t_{x}^{2}}\right)-\cos \left(\frac{2 \pi a \sin \theta_{0}}{\gamma \lambda} t_{y}+2 \psi\right)\right] \tag{1.4}
\end{array}
$$

where $a$ is the slit size, $\theta_{0}$ is the target tilt angle (relative to velocity vector), $t_{x}=\gamma \theta_{x}, t_{y}=\gamma \theta_{y}, \psi=\arctan \left(\frac{t_{y}}{\sqrt{1+t_{x}^{2}}}\right)$ - more convenient shortcuts [43], $\theta_{x}, \theta_{y}$ are the radiation emission angles. Energy losses for DR generation are negligibly small and the beam might be used for further users applications. Thus, diagnostic with DR is non-


Figure 1.9: Principle of Diffraction Radiation generation process. a) - normal incidence of charged particle beam, b) - oblique incidence usually used for beam diagnostic.
destructive. Important works in this field were conducted at CLIC Test Facility (CTF3, CERN), where Coherent Diffraction Radiation signal was registered using Michelson interferometer [44-46] and at TESLA test facility (TTF), where coherent diffraction radiation was registered using Martin-Pupplet interferometer. With the possibility to reconstruct bunch length from DR effect - authors also highlighted disadvantages of this method, such as a low signal and high influence from background radiation.

### 1.4.4 Smith-Purcell Radiation

Smith-Purcell radiation process, which is very close to Diffraction radiation, was observed by Smith and Purcell experimentally in 1953 [20]. I was shown that when an electron moves near metallic grating it induces the electromagnetic radiation. During this work authors have also observed a strong correlation between the wavelength of emitted radiation and observation angle (see Fig. 1.10(a)) and derived dispersion relation:

$$
\begin{equation*}
\lambda=d\left(\beta^{-1}-\cos \theta\right) \tag{1.5}
\end{equation*}
$$

where $\lambda$ is the wavelength, $d$ is the period of grating structure, $\theta$ is the angle of emitted radiation.

Coherent Smith-Purcell radiation (CSPR) was first observed in [47]. In terms of longitudinal beam profile diagnostics with CSPR, the most important experiments are are performed by the same scientific group in 2014 [48], where authors used an array of 11 detectors to detect CSPR spectral-angular distribution (see Fig. 1.10(b)).


Figure 1.10: LEFT - Schematic of the Smith-Purcell radiative process; RIGHT - experimental setup for CSPR observation.

The main advantages of using CSPR are the possibility of single-shot diagnostic [49], the possibility to choose the wavelength region of the emitted radiation by choosing grating periodicity (according to 1.5 ), the absence of any external spectrometer, and the possibility to perform non-invasive diagnostics. Difficulties of such diagnostic are mainly caused by the low intensity of CSPR and, therefore, a large influence of coherent background radiation. To solve this problem, one can use the longer gratings, high quality filters, light concetrators, and specific post-processing techniques [50].

### 1.4.5 Vavilov-Cherenkov Radiation and Cherenkov Diffraction radiation

The next polarization radiation mechanism is the well-known Vavilov-Cherenkov radiation (VCR) induced by a charged particle passing through a medium with a speed higher than the speed of light in this medium. The first article about this discovery was published in 1934 by P.A. Cherenkov and V.S. Vavilov [21]. In 1937 this effect was described by I. E. Tamm and I. M. Frank [51] and it was shown that VCR cone is defined by the following condition:

$$
\begin{equation*}
\cos (\theta(\lambda))=\frac{1}{\beta n(\lambda)} \tag{1.6}
\end{equation*}
$$

where $\theta(\lambda)$ is the VCR radiation angle measured from particle velocity vector, $n(\lambda)$ is a medium refractive index.

At the heart of VCR is the process of atomic polarization. If the particle moving at a speed higher than the speed of light in this medium, the energy transferred to the atoms has no time to be returned to the particle. The VCR intensity scales proportionally with the radiator length [51], allowing the possibility to use larger radiators for the diagnostics purposes. The coherent emission of Vavilov-Cherenkov radiation by a bunch of charged particles was studied theoretically by Danos in 1955 [52]. Later this led to the creation of Cherenkov masers [53, 54]. The first experimental observation of coherent Vavilov-Cherenkov radiation (CVCR) from short bunches was made by Ciocci [55] and Ohkuma in 1991 [56]. Its capability to produce high output powers at millimeter or submillimeter wavelengths has inspired several groups to develop Dielectric Wakefield Acceleration (DWA) [57].

In 1955 Linhart [22] and then in 1957 B. M. Bolotovsky [23] considered the case when Vavilov-Cherenkov radiation is generated without interaction of electrons with the medium. It was shown that the atoms close to the target surface can be polarized by the electromagnetic field of moving particles. The typical size of the charged particle electromagnetic field depends on observation wavelength $\lambda$ and the beam energy and typically scales as $\gamma \lambda / 2 \pi$. Nowadays, the term "Cherenkov Diffraction Radiation" (ChDR) and "Coherent Cherenkov Diffraction Radiation" (CChDR) is widely used to describe this concept. It refers to the analogy that Diffraction radiation is the case of non-invasive Transition Radiation.

The use of Coherent Cherenkov radiation combines such advantages as non-invasive diagnostics, high intensity (when using larger radiators), and the possibility to extract radiation with specific angle (away from the particle beam trajectory) by choosing proper refractive index (see eq. 1.6), to limit the background radiation.

Several papers regarding the exploitation of CChDR for beam diagnostics and concerning this work were published by our group [58-62].


Figure 1.11: Schematic drawing of typical ChDR generation process from a prismatic dielectric target. Yellow line - charged particle velocity direction. $\theta^{\prime}$ - ChDR emission angle inside of material, according to $1.6 ; \theta-\mathrm{ChDR}$ emission angle from material to other medium (for example vacuum or air), according to Snells's law.

### 1.5 Motivation for using CChDR for longitudinal diagnostic on modern machines

Summary of all pros and cons of different radiative mechanisms for longitudinal profile diagnostic exploiting coherent radiation is shown in the Table 1.2. Cherenkov Diffraction radiation is a relatively new and barely studied mechanism, which needs to be properly researched and tested at modern facilities. Diagnostics with Cherenkov Diffraction radiation is non-invasive and the possible distance between beam particles and the radiator scales with $\gamma \lambda / 2 \pi$ product, where $\lambda$ is an observing wavelength. For example, if we want to observe CChDR on CLARA machine operating with the beam energy of 35 MeV and bunch length of $\approx 300 \mathrm{fs} \operatorname{RMS}(\gamma=70, \lambda \approx 1 \mathrm{~mm})$, the $\gamma \lambda / 2 \pi$ product will be about 1.1 cm , which gives us flexibility for building the experimental setup. Another advantage of ChDR is a very specific angle of radiation (see Eq. 1.6), which depends on the target refractive index $n$ and can be adjusted by using radiators of different material and shape. It is also important that Cherenkov Radiation is generated along the whole radiator surface and the radiated intensity scales with radiator length. To our knowledge and despite its great potential, Coherent Cherenkov Diffraction Radiation has not been exploited for beam diagnostic purposes. Even though many groups working with short bunches use coherent radiation, the detection systems
are mainly based on Coherent Transition Radiation, Coherent Diffraction Radiation, Coherent Smith Purcell Radiation, or Electro-Optical Sampling.

The Coherent Cherenkov Diffraction Radiation effect is at the heart of this work, so it will be researched with details in section 2.2. In order to emphasize the possibility to use the CChDR as a new method for longitudinal diagnostics, we used a wellstudied CTR as a gauge mechanism which is reviewed in section 2.1. The experimental setup described in Chapter 3 was designed in order to obtain the result of longitudinal diagnostic from CChDR and CTR during the same accelerator run. In Chapter 4 dielectric properties of the radiator used for ChDR generation are studied and discussed. Chapter 5 finalizes this work with the comparison of diagnostic results using CChDR and CTR.

Table 1.2: Comparison of radiative processes for diagnostic with coherent radiation

| Radiative process | PROS | CONS |
| :---: | :---: | :---: |
| CSR | - No need for an extra setup to produce SR in circular machines. <br> - Possibility to obtain very high intensity of radiation. | - Impossible to extract CSR without coherent background. <br> - Can not be used in linacs. <br> - Difficulty to predict a single electron spectrum. |
| CTR | - High intensity. <br> - Possibility extract radiation under specific angle and avoid background radiation. <br> - Theoretically well studied and using for beam diagnostic since 1994. | - Always invasive. <br> - High influence of background radiation. |
| CDR | - Diagnostic is completely noninvasive | - High influence of background radiation. |
| CSPR | - Possibility to choose wavelength region of emitted radiation. <br> - Could be non-invasive. <br> - Could be used without external spectrometer. | - High influence of background radiation. |
| ChDR | - Intensity scales with radiator length. <br> - Radiation always emitted under specific "Cherenkov angle" which can be controlled in order to avoid background. <br> - Non-invasive (not cutting the beam) | - High influence of background radiation. |


\section*{|  |
| :---: |
| Chapter |}

## Theory of polarization radiation

### 2.1 Theory of Transition Radiation

In this section, Transition Radiation angular characteristics will be considered starting with the most simplified case, when BTR induced by relativistic electron crossing a target at a normal incidence angle and it is observed in the far-field. Observation in far-field zone, in simple words, means that the contribution of the radiation source size into the BTR spot size in the detector plane is not significant and can be neglected (i.e. point size source). To use the far-field zone approximation the distance $a$ from the target to detector must exceed the parameter $\gamma^{2} \lambda$ (see eq. 2.5). The so-called pre-wave zone takes place when $a<\gamma^{2} \lambda$ and radiation source size should be considered as not point-like, which leads to the phase advance $\psi$ of the photons emitted from different points on the radiator plane. The model takes into account the pre-wave zone effect will be shown in subsection 2.1.2.

In subsection 2.1.3, the most general case including pre-wave zone effect and oblique electron incidence angle will be shown. The necessity to take into consideration these moments will be shown in subsection 2.1.4, where BTR characteristics are studied for the real experimental layout used during experimental work at CLARA. Finally, BTR single electron spectrum will be shown and discussed. BTR single electron spectra calculated for experimental layout built at CLARA facility will also be demonstrated.

One should note, that for calculating TR characteristics I considered models based
on the classical Ginzbug-Frank (Pseudo-photon method) approach, and not using Polarization Current Approach described in section 2.2. The first reason is that we use TR as a gauge method in our studies, thus the calculation should have a different approach. The second reason is that to our knowledge there is no PCA model for TR accounting for the pre-wave zone effect.

### 2.1.1 Backward transition radiation in the far-field approximation

TR process was first calculated by Vitaly Ginzburg and Ilya Frank in 1945 [18]. Assuming a perfectly conducting and infinite size target, the spectral energy density will be:

$$
\begin{equation*}
\frac{d^{2} W_{T R}}{d \omega d \Omega}=\frac{\beta^{2} q^{2}}{4 \pi^{3} \varepsilon_{0} c} \frac{\sin ^{2} \theta}{\left(1-\beta^{2} \cos ^{2} \theta\right)^{2}} \tag{2.1}
\end{equation*}
$$

where $\theta$ is the angle between the particle velocity and generated TR. The most detailed derivation of this equation can be found in [18], [42] and with the most detailed explanation in [63]. Geometry of BTR generation in the far-field zone is shown in Fig. 2.1.


Figure 2.1: Backward Transition Radiation for geometry with normal electron incidence observing in the far-field zone. Here $a$ - is the distance from radiator to the observation plane. $\theta_{x}$ and $\theta_{y}$ - are the observation angles measured from the specular reflection direction, $\theta$ determines angle between $z$ coordinate and specular reflection in polar system. $u$ and $v$ indicates cartesian coordinate system in detector plane, U and V detector dimensions.

Using the small angle approximation and assuming that $\beta \approx 1-1 / 2 \gamma^{2}$ spectral angular distribution can be written as:

$$
\begin{equation*}
\frac{d^{2} W_{T R}}{d \omega d \Omega}=\frac{\gamma^{4} q^{2}}{4 \pi^{3} \varepsilon_{0} c} \frac{\theta^{2}}{\left(1+\gamma^{2} \theta^{2}\right)^{2}} \tag{2.2}
\end{equation*}
$$

From equation 2.2 it can be seen that there is no radiation when $\theta=0$ and that the intensity reaches its maximum at $\theta \approx \pm 1 / \gamma$. And since geometry for the normal angle discussed here gives us azimuthal symmetry, we can write spectral angular distribution for two polarization components:

$$
\begin{equation*}
\frac{d^{2} W_{T R}}{d \omega d \Omega}=\frac{d^{2} W_{T R_{x}}}{d \omega d \Omega}+\frac{d^{2} W_{T R_{y}}}{d \omega d \Omega}=\frac{q^{2}}{4 \pi^{3} \varepsilon_{0} c} \frac{\theta_{x}^{2}+\theta_{y}^{2}}{\left(\gamma^{-2}+\theta_{x}^{2}+\theta_{y}^{2}\right)^{2}} \tag{2.3}
\end{equation*}
$$

If we replace $\theta_{x, y}$ with $\pm 1 / \gamma$ the intensity will have a maximum at these angles with a value:

$$
\begin{equation*}
I_{\max }^{T R}=\frac{\alpha \gamma^{2} \hbar}{4 \pi^{2}} \tag{2.4}
\end{equation*}
$$

where $\alpha$ is a fine structure constant, $\hbar$ is a reduced Plank constant. All simulations on BTR angular distribution presented in this thesis are normalized by $I_{\max }^{T R}$.


Figure 2.2: Horizontal component of Transition Radiation spectral angular distribution from a single particle at a normal incidence. Results for different beam energies are presented.


Figure 2.3: CTR angular distribution in two-dimensional representation

### 2.1.2 Model of backward transition radiation with accounting for the pre-wave zone effect

Pre-wave zone effect for TR was studied both theoretically [64], [65] and experimentally [66], [67]. When a particle with effective electron field radius treated as $R=\frac{\gamma \lambda}{2 \pi}$ (see for example [42] or can be derived from equation bellow) crosses the boundary, it induces the Transition Radiation source with the same spot radius $R$. To avoid the influence of this spot size it is necessary to choose such a distance between target and detector that with radiation divergence of $1 / \gamma \mathrm{TR}$ source might be considered as a point-like:

$$
\begin{equation*}
\frac{a}{\gamma} \gg \frac{\gamma \lambda}{2 \pi} \Rightarrow a \gg \frac{\gamma^{2} \lambda}{2 \pi} \tag{2.5}
\end{equation*}
$$

If condition 2.5 is not satisfied, the radiation spectral angular distortion will happen at the distance $a \leq \gamma^{2} \lambda / 2 \pi$ due to the radiation phase difference $\Delta \psi$ from different points at the source (see 2.4 caption). It should be noted here, that, for example, for The Final Focus Test Beam (FFTB) facility at Stanford Linear Accelerator Center (SLAC) [68]
the distance should exceed 274 meters $(\lambda=500 \mathrm{~nm}, E=30 \mathrm{GeV})$, which is impossible to achieve in practice. For the CLARA machine (Phase 2) this parameter is 30 cm , which is more realistic. At the same time, in practice, the radiator dimensions itself could be less than effective electron field radius $R$. To take all such peculiarities into account, the effect of the pre-wave zone should be considered upfront.


Figure 2.4: Backward Transition Radiation for geometry with normal electron incidence observing in the pre-wave zone. Here $a$ is the distance between the radiator and the observation plane; $u$ and $v$ indicate the cartesian coordinate system in the detector plane, U and V are detector dimensions. $x_{s}$ and $y_{s}$ indicate cartesian coordinate system in a source plane, X and Y are the source dimensions; $r_{1}, r_{2}, r_{n}$ are the vectors of the photons propagating from different target points into the same point $P(v, z)$ in the detector plane. Waves emitted by points $O(x, y)$ and $S(x, y)$ arrive at any point P with a phase difference $\Delta \psi=k\left(r_{1}-r_{2}\right)$, where $k=2 \pi / \lambda$ is the wave vector.

For calculation of the BTR characteristics in the pre-wave zone, we will use the approach described in [65]. Each radiator point can be counted as an elementary source (Fig. 2.4). In this case, two polarization components $E_{x, y}$ can be presented as a
superposition of all k'th sources $E_{x, y}^{k}\left(x_{s}, y_{s}\right)$ at a certain distance $r$ taking into account the phase difference $\Delta \psi$ :

$$
\begin{equation*}
E_{x, y}=\frac{1}{4 \pi^{2}} \iint E_{x, y}^{k}\left(x_{s}, y_{s} \frac{e^{i \psi}}{r} d y_{s} d x_{s}\right. \tag{2.6}
\end{equation*}
$$

where $x_{s}$ and $y_{s}$ are the position of an elementary source in the radiator plane. Amplitude of an elementary sources for two different polarization components can be presented as [42]):

$$
\begin{align*}
& E_{x}^{k}=\frac{i e k}{4 \pi^{2} \gamma \varepsilon_{0}} \frac{x_{s}}{\sqrt{x_{s}^{2}+y_{s}^{2}}} K_{1}\left(\frac{k}{\gamma} \sqrt{x_{s}^{2}+y_{s}^{2}}\right) \\
& E_{y}^{k}=\frac{i e k}{4 \pi^{2} \gamma \varepsilon_{0}} \frac{y_{s}}{\sqrt{x_{s}^{2}+y_{s}^{2}}} K_{1}\left(\frac{k}{\gamma} \sqrt{x_{s}^{2}+y_{s}^{2}}\right) \tag{2.7}
\end{align*}
$$

where $K_{1}$ is the first order McDonald function (also known as modified Bessel function).
The phase advance of the radiation moving from radiator plane to the detector plane is (see Fig. 2.4):

$$
\begin{equation*}
\frac{e^{i \psi}}{|\vec{r}|}=\frac{e^{i k|\vec{r}|}}{|\vec{r}|}=\frac{\exp \left(i k \sqrt{\left.a^{2}+\left(x_{s}-u\right)^{2}+\left(y_{s}-v\right)^{2}\right)}\right.}{\sqrt{\left.a^{2}+\left(x_{s}-u\right)^{2}+\left(y_{s}-v\right)^{2}\right)}} \tag{2.8}
\end{equation*}
$$

Equation 2.8 can also be simplified by knowing that parameters $\left(x_{s}-u\right) / a$ and $\left(y_{s}-v\right) / a$ responsible for the angles of the radiation generation and these angles are of order $\gamma^{-1} \ll 1$ [65]:

$$
\begin{equation*}
\frac{e^{i \psi}}{|\vec{r}|}=\frac{e^{i k|\vec{r}|}}{|\vec{r}|}=\frac{\exp \left(i k \sqrt{\left.a^{2}+\left(x_{s}-u\right)^{2}+\left(y_{s}-v\right)^{2}\right)}\right.}{a} \tag{2.9}
\end{equation*}
$$

