Party system polarization and the effective number of parties

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Abstract

Polarization is a key characteristic of party systems, but scholars disagree about how polarization relates to the number of parties in a system. Different authors find positive, negative, or null relationships. I claim that when polarization is measured using the weighted standard deviation of standardized party positions, seat-level polarization is equal to $\frac{N_S - 1}{\frac{1}{\sqrt{2}} + N_S - 1}$, where N_S is the effective number of seat-winning parties. This relationship is what one would expect if parties were drawn randomly from a super-population with an effective sample size somewhere between the effective and raw number of parties. I test this claim using multiple datasets which report party positions and seat shares, before extending my analysis to consider vote-level polarization, the range of positions, and polarization in presidential and parliamentary regimes. My work extends the Taageperaan research agenda of building interlocking networks of equations relating key quantities of electoral and party systems. (10,557 words)

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1. Introduction

Party systems can be classified by the number of parties in the system and the extent of disagreement between those parties (Sartori 2005). Where parties with large vote- or seat-shares hold positions on some issue(s) which are far apart, we refer to the system as more *polarized* than a party system where large parties hold similar positions or where parties with extreme positions have only small vote- or seat-shares. Dalton (2008, 908) describes the number of parties and degree of polarization as providing the "quantity and quality" of party systems and concludes that "polarization can vary nearly independently of the number of parties." Many would agree with this assessment (Adams and Rexford 2018): although some studies have found an unconditional **positive** association between the number of parties and polarization (Andrews and Money 2009; Matakos, Troumpounis, and Xefteris 2016; Bol et al. 2019), others have found a **negative** relationship (Dow 2011), a **conditional** relationship (Curini and Hino 2012), or **no** statistically or substantively significant relationship (Gross and Sigelman 1984; Dalton 2008).

Given disagreement about the *direction* of the relationship between the number of parties and polarization, it may seem hubristic to claim to be able to specify not only the direction of the relationship, but also the *specific functional form* of that relationship. Nevertheless, that is what I attempt in this paper. I follow the approach described in Taagepera (2008) and applied to great effect in Shugart and Taagepera (2017): I construct a model of party system polarization which respects the ranges of input and output variables; respects "anchor points" where unique values of the number of parties force a particular value of polarization; and which eschews substantive knowledge of party positioning strategies in favor of premises based on minimal assumptions. The resulting quantitatively predictive logical model (Taagepera 2008) is a parsimonious nonlinear model connecting a single input variable to a single output variable. It can therefore be used as part of a chain of interlocking models to derive further expectations about the relationship between electoral systems and polarization.

I make four claims. *First*, I argue that party positions, since they have no natural zero point, should be analyzed as standardized measures (i.e., measures with a mean of 0 and standard deviation of 1). *Second*, I argue that we should use as our measure of polarization the weighted standard deviation of party positions, which is linearly related to the index Dalton (2008) put forward for analyzing the polarization of party positions on a 0–10 scale. *Third*, I argue that (standardized) party positions can be conceptualized as though they were identically-distributed draws from a notional super-population of parties. *Fourth*, I argue that the polarization of positions among parties in a party system stands in the same relationship to polarization in the super-population as a sample standard deviation does to the population standard deviation. The polarization of party positions is therefore a downward-biased estimator of the polarization (standard deviation) of the super-population, where the degree of bias depends on the number of parties drawn from the super-population. The relationship between the number of parties and polarization is therefore identical in functional form to small-sample corrections for standard deviations.

These claims neglect or are contrary to findings about party formation and party positioning. Parties are not random draws from a super-population, but the result of political entrepreneurs' creative acts (De Vries and Hobolt 2020). Size-ordered party positions are not identically distributed if "small... parties have an incentive to emphasize more extreme positions" (Abou-Chadi and Orlowski 2016, 870). My claims should therefore be regarded as simplifying assumptions which enable me to identify a particular functional form which respects logical constraints whilst providing a good fit to data. The relationship set out here is **compatible** with predictions based on theories of party positioning ("centrifugal incentives increase with the size of the candidate field": Andrews and Money (2009), 807), but does not lend support to any particular theory.

I begin by defining party system polarization and reviewing how it, and party positions, have been measured. I also review some of the claims which have been made regarding polarization, noting where authors have employed particular measurement strategies. I then set out a theory connecting the number of parties to polarization based on anchor points and analogies with small sample statistics. I then return to the issue of measurement, and review datasets from four different projects measuring party positions: the Manifesto Project Database (MARPOR), the Comparative Study of Electoral Systems (CSES), V-Party, and ParlGov. Using these datasets, I estimate separate nonlinear models of party-system polarization, and show that the coefficients in these models are consistent with theoretically predicted values. I then extend my analyses to include polarization of vote-winning parties, polarization as measured by the range of (top) party positions, and polarization in presidential and parliamentary systems. I conclude by noting how this theory accounts for some conflicting findings and by reflecting on the place of these findings within an overall research program which emphasizes parsimony and interlocking connections.

2. Definitions, measures, and claims made

As a characteristic of many different social structures, polarization can be difficult to analyze axiomatically (Esteban and Ray 1994). DiMaggio, Evans, and Bryson (1996) describe two key intuitions regarding opinion polarization: that it is related to the *dispersion* of opinion, and that it is related to "the extent to which opinions move toward separate *modes*" (693, emphasis added). This is also true for the polarization of party systems, except that it is party positions rather than opinions which are polarized. Discussion of both dispersion and modality is present in classic texts on party systems: the post-war Italian party system qualified as an example of "polarized pluralism" not only because of the presence of parties with very extreme positions (Communists and neo-fascists), but also because these extreme parties — together with parties of the center — formed opposing poles which exerted centrifugal force on intermediate parties (Sartori 2005, 121). Though Sartori might not have described it so, the distribution of party opinion was characterized by three distinct and distant modes, with each mode corresponding to more than one party.

Although some studies have retained this emphasis on modality (usually by incorporating additional information about distributions of voters or electoral districts: Rehm and Reilly (2010)), most studies of party-system polarization have emphasized the *dispersion* of party positions along a single dimension. A variety of dispersion-based measures have been proposed — including range-based measures (Andrews and Money 2009; Sørensen 2014; Matakos, Troumpounis, and Xefteris 2016) and measures based on the presence of (anti-system) parties at extreme ends of the distribution (Warwick 1992) — but the most common measure is the weighted standard deviation of party positions, or a variant thereof (Taylor and Herman 1971; Sigelman and Yough 1978; Gross and Sigelman 1984). The measure Dalton (2008) proposed has become particularly popular among researchers who use party positions from the CSES. The measure involves calculating differences between each party's position (LR_i) on a 0-10 scale, and the weighted system average ($\bar{LR}$). These differences are then standardized by dividing them by half the range of the data. The standardized differences are then squared and weighted in proportion to each party's share of the vote (v_i). The weighted squared differences are added together before the square root is taken:

$$\text{Dalton's index} = \sqrt{\sum_{i}^{N} v_i \cdot \left(\frac{LR_i - \bar{LR}}{5}\right)^2} \tag{1}$$

The resulting measure has a minimum of 0 and a maximum of 10. Because Dalton's index differs from the weighted standard deviation by dividing differences by 5, Dalton's index is a simple multiple of the *weighted uncorrected sample standard deviation.* Ordinarily the sample standard deviation would be denoted by s; however, since s can also be used to represent a seat share (for example: the seat shares of the top two parties s_1, s_2), I use ς to refer to this weighted sample quantity. Because polarization can be calculated by summing differences for all seat-winning parties (N_{S0}) or all vote-winning parties (N_{V0}), I use subscripts to indicate polarization at the level of seats (ς_S) or votes (ς_V).

$$\varsigma_S = \sqrt{\sum_{i}^{N_{S0}} s_i \cdot \left(LR_i - \bar{LR}\right)^2} \tag{2}$$

I use the weighted standard deviation of party positions as my measure of polarization for several reasons. First, the most common measurement based on modality (Esteban and Ray 1994) has the undesirable characteristic that polarization can decrease when a party moves away from the system average position, if by moving towards the centre it contributes to making a clearer mode or "pole" in the system.[2] I take this violation of Dalton's transfer principle to be a strong argument against mode-based measures, given the way polarization has been used in the literature. Second, standard-deviation based measures of dispersion

[2] See supplementary information A2 for a worked example.

are the most common dispersion-based measures of polarization used in the literature (Schmitt 2016). They also avoid the weaknesses of more intuitive measures such as the range, which are sensitive to the inclusion of small extremist parties. Readers who prefer to use the range can see an extension of the theory for the range of positions later in the article.

