Comment on "Unified Formalism of Andreev Reflection at a Ferromagnet/Superconductor Interface" by T. Y. Chen, Z. Tesanovic, and C. L. Chien

A recent paper of Chen *et al.* [1], claims to have derived an allegedly, previously unavailable "unified" Andreev Reflection (AR) formalism for an arbitrary spin polarization P that recovers earlier results as its special limits. In this Comment we show that, contrary to this claim, there are numerous works correctly solving the problem formulated in Ref. [1] for an arbitrary P [2–7], while Ref. [1] fails to correctly incorporate $P \neq 0$ effects and violate basic physical principles.

In the $P = 0$ limit the approach of Ref. [1] is identical to the one dimensional (1D) BTK model [8] for nonmagnetic metals, while for $P \neq 0$ the only difference is postulating an AR wavefunction that has an additional evanescent wave contribution parameterized by a decay constant α, $\psi_{AR} = a \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \exp[(\alpha + i)q^+ x]$, with $\alpha = 2\sqrt{P/(1-P)}$. However, this wavefunction violates the charge conservation: the corresponding charge current density is $j_{AR}(x) \propto \mathrm{Im}[\psi_{AR}^* \nabla \psi_{AR}] \propto q^+ |a|^2 \exp[2\alpha q^+ x]$. In the F-region, for any $0 < \alpha < \infty$, *i.e.*, $0 < P < 1$, the divergence of the *total* current is finite and thus the total charge is not conserved (in particular, for $0 < x \lesssim 1/2\alpha q^+$), signaling that ψ_{AR} is unphysical. Even for the $P = 1$ half-metal (HM) state, where the above expression correctly gives $j_{AR} = 0$, it is still incorrect. For a HM the AR electron decays with a finite (not infinitely small, as postulated in Ref.[1]) penetration depth, which depends on the electronic structure details, primarily the size of the gap in the nonmetallic spin channel [9].

The rationale for inventing a new ψ_{AR} [1] was based on the premise that: (a) the result in Ref. [3] was derived only in the extreme case of a fully-polarized HM with $\tilde{\psi}_{AR} = a \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \exp(\kappa x)$ and a nonmagnetic metal with $\tilde{\psi}_{AR} = a \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \exp(ikx)$, while postulating that for an intermediate P the current will be a linear combination of these cases, (b) no prior work had treated a $0 < P < 1$ case, and (c) one can meaningfully define P of an individual electron. Regarding (a-b), the derivation in Ref. [3], as well as in other works [4–7], is rigorous for an arbitrary P. The last point, (c) is particularly misleading. In fact, one cannot define a BTK model in 1D with an arbitrary P because any given electron in a metal Andreev-reflects either into a propagating, or into an evanescent wave. Finite P simply means that some current-carrying electrons belong to the former group and the others to the latter; one can only define transport spin polarization in a multielectron system, where the numbers of electron states at the Fermi surface (conductivity channels) for the two spin directions differ. If the electron wavevector $\mathbf{k}$ is decomposed into a non-conserved k_x (normal to the F/S interface) and conserved [10] 2D $\mathbf{k}_\parallel$, then, after the usual quantization of $\mathbf{k}_\parallel$, one finds that the total number of states available for transport in the x direction (*i.e.,* the number of the conductivity channels) for a given spin direction is proportional to the total area of the Fermi surface projection on the interface plane, given by the Fermi surface average of the Fermi velocity, $n = \langle N(E_F) v_{Fx} \rangle$ [3]. After summation over *all* electronic states the total current *turns out* to be a linear combination (with the weights defined by P), of the solutions of the 1D BTK model with $P = 0$ and $P = 1$. This was not postulated, but derived, in numerous papers (see the discussion in Ref. 7).

In contrast to the "universal" P in the Ref. [1], independent of electronic mass, Fermi wavevectors, or any band structure at all, the real transport spin polarization for AR spectroscopy depends on the *overall* Fermi surface properties. Moreover, there is no unique spin polarization (even for a uniform bulk metal), as it depends on an experimental probe. In fact, the definition used in Ref. [1] (neglecting the Fermi velocities) does not correspond to the AR experiments, but rather to spin-polarized photoemission.

To summarize, Ref. [1] has misinterpreted or ignored previous works where the posed problem has been correctly solved for an arbitrary P, and attempted to supplant the existing solution with an incorrect formula, postulating an unphysical wavefunction for the reflected electron, which does not conserve charge and has an incorrect HM limit. They proceeded by calculating the current due to ψ_{AR} at $x = 0$, even though the actual current is measured far away from the interface (where they would have gotten zero Andreev contribution for any P and grave disagreement with the experiment). Furthermore, they have arbitrarily decided that the penetration depth for an electron inside the band gap in the transport-spin-minority channel is uniquely defined by the spin polarization (these two quantities are in fact unrelated). As a result, they arrived at a formula that for their own experimental data provides a fit that is essentially identical to that obtained by using Ref. [3] (the difference is below potential errors introduced by the BTK approximations of a step-shaped pairing potential and spin-independent δ-shaped barrier [11]).

Finally, we note that the inclusion of inelastic scattering using a phenomenological finite $\Gamma \neq 0$, another claimed novelty of Ref. [1] has already been previously employed for AR with arbitrary P [6, 12].

It may be that the formulas contrived in Ref. [1] fit a particular experiment. However, there is a maxim attributed to Niels Bohr, that there exists an infinite number of incorrect theories that correctly describe the finite number of experiments.

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[9] Ref. [3], as well as some others, eventually uses the limit of the infinitely small penetration depth, because the final result depends on this parameter very little, but it is important to use in the derivation a physically meaningful wavefunction that allows for a finite penetration depth. This inconsistency in Ref. [1] is a part of a bigger problem of using the same k_F for both spin channels, despite the fact that as $P \to 1$ one of the two $k_F^{\pm}$ gradually vanishes [2, 4, 5, 7].

[10] For a ballistic flat interface. The generalization onto a diffusive case, where only the total number of the conductivity channels for each spin direction, but not the individual $\mathbf{k}_{\parallel}$ are conserved, is straightforward [3].

[11] For other parameters [see their Fig. 2(c)] their approach leads to unphysical kinks at zero bias and unphysical notches near the gap voltage.

[12] Other authors observed that instead of adding a Γ, at an extra computational cost, one can simply artificially increase the temperature in the original expressions, since the effect of the two types of broadening on the conductance curves is essentially the same, see, *e.g.*,Y. Bugoslavsky, Y. Miyoshi, S. K. Clowes, W. R. Branford, M. Lake, I. Brown, A. D. Caplin, and L. F. Cohen, Phys. Rev. B **71**, 104523 (2005)