Substituting Eq. 2.9 and 2.7 into 2.6 we have radiation intensity in the observation plane:

$$
\begin{array}{r}
\Re\left\{E_{x, y}(v, u)\right\}=\frac{e k}{16 \pi^{4} \gamma \varepsilon_{0}} \int_{-X / 2}^{X / 2} \int_{-Y / 2}^{Y / 2} \frac{x_{s}, y_{s}}{\sqrt{x_{s}^{2}+y_{s}^{2}}} K_{1}\left(\frac{k}{\gamma} \sqrt{x_{s}^{2}+y_{s}^{2}}\right) \times \\
\times \frac{\cos \left(k \sqrt{\left.a^{2}+\left(x_{s}-u\right)^{2}+\left(y_{s}-v\right)^{2}\right)}\right.}{a} d x_{s} d y_{s} \\
\Im\left\{E_{x, y}(v, u)\right\}=\frac{e k}{16 \pi^{4} \gamma \varepsilon_{0}} \int_{-X / 2}^{X / 2} \int_{-Y / 2}^{Y / 2} \frac{x_{s}, y_{s}}{\sqrt{x_{s}^{2}+y_{s}^{2}}} K_{1}\left(\frac{k}{\gamma} \sqrt{x_{s}^{2}+y_{s}^{2}}\right) \times  \tag{2.10}\\
\times \frac{\sin \left(k \sqrt{\left.a^{2}+\left(x_{s}-u\right)^{2}+\left(y_{s}-v\right)^{2}\right)}\right.}{a} d x_{s} d y_{s}
\end{array}
$$

Separation of exponent on Real and Imaginary parts via Euler's formula is necessary for proper numerical calculation as integration over complex numbers is a difficult task for most programming languages. Two polarization components can be presented then as:

$$
\begin{equation*}
\left|E_{x, y}\right|=\sqrt{\Re\left\{E_{x, y}(v, z)\right\}^{2}+\Im\left\{E_{x, y}(v, u)\right\}^{2}} \tag{2.11}
\end{equation*}
$$

The spectral-angular distribution of the BTR can be obtained as follows:

$$
\begin{equation*}
\frac{d^{2} W_{T R}}{d \omega d \Omega}=16 \pi^{3} \varepsilon_{0} k^{2} a^{2}\left[\left|E_{x}\right|^{2}+\left|E_{y}\right|^{2}\right] \tag{2.12}
\end{equation*}
$$

It is also important to mention that $\sqrt{x_{s}^{2}+y_{s}^{2}}$ coefficient in denominator of Eq. 2.10 will create a singularity point when $x_{s}^{2} \approx y_{s}^{2} \approx 0$. To avoid it I recommend to create an imaginary slit in the horizontal or vertical plane of the radiator and substitute the equation 2.10 with (example for the real part):

$$
\begin{array}{r}
\Re\left\{E_{x, y}(v, u)\right\}=\frac{e k}{16 \pi^{4} \gamma \varepsilon_{0}} \int_{-X / 2}^{X / 2}\left[\int_{-Y / 2}^{-\lambda / 2} \frac{x_{s}, y_{s}}{\sqrt{x_{s}^{2}+y_{s}^{2}}} K_{1}\left(\frac{k}{\gamma} \sqrt{x_{s}^{2}+y_{s}^{2}}\right) \times\right. \\
\times \frac{\cos \left(k \sqrt{\left.a^{2}+\left(x_{s}-u\right)^{2}+\left(y_{s}-v\right)^{2}\right)}\right.}{a} d y_{s}+ \\
\quad+\int_{\lambda / 2}^{Y / 2} \frac{x_{s}, y_{s}}{\sqrt{x_{s}^{2}+y_{s}^{2}}} K_{1}\left(\frac{k}{\gamma} \sqrt{x_{s}^{2}+y_{s}^{2}}\right) \times  \tag{2.13}\\
\left.\times \frac{\cos \left(k \sqrt{\left.a^{2}+\left(x_{s}-u\right)^{2}+\left(y_{s}-v\right)^{2}\right)}\right.}{a} d y_{s}\right] d x_{s}
\end{array}
$$

where $-\lambda / 2$ to $\lambda / 2$ is the slit dimensions (chosen to be smaller than a wavelength).

The model described here takes into account the following parameters: beam energy, target dimensions, the distance between target and detector. Bellow several results of BTR simulation with simulation parameters from table 2.1 are shown.

Table 2.1: Simulation and experimental parameters

| Parameters | Value |
| :--- | :---: |
| Lorentz - factor $\gamma$ | 100 |
| Radiation frequency $f$ | $[0.5,1,1.5] T H z$ |
| Target dimensions $X=Y$ | 2 cm |
| Distance between radiator and <br> observation plane | $10 \frac{\gamma^{2} \lambda}{2 \pi}, \frac{\gamma^{2} \lambda}{2 \pi}, 0.1 \frac{\gamma^{2} \lambda}{2 \pi}$ |

Fig. 2.5 illustrates the calcuated BTR spatial-angular distribution for different distances $a$ between radiator and observation planes. The case for $a=10 \frac{\gamma^{2} \lambda}{2 \pi}$ corresponds to the far-field zone, $a=\frac{\gamma^{2} \lambda}{2 \pi}$ is the edge of far-field and pre-wave zones and $a=0.1 \frac{\gamma^{2} \lambda}{2 \pi}$ is the deep pre-wave zone. The important thing to be noted here is that in the pre-wave zone, not only diffraction/distortion effects appear, but also dependance of maximum peak position on distance $a$ (see Fig. 2.6).


Figure 2.5: The horizontal component of Transition Radiation spectral angular distribution from a single particle at a normal incidence. Results for different distances $a$ between target and detector are presented; x values normalized on $a$ to suits 3 plots in one axes; radiation frequency $f=1 T H z$


Figure 2.6: Dependence of the BTR peak position on a distance between the target and the observation plane. y axes values normalized on $a$ to show only influence of zone effect. $x$ axes values normalized on $\frac{\gamma^{2} \lambda}{2 \pi}$ by analogy with Fig. 2.5.

Another feature of the model with the pre-wave zone effect is the appearance of frequency dependence. In Fig. 2.7 BTR angular distributions for different radiation frequencies are shown. Decreasing the frequency, the value of the pre-wave zone condition $\left(\frac{\gamma^{2} \lambda}{2 \pi}\right)$ decreases as well. Thus, at higher frequencies, the angular distribution will be significantly different from the one with a maximum at $1 / \gamma$.

Fig. 2.8 two dimensional angular distribution for deep pre-wave zone case ( $a=$ $\left.0.1 \frac{\gamma^{2} \lambda}{2 \pi}\right)$ is shown.

### 2.1.3 Model of backward transition radiation with accounting for the pre-wave zone effect and the electron incidence angle

As it was shown in pioneer work [18], in case of normal incidence the radiation is radially polarized. In works $[18,69]$ the oblique incidence (when a particle velocity vector has an angle $\theta_{o}$ with z axis) was researched and it was shown that the longitudinal polarization component starts contributing. Nowadays, the oblique incidence is common for most TR experiments, since with $\theta_{o}=45^{\circ}$, BTR will be radiated perpendicular to the


Figure 2.7: BTR spatial distribution in the pre-wave zone for different radiation frequencies. Here $a=0.05 \mathrm{~m}$. For a far-field zone peak position should be in $a / \gamma=5 e^{-5}$.


Figure 2.8: Two dimensional spatial distribution of BTR in the pre-wave zone; $a=$ $0.1 \frac{\gamma^{2} \lambda}{2 \pi}$; radiation frequency $f=1 T H z$
beam trajectory, which simplifies the radiation detection. According to [70] real and


Figure 2.9: Backward Transition Radiation at $\theta_{o}$ incidence. BTR will be generated under the angle of particle incidence specular reflection. The angle of radiation maximum $1 / \gamma$ will be now counted relative to this angle of specular reflection. For example, with $\theta_{o}=45^{\circ}$ radiation will reflect with $45^{\circ}$ relative to $z$ axis and $90^{\circ}$ relative to the beam pass, which is using in most experiments based on BTR effect. Some volume was added to the radiator and detector planes for a more obvious perspective.
imaginary parts of two orthogonal polarization components for the case of oblique particle incidence and taking into account the pre-wave zone effect can be written as:

$$
\begin{array}{r}
\Re\left\{E_{x}(v, u)\right\}=\frac{e k}{16 \pi^{4} \gamma \varepsilon_{0}} \int_{-X / 2}^{X / 2} \int_{-Y / 2}^{Y / 2} \frac{x_{s}}{\sqrt{x_{s}^{2}+y_{s}^{2}}}\left[K_{1}\left(\frac{k}{\gamma} \sqrt{x_{s}^{2}+y_{s}^{2}}\right) \frac{\cos \psi}{a}\right] d x_{s} d y_{s} \\
\Im\left\{E_{x}(v, u)\right\}=\frac{e k}{16 \pi^{4} \gamma \varepsilon_{0}} \int_{-X / 2}^{X / 2} \int_{-Y / 2}^{Y / 2} \frac{x_{s}}{\sqrt{x_{s}^{2}+y_{s}^{2}}}\left[K_{1}\left(\frac{k}{\gamma} \sqrt{x_{s}^{2}+y_{s}^{2}}\right) \frac{\sin \psi}{a}\right] d x_{s} d y_{s} \\
\Re\left\{E_{y}(v, u)\right\}=\frac{e k}{16 \pi^{4} \gamma \varepsilon_{0}} \int_{-X / 2}^{X / 2} \int_{-Y / 2}^{Y / 2} \frac{y_{s}}{\sqrt{x_{s}^{2}+y_{s}^{2}}}\left[K_{1}\left(\frac{k}{\gamma} \sqrt{x_{s}^{2}+y_{s}^{2}}\right) \frac{\cos \psi}{a} \cos \theta_{o}+\right. \\
\left.+\frac{1}{\gamma} K_{0}\left(\frac{k}{\gamma} \sqrt{x_{s}^{2}+y_{s}^{2}}\right) \frac{\cos \psi}{a} \sin \theta_{o}\right] d x_{s} d y_{s}  \tag{2.14}\\
\Im\left\{E_{y}(v, u)\right\}=\frac{e k}{16 \pi^{4} \gamma \varepsilon_{0}} \int_{-X / 2}^{X / 2} \int_{-Y / 2}^{Y / 2} \frac{y_{s}}{\sqrt{x_{s}^{2}+y_{s}^{2}}}\left[K_{1}\left(\frac{k}{\gamma} \sqrt{x_{s}^{2}+y_{s}^{2}}\right) \frac{\sin \psi}{a} \cos \theta_{o}+\right. \\
\left.-\frac{1}{\gamma} K_{0}\left(\frac{k}{\gamma} \sqrt{x_{s}^{2}+y_{s}^{2}}\right) \frac{\sin \psi}{a} \sin \theta_{o}\right] d x_{s} d y_{s}
\end{array}
$$

where $\psi$ defined in Eq. 2.8 and $\theta_{o}$ is the target tilt angle shown in Fig. 2.9. Substituting 2.14 in to Eq. 2.11 and Eq. 2.12 we can calculate angular distribution for a different incidence angles. It can be seen from equation 2.14 that additional term in $E_{y}$ is inversely proportional to $\gamma$, thus effect of oblique incidence will be noticeably different for different particle energies and this effect should be investigated.

In figures 2.10 and 2.11 spatial spectral distribution for different incidence angles of charged particle with $\gamma=10$ is presented. Figures 2.10 demonstrate spatial distribution over vertical dimension and figure 2.11 demonstrate 2 dimensional spatial distribution. It is clearly seen that on such a low energies additional component will cause a distortion of angular distribution over vertical dimension. Comparing these dependencies with the one for $\gamma=100$ (Fig. 2.12) it is easy to notice that for ultra-relativistic particles effect of asymmetry due to oblique incidence can be neglected. The asymmetry gets smaller with increase of gamma because effective electron field radius $R$ starts to be comparable with the radiator size and effect of polarization asymmetry is washed out.


Figure 2.10: Comparison between BTR spatial distributions for a different charged particle incidence angles $\theta_{0} . \quad \gamma=10 ; a=\frac{\gamma^{2} \lambda}{2 \pi}$ (edge of pre-wave zone); $f=1 T H z$; radiator dimensions $X=2 \cos \theta_{0}, Y=2 \mathrm{~cm}$;


Figure 2.11: BTR spatial distribution at $\theta_{0}=45^{\circ}$. Asymmetry of the vertical component is clearly notable. For this simulation $\gamma=10 ; a=\frac{\gamma^{2} \lambda}{2 \pi}$ (edge of pre-wave zone); $f=1 T H z$; radiator dimensions $X=2 \cos \theta_{0}, Y=2 c m$;


Figure 2.12: Comparison between BTR spatial distributions for different charged particle incidence angles $\theta_{0} . \gamma=100 ; a=\frac{\gamma^{2} \lambda}{2 \pi}$ (edge of pre-wave zone); $f=1 T H z$; radiator dimensions $X=2 \cos \theta_{0}, Y=2 \mathrm{~cm}$;

### 2.1.4 Single electron spectrum of BTR in the pre-wave zone

Now, a single electron spectrum can be obtained by integrating BTR distributions over detector aperture:

$$
\begin{equation*}
\frac{d W_{T R}}{d \omega}=\int_{0}^{\Omega} \frac{d^{2} W_{T R}}{d \omega d \Omega} d \Omega \equiv \int_{-U / 2}^{U / 2} \int_{-V / 2}^{V / 2} \frac{d^{2} W_{T R}}{d \omega d u d v} d u d v \tag{2.15}
\end{equation*}
$$

In this subsection different dependencies which may be of interest in preparing the experimental setup are shown. Parameters used in simulations and presented different configurations of the system are shown in table 2.2.

Figure 2.13 shows the single electron spectra for three different target dimensions and constant detector aperture $U=V=4 \mathrm{~cm}$, the distance between target and radiator $a=1 \mathrm{~m}$. In the long wavelengths region the radiation intensity is significantly suppressed by two reasons. First, the electron field radius $R$ is comparable, or even, larger than the transverse target dimensions, which leads to the diffraction effects and intensity suppression. Second, due to finite detector dimensions, low frequencies do not fit into the detector aperture. For example, lets consider effective electron field

Table 2.2: Simulation parameters for different frequency dependencies

| Parameters | Value |
| :--- | :---: |
| Lorentz - factor $\gamma$ | 100 |
| Radiation frequency $f$ | $[0-1.5] T H z$ |
| Target dimension $Y$ | $1,5,10 \mathrm{~cm}$ |
| Target dimension $X$ | $Y \cos \theta_{0} \mathrm{~cm}$ |
| Detector aperture $U=V$ | $1,4,5 \mathrm{~cm}$ |
| Distance between radiator and | $5,10,100 \mathrm{~cm}$ |
| observation plane $a$ |  |

radius for 100 GHz frequency $R=\frac{\gamma \lambda}{2 \pi}=\frac{100 * 3 \mathrm{~mm}}{2 \pi} \approx 50 \mathrm{~mm}$. In this case, 100 GHz frequency from the smallest radiator of $10 \times 10 \mathrm{~mm}$ (blue curve in Fig. 2.13) will be fully suppressed. For the radiator with larger sizes (red curve) suppression effect in low-frequency region is less perceptible than for a small radiator. For the frequencies less than 10 GHz another effect will take place, which is finite detector aperture -4 x 4 cm . In real experiments we should also take into account all additional optics apertures.


Figure 2.13: Transition radiation single electron spectrum for different target dimensions. Distance between target and detector $a=1 \mathrm{~m}$ (pre-wave zone for some observing frequencies); Detector aperture $4 x 4 \mathrm{~cm}$. For longer wavelengths radiation is suppressed due to finite target and detector apertures

Figure 2.14 shows another interesting dependence. Here, the red curve is a spectrum at the distance of $a=1 \mathrm{~m}$ between the target and the detector. At lower frequencies, its behavior is explained by Figure 2.7 (Intensity peak at lower frequencies shifts forward on larger angles). Prevailing of normalized intensity of red curve ( $a=1 \mathrm{~m}$ ) over blue one $(a=0.03 \mathrm{~m})$ on higher frequencies can be explained by Figure 2.5: 1 meter is a farfield zone so no diffraction/distortion effects will affect red curve and higher frequencies.


Figure 2.14: Transition radiation single electron spectrum for different distances between target and detector. Target size is $Y=50 \mathrm{~mm}, X=35 \mathrm{~mm}$; Detector aperture $4 \times 4 \mathrm{~cm}$.

In Figure 2.15 effect of lower frequency cutoff for detector with different size is shown. In this case, the intensity suppression for smaller detectors is explained by spectral-angular distribution of the emitted radiation (not all frequencies fit into the smaller detector aperture).

As a conclusion to this subsection, it needs to be said that numerical calculation of the spectral-spatial distributions and single electron spectra of emitted radiation is of great importance for diagnostics with coherent radiation. First of all, it is required to derive the form factor of charged particles bunch. Moreover, it is necessary to make all preliminary calculations of spectral-spatial characteristics for experimental setup design and construction. Optical elements and experimental layout components were chosen


Figure 2.15: Transition radiation single electron spectrum for different detector dimensions. Target size is $Y=50 \mathrm{~mm}, X=35 \mathrm{~mm}$; Distance between target and detector $a=1 \mathrm{~m}$. For longer wavelengths radiation is suppressed due to larger emission angles for these wavelengths and finite detector aperture.
after collaborative discussion and accounting for the CTR spectral-spatial properties discussed here.

### 2.1.5 TR single electron spectrum properties for the experiment conducted at CLARA facility

To reconstruct the longitudinal bunch profile from TR coherent spectrum we will need TR single electron spectra calculated for the exact experimental layout used on CLARA. These parameter are: $\gamma=70$, Target diameter $d$ is 5 cm , angle between radiator surface and beam direction $\alpha$ is $45^{\circ}$, detector aperture $D$ is 5 cm , distance between radiator and detector is $d=100 \mathrm{~mm}$. Result of the calculations in THz region shown in Fig. 2.16.

### 2.2 Theory of Cherenkov Diffraction Radiation

The main task of this section is to study spectral-spatial properties of ChDR in common and for CLARA experiments in particular. First of all, in subsection 2.2.1, we will


Figure 2.16: Simulation results of transition radiation single electron spectrum for experimental setup installed at the CLARA facility.
review the Polarization Current Approach (PCA) as it is the most general theory, which allows to solve the task and calculate Cherenkov Diffraction Radiation spectralspatial properties and account for other radiation types induced in the radiator (e.g. Diffraction Radiation). Then we will discuss advantages and disadvantages of radiators with different geometries. By analogy with section 2.1 we will investigate dependencies of ChDR spatial distribution on the impact parameter, the target dimensions, the angle between particle and radiator, the electron energy, and the observing frequency. Such dependencies are crucial not only for CLARA experimental setup but for all typical diagnostics layouts. In this section, we will also investigate all dependencies for a single electron spectrum. In conclusion, simulations for CLARA experimental setup will be shown.

### 2.2.1 Polarization Radiation and Polarization Current Approach

Diffraction Radiation, Smith-Purcell Radiation, Transition Radiation, Cherenkov Radiation, Cherenkov Diffraction Radiation may arise when a charged particle moves nearby a spatially inhomogeneous condensed medium or inside it. Whilst it was believed that all these types of radiation arise during a uniform and rectilinear motion (direction of the velocity remains constant and the path is a straight line) of the charged particle, each of them was considered independently due to complexity in generation
and observation geometry, which can be explained by historical reasons and theoretical approaches used to interpret the observed effects. For example, TR and CR were considered as a process of annihilation of a charge and its image (charge with an opposite sign) moving towards each other, or as a process of bremsstrahlung [18, 71]. Diffraction radiation was described using the closely related to it phenomenon of optical diffraction based on Huygens-Fresnel principles [42, 72, 73]. Smith-Purcell radiation was considered as a process of a charge field scattering in a periodic structure [74-77]. Another method called surface current approach was applied to generalize theory of $\operatorname{DR}$ and TR [78]. The considered methods have proved themselves well in relation to a particular type of radiation with a specific frequency range and specific energy of a charged particle, thus it was impossible to take into account the contribution of other radiation types.