As Dalton's index illustrates, measures of party-system polarization can be tightly bound up with measurements of party position. Measurements of party positions in comparative research are typically based on mass surveys, expert surveys, or quantitative content analysis (Benoit and Laver 2006, Ch. 3). In the CSES studies Dalton (2008) uses, mass-survey respondents are asked to place parties on a 0–10 left-right scale. Expert surveys have used longer and shorter response scales: Benoit and Laver (2006) asked experts to place parties on a 1–20 scale, whilst the V-Party project (Lührmann et al. 2020) asks experts to place parties on a fully labeled ordinal scale between 0 and 6.[3] The estimates of party position produced by the Manifesto Research Project (MARPOR) are based on counts of quasi-sentences from party manifestos; the RILE ("RIght-LEft") index is the percentage of quasi-sentences which fall in one of thirteen right-leaning categories minus the percentage of quasi-sentences which fall into one of thirteen left-leaning categories. As such, this measure is bounded between -100 and +100, and has a a natural zero point. The RILE scale, however, has been criticized on the grounds that what matters is not the "*absolute quantity* of sentences [coded as left or right], but rather their relative *balance*" (Lowe et al. 2011, 131, emphases in original), and that taking the difference in proportions — rather than using ratios — creates a bias toward centrism in the left-right scores (Gemenis 2012). Alternative estimators based on the same counts of quasi-sentences (e.g., Lowe et al. 2011) are not intended to be ratio-level measurements.

Measurement of the number of parties is much less varied than measurement of either polarization or party positions. Almost all researchers use the effective number of Laakso and Taagepera (1979), calculated either on the basis of party seat shares (N_S) or vote shares (N_V):

$$N_S = \frac{1}{\sum_{i=1}^{N_{S0}} s_i^2}, \quad N_V = \frac{1}{\sum_{i=1}^{N_{V0}} v_i^2}$$

The effective number is more relevant than the raw number of vote or seat-winning parties (respectively, N_{V0} and N_{S0}), although the two quantities are linked: $N_S = N_{S0}^{2/3}$ (Shugart and Taagepera 2017, 149). These measures are parts of a broader research agenda designed to construct "logical models" of electoral systems, or models which yield logically possible predictions for all possible input values, and which go beyond directional theories to predict how much the output variable should change given a change in the

[3] The scale used to **elicit** party positions does not need to be the same as the scale used for subsequent analysis: the 0–6 ratings given by V-Party raters are inputs to a multi-rater measurement model which produces estimates of party positions which are (very approximately) normally distributed with a mean of 0 (Pemstein et al. 2020).

input variable (Taagepera 2008; Colomer 2017). This research agenda has established laws or stylized facts relating the effective number of seat- and vote-winning parties to the seat product (average district magnitude M times assembly size S), and has been extended to include the raw number of governing parties (Colomer 2017). Some of these relationships are set out in Table 1, together with the notation I use.

Table 1: Notation and relationships used

Input variables	**Parameters**
N_{V0}	k
Count or raw number of vote-winning parties	Shape parameter (value of $N_V - 1$ at which ς is half the asymptote)
N_V	d
Effective number of vote-winning parties	Asymptote parameter (value of ς with infinite parties)
N_{S0}, N_S	
Count and effective number of seat-winning parties	
v_i, s_i	
Vote and seat share of the i-th party	
MS	
Seat product, or assembly size times district magnitude	
Outcome variables	**Relationships used**
ς_v, ς_s	Effective numbers from counts
Seat- and vote-level polarization	$N_S = N_{S0}^{2/3}, N_V = N_{V0}^{2/3}$
R_v	Seat to vote-winning parties
Range of positions amongst all vote-winning parties	$N_V = (N_S^{3/2} + 1)^{2/3}$
R_v^*	Effective number from seat product
Range of positions amongst top N_V vote-winning parties	$N_S = MS^{1/6}$

Given the many different ways of measuring the key quantities involved, it is perhaps unsurprising that the literature has failed to reach firm conclusions about the relationship between the number of parties and party-system polarization — and *a fortiori*, the relationship between the electoral system and polarization. The literature can be divided on the basis of the relationship identified:

- **positive relationship:** Andrews and Money (2009) find that party-system dispersion (the log of the range of positions of the top N_S parties rounded to the nearest whole number) increases in the (logged) effective number of parties, across both economic and social dimensions. Matakos, Troumpounis, and Xefteris (2016) find that the effective number of parties has a linear effect on the range of the party system, but not on polarization as measured by Dalton's index.

- **negative relationship:** Dow (2011), using a measure which expresses party positions relative to citizen positions, finds an "anomalous" negative effect of the effective number of parties on "party system extremism" (Alvarez and Nagler 2004) when controlling for the proportionality of the electoral system.
- **no relationship:** Ezrow (2008) finds no significant effect of the effective number of parties in cross-country regressions using (alternately) expert, citizen, and manifesto-based party placements, and weighted and unweighted measures of party-system extremism.
- **conditional relationship:** Curini and Hino (2012) find that where single-party majority government is common, increasing the effective number of parties results in an increase in polarization; but where minority or coalition government is the norm, increases in the effective number of parties result in decreasing polarization.

These articles often include variables (electoral system; propensity to coalition government) which are either "up" or "down-stream" of the effective number of parties, presenting a risk of post-treatment bias. Most articles also test for a *linear* relationship between the effective number of parties and polarization (Andrews and Money (2009) is an exception). It is therefore possible that conflicting findings have emerged because authors have used inappropriate functional forms for the relationship between the effective number of parties and polarization. In the next section, I discuss a possible functional form.

3. Theory

Here I set out a theory connecting the polarization of vote-winning parties to the effective number of vote-winning parties. Although this theory is cast in general terms, it relies on a particular operationalization of polarization as a weighted standard deviation, and at times relies on summary statistics which are only described fully in the following section. I start with seat- rather than vote-winning parties for two reasons. My first reason is a practical one: the degree of missingness of left-right positions is much lower for seat-winning than for vote-winning parties. As such, I can be more confident that any results I find for seat-winning parties are not an artifact of missingness. My second reason is that Shugart and Taagepera (2017) have shown how greater progress can be made by starting with quantities related to seats (and in particular, the seat product) than quantities related to votes.

In characterizing the relationship between the effective number of seat-winning parties N_S and my measure of polarization ς_S, I consider five different aspects: (1) the range of the input and output variables; (2) the presence of anchor points; (3) the possible asymptotes of any relationship; (4) the choice of functional form; and (5) the shape of the chosen functional form. Considering (1) — the range of the input and output variables — is necessary to avoid "predict[ing] absurdities, even under extreme circumstances" (Taagepera

2008, 23). The input variable, N_S, has a lower bound of 1 but no *a priori* upper bound. The output variable, ς_S, has a lower bound of zero (achieved when all seat-winning parties have the same position), but no *a priori* upper bound. Even when party positions are scaled to have a standard deviation of 1, it is still possible (if unlikely) for two vote-winning parties with an even share of the vote to have positions of -5 and +5 ($\varsigma_S = 5$). Whatever the relationship, it must not predict negative values. From this, it follows that the relationship between N_S and ς_S is unlikely to be a linear additive relationship but, rather, is likely to be nonlinear and/or multiplicative.

Considering (2) — the anchor points of the relationship, or points where the value of the input variable predicts a unique value of the output variable (Taagepera 2008, 34) — there is just one anchor point, the point $(1, 0)$. If each party has a single position in space, then the polarization of a party system with just one party must be equal to 0. Because functions that go through the origin $(0, 0)$ are easier to work with than functions which go through $(1, 0)$, I re-define my input variable as $N_S - 1$ rather than N_S.

An infinite number of functions produce outputs in the range $[0, \infty]$ and pass through the origin $(0, 0)$. Considering variable ranges and anchor points cannot therefore determine the relationship between N_S and ς_S. One way forward involves considering (3): the likely value of ς_S at infinitely large values of N_S, or the asymptote of the function. In characterizing this asymptote, I have found it helpful to think of the population of parties in a country as being a random draw from a super-population of parties. How might we characterize the distribution of party positions in this (notional) super-population? One option is to appeal to the (actually existing) *global* distribution of party positions — or at least to the distribution of party positions encompassed by the data at hand. Characterizing **this** distribution might also be hard — except, of course, that this distribution (because it depends on a standardized variable) is known to have a mean of 0 and a standard deviation of 1. If — by construction — the standard deviation of party positions is 1, then with an infinite number of unweighted parties, the measure of party polarization set out here ought also to approach the standard deviation of party positions. Our expectation, therefore, is that as the number of parties approaches infinity, polarization should tend to 1.

Having identified two points — an anchor point at $(0, 0)$ and a looser expectation at $(\infty, 1)$ — I now turn to consider (4): the functional form of the relationship between N_S and ς_S. Because I have conceptualized parties as a draw from a notional super-population, I base my functional form on a correction applied when moving from a sample standard deviation to a population standard deviation. As is well known, the uncorrected sample standard deviation is a biased estimate of the population standard deviation. The degree of bias depends on the size of the sample: the smaller the sample, the greater the downward bias. At the lower limit, the bias is infinite: with just one observation, the sample standard deviation is 0 (or alternatively is undefined). Researchers can obtain a less biased estimate of the population standard deviation by applying

Bessel's correction:[4] multiplying the sample standard deviation by $\frac{N}{N-1}$. In the less likely situation in which a researcher knows the population standard deviation and wishes to estimate the sample standard deviation for a sample of size N, she would instead multiply the population standard deviation by $\frac{N-1}{N}$. We might therefore model seat-level polarization as:

$$\varsigma_S = d\frac{N_S - 1}{N_S} \tag{3}$$

where d is the standard deviation of party positions in the super-population, and which (because party positions have been standardized) should be close to one. The parameter d also gives the asymptote of the function.

