

# Melt Redistribution by Pulsed Compaction within UltraLow Velocity Zones

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## Abstract

This article investigates the melt distribution and resultant seismic signature within UltraLow Velocity Zones (ULVZs) forced by pulsed compaction at the mantle-ULVZ interface. Transient flow in the ambient mantle causes periodic compaction in the ULVZ matrix. For a neutrally buoyant melt, an initially uniform melt distribution is modified by the formation of a thin, decompacting, melt-rich layer near the top and a wide, melt-poor, compacting layer near the bottom. Such a structure is reflected in large reductions in  $S$  and  $P$  wave velocities near the top and smaller reductions near the bottom of the ULVZ. A dense melt pools near the bottom of the ULVZ, leading to larger reductions in seismic wave speed near the bottom. The magnitude of melt segregation in the decompaction layer is controlled by the viscosity of the ULVZ matrix in a nonlinear fashion. At high ULVZ viscosities, the compaction length becomes substantially larger than the dimension of thin

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ULVZs, leading to a reduction in the magnitude of melt segregation in the decompaction layer. In a ULVZ of matrix viscosity  $10^{20}$  Pas containing an average melt volume fraction of 0.05, formation of decompacting, melt-rich layers reduce the  $S$  and  $P$  wave velocities by 25% and 8%, respectively. Vertical variation in seismic velocity reduction within the ULVZ column is a consequence of melt redistribution by compaction, rather than variation of melt microstructure within the ULVZ.

*Keywords:* Core-Mantle Boundary; Two-Phase Flow; ULVZ; Compaction; microgeodynamics

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## 1. Introduction

A number of thin, dense UltraLow Velocity Zones (ULVZs), characterized by low seismic shear wave speed appear on the mantle side of the Earth's Core-Mantle Boundary (CMB). The ULVZs, which are up to 10% denser than the surrounding mantle, are characterized by differential reductions of  $S$  (up to 30%) and  $P$  (up to 10%) wave velocities (Rost et al., 2010, 2006; Williams and Garnero, 1996). The elevated density and body wave speed reduction within the ULVZs indicates that the ULVZs are chemically anomalous compared to the surrounding lower mantle. Such chemical anomaly can arise from a neutrally buoyant interstitial melt hosted in an iron-rich solid matrix (Hernlund and Jellinek, 2010; Ohtani and Maeda, 2001; Stixrude and Karki, 2005). A phase equilibria study by Fiquet et al. (2010) suggests that fertile peridotite reaches its solidus at 4180 K and 135 GPa, implying the likely presence of partial melting within the ULVZ.

The chemically anomalous ULVZs are also dynamically coupled to flow

16 in the surrounding mantle. A number of recent studies demonstrate the two-  
17 way nature of this coupling. First, the presence of a ULVZ-like, thin, dense,  
18 and low-viscosity layer can anchor mantle plumes to the CMB, and con-  
19 tribute to the longevity of plumes (Jellinek and Manga, 2004). Second, man-  
20 tle motion-induced stirred compaction within the dense ULVZ redistributes  
21 nearly neutrally buoyant melt (Hernlund and Jellinek, 2010). Third, near the  
22 margin of Large Low Shear Velocity Provinces (LLSVPs), ULVZ-like struc-  
23 tures break-up, coalesce, and are mobilized by circulation internal to the  
24 LLSVPs (McNamara et al., 2010). Finally, the curvature and topography  
25 of the ULVZ-mantle interface results from dynamic interaction between the  
26 mantle and the ULVZ, and is modulated by the density and viscosity of the  
27 ULVZ material (Bower et al., 2011; Hier-Majumder and Revenaugh, 2010).

28 Such transient variation in the ambient mantle flow around a partially  
29 molten ULVZ will also redistribute melt by compacting the matrix, leading  
30 to spatial and temporal variations in effective elastic properties. In a study  
31 of anomalous velocities of the core-reflected *ScP* phase, Rost et al. (2006)  
32 observed a downward increase in seismic wave speed within the ULVZ. They  
33 suggested that such an increase likely arises from a change in the melt mi-  
34 crostructure from tubules near the top to spherical pockets near the bottom  
35 of the ULVZ. Such a conclusion would also imply that the thermodynamic  
36 forces that control the melt microstructure, must also display a corresponding  
37 variation. The source of such a variation, however, is not clear. In addition  
38 to internal variations, forced by transient coupling with mantle flow, seismic  
39 signature of different ULVZs will vary based on the nature of the surround-  
40 ing mantle flow. While Hernlund and Jellinek (2010) studied the effect of

41 an imposed, steady-state matrix velocity on redistribution of melt within a  
42 ULVZ, the role of transient compaction on melt redistribution and the seismic  
43 signature within ULVZs has not yet been studied.