Theoretical [79-84] and experimental [85-88] works have demonstrated the unified nature of these types of radiation and considered them as a particular cases of Polarization Radiation ${ }^{1}$. Subsequently, a new approach for the calculation of polarization radiation characteristics was developed. This method is known as Polarization Current Approach and allows to calculate angular and spectral characteristics of polarization radiation emitted by uniformly moving charged particle in arbitrary shape medium. PCA is based on the hypothesis that when uniformly moving charged particles move nearby or inside the medium - the atomic structure of this medium is polarized by its electric field. The induced polarization vector changes during this interaction in magnitude and direction, thereby leading to polarization currents density change. Thus, we can say that Polarization/Depolarization of the atomic medium is the cause of polarization currents existence and, therefore, the existence of changing electromagnetic fields.

The most detailed description for the Polarization Current Approach in the case of Cherenkov diffraction radiation is given in A. Konkov PhD thesis in Russian [79], where all peculiarities, such as the influence of spatial dispersion and magnetic moment

[^0]were taken into account. Here, the basics of derivation procedure ${ }^{2}$.
When a charged particle moves nearby or inside the medium, the electric field of this particle $\mathbf{E}_{\mathbf{0}}(\mathbf{k}, \omega)$ polarizes atoms and molecules of the medium. As a result, a field of polarization radiation $\mathbf{E}_{\mathbf{p o l}}(\mathbf{k}, \omega)$ arises and propagates through the target material leading to a secondary polarization [80]. For this reason, the density of the polarization current $\mathbf{j}_{\text {pol }}(\mathbf{k}, \omega)$ will linearly depend on both the external field (which caused the primary polarization) and the field of polarization radiation (which is the source of secondary polarization) ${ }^{3}$ :
\[

$$
\begin{equation*}
\mathbf{j}_{\mathbf{p o l}}(\mathbf{k}, \omega)=\sigma(\mathbf{k}, \omega)\left(\mathbf{E}_{\mathbf{0}}(\mathbf{k}, \omega)+\mathbf{E}_{\mathbf{p o l}}(\mathbf{k}, \omega)\right)=\mathbf{j}_{\mathbf{0}}(\mathbf{k}, \omega)+\sigma(\mathbf{k}, \omega) \mathbf{E}_{\mathbf{p o l}}(\mathbf{k}, \omega) \tag{2.16}
\end{equation*}
$$

\]

Here and after we will use frequency-domain wave equations and consider the medium as uniform, isotropic (no spatial dispersion), and non-magnetic ( $\mu=1$ ). Also, a Gaussian system of units is used. Conductivity $\sigma(\mathbf{k}, \omega)$ is a complex function of permittivity and posses some frequency dispersion:

$$
\begin{equation*}
\sigma(\mathbf{k}, \omega)=\frac{i \omega}{4 \pi}(\varepsilon(\mathbf{k}, \omega)-1) \tag{2.17}
\end{equation*}
$$

Maxwell equations for the time Fourier transforms with polarization current inside medium is:

$$
\begin{gather*}
\nabla \times \mathbf{H}_{\mathbf{p o l}}(\mathbf{k}, \omega)=\frac{4 \pi}{c} \mathbf{j}_{\mathbf{p o l}}(\mathbf{k}, \omega)-\frac{i \omega}{c} \mathbf{E}_{\mathbf{p o l}}(\mathbf{k}, \omega)  \tag{2.18}\\
\nabla \times \mathbf{E}_{\mathbf{p o l}}(\mathbf{k}, \omega)=\frac{i \omega}{c} \mathbf{H}_{\mathbf{p o l}}(\mathbf{k}, \omega)  \tag{2.19}\\
i \nabla \cdot \mathbf{E}_{\mathbf{p o l}}(\mathbf{k}, \omega)=4 \pi \rho^{p o l}(\mathbf{k}, \omega)  \tag{2.20}\\
\nabla \cdot \mathbf{H}_{\mathbf{p o l}}(\mathbf{k}, \omega)=0 \tag{2.21}
\end{gather*}
$$

To determine the magnetic field of the polarization radiation $\mathbf{H}_{\text {pol }}(\mathbf{k}, \omega)$, we need to apply $\nabla$ operator to left and right parts of equation 2.18 and then substitute 2.2.2 to 2.18 .

[^1]Then we can simplify $\nabla \times\left(\nabla \times \mathbf{H}_{\text {pol }}(\mathbf{k}, \omega)\right)$ using Eq. 2.37 and using Helmholtz equation $\left(\nabla^{2} a=-k^{2} a\right)$. Thus the magnetic field of the polarization radiation $\mathbf{H}_{\mathbf{p o l}}(\mathbf{k}, \omega)$ can be written as:

$$
\begin{equation*}
\left(k^{2}-\frac{\omega^{2}}{c^{2}}\right) \mathbf{H}_{\mathbf{p o l}}(\mathbf{k}, \omega)=\frac{4 \pi i}{c} \nabla \times \mathbf{j}_{\mathbf{p o l}}(\mathbf{k}, \omega) \tag{2.22}
\end{equation*}
$$

Since appearing of polarization current inside the material leads to changing of total electric field effecting on every atom and molecule (as it seen in 2.16), authors of [90] propose to perform "renormalisation" of electric field. Mathematically such renormalisation will be expressed in replacement of wavenumber $\frac{\omega}{c}$ and polarization current density $\mathbf{j}_{\text {pol }}$ on wavenumber for particular medium $\frac{\omega}{c} \sqrt{\varepsilon(\mathbf{k}, \omega)}$ and current density induced by charged particle itself $\mathbf{j}_{\mathbf{0}}(\mathbf{k}, \omega)=\sigma(\mathbf{k}, \omega) \mathbf{E}_{\mathbf{0}}(\mathbf{k}, \omega) \mathbf{e}$ accordingly (more details in $[79,90])$. Here $\mathbf{e}$ is a unit vector. Thus we can replace 2.22 with:

$$
\begin{equation*}
\left(k^{2}-\frac{\omega^{2} \sqrt{\varepsilon(\mathbf{k}, \omega)}}{c^{2}}\right) \mathbf{H}_{\mathbf{p o l}}(\mathbf{k}, \omega)=\frac{4 \pi i}{c} \nabla \times \sigma(\mathbf{k}, \omega) \mathbf{E}_{\mathbf{0}}(\mathbf{k}, \omega) \mathbf{e} \tag{2.23}
\end{equation*}
$$

Further we can solve this equation with using Green Function of Helmholz Operator $\left(\nabla^{2}+k^{2}=\frac{-\exp (-i k r)}{4 \pi r}\right)$ and set the radiator volume boundaries $V_{r a d}$ :

$$
\begin{equation*}
\mathbf{H}_{\mathbf{p o l}}(\mathbf{r}, \omega)=\nabla \times \frac{1}{c} \int_{V_{\text {rad }}} \sigma(\mathbf{k}, \omega) \mathbf{E}_{\mathbf{0}}(\mathbf{k}, \omega) \mathbf{e} \frac{\exp \left(i \sqrt{\varepsilon(\omega)}\left|\mathbf{r}^{\prime}-\mathbf{r}\right| \frac{\omega}{c}\right)}{\left|\mathbf{r}^{\prime}-\mathbf{r}\right|} d^{3} r^{\prime} \tag{2.24}
\end{equation*}
$$

By taking rotor from equation 2.24 (first derivative of integrand over $\left|\mathbf{r}^{\prime}-\mathbf{r}\right|$ ) we receive:

$$
\begin{equation*}
\mathbf{H}_{\mathbf{p o l}}(\mathbf{r}, \omega)=\frac{1}{c} \int_{V_{\text {rad }}} \sigma(\mathbf{k}, \omega) \mathbf{E}_{\mathbf{0}}(\mathbf{k}, \omega) \mathbf{e}\left(i \sqrt{\varepsilon(\omega)} \frac{\omega}{c}-\frac{1}{\left|\mathbf{r}^{\prime}-\mathbf{r}\right|}\right) \frac{\exp \left(i \sqrt{\varepsilon(\omega)}\left|\mathbf{r}^{\prime}-\mathbf{r}\right| \frac{\omega}{c}\right)}{\left|\mathbf{r}^{\prime}-\mathbf{r}\right|} d^{3} r^{\prime} \tag{2.25}
\end{equation*}
$$

From 2.25 it is seen that the integrand makes the main contribution only at distances $\left|\mathbf{r}^{\prime}-\mathbf{r}\right| \gg \lambda / \varepsilon(\omega)$, which is also called the wavezone. Thus, at the observation distances $r \gg \lambda$, assuming $1 /\left|\mathbf{r}^{\prime}-\mathbf{r}\right|=1 / r$, we can write magnetic field as:

$$
\begin{equation*}
\mathbf{H}_{\mathbf{p o l}}(\mathbf{r}, \omega)=\frac{i}{c} \frac{\exp (i r \sqrt{\varepsilon(\omega) \omega / c})}{r} \mathbf{k} \int_{V_{\text {rad }}} \sigma(\mathbf{k}, \omega) \mathbf{E}_{\mathbf{0}}(\mathbf{k}, \omega) \exp \left(-i \mathbf{k r}^{\prime}\right) d^{3} r^{\prime} \tag{2.26}
\end{equation*}
$$



Figure 2.17: PCA geometry description. Charged particle moving along $z$ axis and normal to $x y$ plane. Target with arbitrary shape and dimensions is placed along the particle trajectory and in $x y$ plane. Distance between target and particle $h$, which is also called impact parameter, can vary from 0 (inside material) to some value. On this figure propagation inside material assuming, so impact parameter is not shown. The direction of propagating $H_{\text {pol }}$ field emitted with angle $\Theta$ relatively to z axis (polar angle) and angle $\phi$ relatively to xyplane (azimuthal angle) is shown with the red arrow and assuming as $\mathbf{r}$ vector. On the target surface magnetic field from medium $\left(H_{m e d}^{\text {pol }}\right)$ will be emitted into vacuum $\left(H_{v a c}^{p o l}\right)$. Relationship connecting $H_{m e d}^{p o l}$ with $H_{v a c}^{p o l}$ and its projections on axis are shown with Eq. 2.27. In the figure polarization vectors of the magnetic field are also shown: vertical polarization component with a green line and horizontal component with a blue line. The particle field assumes to be radially polarized which is leads to a radially polarized vertical polarization component. Since geometry is not azimuthally symmetric part of horizontal polarization will be "cut" which leads to linear polarization of the horizontal polarization component. This effect will be shown in the section below. Total field emitted in vacuum should be $H^{p o l}=H_{x}^{p o l}+H_{z}^{p o l}+H_{x}^{p o l}$. Also it can presented as $H^{p o l}=H_{\perp}^{p o l}+H_{\|}^{p o l}$. In our case projection of Horizontal polarization planes. For this geometry vertical polarization component will be described as $H_{\perp}^{p o l}=H_{y} \sin (\phi+90)+H_{x} \cos (\phi+90)+H_{z}(=0)$ and horizontal component will be $H_{\|}^{p o l}=\sqrt{H_{z}^{2}+\left(H_{y} \sin (\phi)+H_{x} \cos (\phi)^{2}\right)}$. This logic is implied in Eq. 2.33.
where $\mathbf{k}=\sqrt{\varepsilon(\omega)}(\omega / c) \mathbf{e} ; \mathbf{e}=$ is a unit vector. The equation 2.26 describes the magnetic field propagating in the medium. However our detector plane is placed in "vacuum". The relationship between the unit vectors in the medium and in vacuum applying Snell's law is:
$\mathbf{e}=\{\sin \Theta \sin \phi ; \sin \Theta \cos \phi ;-\cos \Theta\}=\frac{1}{\sqrt{\varepsilon(\omega)}}\left\{\sin \theta \sin \phi ; \sin \theta \cos \phi ;-\sqrt{\varepsilon(\omega)-\sin ^{2} \theta}\right\}$
Here, $\Theta$ is the radiation polar angle in the medium; $\theta$ is the radiation polar angle in vacuum; $\phi$ is an azimuthal angle. The particle velocity vector is along $z$ axis. Plus or minus sign in front of z component determines forward and backward radiation cases.

In order to connect electric field in vacuum $\mathbf{E}_{\text {vac }}^{\text {pol }}$ and magnetic field in medium $\mathbf{H}_{\text {med }}^{\text {pol }}$ we need to use reciprocity principle described in [63, 90].

$$
\begin{equation*}
\left|\mathbf{E}_{\text {vac }}^{\text {pol }}\right|=\left|\frac{1}{\sqrt{\varepsilon(\omega)}} \mathbf{E}_{\text {med }}^{\text {pol }}\right|=\left|\frac{1}{\varepsilon(\omega)} \mathbf{H}_{\text {med }}^{\text {pol }}\right| \tag{2.28}
\end{equation*}
$$

To determine the electric field in vacuum $\mathbf{E}_{\text {vac }}^{\text {pol }}$, the magnetic field strength in the medium $\mathbf{H}_{\text {med }}^{\text {pol }}$ should be decomposed into polarization components relative to the plane of incidence of the wave[80]:

$$
\begin{align*}
& \left.\left|\mathbf{H}_{\text {med }}^{\text {pol }}\right|=\mid H_{\|}^{\text {pol }} \text { (med }\right)\left.\right|^{2}+\left|H_{\perp}^{\text {pol }(\text { med })}\right|^{2} \\
& \mid H_{\perp}^{\text {pol }} \text { (med) }\left.\right|^{2}=\left|f_{H}\right|^{2}\left|H_{\perp}^{\text {pol }}\right|^{2}  \tag{2.29}\\
& \left|H_{\|}^{\text {pol }(\text { med })}\right|^{2}=\left|\sqrt{\varepsilon(\omega)} f_{E}\right|^{2}\left|H_{\|}^{\text {pol }}\right|^{2}
\end{align*}
$$

$\left|H_{\perp}^{\text {pol }}\right|$ corresponds to the polarization component of magnetic field perpendicular to the plane, and its vector projection can be written as (here and after geometry with radiator above $z$ axis is implied):

$$
\begin{equation*}
\left|H_{\perp}^{\text {pol }}\right|=\left|H_{x}^{\text {pol }} \cos \phi\right|+\left|H_{y}^{\text {pol }} \sin \phi\right| \tag{2.30}
\end{equation*}
$$

And $\left|H_{\|}^{\text {pol }}\right|$ corresponds to the the polarization component of magnetic field parallel to
the plane(remembering $\cos (x+90)=\sin (x), \sin (x+90)=-\cos (x))$ :

$$
\begin{equation*}
\left|H_{\|}^{\text {pol }}\right|=\mid \sqrt{\left(H_{z}^{\text {pol }}\right)^{2}+\left(H_{x}^{\text {pol }} \sin \phi+H_{y}^{\text {pol }} \cos \phi\right)^{2}} \tag{2.31}
\end{equation*}
$$

Fresnels coefficients are [80]:

$$
\begin{gather*}
f_{H}=\frac{2 \varepsilon(\omega) \cos \theta}{\varepsilon(\omega) \cos \theta+\sqrt{\varepsilon(\omega)-\sin ^{2} \theta}} \\
f_{E}=\frac{2 \cos \theta}{\cos \theta+\sqrt{\varepsilon(\omega)-\sin ^{2} \theta}} \tag{2.32}
\end{gather*}
$$

Finally, substituting 2.29 into 2.28 we can write equation for spectral angular density of polarization radiation ${ }^{4}$ :

$$
\begin{gather*}
\frac{d^{2} W}{d \omega d \Omega}=c r^{2}\left|\mathbf{E}_{\mathbf{v a c}}^{\text {pol }}\right|^{2}=\frac{c r^{2}}{|\varepsilon(\omega)|^{2}}\left(\left|f_{H}\right|^{2}\left|H_{\perp}^{\text {pol }}\right|^{2}+\left|\sqrt{\varepsilon(\omega)} f_{E}\right|^{2}\left|H_{\perp}^{\text {pol }}\right|^{2}\right)= \\
=\frac{c r^{2}}{|\varepsilon(\omega)|}\left[\left|f_{H}\right|^{2}\left|H_{x}^{\text {pol }} \cos \phi\right|+\left|H_{y}^{\text {pol }} \sin \phi\right|^{2}+\right.  \tag{2.33}\\
\left.+\left|\sqrt{\varepsilon(\omega)} f_{E}\right|^{2} \mid \sqrt{\left(H_{z}^{\text {pol }}\right)^{2}+\left(H_{x}^{\text {pol }} \sin \phi+H_{y}^{\text {pol }} \cos \phi\right)^{2}}{ }^{2}\right]
\end{gather*}
$$

### 2.2.2 ChDR from a dielectric prism

To find the exact solution of spectral angular distribution for geometry with prismatic target we must supplement equation for magnetic field 2.26 with fourier component of electric field $\mathbf{E}_{\mathbf{0}}\left(k_{x}, y^{\prime}, z^{\prime}, \omega\right)^{5}$ :

$$
\begin{gather*}
\mathbf{E}_{\mathbf{0}}\left(k_{x}, y^{\prime}, z^{\prime}, \omega\right)=\frac{i e}{2 \pi v K} \exp \left[i \frac{\omega}{v} z^{\prime} \cos \alpha\right] \exp \left[-i \frac{\omega}{v}\left(y^{\prime}+h\right) \sin \alpha\right] \times \\
\times\left\{\gamma \beta e_{x} \sqrt{\varepsilon(\omega)},-\gamma^{-1} \sin \alpha-K \cos \alpha, \gamma^{-1} \cos \alpha-K \sin \alpha\right\} \times  \tag{2.34}\\
\times \exp \left[\frac{\omega}{v \gamma} z^{\prime} K \sin \alpha\right] \exp \left[\frac{w}{v \gamma} K \cos \alpha\right]
\end{gather*}
$$

Here, notation $K=\sqrt{1+\left(\gamma \beta e_{x}\right)^{2} \varepsilon(\omega)}$ has been used. $e_{x}$ is given in Fig. 2.27, $\gamma$ is the Lorenz factor, $\beta=v^{2} / c^{2}$, where $v$ is the particle speed, $c$ is the speed of light. In

[^2]this geometry only $y^{\prime}$ and $z^{\prime}$ components are considered, since, the target is assumed to be infinite along x axis. Impact parameter $b=h \cos (\delta)$, is the distance between target surface and particle, $\delta$ is the angle between target interface and particle direction, $h$ is the distance between target and rotation point. The schematic view of radiation geometry shown in Figure 2.18.


Figure 2.18: The schematic view of the radiation geometry for a charged particle moving in the vicinity of the triangular dielectric target.