This equation can be generalized by including an extra parameter in the denominator:

$$\varsigma_S = d\frac{N_S - 1}{N_S - 1 + k} \tag{4}$$

The parameter k is a shape parameter, where **lower** values of k indicate **higher** initial slopes (see Figure 1(a)). The parameter k has a secondary interpretation as the value of the input variable at which the output is half its asymptotic value. This equation is known variously as the Michaelis-Menten equation, the Monod equation, or the Hill equation.

If party positions are a random draw from a standard normal distribution, and if the effective number of parties is analogous to sample size, then we should expect that the value of the asymptote parameter d will be one, and the shape parameter k should also be close to one (since this gives the inverse of Bessel's correction). What if the effective number of parties is not analogous to sample size? What if, instead, the raw number of parties plays that role? In this case, we could either use the raw number of parties as our input variable or, alternatively, retain the effective number of parties as our input variable and exploit known relationships between the raw and effective number of parties. I take this second route. Figure 1(b) shows two Michaelis-Menten curves, one using the effective number of parties (the "effective curve," plotted using a solid line) and one using the raw number of parties, where — following Shugart and Taagepera (2017), 149 — the raw number of parties is equal to the effective number raised to the power of $\frac{3}{2}$ (the "raw curve," plotted using a dotted line). The raw curve is higher than the effective curve at every point, which is exactly what we would expect given that the input value is always higher. Rather than using two equations with different inputs, we can try to approximate the raw curve with $k = 1$ by keeping the effective number of parties as our input and adjusting the value of k to match the value of the raw curve at each point. We can do this in two different ways. First, we can set the two curves equal to each other:

[4]Strictly speaking, Bessel's correction is a correction for the sample variance. There is no general correction for the standard deviation. Applying Bessel's correction yields a less-biased estimate, but does not eliminate the bias.

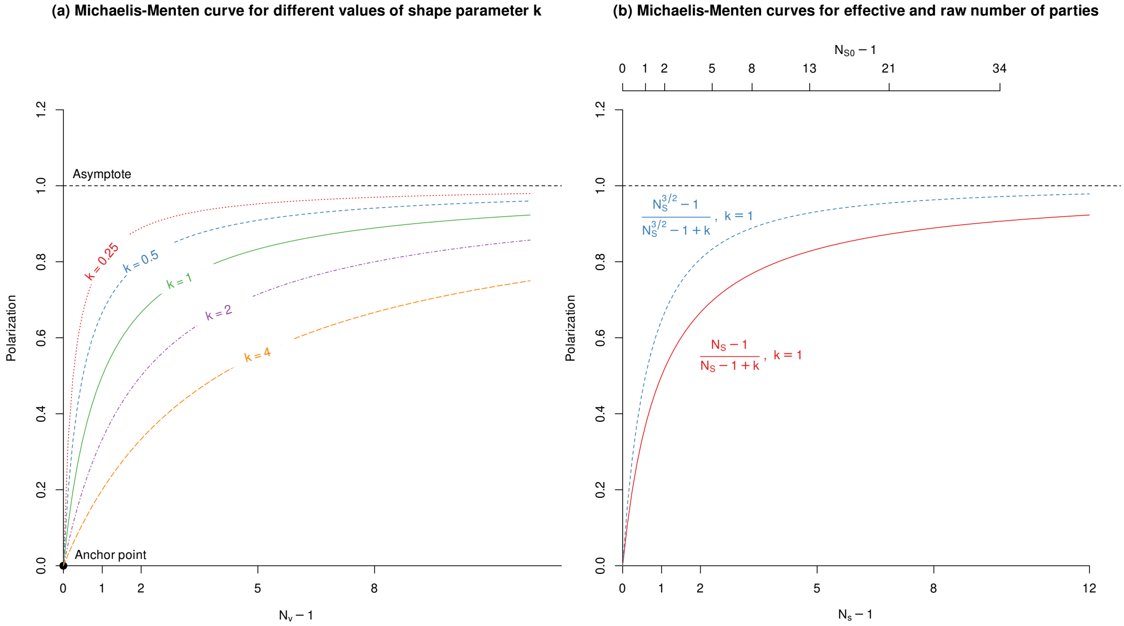

Figure 1: Different Michaelis-Menten curves. Figure 1(a) shows Michaelis-Menten curves for different values of the shape parameter k. Figure 1(b) shows Michaelis-Menten curves with the same value of k, but alternately using the effective number of seat-winning parties N_s (primary axis) and the raw number N_{S0} (secondary axis).

$$\frac{N_S^{3/2} - 1}{1 + N_S^{3/2} - 1} = \frac{N_S - 1}{k + N_S - 1} \tag{5}$$

and solve for a representative value of N_S. Since the median value of N_S in the datasets I use is close to 3, I use that value, and solve to find that $k = 0.5$. Second, we can set up a sequence of input values of N_S and find, for these input values, the value of k which minimizes the sum of squared differences between these two curves. For plausible input values, the exact value of k which minimizes the difference between the two curves is very slightly smaller than 0.5, at 0.4725.[5]

There are, therefore, reasons to expect that the polarization of party systems will follow a Michaelis-Menten equation with either $k = 1$ (if the effective number of parties is analogous to sample size) or $k \approx 0.5$ (if instead the raw number of parties is analogous to sample size). Since there are no good reasons to prefer one of these two values, our best guess is the geometric mean[6] of these two values, or $k = \sqrt{0.5 \times 1} = \frac{1}{\sqrt{2}} \approx 0.707$.

This reasoning is based on an heuristic device in which party positions are treated as identically distributed draws from a super-population with known standard deviation. This is not the same as requiring party positions to *independent* and identically distributed (iid). Consider the claim that the party positions

[5]The code used is given in the supplementary information (A3).

[6]For reasons why the geometric mean is often preferable to the arithmetic mean, see Taagepera (2008) pp. 120-129.

X_1, X_2 and X_3 are distributed according to the following multivariate normal distribution:

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & -0.67 & 0 \\ -0.67 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right)$$

The (marginal) distribution of X_1 is the same as the marginal distributions of X_2 and X_3 respectively: all are distributed $N(0,1)$. However X_2 is not independent of X_1, since the two are negatively correlated. Nor are these positions exchangeable: the relationship between X_1 and X_2 is not the same as the relationship between X_1 and X_3. Party positions could be correlated in this way without affecting the central point that polarization in actually-existing party systems, as an analogue to sample standard deviation, is generally an underestimate of a notional super-population. This central point would not work if, for example, the positions of X_1 and X_2 followed a less dispersed $N(0, 0.5)$ distribution in contrast with a wider distribution for X_3.

This reasoning gives an expectation for polarization of **seat**-winning parties. Suppose that we take this relationship for seat-winning parties as fundamental, and that we use this relationship to formulate expectations for **vote-winning** parties. Curiously, seat-level polarization is very similar to vote-level polarization: in ParlGov data, a regression of seat-level polarization on vote-level polarization gives an intercept of -0.007 and a coefficient of 1.007. The fact that seat- and vote-level polarization are so similar suggests we cannot simply replace (in equation 3) the effective number of vote-winning parties with the effective number of seat-winning parties. Instead, I once proceed by setting two equations equal to each other and solving for k. I set:

$$\frac{N_S - 1}{\frac{1}{\sqrt{2}} + N_S - 1} = \frac{N_V - 1}{k + N_V - 1} \tag{6}$$

where the left-hand side of the equation represents the hypothesized relationship for seat-level polarization, and the right hand side the same functional form, except that k is allowed to vary. We can substitute in the relationship between seat- and vote-winning parties (Table 1) to give:

$$\frac{N_S - 1}{\frac{1}{\sqrt{2}} + N_S - 1} = \frac{\left(N_S^{3/2} + 1\right)^{2/3} - 1}{k + \left(N_S^{3/2} + 1\right)^{2/3} - 1} \tag{7}$$

We can now find values of k which preserve the equality for a given value of N_S. Here, I set N_S to a representative value of 3, and find that $k = 0.839$, which I approximate as $\frac{5}{6}$. As before, I obtain similar results if I supply a range of plausible values for N_S and use numerical optimization.

There are, therefore, reasons to expect that the seat-level polarization of party systems will follow a Michaelis-Menten equation with parameter $k = \frac{1}{\sqrt{2}}$, and that vote-level polarization will follow the same equation, but with a value of $k = \frac{5}{6}$. Simulation studies in the supplementary information show that when party positions are drawn from a $N(0, 1)$ distribution and merged with actually existing seat shares, the simulated data closely fits the theoretical prediction.[7] The test of the theory comes not from simulated, but from actual data, which I describe now.

4. Data

The data sources I utilize here use different methods (content analysis, expert judgment, mass judgment, and aggregates of these methods) to estimate party positions, and have different coverage. By testing my theory on multiple datasets which measure party positions in different ways, I am able to demonstrate the robustness of my theory — particularly when (as is the case here) the correlations between measurements of the same concepts across datasets are not strong.[8] In describing the datasets, I refer to the proportion of featured elections which took place in systems with an elected head of state, and the average level (on a 0–1 scale) of programmatic competition in these elections. These measures are taken from the V-Dem project (Coppedge et al. 2021). The measure of whether a system has an elected head of state (`v2ex_elechos`) is a simple dummy variable. The measure of programmatic competition (`v2psprlnks_ord`) was originally on a five-point ordinal scale: I collapse this into a dummy variable which has a value of 1 if competition in each country in each year falls into the top, "programmatic" category ("Constituents respond to a party's positions on national policies, general party programs, and visions for society"). All data sets have been filtered to include only democratic elections (defined as years when the score on the V-Dem polyarchy index was greater than 0.5) in which the cumulative vote share of featured parties was greater than 80% and less than or equal to 100%, and where the cumulative seat share of featured parties was also greater than 80% and less than or equal to 100%. (In some datasets vote shares can exceed 100% because parties can run as part of electoral alliances which are also reported).