Now, putting 2.34 into 2.26 and integrating over target dimensions $x^{\prime}=[0: a]$ and $z^{\prime}=[0: a / \cot \varphi]$ we can express vertical and horizontal polarization components [80]:

$$
\begin{gather*}
\frac{d^{2} W_{\|}}{d f d \Omega}=\frac{a}{2 \pi^{2}} \frac{\hbar f^{2}}{c} \frac{\cos ^{2}(\pi / 2-\theta-\alpha)}{k^{2}|P|^{2}}\left|\frac{\varepsilon(f)-1}{\varepsilon(f)}\right|^{2} \times \\
\times \left\lvert\, 1-\exp \left[-i a \frac{2 \pi f}{\beta c}(P+\Sigma \cot \varphi) \sin \varphi\right]-\frac{P \exp \left[i \frac{2 \pi f}{\beta c} \Sigma a \cos \varphi\right]}{P+\Sigma \cot \varphi}+\right. \\
+\frac{P^{2}+\Sigma^{2} \cot ^{2} \varphi}{P^{2}-\Sigma^{2} \cot ^{2} \varphi} \exp \left[-i a \frac{2 \pi f}{\beta c} P \sin \varphi\right]-\left.\frac{\Sigma \cot \varphi \exp \left[-i a \frac{2 \pi f}{\beta c} P \cos \varphi\right]}{P-\Sigma \cot \varphi}\right|^{2} \times  \tag{2.35}\\
\times \frac{\exp \left[-2 \frac{2 \pi f}{\gamma \beta c}(h+a \cos \varphi) K \cos \alpha\right]}{1-\beta^{2} \sin ^{2}(\theta-\alpha)+\beta^{2} \sin ^{2} \alpha\left(1-\cos ^{2}(\theta-\alpha) \sin ^{2} \phi\right)+2 \beta \sin \alpha \cos \theta-\alpha \cos \phi} \times \\
\left.\times\left|\frac{\varepsilon(f)}{\varepsilon(f) \sin (\theta-\alpha)+U}\right|^{2} \right\rvert\, \cos \alpha\left(\gamma^{-1} \cos \theta-\alpha-i K U \cos \phi\right)+ \\
+\sin \alpha\left(i K \cos \theta-\alpha+\gamma^{-1} U \cos \phi\right)-\gamma \beta U \cos \theta-\left.\alpha \sin ^{2} \phi\right|^{2}
\end{gather*}
$$

$$
\begin{gathered}
\frac{d^{2} W_{\perp}}{d f d \Omega}=\frac{a}{2 \pi^{2}} \frac{\hbar f^{2}}{c} \frac{\cos ^{2}(\pi / 2-\theta-\alpha)}{k^{2}|P|^{2}}\left|\frac{\varepsilon(f)-1}{\varepsilon(f)}\right|^{2} \gamma^{2} \sin ^{2} \phi\left|\frac{\sqrt{\varepsilon(f)}}{\sin (\theta-\alpha)+U}\right|^{2} \times \\
\times\left(\cos ^{2}(\theta-\alpha)+|U|^{2}\right) \left\lvert\, 1-\exp \left[-i a \frac{2 \pi f}{\beta c}(P+\Sigma \cot \varphi) \sin \varphi\right]-\frac{P \exp \left[i \frac{2 \pi f}{\beta c} \Sigma a \cos \varphi\right]}{P+\Sigma \cot \varphi}+\right. \\
+\frac{P^{2}+\Sigma^{2} \cot ^{2} \varphi}{P^{2}-\Sigma^{2} \cot ^{2} \varphi} \exp \left[-i a \frac{2 \pi f}{\beta c} P \sin \varphi\right]-\left.\frac{\Sigma \cot \varphi \exp \left[-i a \frac{2 \pi f}{\beta c} P \cos \varphi\right]}{P-\Sigma \cot \varphi}\right|^{2} \times \\
\times \frac{\exp \left[-2 \frac{2 \pi f}{\gamma \beta c}(h+a \cos \varphi) K \cos \alpha\right]}{1-\beta^{2} \sin ^{2}(\theta-\alpha)+\beta^{2} \sin ^{2} \alpha\left(1-\cos ^{2}(\theta-\alpha) \sin ^{2} \phi\right)+2 \beta \sin \alpha \cos \theta-\alpha \cos \phi} \times \\
\times\left[1-\beta^{2} \sin ^{2}(\theta-\alpha)+2 \beta \gamma^{-} 1 \sin \alpha \cos \theta-\alpha \cos \phi-\gamma^{-2} \sin ^{2} \alpha\left(K^{2}-\gamma^{-2}\right)\right]
\end{gathered}
$$

$\alpha=1 / 137$ is the fine structure constant, $f$ is the radiation frequency, $\varepsilon(f)$ is the target dielectric permittivity (complex permittivity will be considered in Chapter 4).

It should be noted here, that for our tasks it is more relevant to count polar and azimuth angles relative to the normal of the target output surface. Thus replacement $\theta-\delta$ has been used (see. insert in Fig. 2.18).

During all further analysis, we will use equations 2.2.2 and 2.2.2. Input parameters for all dependencies presented in this chapter are shown in table 2.3.

Table 2.3: Simulation parameters fo ChDR

| Parameters | Value |
| :--- | :---: |
| Lorentz - factor $\gamma$ | $[50,100,1000]$ |
| Radiation frequency $f$ | $[0.5,1,1.5] \mathrm{THz}$ |
| Target base length $a$ | $[1,2,5] \mathrm{cm}$ |
| Angle of the prism $\phi$ | $30^{\circ}$ |
| Dielectric permittivity $\varepsilon(f)$ | See eq. about snell law |
| Impact parameter $h$ | $[1,2,5] \mathrm{mm}$ |
| Detector diameter $D$ | $[2,5,10] \mathrm{cm}$ |
| Distance between radiator and | 10 cm |
| observation plane $d$ |  |
| Detector angular acceptance | $[11.4,28,53]$ degrees |
| $A=2 \arctan (D /(2 d)$ |  |

The expressions derived for the spectral-angular distribution of PR take into account both DR and ChDR , which can be seen from Figure 2.19. ChDR peak is concentrated around the angle of $\approx 40^{\circ}$, which coincides with Cherenkov Angle. All side blobs are the effect of diffraction of finite target boundaries (see Eq.2.32). With the red line, the possible detector angular acceptance of 28 degrees is shown to demonstrate that ChDR peak is properly isolated from DR peak which is around $0^{\circ}$. In complex target geometry like this, DR can experience several internal reflections inside the material and leave the target with the same as ChDR angle. Multiple reflections are not taken into account in this model, however, we propose that DR will leave target aside from ChDR (see Fig. 2.18)

Polarization components for the case of geometry with azimuthal asymmetry are different: vertical component (electric field vector is orthogonal to the radiation plane) is the same as the vertical component of radially polarized electron field, whereas hori-
zontal component (electric-field vector is parallel to the radiation plane) is only half of the horizontal component of radially polarized electron field (see "radiation plane" in Fig. 2.17). Angular distributions of different ChDR polarization components induced by a single particle are shown in Figures 2.20.


Figure 2.19: Angular distribution of polarization radiation generated by a charged particle with energy $\gamma=100$ moving near a dielectric wedge. Parameters are $a=2$ $\mathrm{cm} ; h=2 \mathrm{~mm}, f=0.5 \mathrm{THz}$. With a red line possible angular acceptance of $28^{\circ}$ is shown. Figure below shown with azimuthal angle $\phi=0^{\circ}$. Both pictures presented in logarithmic scale. The ChDR peak which will be used for diagnostic is located at $40^{\circ}$

### 2.2.3 Analysis of ChDR angular properties

In this section different angular dependencies, which help understand the ChDR generation mechanism, are presented. Such analysis is also crucial for choosing the experimental layout, radiator geometry, and optical elements. In Fig. 2.21(a) ChDR angular distributions for different electron energies are presented. It can be seen that for the energy region of our interest the peak angle keeps unchanged due to relativistic energies. In Fig 2.21(b) the dependence of ChDR peak position versus electron energy is shown with a red line. The blue line shows the radiation intensity emitted by a single electron with frequency $f=0.5 \mathrm{THz}$ and observed by detector plane with angular


Figure 2.20: Angular distribution of ChDR horizontal (a) and vertical (b) polarization components in two-dimensional representation. Due to azimuthal assymmetry horizontal polarization component is "cut". However it has higher energy density, since the part of electron field which induced is is closer to the target. Parameters are $\gamma=100$; $a=2 \mathrm{~cm} ; h=2 \mathrm{~mm} ; f=0.5$ THz.
acceptance $A=28^{\circ}$ is shown. Unlike peak angle, the emitted intensity has a strong dependence on $\gamma$ due to fixed distance between target and electron ( $h=2 \mathrm{~mm}$ ) and changing of effective electric field size which is scaled with $\gamma \lambda$.

Next step - is to investigate the influence of ChDR observation frequency (Fig. $2.22(\mathrm{a}))$. First of all, one needs to pay attention to lower frequencies when the wavelength starts to be comparable with the target dimensions. In this case, the peak angle position and intensity depend on the diffraction effects at the boundaries. At higher frequencies, the radiation intensity decreases due to the change of effective electron field size which is scales with $\gamma \lambda$.

In the figure $2.23(\mathrm{a}) \mathrm{ChDR}$ angular distributions for different values of impact parameter are shown. It can be seen that for the same frequency, peak position of emitted radiation does not change with the variance of $h$ parameter. Dependence of radiated intensity on impact parameter $h$ follows from the exponential decay of electron field radius with $\approx \frac{\gamma \lambda}{2 \pi}$.


Figure 2.21: a) ChDR angular distribution for different electron energies. Peak angle keeps unchanged for relativistic electrons; b) Red line - dependence of ChDR peak position on electron energy (which corresponds with the classic Tamm-Frank equations (see 1.6), blue line - dependence of ChDR intensity on electron energy. To calculate ChDR intensity angular distribution was integrated over angular acceptance $A=28^{\circ}$. Parameters for this simulation are $a=2 \mathrm{~cm} ; h=2 \mathrm{~mm} ; f=0.5 \mathrm{THz}$.


Figure 2.22: Radiation angular distribution along polar angle at the different observation frequencies. At lower frequencies the wavelength starts to be comparable with target dimensions and ChDR degrades. Parameters for this simulation are $\gamma=100$; $a=2 \mathrm{~cm} ; h=2 \mathrm{~mm}$.

### 2.2.4 Analysis of ChDR spectrum properties

First of all, considering ChDR single electron spectrum we should pay attention to intensity suppression on lower and higher frequencies. Suppression on lower frequencies is caused by diffraction effects due to finite target size, which is for our cases starts on frequencies lower than $200 G H z(\lambda=1.5 \mathrm{~mm})$. Suppression on higher frequencies follows from the exponential decay of electron field radius with $\approx \frac{\gamma \lambda}{2 \pi}$, i.e. electron is too far away from the dielectric to generate low wavelengths electromagnetic fields. These suppression effects can be seen in all figures presented in this section.



Figure 2.23: ChDR angular distribution for different impact parameters (a) and dependence of maximum radiation intensity on impact parameter (b). In the figure bellow with a red line $\frac{\gamma \lambda}{2 \pi}$ coefficient is shown. $\gamma=100 ; a=2 \mathrm{~cm} ; f=0.5 \mathrm{THz}$

One of the ChDR advantages is the possibility to build a non-invasive diagnostic system. The $\gamma \lambda / 2 \pi$ parameter is very important for such construction since it will have


Figure 2.24: Schematic view to explain Fig. 2.23
an influence not only on ChDR intensity but also on emitted spectral properties. In simple words we can say, that most radiation will be generated within an effective spot size of $R=\gamma \lambda / 2 \pi$ radius [93, 94] and this spot size must be larger than the distance between the particle and target $h$. If the target is placed on the edge of this spot - only the long-wavelength component will be generated, and if the target is placed close to the electron - the short-wavelength component will also be there. This effect can be seen in figure 2.25 where spectra for different distances between particle and target are shown. With smaller $h$ we have higher intensity because of the high-frequency component. This information should be used to find a compromise between the geometry of the experimental setup and the possibility to detect higher frequencies.


Figure 2.25: ChDR spectrum for different impact parameters $\gamma=100 ; a=2 \mathrm{~cm}$; $A=53^{\circ}$

Another ChDR advantage for diagnostic purposes is the fact that ChDR intensity
scales with the target length. Thus, it is important to investigate the dependency of the ChDR on this characteristic. Figure 2.26 shows that overall relative intensity for the same frequencies scales proportionally with the target base length $L$. Also, it can be seen that with increasing of target base length $L$ (i.e. increasing of target size) the influence of the low-frequency suppression due to finite target sizes is decreased. From this perspective, the use of a larger target is profitable, however, we did not take into account the dielectric absorption yet. It will be properly discussed in chapter 4 , where dielectric properties of the Teflon material will be experimentally obtained and implemented into the described model.


Figure 2.26: ChDR spectrum for different target base lengths. $\gamma=100 ; A=53^{\circ}$; $h=5 \mathrm{~mm}$

### 2.2.5 ChDR single electron spectrum properties for the experiment conducted at CLARA facility

During this section, we did not cover an essential detail for the ChDR spectral characteristics - dielectric properties of radiator material. It will be investigated separately in Chapter 4, and only after accounting Teflon extinction coefficient, ChDR single electron spectra for the experiment conducted at the CLARA facility should be presented in Fig. 4.10.

### 2.3 Theory of longitudinal bunch length reconstruction via coherent radiation

Coherent radiation appears when the radiation wavelength is comparable to, or longer than the bunch length of a charged particle beam, that interacts with the radiator. Full coherence means that the radiation fields of all N electrons add constructively. In this case, the radiation photon yield is proportional to the number of electrons squared. By measuring the radiation spectrum in the wavelength range where the incoherent radiation is transformed into the coherent one, we can deduce information about the longitudinal bunch form factor and, subsequently, reconstruct the longitudinal beam profile.

### 2.3.1 Coherent Radiation from charged particle bunches

Electromagnetic radiation emitted by a bunch of charged particle can be defined as the superposition of the radiation field emitted by each electron in the bunch:

$$
\begin{equation*}
I(\omega)=|E(\omega)|^{2}=\left|E^{0}(\omega)\right|^{2}\left|\sum_{j=1}^{N_{e}} \exp \left(i w t_{j}\right)\right|^{2}=I^{0}(\omega)\left|\sum_{j=1}^{N_{e}} \exp \left(i w t_{j}\right)\right|^{2} \tag{2.38}
\end{equation*}
$$

where $|E(\omega)|$ is the total electron field field from a bunch of $N$ electrons, $\left|E^{0}(\omega)\right|$ is the electron field from a single electron, $I^{0}(\omega)$ it the radiation intensity from a single electron, $\omega$ is the angular frequency, $t_{j}$ is the phase delay of a $j^{t h}$ electron. In [95] shown that for the wavelengths much shorter than the electron bunch length the factor $-w\left(t_{i}\right)$ goes up and the phase difference grows up as well. The radiation fields, in this case, add incoherently and we can write the total radiation intensity as the intensity of N independent electrons:

$$
\begin{equation*}
I(\omega)=N I^{0}(\omega) \tag{2.39}
\end{equation*}
$$

For the wavelengths longer than the bunch length, phase difference goes down and, therefore, the electric fields add in phase and we can write the total radiation intensity as:

$$
\begin{equation*}
I(\omega)=N I^{0}(\omega)+N(N-1) I^{0}(\omega) \tag{2.40}
\end{equation*}
$$

Result of equation 2.39 and 2.40 is also demonstrated in Fig 2.27. When the distance between the electrons is comparable to or smaller than the radiation wavelength, the generated radiation will be in phase. Light intensity generated from two electrons will be: $I_{\text {coh }}=\left|a_{1}+a_{2}\right|^{2}=4|a|^{2}$, or, for the case of N electrons: $I_{\text {coh }}=N^{2}|a|^{2}$. For the wavelengths smaller then this distance, photons are not in phase, and the radiation intensity can be written as: $I_{\text {incoh }}=a_{1}^{2}+a_{2}^{2}=2|a|^{2}$, or, for the case of N electrons: $I_{\text {incoh }}=N|a|^{2}$.


Figure 2.27: Schematic of incoherent and coherent radiation produced by a charged particle. $a_{1}$ and $a_{2}$ are the electric field amplitudes. Figure taken from Pavel Karataev lectures

For the real bunch with collection of electrons independently distributed in continuum limit we can rewrite eq. 2.38 as:

$$
\begin{equation*}
I(\omega)=I^{0}(\omega)\left[N+N(N-1)\left|\int \rho(z) \exp \left(i \frac{\omega}{c} z\right) d z\right|^{2}\right]=I^{0}(\omega)[N+N(N-1) F(\omega)] \tag{2.41}
\end{equation*}
$$

Where $F(\omega)=\left|\int \rho(z) \exp (i(\omega / c) z) d z\right|^{2}$ is the longitudinal bunch form factor and represents the Fourier transform of the normalized bunch distribution $\rho(z)$ squared. In simple words, the form factor is a measure of the degree of coherence and its value is determined by how the wavelength is related to the bunch length. The value of the bunch form factor varies from zero at wavelengths $\lambda \ll \sigma$ (incoherent limit) to unity at $\lambda \gg \sigma$, where $\sigma$ is the bunch length RMS. In this thesis, only the longitudinal form factor is considering because in a relativistic case the contribution of the transverse beam size to the degree of coherence is negligible [81].

By measuring the coherent spectrum emitted by a bunch of charged particles and knowing the spectrum emitted by a single electron, we can derive information about the longitudinal form factor. Form factor in turn can provide information about longi-
tudinal profile through the inverse Fourier transform expression. Since the form factor is Fourier transform of a square of the function, the inverse Fourier transform of the square-root gives only the even component of the distribution function $(f(-\omega)=f(\omega))$ and we can use cosine Fourier transform:

$$
\begin{equation*}
\rho(z)=\frac{1}{\pi c} \int_{0}^{\infty} \sqrt{F(\omega)} \cos \left(\frac{\omega z}{c}\right) \tag{2.42}
\end{equation*}
$$

### 2.3.2 Kramers-Kronig analysis

The use of interferometric methods such as Martin-Pupplet interferometer described in this thesis leads to the fact that only intensity measurements of the coherent spectrum are possible, and the form factor phase remains unknown. Thus, we can not retrieve the longitudinal bunch profile with just inverse Fourier transform of form factor, since many different bunch shapes are yielding very similar spectra. However, the phase can be recovered using so called phase-retrieval techniques such Kramers-Kronig [26] analysis, Blaschke phase retrieval [27-29], or more novel approaches such as phase-constrained iterative method [30].

For this work, Kramers-Kronig procedure which proposed and described by Lai and Sievers in [26] will be used. This procedure was proposed to retrieve a phase and bunch structure knowing only the radiated spectrum. The complex form factor amplitude $\hat{F}(\omega)$ with real and imaginary parts in terms of magnitude $\sqrt{F(\omega)}$ and phase $\psi(\omega)$ will be:

$$
\begin{equation*}
\hat{F}(\omega)=\int_{0}^{\infty} \rho(z) \exp \left(i \frac{\omega z}{c}\right) d z \equiv \sqrt{F(\omega)} \exp i(\psi(\omega)) \tag{2.43}
\end{equation*}
$$

The minimal phase $\psi(\omega)$ can be obtained according procedure described in [26]:

$$
\begin{equation*}
\psi(\omega)=-\frac{2 \omega}{\pi} \int_{0}^{\infty} \frac{\ln (\sqrt{F(x)} / \sqrt{F(\omega)})}{x^{2}-\omega^{2}} d x \tag{2.44}
\end{equation*}
$$

Here, $x$ is the angular frequency variable running through the whole integration range (note, when $x=\omega$, there is singularity appear, so a slight interval should be used for numerical calculations). Thus, the bunch profile distribution with taking into account
phase shift will be:

$$
\begin{equation*}
\rho(z)=\frac{1}{\pi c} \int_{0}^{\infty} \sqrt{F(\omega)} \cos \left(\psi(\omega)-\frac{\omega z}{c}\right) d \omega \tag{2.45}
\end{equation*}
$$

### 2.3.3 Low frequency and high frequency extrapolation

In practice, it is impossible to measure radiation spectrum in all frequency region. For example, detected wavelengths region could be affected by diffraction effects in the finite size dielectric, detector properties, absorption lines of optical elements or absorption line of humid air, and so on. It is obvious that the detection system should provide as much spectral information about coherent radiation as possible, however, if instrumental limitations were achieved, there is a possibility to extrapolate the form factor towards lower and higher frequencies. For example, such an extrapolation procedure was proposed in [44] where authors worked on bunch profile reconstruction with Diffraction Radiation.

Consider some theoretical example of gaussian bunch with the "tail" or "decherping" effect, which can be described as combination of two gaussian curves:

$$
\begin{equation*}
\rho(z)_{\text {ideal }}=\frac{\exp \left(-z^{2} / 2 \sigma_{1}^{2}\right)}{4 \sqrt{2} \pi \sigma_{1}}+\frac{3 \exp \left(z-z_{0}\right)^{2} / 2 \sigma_{2}^{2}}{4 \sqrt{2} \pi \sigma_{2}} \tag{2.46}
\end{equation*}
$$

where $\sigma_{1}=0.3 \mathrm{~mm}$ and $\sigma_{2}=0.45 \mathrm{~mm}$ represent tale and core lengths in RMS and $z_{0}=1.5 \mathrm{~mm}$ represent the offset between the bunches. The longitudinal bunch distribution described by this equation is shown in Fig. 2.28(a). For a Gaussian bunch profile form factor should be exponentially decreasing function, however for the bunch with collateral peak form factor shape will change and demonstrate complex behaviour at higher frequencies. Following equation 2.42 the form factor of this bunch can be expressed in analytical form [45]:

$$
\begin{equation*}
F(\omega)_{\text {ideal }}=\frac{1}{16}\left(\exp \left(-\sigma_{1}^{2} k\right)+9 \exp \left(-\sigma_{2}^{2} k\right)+6 \exp \left(-\frac{k\left(\sigma_{2}^{1}+\sigma_{2}^{2}\right)}{2}\right) \cos \left(k z_{0}\right)\right. \tag{2.47}
\end{equation*}
$$

where $\mathrm{k}=\omega / c$. The calculated form factor is presented in Fig.2.28(b) ${ }^{6}$. Let's assume

[^3]that the green line is the real data set obtained using interferometry. The black dashed vertical lines represent the spectral range that is possible to detect (as an example, from 20 to 150 GHz$)$. Blue and red lines are extrapolated data towards lower and higher frequencies.


Figure 2.28: Example of non-Gaussian bunch distribution calculated from Eq. 2.46 (a) and corresponding form factor (b) with applied low (blue line) and high (red line) frequency extrapolation.

The extrapolation towards lower frequencies should satisfy the condition: $\mathrm{F} \rightarrow 1$ as $\omega \rightarrow 0$ and match the smallest possible frequency point from experimental data. The following extrapolation function is suggested in [44]:

$$
\begin{equation*}
F(\omega)_{\text {small }}=\exp \left(-\alpha \omega^{2} / c^{2}\right) \tag{2.48}
\end{equation*}
$$

where $\alpha$ is the adjustment constant: extrapolation should match the smallest possible frequency point from the experimental data set.

The extrapolation towards the larger frequencies should satisfy the condition: F $\rightarrow 0$ as $\omega \rightarrow \infty$ :

$$
\begin{equation*}
F(\omega)_{\text {large }}=\exp \left(-\beta \omega^{2} / c^{2}+\gamma \omega / c+\delta\right) \tag{2.49}
\end{equation*}
$$

where $\beta, \gamma, \delta$ are chosen to smoothly join the larger frequencies. $F(\omega)_{\text {large }}$ should also match the experimental data at the largest frequencies. Extrapolation to higher frequencies should be precise since the overall bunch length is sensitive to high-frequency
components. Thus, both the first and second derivatives of $F(\omega)_{\text {large }}$ should match the first and second derivatives of experimental data at the point of contact.

In Fig. 2.29(a) minimal phase as a result of Kramers-Kronig analysis (see Eq.2.44) is shown. A blue line represents a minimal phase for a theoretical form factor (Eq. 2.28) and a red line represents a minimal phase for an extrapolated form factor following extrapolation procedure [44]. Fig. 2.29(b) shows the comparison of longitudinal profile plotted from Eq. 2.46 (blue line) and longitudinal profile obtained via reconstruction of extrapolated form factor and Kramers-Kronig analysis (Eq. 2.45 and Eq. 2.44).


Figure 2.29: Fig a: Minimal phase $\psi(\omega)$ calculated using Eq. 2.44 for a theoretical form factor from Eq. 2.28(b) (blue line) and for an extrapolated form factor (red line). Fig b: Longitudinal profile plotted using Eq. 2.46 (blue line) and reconstructed longitudinal profile using extrapolated form factor and Eq. 2.45 and Eq. 2.44 (red line).

### 2.4 Summary

In this section, we have considered all the necessary theoretical information about Cherenkov Diffraction Radiation and Transition Radiation. Theoretical knowledge about single electron spectral angular distribution is of great importance for experimental setup preparation since it allows us to properly choose experimental layout geometry, detection system, and optical elements. We have also covered the theory of reconstruction of the longitudinal beam profile was also considered, which will be used in Chapter 5 with experimental data obtained on the CLARA accelerator.


## Experimental setup

In this Chapter, the experimental setup for noninvasive longitudinal beam profile diagnostic built at the CLARA electron accelerator facility is described. At the heart of this setup is the idea to provide longitudinal diagnostics using two mechanisms in parallel: Coherent Cherenkov Diffraction Radiation and Coherent Transition Radiation.

The whole experimental setup can be divided in three parts:

- Setup for generation of sub-THz ChDR and CTR radiation installed in the vacuum chamber. It also includes optical elements for extraction of THz radiation out of the chamber;
- Spectroscopy setup for THz radiation spectrum measurement installed outside of the chamber. For spectroscopy, the Martin-Pupplet interferometer was chosen as the most efficient instrument in sub- THz region.
- Data acquisition system. This part includes: processing of ChDR and CTR signals obtained from THz detectors; normalized interferogram and spectra obtaining; longitudinal beam profile reconstruction according to the prescriptions described in Chapter 2.

As a result after two consecutive scans for ChDR and CTR, we should have two different spectra and two longitudinal profiles for analysis and comparison. These three different parts along with the CLARA machine overview will be presented in this Chapter.

### 3.1 CLARA machine overview

CLARA (Compact Linear Accelerator for Research and Applications) [96] is in its Phase 1 of construction. Machine equipped with 10 Hz UV frequency photo-injector providing up to 100 pC charge, 3 GHz RF gun accelerating electrons up to 4 MeV , and a linac which accelerates electron beams up to 50 MeV (this part of the construction is also called CLARA front end). The facility layout assumes that the beam after CLARA front end can either be transported directly to the CLARA beamline (under development from January 2019) or, using quadrupole triplet and Lozenge dipole (dog-leg section), to already existing VELA beamline (Figure 3.1). During our work, the electron bunches were compressed in a dog-leg system, transported to the VELA beamline, and then to the Beam Area 1 (BA1).


Figure 3.1: Layouts of CLARA front end facility (top) and VELA beam line (bottom), showing CLARA Phase 1 and connection to VELA via the C2V dog-leg. The VELA beamline continues to BA1 through a 2 m thick concrete shield wall. BA1 will be shown in figure 3.3

### 3.1.1 Motivation for longitudinal diagnostic at CLARA

During the time of these experiments at CLARA/VELA, despite the possibility to provide various bunch compression parameters for different user applications, no dedicated longitudinal diagnostics were available. However, a CTR monitor with the main goal to observe the relative influence of different bunch compression parameters via CTR signal was already in commissioning plans. Diagnostics based on CTR spectrum analysis were also considered, and space for interferometry setup was reserved.

Since frequency domain diagnostics with CTR and CChDR have the same nature, it was decided to build diagnostic measurement in a such way, that both effects could be measured after minor adjustment. Thus, several scientific and technical problems have been solved: conduction of the experiments on non-invasive diagnostics with CChDR ; study of CChDR physical properties; comparison of diagnostics results with well-known CTR; investigation of bunch compression parameter influence on longitudinal beam properties for the CLARA facility.

### 3.1.2 Beam properties on CLARA

For this experiment, important parameters are energy, transverse size, current, and bunch length. For example, current fluctuation will induce quadratic change in coherent radiation signal. Transverse size and position jitter are important for precise target positioning. Bunch length needs to be precisely controlled to achieve reproducible results. Energy has an influence on both TR and ChDR intensity spectral angular properties (see Chapter 2). Due to active commissioning, the main accelerator parameters were lowered down to $\approx 70 p C$ charge and 35 MeV to provide reproducible beam on daily basis. Transverse profile obtained by YAG screen installed near interaction point shown in figure 3.2 and was of $\sigma_{x}=(122 \pm 3) \mu m$ and $\sigma_{y}=(116 \pm 3)$ RMS. Longitudinal bunch compression for CLARA discussed in section 1.1.3 and simulated bunch length $\sigma_{t}$ for a different bunch compression parameters shown in figure 1.3. The most effective compression should happen at $10^{\circ}$ off-crest phase and $\sigma_{t}=300 \mathrm{fs}$ was expected.


Figure 3.2: Transverse profile on YAG screen installed near interaction point. RMS values for the gaussian function are $\sigma_{x}=(122 \pm 3) \mu m$ and $\sigma_{y}=(116 \pm 3)$

### 3.2 CLARA Beam Area 1 (BA1)

BA1 is a special user facility area developed for experiments with beam-mater interaction in the 2 meters long vacuum chamber. The vacuum chamber is equipped with a motorized support system for user devices. Also, a set of standard diagnostics (including energy spectrometer, yttrium-aluminum-garnet (YAG) screens on motorized stages, and beam position monitors) is installed inside. Fig. 3.3 shows the schematic drawing and photos of BA1 installations. Along the left side of the vacuum chamber, three cameras are installed to measure the beam spot size along the beamline. The right side vacuum viewport is equipped with a z-cut quartz window. Radiation is extracted through the quartz window to be measured with different optical instruments (detectors, interferometer, etc.)


Figure 3.3: Top: Schematic drawing and photos of BA1 installations; Bottom: photos of BA1 vacuum chamber (a) and diagnostics table (b)

### 3.3 Setup for ChDR and CTR generation

The main goal of this experiment is to compare results obtained with CChDR and CTR diagnostic during the same machine run. Thus, it was important to develop an experimental layout in a way that provides fast and noninteractive switching between targets. Also, working with THz radiation it was important to retain the consistency of using optical elements for radiation transport.

A schematic diagram and 3D model of the setup are shown in Fig. 3.4. Both ChDR (2) and TR (7) targets are installed on the same platform with two degrees of translational freedom - vertical (4) and horizontal (1). Two ChDR targets were
placed on a platform (3) and CTR target (9) was mounted on an optical holder. A YAG screen was also installed on this platform (not shown in the scheme). Figure 3.5 shows the photo of the assembled setup.

In one scenario we can displace the platform in a position when the beam of charged particles (yellow line) interacts with the TR target. In this case, BTR (red arrows) is induced. Then radiation is reflected by a steering mirror (8), then collimated by an off-axis parabolic mirror (7) (it was installed at 101.6 mm from the CTR target which is its focal length) and delivered to the detection system via output quartz window. The parabolic mirror was the Thorlabs MPD244-M01-45 with an off-Axis angle of $45^{\circ}$, diameter of $2^{\prime \prime}$, and reflected focal length of $4^{\prime \prime}$. The parabolic mirror was installed on a horizontal translational stage (5) and rotational stage (6). The main purpose of the rotational stage (6) is precise radiation steering from TR and ChDR targets as they propagate to mirror (7) at different angles. The main purpose of the translational stage (5) is to adjust the distance between mirror and ChDR when the dependence on impact parameter (h) is investigated. Another scenario is when the platform is displaced in a position when a beam of charged particles passes by the ChDR target (2). In this case, CChDR (blue arrow) is emitted at $\approx 45^{\circ}$ relative to the trajectory of the charged particle beam. CChDR then propagates to the parabolic mirror and steered towards the detection system.

### 3.4 Martin-Pupplet interferometer

The Martin-Paplett Interferometer is one of the most effective instruments to measure the radiation spectrum in the millimeter range. The main advantage of it is the design with a polarisation splitter, which provides flatter efficiency of use, as well as the possibility to use two detectors and eliminate the influence of external noises and current fluctuations. In this section design and principle of operation of MPI will be discussed. Also, a precise alignment of this instrument using a test THz source and a THz camera for visualization will be described.

To start the discussion of the MPI operation principle we need first to research its base components - grid polarizers and rooftop mirrors.


Figure 3.4: Setup inside vacuum chamber: 1 - horizontal positioning stage, 2 - CChDR targets, 3 - platform for CChDR, 4 - vertical positioning stage, 5 - horizontal translational stage for concave mirror, 6 - rotational stage, 7 - concave mirror, 8 - CTR radiator, 9 - CTR steer mirror. Result of single electron simulation for this particular experimental layout shown in Fig. 4.9


Figure 3.5: Photo of the experimental setup inside a vacuum chamber.

## a) Wire grid polarizer

A wire grid polarizer consists of many thin wires separated by a small distance between them. These wires strung parallel to each other in a plane. In the Martin-Pupplet interferometer, we use it as an input polarising gird, a beam splitting grid, and a recombining grid. All wire grids in our case were made from $10 \mu m$ thick wires with $25 \mu \mathrm{~m}$ spacing, producing polarisation of $>90 \%$ across the bandwidth of $0.1-5 \mathrm{THz}$. Polarizers aperture was 75 mm . When light enters the wire grid plane, the component of the electric field $E$ which is parallel to wires will induce current inside them. The collection of wires will act as a mirror for this electric field component and will reflect the light. On the other hand, the component of the electric field $E$ which oscillates perpendicular to wires will not be affected by the wire grid and will be transmitted. Thus, the wire grid could be used as a polarizer.

If the light is already polarized, for example, horizontally and it is incident to a wire grid polarizer oriented at an angle of $45^{\circ}$ to this polarization, half of the light will
transmit and half of the light will reflect. This is because horizontally polarized light can be seen as a superposition of light with an electric field vector oriented at $+45^{\circ}$ and $-45^{\circ}$ with equal magnitudes. Thus, the wire grid polarizer can act as a splitter and both transmitted and reflecting light will be polarized to $\pm 45$ degrees relative to initial polarization.

## b) Rooftop mirror

A rooftop mirror is a combination of plane metal surfaces which are placed at $90^{\circ}$ relatively to each other. The line of intersection between these metal surfaces is called the "roofline". In our case, the aluminium rooftop mirrors were designed to have a 50 mm clear aperture. The rooftop mirror reflects the light which is incident perpendicular


Figure 3.6: Transformation of the electric field vector as it reflects on a rooftop mirror. Figure taken from [97].
to the roofline back in the direction of incidence while it also rotates the polarization by an angle of $2 \theta$, where $\theta$ is the angle between the roofline and the incident polarization direction (see Fig. 3.6). If $\theta$ is $\pm 45$ degrees (as it should be after beam splitter), then
the shift in polarization is $\pm 90$ degrees $^{1}$.
In simple words the purpose of rooftop mirror can be described as follows: the light which initially was reflected at wire grid splitter should pass it after reflection on rooftop mirror, and the light which initially passed the splitter should be reflected by it after reflection on rooftop mirror (see Fig. 3.7). Thus, the light from two interferometer hands can be recombined after the wire grid splitter and initial vertical and horizontal polarization components will be shifted by $90^{\circ}$ relatively to each other.

### 3.4.1 Principle of Martin-Pupplet interferometer

Scheme of Martin-Pupplet interferometer shown in Fig 3.7. After the radiation passes through the polarizer P1 - all components of the electric field perpendicular to the polarizer grid pass further, while the parallel components cause reverse currents in the grid and reflect back. At the output of the polarizer, this radiation can be described as follows:

$$
\begin{equation*}
E(t, x) \mathbf{u}_{\mathbf{h}}=E_{0} \sin (\omega t-k x) \mathbf{u}_{\mathbf{h}} \tag{3.1}
\end{equation*}
$$

where $\mathbf{u}_{\mathbf{h}}$ - horizontal unit vector, $\mathbf{k}$ - radiation vector ( $\mathbf{u}_{\mathbf{h}} \perp \mathbf{k} \perp \mathbf{u}_{\mathbf{v}}$ ), $\mathbf{k} x$ - phase shift. Unit vectors are here to highlight the separation between horizontal and vertical polarization parts. If we take $\mathbf{k} x=0$, we can write:

$$
\begin{equation*}
E(t) \mathbf{u}_{\mathbf{h}}=E_{0} \sin (\omega t) \mathbf{u}_{\mathbf{h}} \tag{3.2}
\end{equation*}
$$

Polarization splitter S will separate radiation into two parts: one will go through and will be polarized as $-45^{\circ}$ relatively to $\mathbf{u}_{\mathbf{h}}$; another will reflect and takes polarization $+45^{\circ}$ relatively to $\mathbf{u}_{\mathbf{h}}$ :

$$
\begin{gather*}
E_{\text {trans }}(t)=\frac{E_{0}}{2} \sin (\omega t)\left(\mathbf{u}_{\mathbf{h}}-\mathbf{u}_{\mathbf{v}}\right) \\
E_{\text {refl }}(t)=\frac{E_{0}}{2} \sin (\omega t)\left(\mathbf{u}_{\mathbf{h}}+\mathbf{u}_{\mathbf{v}}\right) \tag{3.3}
\end{gather*}
$$

which then travel along their respective arms and arrive back at the beam splitter with a phase offset, $\omega \tau$, that depends on the path length difference (phase shift) $2 \Delta x$. The time taken to cover the distance is $\tau=\frac{2 \Delta x}{c}$, where $c$ is the speed of light. The

[^4]

Figure 3.7: Schematic view on MPI: P, P2 - polarisers, S - polarisation spliter, MM - movable roof mirror, FM - fixed roof mirror, PD1 and PD2 - pyroelectric detectors (Gentec THz51); additional pyroelectric detector PD3 was used to monitor the noise in the environment and provide a shot-to-shot background correction. The wire grids transmit polarisation perpendicular to the wire orientation, therefore the field after the polariser P is horizontally polarised;
pass difference $\Delta x$ is controlled by a movable mirror installed on a translational stage. After reflection on the roof mirrors (FM and MM) Eq. 3.3 transforms to:

$$
\begin{gather*}
E_{\text {trans }}^{\prime}(t)=\frac{E_{0}}{2} \sin (\omega t)\left(\mathbf{u}_{\mathbf{h}}+\mathbf{u}_{\mathbf{v}}\right) \\
E_{\text {refl }}^{\prime}(t)=\frac{E_{0}}{2} \sin (\omega t-\omega \tau)\left(\mathbf{u}_{\mathbf{h}}-\mathbf{u}_{\mathbf{v}}\right) \tag{3.4}
\end{gather*}
$$

Now, radiation that was before reflected on the splitter will pass through it, and radiation which was transmitter - will be reflected. In the region between the beam splitter and the recombiner we will have:

$$
\begin{equation*}
E_{\text {tot }}(t)=E_{\text {refl }}^{\prime}(t)+E_{\text {trans }}^{\prime}=\frac{E_{0}}{2}\left[(\sin (\omega t-\omega \tau)+\sin (\omega t)) \mathbf{u}_{\mathbf{h}}+(\sin (\omega t-\omega \tau)-\sin (\omega t)) \mathbf{u}_{\mathbf{v}}\right] \tag{3.5}
\end{equation*}
$$

As a result, in the region between the beam splitter and second polarizer, we have co-existence of two perpendicularly polarised fields with a (variable) phase delay. The phase delay will determine the amplitude of the electric field and therefore, will determine an interferogram pattern. We can simplify it using sum-to-product identity (and removing unit vectors):

$$
\begin{equation*}
E_{\text {tot }}(t)=E_{\text {refl }}^{\prime}(t)+E_{\text {trans }}^{\prime}=E_{0} \sin \left(\omega t-\frac{\omega \tau}{2}\right) \cos \left(\frac{\omega \tau}{2}\right)+E_{0} \cos \left(\omega t-\frac{\omega \tau}{2}\right) \sin \left(\frac{\omega \tau}{2}\right) \tag{3.6}
\end{equation*}
$$

And we can write two different polarization component after the second polarizer splitter as:

$$
\begin{align*}
& E_{v}=E_{0} \sin \left(\omega t-\frac{\omega \tau}{2}\right) \cos \left(\frac{\omega \tau}{2}\right)  \tag{3.7}\\
& E_{h}=E_{0} \cos \left(\omega t-\frac{\omega \tau}{2}\right) \sin \left(\frac{\omega \tau}{2}\right)
\end{align*}
$$

Since radiation intensity is proportional to the square of electric field and that detectors will measure time-averaged field intensity (without phase shift) we can write:

$$
\begin{equation*}
I_{h, v}(\tau) \propto \frac{1}{2 T} \int_{0}^{T} E_{t o t}^{2}(t) d t \tag{3.8}
\end{equation*}
$$

After integrating we can write light intensity for different polarizations as:

$$
\begin{align*}
& I_{h}(\tau) \propto E_{0}^{2} \cos ^{2} \frac{\omega \tau}{2}  \tag{3.9}\\
& I_{v}(\tau) \propto E_{0}^{2} \sin ^{2} \frac{\omega \tau}{2}
\end{align*}
$$

which are anti-correlated and, as a sum, directly proportional to the field intensity after the first polariser. We can now use "normalized difference interferogram" $\delta(\tau)$ to avoid all simultaneous fluctuations in the detected intensity:

$$
\begin{equation*}
\delta(\tau)=\frac{I_{h}-I_{v}}{I_{h}+I_{v}} \tag{3.10}
\end{equation*}
$$

### 3.4.2 MPI components

The optical elements used in the MP-I design were as follows: protected gold THz transport mirrors (Tydex LLC.), 2 TPX lenses (Tydex LLC.), wire grid polarisers (Specac Ltd.), and gold-coated aluminum rooftop mirrors (LBP Optics Ltd.).