4.1. Comparative Study of Electoral Systems (CSES)

The CSES is a multinational collaboration through which researchers conduct national election studies using a common set of questions asked of survey respondents. One set of questions asks survey respondents to position parties on a 0–10 scale, where 0 means the left and 10 means the right. Different national surveys ask respondents to position a different number of parties, but that number is never greater than 9, and has a median of 6 and a minimum of 2. These mass judgments have been used as the basis for estimates of party positions, either by taking a simple or weighted average of respondent estimates, or by more complicated

[7]See supplementary information (section A4).

[8]The correlation between ParlGov and V-Party scores is the highest ($r = 0.62$); the correlation between ParlGov and Marpor scores is the lowest ($r = 0.10$).

scaling models which take account of different perceptual biases (Carroll and Kubo 2018). These CSES-derived estimates of party positions can, when combined with the auxiliary information on seat share and vote share recorded in the CSES data sets (which is present for up to nine parties), be used to measure seat- or vote-level polarization. I take the weighted average of respondents' estimates as my estimate of each party's position. The weights I use are demographic weights. Where demographic weights are not available for a particular election I take the unweighted average.

The CSES data covers 106 elections in 42 unique countries. Prior to standardization, the (unweighted) mean of left-right party positions was 5.08 (SD: 1.8). The median value of seat-level polarization is 0.95 (IQR: 0.73–1.16; min-max: 0.12–1.85). The median effective number seat-winning parties is 3.49 (IQR: 2.72–4.5; min-max: 1.94–6.74). 46 percent of featured elections took place in systems with an elected head of state, and 66% of elections were contested on the basis of programmatic competition between parties.

4.2. The Manifesto Project (MARPOR)

MARPOR, is the successor to the Comparative Manifestos Project and the Manifesto Research Group. The project compiles and codes party manifestos (or other programmatic statements) for a broad range of democratic countries. The expectation is that the project will code the manifestos for all parties which win a seat in a given election, though there are exceptions for certain regions (in Eastern Europe the threshold for inclusion is two seats) and for certain parties (parties which were historically important in virtue of government participation, but which lost all their seats, are often coded). As noted earlier, the project measures a party's left-right position as the proportion of quasi-sentences in the document which fall into one of 13 left-leaning categories, minus the proportion which fall into one of 13 right-leaning categories. As my estimate of each party's position I take the measure proposed by Lowe et al. (2011): a party's position is the log of one-half plus the count of right-leaning quasi-sentences, minus the log of one-half plus the count of left-leaning quasi sentences. I construct measures of seat- and vote-level polarization using MARPOR-supplied information on party seat- and vote-shares.

The MARPOR data covers 613 elections in 48 unique countries. Prior to standardization, the (unweighted) mean of left-right party positions was -0.16 (SD: 1.02). The median value of seat-level polarization is 0.6 (IQR: 0.41–0.86; min-max: 0.03–2.61). The median effective number of seat-winning parties is 3.23 (IQR: 2.5–4.43; min-max: 1.32–10.85). 35 percent of featured elections took place in systems with an elected head of state, and 60% in systems with programmatic competition.

4.3. ParlGov

ParlGov (Döring and Manow 2020) collects comprehensive information on electoral and governmental outcomes in a number of parliamentary and semi-presidential regimes. Information is recorded for all elections and cabinets after 1945 or after full democratization, and for a limited number of countries from

1900. Parties are included if they won more than 1% of the vote or two seats or more. Party positions in ParlGov are "time-invariant unweighted mean values of information from party expert surveys on a 0 to 10 scale." The left-right positions rely on estimated reported in Castles and Mair (1984), Huber and Inglehart (1995), Benoit and Laver (2006) and (for the 1999, 2002 and 2006 editions) Bakker et al. (2015).

The ParlGov data covers 741 elections in 37 unique countries. Prior to standardization, the (unweighted) mean of left-right party positions was 5.18 (SD: 2.27). The median value of seat-level polarization is 0.77 (IQR: 0.65–0.89; min-max: 0.14–1.49). The median effective number of seat-winning parties is 3.42 (IQR: 2.57–4.56; min-max: 1.54–10.86). As the name of the data-set suggests, relatively few of the elections featured in the ParlGov data took place in systems with an elected head of state (25%), but the average level of programmatic competition across all elections was second highest (at 63%).

4.4. V-Party

Some of the infrastructure established for ParlGov is also present in V-Party, a project related to the well-known V-Dem project. The V-Party project asks experts to rate parties on several criteria in a range of (democratic and non-democratic) countries. V-Party expert codings cover 169 countries: within those countries, the project covers parties with vote share greater than 5%. On the specific issue of left-right positions, experts are asked to rate parties on a 0–6 scale, and these ratings form the input to a multi-rater measurement model. The output of this model is approximately normally distributed, but before (further) standardization, the mean left-right position was -0.02 (SD: 1.48).

The V-Party data covers 677 elections in 102 unique countries. The median value of seat-level polarization is 0.8 (IQR: 0.58–0.96; min-max: 0–1.62). The median effective number of seat-winning parties is 3.04 (IQR: 2.3–4.21; min-max: 1–9.38). 47 percent of featured elections took place in systems with an elected head of state. The V-Party data features the lowest levels of programmatic competition (40%).

4.5. Treatment of missing data

These datasets may be incomplete in two ways. First, datasets may have an incomplete list of parties which won votes or won seats. In part, I deal with this by only including elections where the sum of seat shares *and* the sum of vote shares was greater than 80% and less than or equal to 100%. I also calculate the effective number of (vote- or seat-winning) parties using the formula given by Taagepera (1997), which extends the formula given in Laakso and Taagepera (1979) to allow for incomplete shares.

Second, the data sets may also be incomplete because parties are present in the data but lack a left-right position. Rates of party position missingness are highest in the V-Party data (25%) and lowest in the MARPOR data (0%). Rates of missingness in the ParlGov and CSES data are closer to the MARPOR data than the V-Party data (4% and 6% respectively). Note that these rates are averages across parties of very different sizes (missingness is less of an issue for larger parties) and that some datasets may have higher rates of missingness because they have a more complete list of parties. I deal with this kind of missingness by using multiple imputation. Specifically, I used Amelia II (Honaker et al. 2011) to generate five multiply-imputed datasets, using as auxiliary variables the identity of the country, the year of the election, the square of the year of the election, whether the country has an elected head of state, the level of democracy (as measured by the V-Dem polyarchy index) and the level of programmatic competition. The summary statistics reported earlier are across-imputation means for medians and interquartile ranges, but across-imputation maxima and minima for the minimum and maximum values.

5. Analysis

An initial test of the plausibility of my theory comes from exploratory visual analysis. Figure 2 shows, for the four datasets, a scatter-plot of the effective number of seat-winning parties (on the horizontal axis) against the measure of polarization ς_S. The horizontal line indicates the expected asymptote of 1; the solid line shows a loess smooth; and the black dashed line shows the theoretical prediction. Across the four datasets, the model predictions most closely match the results from the local smoother for the ParlGov and V-Party datasets. The fit is worse for the MARPOR and CSES data, but the average error is different: levels of polarization in the MARPOR data are lower than would be predicted, while levels of polarization in the CSES data are higher than would be predicted.

To test the theory more precisely, I estimate separate nonlinear models using Bayesian methods with the software package `brms` (Bürkner 2017). I estimate **separate** models for each data-set as a way of showing robustness to different sample compositions and methods for estimating party positions. The theory suggests **nonlinear** models, and there is no satisfactory way to transform the Michaelis-Menten equation into an equation that is linear-in-parameters.[9] **Bayesian** methods are appropriate given that the data cannot sensibly be regarded as a random sample from a larger population, but rather aim to provide exhaustive coverage of democratic elections in a progressively larger group of countries. Specifically, I estimate a Gamma regression, since the Gamma distribution is a positive-valued distribution which allows zero values (Zorn 2000; Lauderdale 2012). I relegate discussion of priors to supplementary material (section A5).

I estimate eight models in total — two for each of the four datasets, where one model includes an additional intercept a and one model omits that intercept (thus matching exactly equation 4). Each model was run on four chains for 3,000 iterations per chain, with the first 2,000 chains dropped as warmup iterations. There were no problems with convergence. The results of the models on seat-level polarization are shown in Table 2. Coefficients which match theoretically predicted values are shown in bold.

[9] Although the equation can be linearized by taking reciprocals of both sides to give the Lineweaver-Burk equation (Lineweaver and Burk 1934), estimates of Lineweaver-Burk equations only give the same estimates as direct estimation of the Michaelis-Menten equation when there is no error in input or output variables; where there is error, the use of reciprocals greatly magnifies the effects of this error (Dowd and Riggs 1965).