As detectors, pyroelectric detectors (Gentec THz-51) were used. Pyroelectric detectors were chosen since it is relatively accessible device with high sensitivity ( $70 \mathrm{kV} / \mathrm{W}$ ) and broadband bandwidth $(0.1-30 \mathrm{THz})$. The machine was operating at a repetition rate of 10 Hz , so the rise time of less then 0.2 s of the pyroelectric detectors was enough. The disadvantages of using pyroelectric detectors was a low signal-to-noise ratio (NEP $=10 \mathrm{nW})$ due to high sensitivity to mechanical and acoustic interference causing vibrations in pyroelectric crystal. For an accelerator hall environment, this problem was of great importance due to the presence of vacuum pumps, air conditioning, RF system, etc. To solve this problem system of three identical pyroelectric detectors was used (see Fig. 3.7: PD1 and PD2 as a detector for gathering THz radiation information and PD3 gathered all background noise present in the environment). Then shot-to-shot background (collected by 3rd pyroelectric detector) subtraction was applied.

### 3.4.3 Martin-Pupplet interferometer precise alignment

MPI design allows to effectively remove the influence of intensity fluctuation using separate detectors on different outputs: two signals in those detectors are anticorrelated
relative to each other and we can use Eq. 3.10 to provide interferogram normalization. But on the other hand, signal ratio of these two signals is of great importance, because resultant electric fields in front of the analyzer P2 should have the same amplitude. Thus, it is necessary to provide the best possible optical alignment of all-optical elements in the interferometer.

During our work, the alignment was performed using CW test THz source and a Terasence camera operating at $10 \mathrm{GHz}-1 \mathrm{THz}$. Figure 3.8 shows the example of using visualization and alignment with the THz camera. Test CW THz source was installed on the other side of the vacuum chamber sending the THz radiation across the chamber area emulating generated radiation (CTR or CChDR). This source is based on a Gunn diode with variable frequency ( $20-40 \mathrm{GHz}$ ) and was equipped with a Militech frequency doubler, amplifier, and tripler which allows us to generate 0.1 mW signal at $120-240$ GHz frequency. The source also has an electrical attenuator and external trigger which was used to synchronize the source and the detector. Test source and MPI input were initially aligned using red laser shown in Fig. 3.5. The output radiation from THz source was collimated using a TPX lens installed a focal distance away inside the vacuum chamber. THz radiation from the test source propagated through the entire system to be detected by the THz camera (TeraSence[99]). The top figure in Fig. 3.8 represents the images with and without the lens as a function of the MPI movable mirror position. These images correspond to the interference pattern shown below.

First of all this, approach allowed us to adjust the direction of photons through the interferometer by observing constructive and destructive interference patterns. Secondly, it was necessary for choosing the optimal focal lengths of lenses and the detector displacement. The interferogram and spectrum collected from the source with the aligned MP-I are shown in Figure 3.9. The exact frequency of the source was measured to be 166 GHz , which is corresponds with THz source settings.


Figure 3.8: Visualization and alignment using THz camera and CW THz source operating at 170 GHz : above - images of transverse profile for different movable mirror positions with and without TPX lenses (Tydex LLC.) required for choosing the optimal detector displacement; bellow - interference pattern corresponding to the images above.

(c) Spectrum

Figure 3.9: Example of MPI data taken after alignment of MPI using the test CW THz source. No apodization or zero-padding was used in the calculation of the spectrum. Spectrum sidelobes are the effect of "spectral leakage" caused by interferogram discontinuity at the edges, it can be fixed by applying windowing techniques. Central lobe width is about 2 GHz FWHM is CW THz source frequency bandwidth.

### 3.5 Data acquisition system

The data acquisition system prepared for this experiment is a full-fledged complex that can be used at other machines, excluding the manipulation system of stepping motors inside of the vacuum chamber. Figure 3.10 shows the structure of this complex.

The THz light signal from all three pyroelectric detectors is collected by NI DAQ "PC-Based module USB-6366" and then enters into the LabView program. This LabView code is a central part that is responsible for the Martin-Pupplet interferometer operation. First, the subtraction of the noise collected by PD3 is performed, and then averaging of 100 pulses collected by PD1 and PD2 detectors at a particular movable mirror position. After that, the mirror moves by the Newport translation stage using Newport "SMC100CC" controller. This process is repeated 80 times with $0.5 \mu \mathrm{~m}$ translational stage resolution to achieve an interferometry length of $L=4 \mathrm{~mm}$. Typically the full interferometry scan takes 15 minutes.

A normalized interferogram is obtained using Eq.3.10. Further, Fast Fourier Transform (FFT) procedure using Python SciPy library takes place to build spectrum of coherent radiation. Single electron spectra for TR and ChDR were calculated in advance since the process of computation is rather time-consuming taking up to 20 min utes. Also, the single electron spectrum takes into account parameters such as a target shape, target dimensions, and others, that are usually not changing during the measurement process. Form-factor extrapolation and Kramers-Kronig analysis are performing according to, subsections 2.3.2 and 2.3.3. Finally, we reconstructed the longitudinal charge profile $\rho(z)$.


Figure 3.10: DAQ system developed for CLARA machine.

## Dielectric properties of the Teflon target

The generation of Cherenkov Diffraction Radiation was considered before for the targets of different shapes and materials. For example, in work [82] the theoretical model of ChDR generation from a rectangular target was studied; in [100] the possibility to produce photons by a particle moving through the hole in cylindrical target was discussed; prismatic target shape was discussed in many papers such as [79, 84, 101]. As a material, commonly used are Teflon (TPFE) [85], Fused Silica (SiO2) [102] and Diamond targets have been used.

The optimal choice of dielectric material and shape will depend on particular experimental goals and restrictions. For example, in work [85] performed at Tomsk Polytechnic University microtron, the minimal size of a ChDR radiator was restricted by the diffraction effects in target: coherency of radiation $\lambda_{\text {min }}$ started from 30 mm for electron bunch with $\sigma=2.2 \mathrm{~mm}$, so large up to 30 cm targets should have been used. The target used for experiments was a prism made out of Teflon with dimensions of $175 \times 175 \times 74 \mathrm{~mm}$. In another work, [102] a flat (rectangular) and prismatic targets made of Fused Silica were used to observe incoherent radiation in the visible spectrum. Using the flat target authors verified the theoretical models and the prismatic target allowed them to observe the angular distribution of Cherenkov Diffraction Radiation and accurately measure its polarization content. Another example is the exploitation of the "knife-edge" shape [103] to accumulate radiation along the target surface. To properly extract radiation from such a peculiar geometry - a diamond target with the refractive
index of 2.1 was used.
In this particular project, the radiator shape and geometry were defined by two main factors. First of all, the dielectric material should be transparent in the THz region with a flat transmission coefficient. Secondly, the geometry should provide easy radiation extraction. These requirements are met by a prismatic target made of Teflon. The one used in this project was manufactured in the Royal Holloway University workshop.

Since all researches mentioned above were not focused on the spectral properties of radiation, dielectric properties of chosen material have not been discussed in details. Remarks were made in [79, 101, 102] concerning the Sellmeier dispersion equation for refractive index calculation at different frequencies. For this work, the spectral characteristics of emitted radiation are crucial, so we need to properly investigate the dielectric properties of used targets in the THz region.

### 4.1 Continuous-wave THz spectroscopy for dielectric properties measurements

To determine the the complex dielectric function $\varepsilon(f)=\varepsilon_{1}+i \varepsilon_{2}$ we need to measure both the amplitude and the phase of the transmitted radiation. Measurements of a phase accurately and without using mechanical delay stage can be done using the frequency domain spectroscopy method, where the combinations of continuous-wave THz source with frequency control and a detector should be used. At Royal Holloway, University of London, we have a system proposed at [104] based on the photomixing effect. In this system, the fiber laser 1 (DFB laser 1) and laser 2 (DFB laser 2) are set up on wavelengths 853 and 855 nm respectively. These lasers have a frequency and power control with a feedback system. After photo mixer structure (PM) we receive a frequency that can be varied between 50 GHz and 1.8 THz . It is so called frequency mixture, or frequency beat. Half of this radiation then goes through the antenna (Source) with a bias voltage applied and another half is going to the 2nd photomixer system (detector). Then, THz radiation from Source goes through the sample and after it mixing in 2nd photomixer. Resulting photocurrent will have a phase shift and
attenuation after sample:

$$
\begin{equation*}
I_{p h} \propto E_{T H z} \cos (\Delta \phi)=E_{T H z} \cos (2 \pi \Delta M f / c) \tag{4.1}
\end{equation*}
$$

where $f$ is the terahertz frequency, $\Delta M$ is the optical path from the source to detector.


Figure 4.1: Setup of continuous-wave terahertz spectrometer.


Figure 4.2: Photos of continuous-wave terahertz spectrometer.

The refractive phase difference $\Delta \phi$ can be retrieved by using mechanical delay stage, or alternatively bymodulating the terahertz frequency $f$, which allows to avoid any mechanically moving parts and reduce scanning time. This method is also known as Controlled Frequency Sweeping Systems. $I_{p h}$ will oscillate with frequency, and the period $\Delta f$ of the interference pattern depends on the optical path difference $M$. For figure 4.3, $M=0.35 \mathrm{~m}$, which corresponds to $\Delta f=0.85 \mathrm{GHz}(2 \pi M f / c=2 \pi$.) The maxima are spaced in frequencies as $f_{\max }^{a i r}=m c / \Delta L$, where $\mathrm{m}=1,2,3 \ldots$


Figure 4.3: Photocurrent for reference measurement and for measurement with sample showing phase shift

For the reference measurement, scheme without target is used ( $n=1$ ). For the sample measurement light will travel pass equivalent to $\Delta L+\left(n-n_{\text {air }}\right) d$. Thus, we can write a relationship between frequency maxima as:

$$
\begin{gather*}
f_{\text {max }}^{\text {air }}=m c / \Delta L  \tag{4.2}\\
f_{\text {max }}^{s a m p l e}=m c /\left(\Delta L+\left(n-n_{\text {air }}\right) d\right)
\end{gather*}
$$

The refractive index can be calculated according to:

$$
\begin{equation*}
\left(n-n_{\text {air }}\right) d=\left(\frac{f_{\max }^{a i r}}{f_{\max }^{s a m p l e}}-1\right) \Delta L \tag{4.3}
\end{equation*}
$$

The Refractive index obtained for a Teflon used in this thesis is shown in Figure


Figure 4.4: Refractive index obtained for a Teflon material used in this thesis. With a red line mean value obtained from measurements of 3 different samples with thickness $d=2 \pm 0.03 \mathrm{~mm}$ is shown.
4.4. The mean value from the measurement of 3 different samples of the same material with thickness $d=2 \pm 0.03 \mathrm{~mm}$ is shown with a red line, with a blue line standard error is shown. Large error at lower frequencies is caused by an unstable laser condition in this region.

Another important parameter is the material extinction coefficient $k(f)$. To find it, we should first find the transmittance $T(f)$ :

$$
\begin{equation*}
T=\left(\frac{I_{p h}^{\text {sample }}}{I_{p h}^{\text {air }}}\right)^{2} \tag{4.4}
\end{equation*}
$$

where $I_{\text {air }}$ and $I_{\text {sam }}$ are the reference measurements and measurements with target. The dependence is quadratic because we observe induced current, produced by the electric field, i.e. $I_{p h} \propto|E|^{2}$. In Figure 4.5 the difference between reference measurements and measurements with 35 mm Teflon sample are shown. The effective frequency resolution is $c /(M)$ and, in our case, was equal to 0.85 GHz .

The ratio of the two dependencies provides a transmission coefficient of the investigated target. The transmission coefficient for 35 mm Teflon sample is shown in figure 4.6.


Figure 4.5: Photocurrent for reference measurement and for measurement with sample. The sample is a Teflon rectangular target with thick of 35 mm . Distance between source and detector is $M=0.3 m$

In this case, the high-frequency oscillations are caused by the system instabilities during frequency scan. The integrator of the lock-in amplifier restarts when the system is tuned to a new THz frequency, which causes fluctuations of the detected photocurrent. To avoid this we can vary integration time from 20 ms to 60 ms . However long integration time will cause accumulation of other uncertainties including vibration, acoustic and thermal interference. The choice between short and long integration time depends on a particular experiment. During this work, we decided to use short integration time and compensate high-frequency oscillation by a higher sample rate and further signal high-frequency filtering. The resulting transmittance $T(f)$ obtained using low-pass Chebyshev filter is shown with a red line in Fig. 4.6.

Knowing the transmittance $T(f)$ (Fig. 4.6) and the refractive index $n(f)$ (Fig. 4.4), we can deduce extinction coefficient $k$ with curve fitting method between the equation A. 12 (see Appendix A):

$$
\begin{equation*}
T=\frac{(1-R)^{2} \exp (-\alpha d)}{[1-\operatorname{Rexp}(-\alpha d)]^{2}+4 \operatorname{Rexp}(-\alpha d) \sin ^{2}(n \omega d / c)} \tag{4.5}
\end{equation*}
$$

where $\alpha=2 k \omega / c$ is the absorption coefficient and R is Reflectance which can be retrieve


Figure 4.6: Transmittance of a 35 mm thick Teflon sample
from equation [105, 106]:

$$
\begin{equation*}
R=\frac{(n-1)^{2}+k^{2}}{(n+1)^{2}+k^{2}} \tag{4.6}
\end{equation*}
$$

The dielectric function $\varepsilon_{1}(f)$ and $\varepsilon_{2}(f)$ obtained from $\varepsilon_{1}(f)=n(f)^{2}-k(f)^{2}$ and $\varepsilon_{2}(f)=2 n(f) k(f)$ an plotted in Figure 4.7.


Figure 4.7: Real and imaginary components of the Teflon dielectric constant. Error bar for refractive index shown in Fig. 4.4

### 4.2 ChDR Spectral-angular properties and single electron spectrum taking into account complex permittivity

The polarization current approach for ChDR generation and, in particular, the model for ChDR generation from the prismatic target were used in several works [79, 84, 101]. However, non of these works considered the imaginary part of the complex dielectric function $\varepsilon(f)=\varepsilon_{1}(f)+i \varepsilon_{2}(f)$, which is correlated with the absorption coefficient of the material. Therefore, a detailed calculation of the ChDR spectra from a dielectric material with properties depending on a radiation wavelength has never been done. From the perspective of using dielectric material as an intense source of electromagnetic radiation, we should remember three main aspects. First, there is no solid-state material equally transparent in a wide frequency region. Second, such materials as PTFE, CWD Diamond, Fused Silica are produced at different facilities following different production procedures and might have unpredictable impurities. And the most important one is that we exploiting a cumulative effect when generating ChDR radiation (intensity scales with distance electric field traveled inside of material), so we will need to use solid material with the length of 1 to 10 centimeters which might lead to critical absorption at some frequencies.

In this section, the importance of taking into account the complex part of the dielectric function for single electron spectra calculation will be demonstrated. For this purpose the model shown in Chapter 2 (in 2.2.2 and 2.2.2) was extended to include dielectric permittivity $\varepsilon(f)$ with an imaginary part which is shown in Figure 4.7. In picture 4.8 the geometry of the Teflon prism used in the experiment is shown. The prism length L is 5 cm , the prism width $\mathrm{P}=4 \mathrm{~cm}$, wedge angle is $49^{\circ}$.

Figure 4.9 shows the single electron spectra for the case with and without taking into account the extinction coefficient. With accounting for extinction coefficient high-frequency suppression of intensity appearing. Even though for this chapter we investigated absorption only in the frequency region of $100-1000 \mathrm{GHz}$, it is expected that the absorption will increase also for higher frequencies (see, for example, [107]). One of the obvious conclusions we can make here, and which will be also discussed in the conclusion section, that the Teflon target might be not suitable for ultra-short


Figure 4.8: The schematic view of the radiation geometry for a charged particle moving in the vicinity of the prismatic dielectric target.
bunch diagnostic and should be investigated carefully, as the signal will be dramatically suppressed at higher frequencies.


Figure 4.9: Comparison of ChDR single electron spectrum with and without imaginary part of dielectric constant accounting

Another interesting moment which we should investigate is the influence of target length on emitted spectra. Parameters for the simulations presented below are exact which were used during experiments on CLARA: $\gamma=70$, wedge angle $\phi$ is $49^{\circ}$, angle
between radiator surface and beam direction $\alpha$ is $0^{\circ}$, impact parameter $h$ is 2 mm , detector aperture $D$ is 5 cm , distance between radiator and detector is $d=100 \mathrm{~mm}$, target base length $L=[25,35,50] \mathrm{mm}$.