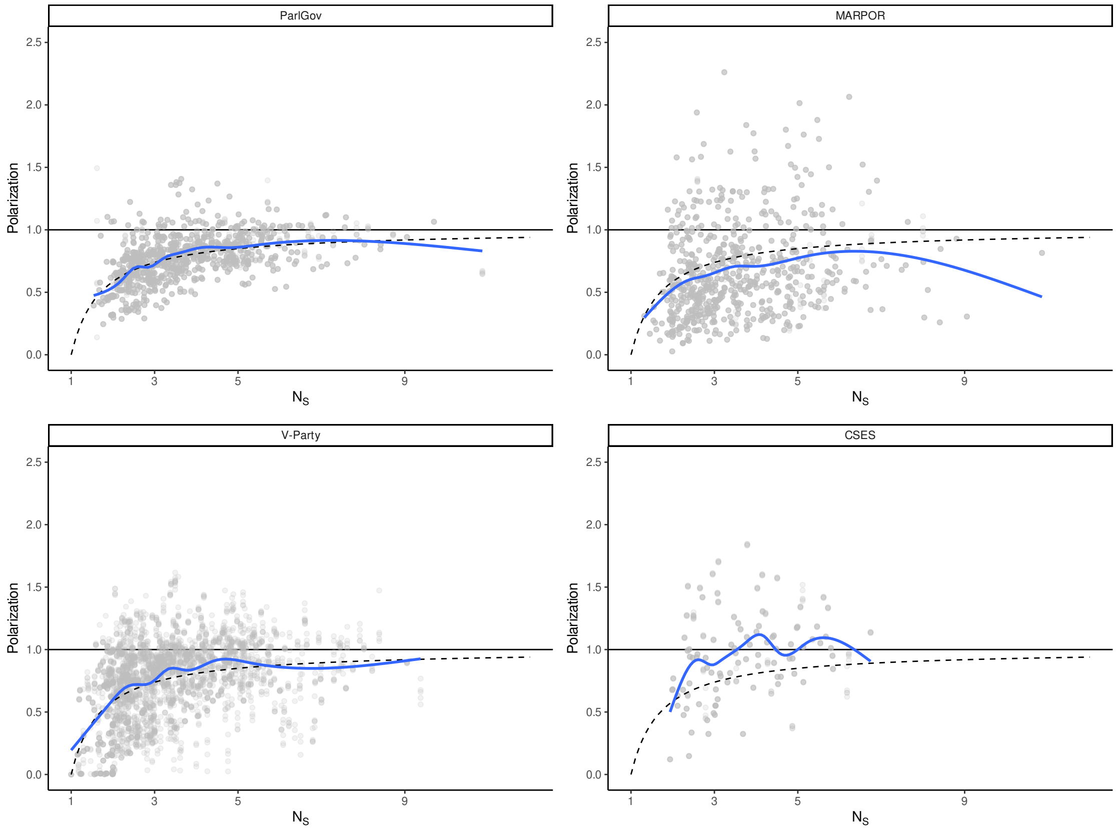

Figure 2: Effective number of seat-winning parties (horizontal axis) against seat-level polarization (vertical axis). Plots from multiply-imputed datasets are plotted on the same graph. Solid blue line gives a generalized additive smooth. Dotted line gives theoretical prediction.

Table 2: Non-linear regression models of polarization of seat-winning parties

	ParlGov		**Marpor**		**V-Party**		**CSES**	
	2-par.	**3-par.**	**2-par.**	**3-par.**	**2-par.**	**3-par.**	**2-par.**	**3-par.**
Shape (k)	0.900	**0.871**	**0.760**	**0.699**	**0.702**	**0.820**	**0.801**	**0.846**
(Expected: 0.707)	[0.749, 1.067]	**[0.417, 1.634]**	**[0.464, 1.111]**	**[0.245, 1.779]**	**[0.471, 0.959]**	**[0.337, 1.584]**	**[0.117, 1.592]**	**[0.034, 2.794]**
Asymptote	1.073	**1.087**	**0.921**	**0.961**	**1.044**	**1.023**	**1.281**	**1.284**
(Expected: 1)	[1.018, 1.130]	**[0.847, 1.415]**	**[0.819, 1.042]**	**[0.645, 1.268]**	**[0.940, 1.158]**	**[0.850, 1.187]**	**[0.994, 1.624]**	**[0.739, 1.841]**
Intercept		**-0.015**		**-0.035**		**0.055**		**0.041**
(Expected: 0)		**[-0.380, 0.292]**		**[-0.405, 0.302]**		**[-0.153, 0.242]**		**[-0.440, 0.515]**
Num.Obs.	741	741	613	613	677	677	106	106
R2	0.324	0.323	0.060	0.059	0.191	0.192	0.104	0.101
looic	-559.874	-556.856	359.614	360.087	599.189	602.099	80.671	80.947

Coefficients are marked in bold where they are consistent with expectations of 0.707, 1, and 0 for shape, asymptote and intercept respectively.

The fit of the models to the data is generally similar when comparing models with intercepts to models without intercepts. In seven of eight models, the coefficients are consistent with the theoretical predictions. The only exception is the two parameter ParlGov model, where the shape and asymptote parameters are higher than their expected values, and where the shape parameter is closer to 1 (a point prediction I considered but rejected) than to $\frac{1}{\sqrt{2}}$. Point estimates of the asymptote are generally too low for the MARPOR data (which has been accused of exaggerating centrism) and too high for the CSES data. Across all two-parameter specifications, the precision-weighted average for the shape parameter is 0.835 (around 18% higher than predicted) and the precision-weighted average for the asymptote is 1.05 (5% higher than predicted). The fact that the asymptote and shape parameter are both slightly higher than predicted does not mean that polarization is systematically greater than the theory predicts. Recall from equation (8) that the asymptote parameter d multiplies $(N_V - 1)$, whilst the shape parameter k divides it. This means that if we simplify the regression equation slightly by fixing the asymptote to one and finding the value of k that best matches the regression equation for a representative value, we get a value of k very close to $\frac{1}{\sqrt{2}}$. For values of N_s

between 2 and 12, the equation $\frac{N_S-1}{N_S-1+\frac{1}{\sqrt{2}}}$ is on average within 1.6% of the value of the regression equation $1.05\frac{N_S-1}{N_S-0.165}$.

Table 3 estimates the same model but for vote-level polarization. As before, coefficients which are consistent with the theoretical predictions are in bold. Fewer coefficients match the theory: in the ParlGov data, both the shape and asymptote are too high, whereas the opposite problem is found in the MARPOR data. Across all specifications, the precision-weighted average for the shape parameter is 0.956 (14% greater than predicted, and closer to 1 than the theoretical prediction of $\frac{5}{6}$); the precision-weighted average for the asymptote is 1.077 (again 7-8% higher than predicted). Once again, the fact that both the asymptote and the shape parameter are higher than predicted means that if we fix the asymptote to one we return close to the theoretically predicted equation $\frac{N_V-1}{N_V-\frac{1}{6}}$

Table 3: Non-linear regression models of polarization of vote-winning parties

	ParlGov		**Marpor**		**V-Party**		**CSES**	
	2-param.	**3-param.**	**2-param.**	**3-param.**	**2-param.**	**3-param.**	**2-param.**	**3-param.**
Shape (k)	1.051	**0.980**	**0.674**	**0.727**	**0.890**	**0.693**	**0.663**	**0.685**
(Expected: 5/6)	[0.883, 1.232]	**[0.499, 1.816]**	**[0.338, 1.064]**	**[0.174, 2.366]**	**[0.637, 1.167]**	**[0.356, 1.257]**	**[0.041, 1.416]**	**[0.008, 2.113]**
Asymptote	1.110	**1.146**	0.891	**0.866**	1.081	**1.220**	**1.195**	**1.185**
(Expected: 1)	[1.062, 1.160]	**[0.860, 1.527]**	[0.791, 0.995]	**[0.514, 1.292]**	[1.001, 1.169]	**[0.905, 1.569]**	**[0.982, 1.460]**	**[0.656, 1.705]**
Intercept		**-0.040**		**0.040**		**-0.151**		**0.033**
(Expected: 0)		**[-0.448, 0.303]**		**[-0.395, 0.450]**		**[-0.535, 0.189]**		**[-0.449, 0.509]**
Num.Obs.	741	741	613	613	678	678	106	106
R2	0.305	0.305	0.041	0.040	0.157	0.161	0.052	0.050
looic	-627.797	-623.702	523.554	524.007	116.433	115.287	63.804	63.915

Coefficients are marked in bold where they are consistent with expectations of 0.839, 1, and 0 for shape, asymptote and intercept respectively.

The results of these models are therefore consistent with the theoretical predictions made. Although these simple models do not explain a large part of the variation we see in seat- or vote-level polarization, this does not count against the proposed relationship. A model can be correct and still have a low R-squared (Taagepera 2008, 46–48), and most variation in polarization is variation “over time within countries that do not change their electoral systems” (Adams and Rexford 2018, 248). The models therefore provide a reasonable basis for making predictions about likely levels of polarization given the effective number of seat- or vote-winning parties.

6. Extensions

Having corroborated my theory in the preceding section, I now consider extensions of the model. I first change the outcome variable from polarization to the *range* of party positions. While the range of party positions is (in my view) not a good measure of polarization, it may be important to know about in its own right given the importance we attach to political extremism. Second, I extend the model to include two covariates — presidentialism and the degree of programmatic competition — which I use to model the shape and the asymptote of the curve, respectively.

6.1. Extension to range of positions

Several authors (Andrews and Money 2009; Bol et al. 2019) have considered the range of positions within a party system. The range of positions may be calculated on the basis of the positions of all (seat- or vote-winning) parties, or among the top n parties, where n is equal to either N_s or N_v rounded to the nearest integer (Andrews and Money 2009). I use R to refer to the range among all parties, and R^* for the range among the top parties.

I have already stipulated that party positions had a mean of 0 and a standard deviation of 1. I now assume that party positions follow a standard normal distribution. By making this further distributional assumption, I can employ the tools of order statistics to identify the largest and smallest values. For a normal distribution, the expected value of the largest value $E(X_{(n)})$ given a sample of size n is given by integrating the probability distribution of $X_{(n)}$:

$$E(X_{(n)}) = \int_{-\infty}^{\infty} nxf(x)F(x)^{n-1}dx \tag{8}$$

where $f(x)$ is the standard normal density function and $F(x)$ the standard normal cumulative density function. The range is given by twice this value. The expected range for $n = 2, 3, 5, 8, 13$ is therefore 1.13, 1.7, 2.33, 2.85, and 3.34. As n tends to infinity, the range tends to ≈ 6.5. This series can be well approximated[10] by the following (linear) equation (Ramirez and Cox 2012):

$$R = 3\sqrt{\log n} - 1.5 \tag{9}$$

[10]The approximation is within 1.25% of the true value for input values between 3 and 20. The error is -2.8% for $x = 2$.

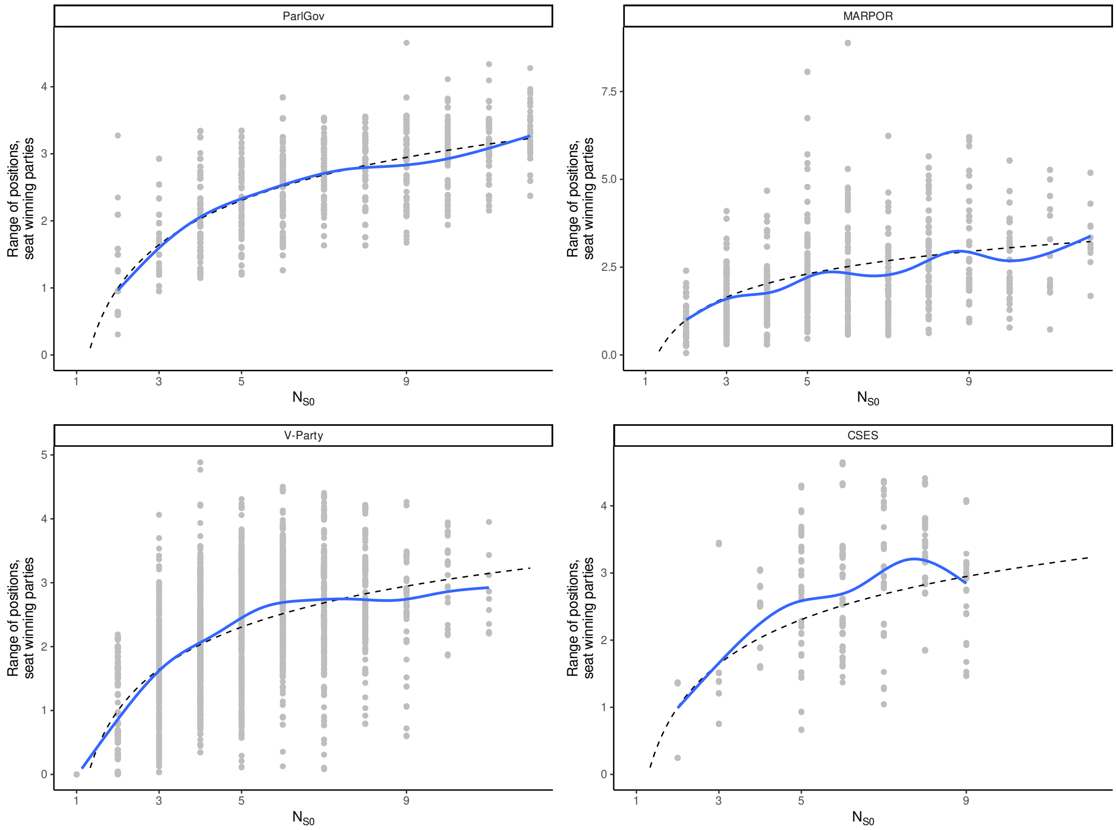

Figure 3: Range plots. Plots from multiply-imputed datasets are plotted on the same graph. Solid blue line gives a generalized additive smooth. Dotted line gives theoretical prediction.

Unlike in the case of polarization, the range of positions in a party system (R) depends much more on the raw count of parties than on the effective number. Figure 3 shows range (on the vertical axis) against the raw count of parties (on the horizontal axis), with a smoothed trend line in solid blue and the dotted line showing equation 14. I hypothesize that the same relationship governs the truncated range (R^*), except with the effective number replacing the raw number.

In Table 4 I estimate models of the range and truncated range of positions of seat-winning parties in a system as functions of the raw and effective number of parties respectively. In both cases I expect the intercept to be close to -1.5, and the coefficient to be close to 3. As in my models of polarization, I estimate separate models for each data-set. I report models of the range of positions of vote-winning parties in the supplementary information.[11] All of the models shown here were estimated using weakly informative N(0, 2.5) priors on both the intercept and coefficients.

The table shows that in seven out of eight models, the coefficient credible intervals are consistent with

[11] Supplementary information (section A6).

Table 4: Linear regression models of range of party positions

	ParlGov		Marpor		V-Party		CSES	
	R	**R***	**R**	**R***	**R**	**R***	**R**	**R***
Intercept	**-1.462**	**-1.174**	**-1.287**	-0.877	**-1.764**	**-1.210**	**-1.411**	**-0.923**
(Expected: -1.5)	**[-1.719, -1.212]**	**[-1.544, -0.787]**	**[-1.894, -0.662]**	[-1.414, -0.328]	**[-2.209, -1.331]**	**[-1.539, -0.867]**	**[-2.910, 0.177]**	**[-2.139, 0.366]**
$\sqrt{\log N_{S0}}$	**2.963**		**2.704**		**3.253**		**3.095**	
(Expected: 3)	**[2.777, 3.148]**		**[2.232, 3.187]**		**[2.904, 3.608]**		**[1.918, 4.253]**	
$\sqrt{\log N_S}$		**2.751**		2.352		**2.744**		**2.947**
(Expected: 3)		**[2.408, 3.086]**		[1.855, 2.845]		**[2.419, 3.042]**		**[1.806, 4.044]**
Num.Obs.	703	703	588	588	676	676	106	106
R2	0.591	0.276	0.171	0.130	0.355	0.333	0.208	0.203
looic	937.255	1542.177	1819.650	1701.877	1512.184	1562.825	261.771	268.979

Coefficients are marked in bold where they are consistent with expectations of -1.5 for the intercept and 3 for the coefficients.

the theoretically predicted values. In the truncated range model using MARPOR data, the intercept is too high and the coefficient too low. As I found in the earlier analysis of polarization, these errors point in the opposite direction, with the consequence that any researcher who uses equation (14) to provide a rough rule of thumb will find that, for typical values of N_s, the predictions are close to the predictions which derive from a regression model. For, say, $N_{S0} = 5$, the prediction from equation 14 is 2.30. The model estimated on Marpor data gives a predicted range of 2.11, while the model estimated on ParlGov data gives a prediction which matches the theoretical prediction almost exactly (2.29).

6.2. Extension to include covariates

The Michaelis-Menten model described in equation (3) can be extended to include additional covariates, but researchers must first specify whether these covariates affect the asymptote or the shape of the curve. I illustrate this by considering two factors which affect the degree of polarization: the programmatic basis of party competition (which affects the asymptote) and presidentialism (which affects the shape). I include these variables because they affect different parts of the model, and because they are not closely related to the effective number of parties.[12]

6.2.1. Programmatic basis of party competition

I have assumed that parties can be characterized primarily by their position in some policy space, and I have operationalized this by using parties' positions on the left-right dimension. This kind of characterization may, however, be a poor guide to some party systems. In some systems, parties compete on the basis of their ability to deliver *targeted* benefits for *clients*. Clientelist party competition is generally understood to be antithetical to programmatic party competition (although parties within the same system may adopt different strategy mixes at different times). In a purely clientelist party system, it would be possible to imagine a large number of parties competing without any programmatic divergence.

Unless clientelism is strongly related to the number of parties (if, for example, only a small number of parties could credibly claim to be reliable brokers of targeted benefits), this means the reasoning above must be amended to take into account varying levels of programmatic competition between parties. I have found it most helpful to think about systems with lower levels of programmatic competition as having a lower asymptote — or, equivalently, being drawn from a separate distribution of parties with lower standard deviation of left-right positions.