Obviously, for the target with larger base length L light will travel longer distance inside the dielectric material. For our case it means, that despite the overall higher intensity, larger frequencies should be suppressed much more in dielectric with larger L. Comparison of three spectra calculated for PTFE targets with different base lengths are shown in figure 4.10 .


Figure 4.10: Comparison of ChDR single electron spectrum for different target base lengths. Complex dielectric constant accounted. Spectrum for the target with a base length of 50 mm , is the one that will be used in Chapter 5 for the form factor obtaining.

### 4.3 Summary

In this chapter, the influence of the imaginary part of dielectric constant on emitted terahertz spectrum was research and explained both experimentally and theoretically. This is of particular importance when we deal with bulk targets for electromagnetic field generation.

The currently existing terahertz spectroscopy systems make it possible to study materials in laboratory conditions. Thus, at the stage of preparation for the experiment,
the transmission properties of the material, as well as the refractive index, should be properly investigated.

The knowledge of dielectric properties allows us not only helps to determine an appropriate material and geometry of the target but also to improve the theoretical analysis required to restore the longitudinal beam profile. As it was shown in Figure 4.10 spectrum emitted from larger targets are particularly susceptible to an absorption influence and disregard of this might lead to a significant misrepresentation of the reconstructed bunch profile.


## Experimental results

In preparing this thesis, the goal was to prove the possibility to perform non-invasive diagnostics of the longitudinal charge profile using coherent ChDR and to compare this method with diagnostic based on coherent TR. In this chapter, the comparison between TR and ChDR interferograms, coherent spectra, form-factors, and reconstructed bunch profiles are presented.

We shall consider results obtained by two consecutive interferometer scans performed with the same machine parameters within 40 minutes. Thus, we assume that the beam parameters remained unchanged throughout these scans.

### 5.1 Interferometry

Pyroelectric detectors are very sensitive to all background contributions in the environment, such as acoustic noises, vibrations, electromagnetic interference because of the broadband response of the pyroelectric crystal. This problem is crucial in the shielded accelerator hall environment where the air conditioning, vacuum pumps, and high power RF systems contribute to acoustic, mechanical, and low-frequency electrical noise respectively. For example, the tests with HeNe laser showed that signal to noise ratio (SNR) is 3 times lower in Beam Area 1 with working accelerator than in a controlled environment of laser laboratory.

The system of multi-shot signal averaging and shot-to-shot background signal subtraction was developed to partially resolve this problem. For background subtraction,
a third pyroelectric detector PD3 was installed in the BA1 (see Fig. 3.7) to acquire all background signals. In Fig. 5.1 (a), as an example, shown CTR signal detected by PD1 (placed after interferometer system and receives both noise and CTR) and the signal from PD3 (receives noise only) are shown. As can be seen, SNR is close to 1 and the useful signal is barely distinguishable. The background subtraction was implemented on a shot-by-shot basis and the results are shown in figure 5.1 (b) with a green line. The useful signal becomes distinguishable, however, there is still a lot of noise spikes caused by the tremor of the pyroelectric crystal itself. This problem was solved with multi-shot averaging (red line). The choice of number of shot depends on the expected scan speed.


Figure 5.1: Pulse processing for THz detection: a) background interference and low intensity THz pulse detected by PD3 and PD1 respectively; b) Subtracted noise signal from the THz signal (green line) and multi-shot averaging of subtracted signal (red line). The signal is of $\approx 5 \mathrm{mV}$

The results of interferometry from pyroelectric detectors PD1 and PD2 are shown in Figure 5.2(a) and 5.2(b).

First of all, it can be noted, that interferograms collected from PD1 and PD2 have different mean values, which is due to the difference in sensitivity between these detectors. We compared the response of all three detectors PD1, PD2, and PD3 from CW THz source with frequencies between $80-220 \mathrm{GHz}$. The response of PD1 and PD3 was the same for all frequencies and a correction factor of 1.3 has been applied to PD2. Such a correction to the measured signal is needed to perform normalization of the total intensity. When we will consider normalized difference interferograms - the


Figure 5.2: Interferograms measured from two different detectors PD1 and PD2 for CChDR (a) and TR (b); average error is showed
correction factor would be already applied.
The standard error was collecting as:

$$
\begin{equation*}
\sigma=\sqrt{\sum_{i}^{N} \frac{\left(x_{i}-\bar{x}\right)^{2}}{N}} / \sqrt{N}=\frac{1}{N} \sqrt{\sum_{i}^{N}\left(x_{i}-\bar{x}\right)^{2}} \tag{5.1}
\end{equation*}
$$

where $x_{i}$ are observed values, $\bar{x}$ is a mean value of N observation, N - is a number of samples (in our case $\mathrm{N}=100$ ). For the majority of measurements, average deviation $\bar{d}$ do not exceed 0.06 mV , or $1.2 \%$ of mean value, which is quite a good result for working with pyroelectric detectors in such a complicated environment. The standard error in these interferograms is mainly determined by the number of shots chosen, stability of various accelerator parameters (current, bunch length) shot-by-shot, background fluctuations, and pyroelectric crystal noise. Long-term (several seconds) fluctuations of accelerator parameters can be eliminated by using normalized difference interferogram 3.10 which was discussed in Chapter 3.

Fig 5.3 shows the influence of the long-going fluctuations and drifts of the total radiation intensity and highlights the use of the Martin-Pupplet interferometer. Longterm fluctuations and drifts affect both PD1 and PD2 signals similarly, whereas the interference patterns for two orthogonal polarizations are anticorrelated. This means that the difference of signals from PD1 and PD2 gives us the total interference pattern
and the sum of PD1 and PD2 signals will represent the signal fluctuation. These fluctuations are shown in Fig 5.3 with a red curve. The vertical dashed lines demonstrate the artifacts caused by signal fluctuations (caused by machine instability) that crucially affect interferogram patterns.


Figure 5.3: Interferograms measured from two different detectors PD1 and PD2 for CChDR (a) and TR (b). Fluctuations as sum of the signals from PD1 and PD2 are shown as the red line. Error estimated for this measurements shown in figure 5.2

Normalized difference interferograms, $\delta(x)$ are shown in Fig. 5.4 were calculated
using equation 3.10. As it is seen, the cross-correlated noise was significantly reduced.


Figure 5.4: Normalized difference interferograms $\delta(x)$ for $\operatorname{CChDR}$ (a) and TR (b); Error estimated for this measurements shown in figure 5.2

### 5.2 Spectrum reconstruction

Normalized difference interferogram, $\delta(x)$, is the Fourier transform of the spectrum. During normalization to the intensity (see Chapter 3 Eq. 3.10) all information regarding absolute power is lost and spectra will be presented in arbitrary units. For the current research, absolute power is not needed, and the relative power spectrum is sufficient. We can determine combined spectrum in arbitrary units as:

$$
\begin{equation*}
I(f)=\int_{-\infty}^{\infty} \delta(\bar{x}) \cos (2 \pi f \bar{x} / c) d \bar{x} \tag{5.2}
\end{equation*}
$$

where $c$ is a speed of light, pass difference $\bar{x}=2 \Delta x$ since the light goes twice of the movable mirror position change. In our case we measured light intensity on the finite travel range of the mirror, thus we have to replace continuous transform with a discrete. Equation of DFT could be written as:

$$
\begin{equation*}
I\left(f_{k}\right)=\frac{1}{N} \sum_{n=1}^{N} \delta\left(\bar{x}_{n}\right) \cos \left(2 \pi f_{k} \bar{x}_{n} / c\right) \tag{5.3}
\end{equation*}
$$

where, $f_{k}$ is $k_{t h}$ frequency, N - is a total number of samples, $x_{n}$ - is a $n_{t h}$ sample. Equation 5.3 could be computed using Fast Fourier Transform (FFT) algorithms. Also can be computed using Discrete Cosine Transform (DCT) algorithm [108]. The resolution of the spectrum (minimum wavelength step) $\Delta f$ is defined as [109]:

$$
\begin{equation*}
\Delta f=\frac{c}{2 L} \tag{5.4}
\end{equation*}
$$

where L is the maximum pass of the radiations in the arms of the interferometer. We use 2 L since the light goes twice the maximum path. In our case $2 L=8 \mathrm{~mm}$ and $\Delta f$ $=37.5 \mathrm{GHz}$.

The highest measurable frequency is $f_{m}$ can be defined as:

$$
\begin{equation*}
f_{m}=\frac{1}{\bar{x}}=\frac{c}{2 x} \tag{5.5}
\end{equation*}
$$

where $\bar{x}$ is the minimal optical pass difference. In our case $\bar{x}=0.05 \mathrm{~mm}$ and $f_{m}$ is equal to 3 THz . Equation 5.5, however, is applicable only for continuous Fourier transform. For the discrete Fourier transform algorithm, the length of the transformed axis of the output is $n / 2+1[110,111]$ we will have maximum frequency of $\Delta f *(n / 2+1)=$ $37.5 * 41 \approx 1.5 \mathrm{THz}$. This is an instrumental limitation.

Figure 5.5 shows the spectra of CChDR and CTR without any post-processing technique applied. For both spectra intensity reduction at frequencies of $0-200 \mathrm{GHz}$ is caused by diffraction effects in radiators of finite dimensions (see Chapter 2) and by diffraction in optical elements (TPX lenses, pyroelectric detectors pinhole). All signal higher than 1 THz is caused by low SNR of measured interferograms. Thus, regions of [0-200] GHz and $[1000+] \mathrm{GHz}$ needs to be extrapolated. Frequency suppression in the region of $[200-1000] \mathrm{GHz}$ is the coherence slope that we detect. Differences between CChDR and CTR spectra are caused by the different nature of these effects, but one can see the effect of humid air absorption line at $\approx 580 \mathrm{GHz}$, which is more explicit in the CTR spectrum. In the next section, some post-processing techniques will be applied to both spectra to improve the analysis accuracy.


Figure 5.5: Spectra of $\operatorname{ChDR}$ (a) and CTR (b) obtained from the interferogram in figure 5.4

### 5.2.1 Interferogram postrprocessing techniques

One of the effects which could crucially affect interferograms is "truncation" or "leakage". When the interferogram pattern breaks off at the boundaries of our measurements, it will affect resulting spectrum [112]. The problem of "leakage" is also connected to the detector sensitivity as the pyroelectric detector can not detect weak signal differences on the edge of the interferogram, where we see sharp-edged peaks or drops caused by statistical fluctuations in intensity rather than a reasonable spectrum. The problem can be partly solved by choosing a window function. In our case, the Blackman-Harris window was chosen as it can provide good side lobe compression and barely affect the main peak ${ }^{1}$. The Blackman-Harris window is shown in Figure 5.6 with a red dashed line.

Another problem is called "the picket fence effect" ${ }^{2}$. It has been mentioned before, that the FFT spectrum is discrete, consisting of estimates of what the spectral level is at specific frequencies. In our case - we produce a spectrum with a 37.5 GHz step. This means there will be peaks or falls that lie between the lines of the FFT analysis. This will also mean, that the peaks in the FFT spectrum will be measured too low in

[^5]

Figure 5.6: CChDR (a) and CTR (b) interferograms together with Blackman-Harris window and zero-filling
level (and the falls will be measured too high) and that the true frequencies of peaks and falls will not be those indicated in the FFT spectrum. The Picket-fence effect can be subdued by adding zeroes to the end of the interferogram before FFT is performed, thereby increasing the number of points per frequency in the spectrum. Thus, zero filling the interferogram has the effect of interpolating the spectrum, reducing the error (improving the photometric accuracy of the data). The Zerro-padding technique should not be substituted with a simple interpolation procedure, because it is still based on the interferogram shape. Although zero-padding increases the number of data points in the spectrum, it cannot increase the physical resolution of the spectrum (because the instrumental line shape is not changed). In our case, we chose a filling factor of 0.5 N , which also means that we will now have 82 points in the spectrum, instead of 41. Zero-padding is shown in Figure 5.6 with a blue dashed line.

Spectra of CChDR and CTR obtained from the post-processed interferogram in Figure 5.6 shown in Figure 5.7 with blue bars. The green line shows "raw" spectra. While the lines of the upper spectrum look badly clipped, the lines in "post-processed" spectra look smooth, which is an effect of zero-filling. The effect of the BlackmanHarris window is expressed in the suppression of high-frequency components, which was a product of pyroelectric noises, but not a reasonable light signal.


Figure 5.7: Spectra of CChDR (a) and CTR (b) obtained from the post-processed interferogram in Figure 5.6.

### 5.2.2 Transfer functions

The obtained spectrum might be affected by several aspects. The whole frequency range below 200 GHz is suppressed by diffraction effects, some frequencies exhibiting might be caused by humid air absorption lines and some by quartz window absorption (which is not noticeable from the presented spectra, but assumed). All these peculiarities should be taken into account and corrected. The combined transfer function is obtained as a product of each separate transfer function:

$$
\begin{equation*}
T_{\text {com }}(f)=T_{\text {air }}(f) T_{\text {quarz }}(f) \tag{5.6}
\end{equation*}
$$

where $T_{\text {air }}(f)$ and $T_{\text {quarz }}(f)$ are the humid air and quartz window transfer functions respectively.

## a) Humid Air absorption lines

In this work, the experimental setup was separated into two parts: the vacuum part and the part installed outside the vacuum chamber, which is affected by humid air absorption. In an ideal experiment, an entire interferometer system should be enclosed and continually purged with dry nitrogen to avoid the strong absorption of THz light by water vapor. For this experiment, however, no such installment was assumed and
we should use a humid air transfer function to normalize the spectra. In Figure 5.8(a) red dashed highlight the main water absorption frequencies are highlighted.


Figure 5.8: a) Influence of humid air absorption; b) Humid air transmission for an optical pass of 1 meter: the red line is a raw transmittance data, and the green line is transmittance convolved with Martin-Pupplet interferometer resolution.

Martin-Pupplet interferometer has a finite spectral resolution and it is smaller than the resolution of the instrument used for obtaining absorption lines. Thus, the transmittance has to be convolved with MPI resolution. It can be done by applying the Blackmann-Harris filter commonly used for [112]:

$$
\begin{equation*}
B H(f)=a_{0}+a_{1} \cos \left(\frac{\pi f_{\max }}{f}\right)+a_{2} \cos \left(\frac{2 \pi f_{\max }}{f}\right)+a_{3} \cos \left(\frac{3 \pi f_{\max }}{f}\right) \tag{5.7}
\end{equation*}
$$

where $f_{\text {max }}$-is the maximum resolvable frequency of interferometer; $f$ - is the frequency with the step equal frequency resolution; Blackmann-Harris window coefficients are: $a_{0}$ $=0.358875, a_{1}=0.48829, a_{2}=0.14128, a_{0}=0.01168$. The corrected transmittance will be:

$$
\begin{equation*}
T_{\text {air.cor }}=T_{\text {air }}(f) \otimes B H(f) \tag{5.8}
\end{equation*}
$$

Fig B.1(b) shows the raw humid air transmittance ${ }^{3}$ and transmittance corrected for Martin-Pupplet resolution. As expected, the corrected function $T_{\text {air.cor }}$ reveals broader features than the original $T_{\text {air }}$, and the depth of narrow absorption lines is reduced. It

[^6]can be seen that absorption lines at frequencies $\approx 0.55 \mathrm{THz}, \approx 0.75 \mathrm{THz}$, and $\approx 1.0 \mathrm{THz}$ might be used for normalization. For the frequencies higher than 1 THz normalization will not make sense due to the low signal. Frequencies higher than 1 THz will then be extrapolated, as will be discussed in section 5.3.

## b) Quartz window

For transportation of both CChDR and CTR out of the vacuum chamber, a large z-cut fused quartz window of 5 mm thick has been used. Therefore, a significant amount of radiation could be absorbed inside the material. The measurement of fused quartz absorption coefficient in mm wavelength region could be found in [114] and shown in Fig. 5.9 (left). Absorption coefficient in this figure is $\left(\alpha=\log _{10}(I o / I) / d=-\log _{10} T / d\right)$, where d is the window thickness, T is transmittance. Figure 5.9 (right) shows the transmittance $T$ calculated for $d=0.5 \mathrm{~cm}$. For this experiment, we should take quartz transmission $T_{\text {quartz }}$ into account and normalize our spectrum on it.


Figure 5.9: Left: THz absorption coefficient for quartz (indicated). The figure is taken from [114]. Right: Transmittance for a window with thickness $d=5 \mathrm{~mm}$.

## c) Diffraction

In the low-frequency region, the main intensity reduction is coming from the diffraction losses due to the finite size of optical elements like TPX lenses, detector input, and, of course, due to the finite size of targets themselves. As it was discussed and demonstrated by single electron spectrum analysis in Chapter 2, intensity suppression caused
by the diffraction in targets of finite size happens at a frequency of 100 GHz for ChDR target and 150 GHz for TR target. The diffraction causes by a small size of optical elements ( 3 cm in diameter TPX lenses and 3 cm in diameter detector windows) in our case should happen in a region less than 200 GHz . In theory, all these effects might be accounted for: first of all, both ChDR and TR models takes into account radiator finite dimensions and that influence is accounted on a single electron spectrum (see Figs); secondly, the combination of the effects caused by diffraction in optical elements with finite dimension might be calculated using Fraunhofer diffraction ${ }^{4}$. However, in practice, we just have a lack of signal to provide proper normalization in this region (see Fig. 5.7). Thus, the signal must be extrapolated to lower frequencies according to the procedure described in Chapter 2, section 4.

## d) Combined transfer function

Now we can write a combined transmission spectrum as the product of individual transmissions quantified in Eq. 5.6. The transfer functions for our experiment are shown in Figure 5.10 with a red line. Thus we can calculate the corrected coherent spectrum as:

$$
\begin{equation*}
I_{\text {corr }}(f)=I_{\text {measured }} / T_{\text {comb }} \tag{5.9}
\end{equation*}
$$

The corrected spectra for ChDR and TR are also shown in Figure 5.10.

### 5.3 Form Factor

As it was shown in Chapter 2, the bunch profile is the inverse Fourier transformation of the form factor, which can be obtained knowing the coherent spectrum and the single electron spectrum:

$$
\begin{equation*}
|F(f)|=\sqrt{\frac{I(f)}{I_{s}(f)}} \tag{5.10}
\end{equation*}
$$

Fig. 5.11 illustrates the comparison between the $|F(f)|$ functions extracted from CTR and CChDR spectra. These form-factors of were obtained using coherent radia-

[^7]

Figure 5.10: Corrected CChDR (a) and CCTR (b) spectra and combined transfer function $T_{\text {comb }}(f)$
tion spectra shown in Fig. 5.10 and single electron spectra shown in 2.16 and 4.10 for CTR and ChDR respectively.


Figure 5.11: $\operatorname{CChDR}$ (a) and CCTR (b) form factors obtained using equation 5.10


Figure 5.12: CChDR (a) and CTR (b) form factors extrapolated using prescriptions from Chapter 3

The low-frequency part ( $<200 G H z$ ) is distorted not only by diffraction effects in the target but also by the diffraction effects caused by the concave mirror, the output quartz viewport, the finite apertures of the interferometer, the mirrors, and detector apertures. These effects are not taken into account in the single electron spectrum and, therefore can not be used for longitudinal profile measurements. At higher frequencies, ( $>1000 G H z$ ) the coherent radiation is significantly suppressed due to finite bunch length. In this case, the apparatus noise generates artifacts in the analysis which are significantly different for CChDR and CTR properties. Therefore, both curves have been extrapolated to lower and higher frequencies from the same points for ChDR and CTR by Gaussian and exponential functions, following the prescriptions described in Chapter 3. The extrapolated parts are shown in Figure 5.12 as green and red bars. In Appendix B, one can find a prescription on how to find what part of the coherency slope we observing.