As noted, my measure of programmatic competition comes from the V-Dem project, and is a simple dummy variable. Note that to ensure that the asymptote remains centered around 1, I scale this variable by subtracting the mean.

6.2.2. Presidentialism

Curini and Hino (2012, 464) suggest that polarization will be lower in presidential systems, holding other things equal, because presidential elections generate centripetal incentives that spill-over to legislative elections. The same prediction can be reached in a different way by positing that polarization in presidential systems is governed by the effective number of presidential parties rather than the effective number of legislative parties. The effective number of presidential parties (N_P) is generally less than the effective number of seat- or vote-winning legislative parties: Shugart and Taagepera (2017) suggest that $N_P = \sqrt{2N_V}$. Parties' positions might be less variable than the effective number of vote-winning parties suggests if one or more vote-winning parties copies the position of another party whose presidential candidate it supports.

To keep the analysis as simple as possible, I use a simple dummy variable which has a value of 1 if the system has an elected head of state, regardless of whether there is also a separate head of government accountable to the legislature.[13] As with my measure of programmatic competition, I center this variable

[12]This is not true for variables like average district magnitude or the seat product. My strong preference is for researchers to build parsimonious chains of interlocking equations rather than including all antecedents of key independent variables.

[13]See supplementary information for an analysis which focuses on pure presidential systems where there is no head of government accountable to the legislature.

by subtracting the mean.

6.2.3. Model specification

The model remains as it was in equation (8), except that I drop the intercept and model the two terms a and k. Specifically,

$$\log(k_i) = \alpha_0 + \beta_0 \text{Presidential}_i \tag{10}$$

and

$$\log(a_i) = \alpha_1 + \beta_1 \text{Programmatic}_i \tag{11}$$

I model the log of these terms because these terms must be strictly positive, and taking the log of these terms ensures positive values for any possible coefficient values. Given that "elected president" and "programmatic competition" are mean-centered, our expectation for α_0 is that it should be roughly $\log(\frac{1}{\sqrt{2}}) = -0.347$, and our expectation for α_1 is that it should be close to 0. My expectation is that both β_0 and β_1 will be positive. I estimate this model using improper uniform priors on α_0, α_1, β_0 and β_1.

6.2.4. Model results

Table 5 shows the results of these models for the four different datasets. As far as the baseline (parliamentary) shape is concerned, all four models give results consistent with the prediction. Across the four models, the effect of presidentialism upon the shape parameter is only clearly distinct from 0 in the CSES and MARPOR datasets. There is therefore only very mixed evidence concerning the role of presidentialism in altering levels of polarization, at least within the framework I have outlined here.

The effects of programmatic competition, in contrast, are much clearer. The coefficient of programmatic competition on the asymptote is positive and clearly distinct from 0 across all four datasets. The magnitude of the coefficient is similar across three of the four datasets, with a much smaller coefficient for the model estimated on the ParlGov data.

These models show how a particular way of thinking about polarization can be extended to allow for the inclusion of additional covariates, provided researchers reflect on whether these covariates change the asymptotic level of polarization or the shape of the number of parties–polarization curve. For example: researchers who wish to model polarization along the secondary dimension of political competition rather than the primary left-right dimension might include, as a covariate of the *asymptote*, the within-polity average level of competition along that dimension. Conversely, researchers interested in the effects of language cleavages might expect the presence of different linguistic groups to attenuate the relationship between the number of parties and left-right competition, leading to a flatter *shape*. Including multiple covariates can become difficult, and can result in very imprecisely estimated parameters. Researchers who intend to test multiple

Table 5: Non-linear regression models of seat-level polarization including additional covariates

	ParlGov	**MARPOR**	**V-Party**	**CSES**
k: Intercept	**-0.149**	**-0.587**	**-0.508**	**-0.891**
(Expected: -0.347)	**[-0.354, 0.040]**	**[-1.155, -0.080]**	**[-0.888, -0.133]**	**[-3.301, 0.398]**
k: Presidential	0.070	0.386	-0.135	0.409
(Expected: +ve)	[-0.083, 0.216]	[0.008, 0.789]	[-0.533, 0.225]	[-0.816, 2.336]
Max: Intercept	**0.033**	-0.217	**-0.081**	**0.030**
(Expected: 0)	**[-0.032, 0.101]**	[-0.355, -0.075]	**[-0.204, 0.034]**	**[-0.236, 0.300]**
Max: Programmatic	0.047	0.169	0.170	0.180
(Expected: +ve)	[0.009, 0.083]	[0.075, 0.266]	[0.077, 0.264]	[0.012, 0.347]
Num.Obs.	703	588	676	106
R2	0.326	0.092	0.228	0.168
looic	-511.426	332.285	592.663	79.059

Coefficients are marked in bold where they are consistent with expectations of log(0.707) = -0.347 and log(1) = 0 for shape and asymptote intercepts respectively.

hypotheses about the determinants of polarization may, therefore, wish to model "surplus polarization" (actual polarization minus the levels of polarization given by $\frac{N_s-1}{N_s-1+\frac{1}{\sqrt{2}}}$) using a linear model.

7. Conclusions

In this paper, I have set out a relationship between the effective number of seat-winning parties and polarization. This relationship does not depend on any particular theories of party positioning: instead it relies on the offensively simple idea that parties can be thought of as though they were drawn from a super-population with a known standard deviation. To empirically test this relationship, I have estimated nonlinear regressions on four different datasets which measure party positions in very different ways. The theory is corroborated by these regressions and by different extensions to model the range of party positions and the inclusion of additional covariates. On average, the polarization of seat-winning parties is given by the equation $\frac{N_S-1}{N_S-1+\frac{1}{\sqrt{2}}}$ and the polarization of vote-winning parties is given by the equation $\frac{N_V-1}{N_V-\frac{1}{6}}$

I believe my findings can help make sense of certain patterns in the literature. Consider first the presence of null findings. If researchers' samples are not a random draw from the population of electoral democracies, but are instead biased toward larger countries with larger assembly sizes and higher effective numbers of parties, then researchers end up modeling the effects of the number of parties at the flattest point of the curve. This dramatically reduces the power of any statistical test to find a relationship. Second, my findings can explain certain other surprising or conditional results in the literature. Dow (2011) finds that the effective number of parties has a negative effect on polarization when controlling for the electoral system, but this negative relationship may be an artefact of modeling a nonlinear process using two related linear terms: one linear term ends up capturing the steep part of the slope, while another linear term is forced negative to capture the marginally decreasing slope. The conditional findings of Curini and Hino (2012) (the effective number of parties promotes polarization, except where there is a "cabinet coalition habit") may arise from a similar process given that the number of governing parties increases as the number of seat-winning parties increases.

These findings have implications for the literature on party systems generally and electoral reform in particular. My work challenges the idea that the number of parties and the distance between them are unrelated characteristics of a party system, as Dalton (2008) suggests. The relationship between the two characteristics is not **strong**, but it does provide a useful guide to expected levels of polarization in party systems with different effective numbers. Because the effective number of vote- or seat-winning parties is strongly related to the seat product, this relationship can also inform electoral reform. Any proposal which would have the effect of increasing the effective number of parties would (in expectation) also increase polarization by a predictable amount.

My findings also matter for the broader Taageperaan research program of creating parsimonious models of quantities related to the electoral system. Much of this research program has been built on considering logical bounds on variables which have necessary upper or lower limits. Party positions are not limited in this way — but I have shown how, through the heuristic device of envisaging party positions as identically distributed draws, a simple yet quantitatively precise model can be constructed. Simple quantitative models are useful for "constitutional engineering": although the link between the seat product and polarization is very indirect,[14] it is possible to calculate the effect of electoral system reform on polarization by using the Seat Product Model in conjunction with the relationship set out here. For example: if the Netherlands ($S = 150$) moved from having a single electoral district ($M = 150$) to the small-magnitude electoral districts recommended by Carey and Hix (2011) (say, $M = 4$), then the effective number of parties would change from ($(150 \times 150)^{1/6} =$) 5.3 to 2.9 (a 45% reduction), and the degree of polarization from 0.86 to 0.73 (a 15% reduction). Although the most likely effects can be calculated by hand, estimation of the uncertainty surrounding these effects is more complicated: simulations from the posterior distribution of parameters in the V-Party 2-parameter model suggest that the 95% credible interval surrounding this discrete change ranges from -0.10 and -0.18 units. Obviously, the down-stream consequences of such a change are even

[14]See supplementary information (section A7).

harder to calculate, and there is doubt about the effect of party system polarization on such closely-related concepts as affective polarization (see Reiljan, 2020, pp. 389-393).

Future research might proceed in two directions. First, research might tackle polarization along multiple issue dimensions, either dimension-by-dimension or in a multivariate analysis. Second, research might extend this model to alternative measures of party system polarization, such as the measure proposed by Esteban and Ray (1994). Since this measure is moderately correlated with the measure used here ($r = 0.51$), this extension seems eminently possible, and may prove more closely associated with related and normatively-loaded concepts such as affective polarization.

References

Abou-Chadi, Tarik, and Matthias Orlowski. 2016. "Moderate as Necessary: The Role of Electoral Competitiveness and Party Size in Explaining Parties' Policy Shifts." *The Journal of Politics* 78 (3): 868–81.