### 5.4 Kramers-Kronig analysis

The reconstruction of the bunch length by direct inverse Fourier transform returns average symmetric particle distribution. The width of this distribution proportional to
the real width, however if the longitudinal profile is asymmetric, the Fourier transform procedure has to account for the initial phase. The whole procedure of obtaining the Minimal Phase using Kramers-Kronig analysis has been described in Chapter 2. Here we used numerical integration to calculate the phase over the frequency range of 0 2 THz from the form factors presented in Figure 5.12. The obtained Kramers-Kronig minimal phase is shown in Figure 5.13. The minimal phase pattern has peaks and falls at extrapolation connection frequencies ( $1 \mathrm{THz)} \mathrm{}. \mathrm{Also} ,\mathrm{it} \mathrm{becomes} \mathrm{non-linear} \mathrm{where} \mathrm{the}$ spectrum is affected by bunch structure and has a linear growth where the form factor close to Gaussian shape.


Figure 5.13: Minimal phases for CChDR (a) and CTR (b) obtained using KramersKronig analysis.

### 5.5 Reconstructed bunch

Fig. 5.14 shows the bunch profile reconstructed according to the prescriptions described in Chapter 2. The negative values appearing close to the bunch tale (CTR bunch at the position of -0.2 mm ) are caused by minimal phase resonance-like behavior around the truncation frequency ( 1 THz ) and nonphysical. The existence of the tales is reasonable since we observed quite distorted coherent spectra both in the case of CChDR and CCTR, including the presence of high-frequency components. The physical resolution of the constructed instrument is limited by the sampling rate of the interferometer,
which is in our case 160 fs .
The best way to provide a numerical evaluation of a bunch length is to perform a curve fitting and regression analysis. Figure 5.14 shows the reconstructed bunches and fitted curve taken from Gaussian function:

$$
\begin{equation*}
g(x)=\frac{1}{\sigma \sqrt{2 \pi}} \exp \left(-\frac{1}{2 \sigma^{2}}(x-\mu)^{2}\right) \tag{5.11}
\end{equation*}
$$

where $\sigma$ corresponds to the RMS function width, $\mu$ is the maximum position. Regression analysis [116] allows us to extract the $\sigma$ value and the standard error of the estimate, which takes into account the average squared distance that the observed values have from the regression line. The obtained values for the samples investigated in this chapter are: ChDR bunch length: $2.084 \mathrm{e}-13 \pm 2.114 \mathrm{e}-15 \mathrm{~s}$ RMS; CTR bunch length: $2.021 \mathrm{e}-13 \pm 2.270 \mathrm{e}-15 \mathrm{~s}$ RMS. The result is in agreement with the expected value of bunch length for this machine. Despite a serious difference in CChDR and CTR spectra


Figure 5.14: The bunch profile reconstruction via CChDR (a) and CTR (b) spectra for the same accelerator settings.
due to differences in single electron spectra, after normalization, the longitudinal form factors look very similar. The reconstructed bunch profiles are shown together in Figure 5.15 and demonstrate a good consistency of the cores of the distributions. The tails are different, which might be the result both of resolution restriction and instabilities that appeared due to longitudinal compression and might vary shot-by-shot. To measure the tails precisely a single shot spectrometer system is needed. The current set-up has
enabled only to diagnose a stable part of the beam. Typical bunch length measurement, including interferometry, FFT, single electron spectrum normalization, and KramersKronig reconstruction takes approximately fifteen minutes. Nevertheless, it can be reduced down to one minute by using detectors with a higher signal-to-noise ratio and an automated software-based procedure.


Figure 5.15: Bunch profile reconstruction (accumulated by 4000 consecutive shots) via CChDR and CTR for the same accelerator settings.

### 5.6 Bunch length for a different compression parameters

The aim of the experimental work was not only to show the longitudinal bunch profile but also to observe the sensitivity of the bunch core length to a small variation of bunch compression parameters. Fig. 5.16 shows the dependence of the bunch length on the linac phase obtained by CChDR and CTR effects, energy modulation technique [117, 118] and ELEGANT code prediction. Results of energy modulation scan and ELEGANT simulation are provided by CLARA facility team. It can be seen that the developed instrument based on the CChDR and CTR effects demonstrates a good consistency with other techniques and expectations. The set of CChDR and CTR scans researched in this chapter corresponds to 8 degrees of RF off-crest phase.


Figure 5.16: Bunch core lengths retrieved by different methods: CChDR scan, CTR scan, ELEGANT code simulation and Energy modulation method

### 5.7 Summary

In this chapter, a complete process of longitudinal bunch profile measurement using coherent transition radiation and coherent Cherenkov diffraction radiation was demonstrated. Bunches from the sub-ps to the ps range have been measured successfully benchmarking new non-invasive diagnostic method based on Coherent Cherenkov Diffraction Radiation, with a more standard but invasive technique such as Coherent Transition Radiation. THz signal generated by the Teflon target was easily detectable and allowed us to provide all necessary steps for the bunch profile reconstruction process. The developed instrument, however, had a limited ability to detect high frequencies of emitted spectra for both ChDR and TR. This influenced the resolution of the measurement. However, there is a lot of possibilities for further detection optimization, which will be discussed in the Conclusion chapter. The current set-up has enabled us to reliably diagnose the stable part of the beam, and extract the bunch core length. The obtained results are in good agreement with simulations performed for the CLARA machine. To be able to measure shot-by-shot variations of the bunch length we need a single shot spectrometer. Nevertheless, the current technique is promising for being used as a non-invasive online bunch length monitor instrument and the use of ChDR target allows to perform it noninvasive.

## Chapter

## Conclusion

To study the longitudinal characteristics of ultrashort beams of charged particles at modern accelerators it is necessary to develop new and non-invasive diagnostic methods. One of the most promising methods for such diagnostics is based on application of Cherenkov Diffraction Radiation, which is generated by the electric field of charged particles moving near a dielectric target.

An experimental setup for generation and detection of the ChDR was developed at the CLARA accelerator. The setup which is based on the Martin-Pupplet interferometer makes possible spectral measurements of radiation emitted by the dielectric target.The signal was detected using pyroelectric detectors, which have a rather low signal-to-noise ratio (SNR). This problem was solved by several technical approaches, such as active noise subtraction by additional pyroelectric detector and using a THz camera for Cherenkov diffraction radiation visualization and MPI precise alignment. Invacuum setup was specially developed to exploit not only ChDR, but also well studied CTR. CTR was used for diagnostics result cross-referencing.

For the beam profile reconstruction from both ChDR and CTR coherent spectra, software based on the Python programming language was specially developed. To perform a complete analysis, the program must perform the following functions:

- Calculation of the single electron spectra for ChDR and TR with specified experimental parameters.
- Post-processing of the obtained interferograms (appodization, zero padding) and
construction of the normalized difference interferogram $\delta(x)$.
- Spectra obtaining (FFT)
- Accounting of the humid air and quartz window transmittance.
- Extrapolation to the low and high frequencies, where diagnostics was impossible due to physical limitations of equipment (low pyroelectric detector SNR)
- Kramers-Kronig analysis
- Final longitudinal beam profile reconstruction.

In Chapter 2, theoretical information about single electron spectral angular distribution and single electron spectra has been extensively covered. It is of great importance for experimental setup preparation, since it allow us to properly choose experimental layout geometry, detection system and optical elements. In addition to that, in Chapter 4 the influence of dielectric properties of Teflon target on emitted ChDR spectrum was investigated. The knowledge of dielectric properties allows us not only to determine an appropriate material and geometry of the target, but also to improve a theoretical analysis from Chapter 2. Experimentally obtained imaginary part of dielectric constant was for the first time accounted for the Polarization current model. Investigation of the target dielectric properties is essential for ChDR generation regardless the material used and neglecting it might lead to the significant misrepresentation of the reconstructed bunch profile.

As a result of this work, Cherenkov Diffraction Radiation method was applied for the first time for the longitudinal beam profile diagnostic and showed a good consistence with a well studied CTR effect.

It is also necessary to indicate the shortcomings of the provided work. One of the most significant is the use of pyroelectric detectors, which have a very low SNR, as a result high-quality spectrum detection at high frequencies (coherence decay) was impossible. Extrapolation of the form factor makes it impossible to qualitatively reconstruct the profile of the charged particle beam, but allows to determine the duration of the beam core. This problem can be solved by using more sensitive detectors, which will
be discussed in the next section. Another disadvantage is the limitation of used optics at frequencies below 200 GHz . In particular, we used TPX lenses with a diameter of 3.5 cm and pyrorlectric detectors with a pinhole diameter of 3 cm . Thus, in the low frequency region the main intensity reduction is coming from the diffraction losses due to the finite size of optical elements. As a result, it was not possible to detect the frequency of full coherence and it was necessary to extrapolate towards the lower frequencies. Overall, proper higher frequencies detection is of greater importance for us since they determine the beam duration and instrumental resolution resolution.

### 6.1 Future work

Since the results of this work has shown the possibility of using Cherenkov Diffraction Radiation to restore the bunch longitudinal profile, it was decided to continue research at the CLARA accelerator. The new series of experiments will mainly focus on the study of the fundamental properties of the Cherenkov Diffraction Radiation: studying the influence of the impact parameter on the intensity, angular distribution and spectral properties of ChDR; research of targets made of different materials and with different geometries; optimization of DAQ process and improving of spectroscopy quality.

The use of quasi-optical Schottky detectors is one of the proposed improvements for spectroscopy. Characteristics of such a detector made by Virginia Diodes [119] are shown in the figure 6.1. For comparison, characteristics of the pyroelectric detector used in this work are shown in the figure 6.2. There is a significant difference in SNR. Pyroelectric detector requires 10 nW of optical signal, to achieve $\mathrm{SNR}=1$, while a quasioptical detector requires 500 pW . But at the same time, the difference in sensitivity is noticeable: a quasi-optical detector has a sensitivity of $500 \mathrm{~V} / \mathrm{W}$, while a pyroelectric has a sensitivity of $70 \mathrm{kV} / \mathrm{W}$.


Figure 6.1: Parameters of the quasi-optical Schottky detector


Figure 6.2: Parameters of the Gentec-51 pyroelectric detector.

To compensate for the difference, it is possible to use a quasi-optical detector in conjunction with a low-noise amplifier. An example of such an amplifier and its parameters are shown in the figure 6.3. An important factor is that modern amplifiers, in


Figure 6.3: Parameters of low-noise signal amplifier SR560
addition to a large gain (up to 50,000 ) have an option of active signal filtering, which is especially important when working with low-energy signals in environment such as accelerator hall. Combination of a quasi-optical detector and a low-noise amplifier presented here is in disposal of our group in the Royal Holloway University and was tested
in laboratory conditions. Also, quasi-optical Schottky detectors has been tested with ultra-short signals in the THz range [120] and was capable to measure radiation with less than 25 ps duration.

## Appendix $A$

Reflection coefficient $r$, absorption coefficient $a$, transmission coefficient $t$ should not be messed with Reflectance $R$, Absorbance $A$ and Transmittance $T$.

## A. 1 Reflection coefficient, Transmission coefficient

Reflection coefficient $r$ is the parameter describing how much of the electromagnetic wave is reflected by the thin medium with no absorption and can be can be derived using Fresnels Equation. For normal incidence it will be [106]:

$$
\begin{equation*}
r=\frac{I_{r}}{I_{0}}=\frac{\left(n_{1}-n_{2}\right)}{\left(n_{1}+n_{2}\right)} \tag{A.1}
\end{equation*}
$$

Transmission coefficient $t$ is the parameter describing how much of the electromagnetic wave is passed a thin medium with no absorption. It can be derived using Fresnel Equation as:

$$
\begin{equation*}
r=\frac{I_{\text {out }}}{I_{0}}=\frac{2 n_{1}}{\left(n_{1}+n_{2}\right)} \tag{A.2}
\end{equation*}
$$

## A. 2 Absorption coefficient

Absorption coefficient $a$ stay aside from reflection coefficient $r$ and transmission coefficient $t$. Both $r$ and $t$ can be measured in laboratory environment directly by measure incident, reflecting or transmitted light. But Absorption coefficient is characteristic of a material itself and can not be measured directly. Absorption coefficient $a$ describes the fractional decrease in intensity with distance in thick absorptive medium and
defined as:

$$
\begin{equation*}
a=\frac{1}{I}=\frac{d I}{d r} \tag{A.3}
\end{equation*}
$$

Absorption coefficient $a$ has a relation with an extinction coefficient $k$, which defines how strongly material absorb at a particular wavelength, as (see. Frederick Wooten Optical properties of solids; Chapter 2 - Maxwell's equations and the dielectric function[121]):

$$
\begin{equation*}
a=2 \omega k / c \tag{A.4}
\end{equation*}
$$

## A. 3 Reflectance, Transmiittance

Reflectance and Transmittance describe wave propagation through an absorptive medium with thickness $d$, as for example in [122]. In this case we should account interaction of light with matter and it is described by a complex reflection coefficient $\hat{n}$ :

$$
\begin{equation*}
\hat{n}=n+i k \tag{A.5}
\end{equation*}
$$

Reflectance $R$ is the parameter describing how much of the electromagnetic wave is reflected by the thick absorptive medium and can be defined as [105, 106, 121]:

$$
\begin{equation*}
R=\frac{\left(n_{1}-n_{2}\right)^{2}+k^{2}}{\left(n_{1}+n_{2}\right)^{2}+k^{2}} \tag{A.6}
\end{equation*}
$$

To calculate a transmittance from a thick absorptive medium we should take into account 3 parts. First part is radiation transmission between air and medium though a front surface with transmittance $T=1-R_{1-2}$ (everything is not transmitted reflected). Radiation intensity after it will be:

$$
\begin{equation*}
I_{1}=I_{0}\left(1-R_{1-2}\right) \tag{A.7}
\end{equation*}
$$

The second part is radiation transmission inside the material, where electromagnetic
field will be attenuated by $\exp (-a d)$. Radiation intensity after it will be:

$$
\begin{equation*}
I_{2}=I_{1} \exp (-a d)=I_{0}\left(1-R_{1-2}\right) \exp (-a d) \tag{A.8}
\end{equation*}
$$

The third part is transmission between medium and air through a rear surface with transmittance $T=1-R_{2-3}$. Radiation intensity after it will be:

$$
\begin{equation*}
I_{3}=I_{0}\left(1-R_{1-2}\right) \exp (-a d)\left(1-R_{2-3}\right) \tag{A.9}
\end{equation*}
$$

Taking $R_{1-2}$ equal to $R_{2-3}$ we receive:

$$
\begin{equation*}
I_{3}=I_{0}(1-R)^{2} \exp (-a d) \tag{A.10}
\end{equation*}
$$

Transmittance $\mathbf{T}$ as the parameter describing how much of the electro-magnetic wave is passed a thick absorptive medium will be:

$$
\begin{equation*}
T=(1-R)^{2} \exp (-a d) \tag{A.11}
\end{equation*}
$$

Equation A. 11 becomes more complicated if we considering multiple internal reflections, coherency of light and long radiation wavelength. The one derived in [123] has been used in this thesis:

$$
\begin{equation*}
T=\frac{(1-R)^{2} \exp (-\alpha d)}{[1-\operatorname{Rexp}(-\alpha d)]^{2}+4 \operatorname{Rexp}(-\alpha d) \sin ^{2}(n \omega d / c)} \tag{A.12}
\end{equation*}
$$

One who confused about spectroscopy terminology and want to know difference between absorbance $(A=1-T-R)$ and absorptance $\left(A=\log \left(I_{0} / I_{1}\right)\right)$ might find useful historical overview in [124].

## B. 1 Form-factor correction constant

It should be noted here, that information from measured frequency region (200 to 1000 GHz ) is not enough to reliably establish where the whole coherency appear (at lower frequencies) or where exactly spectrum is fully decay (at high frequencies). In future this problem should be solved instrumentally - using more sensitive detectors for higher frequencies, or avoiding diffraction losses by proper choosing of optical elements and target. Now, however, we need do an estimation of what part of coherency slope we observing and use some correction constant $k$. The best way to determine what correction constant we need - is to compare our form factor with both low frequency and high frequency extrapolation "donors" and find the right correction constant so the form factor correspond with Gaussian slope an exponential decay as it proposed in [44]. This is a compulsory measure caused by deficiencies in the experimental system. The use of a very wide range of extrapolation is a significant drawback and will lead to the fact that the restored bunch will be close to the Gaussian form, even if in reality this is not quite the case.


Figure B.1: Examples of $\mathrm{k}=1.4$ "wrong" (a) and $\mathrm{k}=1.07$ "right" (b) correction constants for ChDR form factor

As an example, two different cases for "wrong" and "right" correction constants for ChDR form factor shown in Figure B.1. In our case this constant was 1.07 for all samples. Such a small number means that we are close to observe stable coherency at lower frequencies, but not yet there.

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[^0]:    ${ }^{1}$ Polarization Radiation term was first used by Amusia in 1974 [89], where he considered Bremsstrahlung of electrons on atoms and concludes that the part of electron energy will be consumed by an "atomic" absorption due to atomic polarization.

[^1]:    ${ }^{2}$ For one who want to use PCA analytical models I want to reference one more time to the next works in English [80, 81, 90, 91] and Russian [79, 82]. I also wish to thank my second supervisor prof. A.P. Potylitsyn and A. Konkov for consultation on PCA theory and help with this chapter.
    ${ }^{3}$ Eq. 2.16 also can be interpreted as the field of polarization radiation is generated by electric dipole moment within effective volume: $\mathbf{P}(\mathbf{k}, \omega)=\frac{\mathbf{j}_{\text {pol }}(\mathbf{k}, \omega)}{(-i \omega)}=\frac{(\varepsilon(\mathbf{k}, \omega)-1)}{4 \pi}\left(\mathbf{E}_{\mathbf{0}}(\mathbf{k}, \omega)+\mathbf{E}_{\mathbf{p o l}}(\mathbf{k}, \omega)\right)$

[^2]:    ${ }^{4}$ Finishing this section i want to reference one more time to the next works in English [80, 81, 90, 91] and Russian [79, 82]. I also wish to thank my second supervisor prof. A.P. Potylitsyn and A. Konkov for consultation on PCA theory.
    ${ }^{5}$ Equation for electric field derivation can be found in Jackson's Classical Electrodynamics [92]. In third edition it is in Chapter 13, paragraph 3, equations 13.21-13.32. Also explained in [80].

[^3]:    ${ }^{6}$ One can also derive analytical form for such equations using "Integral of a Gaussian function" rule: $\int e^{\left(-a x^{2}+b x\right)} d x=\sqrt{\frac{\pi}{a}} e^{\frac{b^{2}}{4 a}}$

[^4]:    ${ }^{1}$ The explanation of how the rooftop mirror can achieve this polarization of $2 \theta$ can be found in [98]

[^5]:    ${ }^{1}$ More about choosing of window function could be found in National Instrument review [113]
    ${ }^{2}$ This name is referencing the idea that looking at the FFT spectrum is like looking at mountains through a picket fence.

[^6]:    ${ }^{3}$ The humidity level was not measured specifically in the lab, so normal humidity condition ( $50 \%$ ) are implied

[^7]:    ${ }^{4}$ A more detailed explanation on how to calculate diffraction losses in finite-size elements can be fount in [115]