Adams, James F, and Nathan J Rexford. 2018. "Electoral Systems and Issue Polarization." In *The Oxford Handbook of Electoral Systems*, edited by Erik S. Herron, Robert J. Pekkanen, and Matthew S. Shugart. Oxford: Oxford University Press.

Alvarez, R Michael, and Jonathan Nagler. 2004. "Party System Compactness: Measurement and Consequences." *Political Analysis* 12 (1): 46–62.

Andrews, Josephine T, and Jeannette Money. 2009. "The Spatial Structure of Party Competition: Party Dispersion Within a Finite Policy Space." *British Journal of Political Science* 39 (4): 805–24.

Bakker, Ryan, Catherine De Vries, Erica Edwards, Liesbet Hooghe, Seth Jolly, Gary Marks, Jonathan Polk, Jan Rovny, Marco Steenbergen, and Milada Anna Vachudova. 2015. "Measuring Party Positions in Europe: The Chapel Hill Expert Survey Trend File, 1999–2010." *Party Politics* 21 (1): 143–52.

Benoit, Kenneth, and Michael Laver. 2006. *Party Policy in Modern Democracies.* Routledge.

Bol, Damien, Konstantinos Matakos, Orestis Troumpounis, and Dimitrios Xefteris. 2019. "Electoral Rules, Strategic Entry and Polarization." *Journal of Public Economics* 178: 104065.

Bürkner, Paul-Christian. 2017. "brms: An R Package for Bayesian Multilevel Models Using Stan." *Journal of Statistical Software* 80 (1): 1–28. `https://doi.org/10.18637/jss.v080.i01`.

Carey, John M, and Simon Hix. 2011. "The Electoral Sweet Spot: Low-Magnitude Proportional Electoral Systems." *American Journal of Political Science* 55 (2): 383–97.

Carroll, Royce, and Hiroki Kubo. 2018. "Explaining Citizen Perceptions of Party Ideological Positions: The Mediating Role of Political Contexts." *Electoral Studies* 51 (2): 14–23.

Castles, Francis G, and Peter Mair. 1984. "Left–Right Political Scales: Some 'Expert' Judgments." *European Journal of Political Research* 12 (1): 73–88.

Colomer, Josep M. 2017. "Politometrics: Quantitative Models of Political Institutions." In *Oxford Research Encyclopedia of Politics.*

Coppedge, Michael, John Gerring, Carl Henrik Knutsen, Staffan I. Lindberg, Jan Teorell, Nazifa Alizada, David Altman, et al. 2021. "V-Dem Country-Year/Country-Date Dataset v11." Varieties of Democracy (V-Dem) Project. `https://www.v-dem.net/en/data/data-version-11/`.

Curini, Luigi, and Airo Hino. 2012. "Missing Links in Party-System Polarization: How Institutions and Voters Matter." *The Journal of Politics* 74 (2): 460–73.

Dalton, Russell J. 2008. "The Quantity and the Quality of Party Systems: Party System Polarization, Its Measurement, and Its Consequences." *Comparative Political Studies* 41 (7): 899–920.

De Vries, Catherine E, and Sara B Hobolt. 2020. *Political Entrepreneurs: The Rise of Challenger Parties in Europe*. Princeton University Press.

DiMaggio, Paul, John Evans, and Bethany Bryson. 1996. “Have Americans’ Social Attitudes Become More Polarized?” *American Journal of Sociology* 102 (3): 690–755.

Döring, Holger, and Philip Manow. 2020. “Parliaments and Governments Database (ParlGov): Information on Parties, Elections and Cabinets in Modern Democracies.” *Development Version*.

Dow, Jay K. 2011. “Party-System Extremism in Majoritarian and Proportional Electoral Systems.” *British Journal of Political Science* 41 (2): 341–61.

Dowd, John E, and Douglas S Riggs. 1965. “A Comparison of Estimates of Michaelis-Menten Kinetic Constants from Various Linear Transformations.” *Journal of Biological Chemistry* 240 (2): 863–69.

Esteban, Joan-Maria, and Debraj Ray. 1994. “On the Measurement of Polarization.” *Econometrica: Journal of the Econometric Society* 62 (4): 819–51.

Ezrow, Lawrence. 2008. “Parties’ Policy Programmes and the Dog That Didn’t Bark: No Evidence That Proportional Systems Promote Extreme Party Positioning.” *British Journal of Political Science* 38 (3): 479–97.

Gemenis, Kostas. 2012. “Proxy Documents as a Source of Measurement Error in the Comparative Manifestos Project.” *Electoral Studies* 31 (3): 594–604.

Gross, Donald A, and Lee Sigelman. 1984. “Comparing Party Systems: A Multidimensional Approach.” *Comparative Politics* 16 (4): 463–79.

Honaker, James, Gary King, Matthew Blackwell, and others. 2011. “Amelia II: A Program for Missing Data.” *Journal of Statistical Software* 45 (7): 1–47.

Huber, John, and Ronald Inglehart. 1995. “Expert Interpretations of Party Space and Party Locations in 42 Societies.” *Party Politics* 1 (1): 73–111.

Kam, Christopher, Indridi Indridason, William Bianco, and others. 2017. “Polarization in Multiparty Systems.”

Laakso, Markku, and Rein Taagepera. 1979. “”Effective” Number of Parties: A Measure with Application to West Europe.” *Comparative Political Studies* 12 (1): 3–27.

Lauderdale, Benjamin E. 2012. “Compound Poisson—Gamma Regression Models for Dollar Outcomes That Are Sometimes Zero.” *Political Analysis* 20 (3): 387–99.

Lineweaver, Hans, and Dean Burk. 1934. “The Determination of Enzyme Dissociation Constants.” *Journal of the American Chemical Society* 56 (3): 658–66.

Lowe, Will, Kenneth Benoit, Slava Mikhaylov, and Michael Laver. 2011. “Scaling Policy Preferences from Coded Political Texts.” *Legislative Studies Quarterly* 36 (1): 123–55.

Lührmann, Anna, Nils Düpont, Masaaki Higashijima, Yaman Berker Kavasoglu, Kyle L. Marquardt, Michael Bernhard, Holger Döring, et al. 2020. “Varieties of Party Identity and Organization (V-Party)

Dataset V1." Varieties of Democracy (V-Dem) Project. `https://doi.org/10.23696/vpartydsv1`.

Matakos, Konstantinos, Orestis Troumpounis, and Dimitrios Xefteris. 2016. "Electoral Rule Disproportionality and Platform Polarization." *American Journal of Political Science* 60 (4): 1026–43.

Nyblade, Benjamin. 2004. "The 'Effective' Number of Issue Dimensions: A Measure with Application to West Europe." In *Annual Meeting of the Midwest Political Science Association, April*, 15–18.

Pemstein, Daniel, Kyle L. Marquardt, Eitan Tzelgov, Yi-ting Wang Wang, Juraj Medzihorsky, Joshua Krusell, Farhad Miri, and Johannes von Römer. 2020. "The V-Dem Measurement Model: Latent Variable Analysis for Cross-National and Cross-Temporal Expert-Coded Data." University of Gothenburg: Varieties of Democracy Institute.

Ramirez, Alfredo, and Charles Cox. 2012. "Improving on the Range Rule of Thumb." *Rose-Hulman Undergraduate Mathematics Journal* 13 (2): 1.

Rehm, Philipp, and Timothy Reilly. 2010. "United We Stand: Constituency Homogeneity and Comparative Party Polarization." *Electoral Studies* 29 (1): 40–53.

Reiljan, Andres. 2020. "Fear and loathing across party lines (also) in Europe: Affective polarisation in European party systems". *European Journal of Political Research* 59 (2): 376-396.

Sartori, Giovanni. 2005. *Parties and Party Systems: A Framework for Analysis.* Colchester: ECPR Press.

Schmitt, Johannes. 2016. "How to Measure Ideological Polarization in Party Systems." In *ECPR Graduate Student Conference.*

Shugart, Matthew S, and Rein Taagepera. 2017. *Votes from Seats: Logical Models of Electoral Systems.* Cambridge University Press.

Sigelman, Lee, and Syng Nam Yough. 1978. "Left-Right Polarization in National Party Systems: A Cross-National Analysis." *Comparative Political Studies* 11 (3): 355–79.

Singh, Shane P. 2012. "Where Do Parties Live? Electoral Institutions, Party Incentives, and the Dimensionality of Politics." *Social Science Quarterly* 93 (4): 950–67.

Sørensen, Rune J. 2014. "Political Competition, Party Polarization, and Government Performance." *Public Choice* 161 (3-4): 427–50.

Taagepera, Rein. 1997. "Effective Number of Parties for Incomplete Data." *Electoral Studies* 16 (2): 145–51.

———. 2008. *Making Social Sciences More Scientific: The Need for Predictive Models.* Oxford: Oxford University Press.

Taylor, Michael, and Valentine M Herman. 1971. "Party Systems and Government Stability." *American Political Science Review* 65 (1): 28–37.

Warwick, Paul. 1992. "Ideological Diversity and Government Survival in Western European Parliamentary Democracies." *Comparative Political Studies* 25 (3): 332–61.

Zorn, Christopher JW. 2000. “Modeling Duration Dependence.” *Political Analysis* 8 (4): 367–80.