44 The nature of time-dependence of the ULVZ compaction is difficult to con-  
45 strain. On the surface, variations in dynamic topography, driven by mantle  
46 flow, can be constrained using various geological and geophysical techniques.  
47 At the CMB, constraining the time dependence of mantle flow is much less  
48 straightforward, as seismic observations only provide the information at the  
49 present time. In the absence of observational constraints, one can describe  
50 the transient forcing on compaction of the ULVZ as a sum of a number of  
51 periodic variations of various frequencies. One can then study the response  
52 of the internal structure of the ULVZ to each individual frequency, over a  
53 range of frequencies. This is the approach taken in this article. The time pe-  
54 riod of such periodic variations should capture relatively rapid gravitational  
55 drainage of dense melts and slower oscillatory mass transport through plume  
56 conduit waves. In a compacting ULVZ matrix, gravitational drainage can  
57 segregate melt, denser than the matrix by a few percents, into a thin layer  
58 near the bottom over a few ka (Hier-Majumder et al., 2006). Numerical and  
59 analog material experiments indicate that mass is transferred in the plume  
60 conduit in periodic, conduit waves with time periods of a few Mas (Olson  
61 and Christensen, 1986; Schubert et al., 1989). The time periods intermediate  
62 to these two time scales are crucial to understand the structural evolution of  
63 the ULVZs in response to the relevant forces. Accordingly, the time periods  
64 of pulsation in this study were chosen to provide a glimpse into the response  
65 of the ULVZ to both short and long term variations.

66 As compaction of the matrix redistributes the melt, the elastic properties  
67 are also modified. Using robust models of effective elastic properties, one can  
68 predict such spatial and temporal variations in the seismic signature. In a  
69 recent microgeodynamic model, Wimert and Hier-Majumder (2012) demon-  
70 strated that the seismic signature of the ULVZs can be explained by only 0.1  
71 volume fraction of melt residing in tubules. In that study, only average wave  
72 speed reduction within the ULVZ was considered. In contrast, in recent mod-  
73 els of coupling between mantle flow and the ULVZ, no robust microstructural  
74 models were used to predict seismic profiles (Hernlund and Jellinek, 2010;  
75 Hier-Majumder and Revenaugh, 2010). This work bridges the gap, by cou-  
76 pling melt redistribution with a microgeodynamic model, providing a first  
77 order prediction on the vertical variation of the seismic profile within the  
78 ULVZ.

79 This article presents numerical results for the transient internal struc-  
80 ture of a partially molten column within the ULVZ, with a time-dependent  
81 mantle forcing. As outlined in Figure 1, the matrix velocity at the ULVZ-  
82 matrix interface is forced to oscillate over a range of frequencies, inducing  
83 a pulsed compaction of the ULVZ matrix. This article simulates the redis-  
84 tribution of both neutrally buoyant and dense interstitial melts within the  
85 ULVZ and the resultant reductions in  $S$  and  $P$  wave velocities, for five differ-  
86 ent viscosities of the ULVZ matrix. This calculation neglects the role of melt  
87 generation (Hewitt and Fowler, 2008; Rudge et al., 2011; Sramek et al., 2006)  
88 and dissolution-precipitation (King et al., 2011; Takei and Hier-Majumder,  
89 2009). Since this calculation is carried out in a one-dimensional column, it  
90 also neglects the effect of lateral gradients of dynamic pressure arising from

91 circulation within the ULVZ (Hernlund and Jellinek, 2010).

## 92 **2. Formulation**

93 The schematic diagram in Figure 1 outlines the problem. The domain  
94 in our formulation represents a column within the ULVZ, as depicted in the  
95 figure. The top of the column represents the mantle-ULVZ interface, and  
96 the bottom represents the CMB. When compacted, melt within this cylin-  
97 drical column can migrate laterally to the other parts of the ULVZ, as if  
98 the curved wall of the cylinder is permeable. In this one dimensional model,  
99 we achieve this effect by prescribing a permeable bottom boundary, as there  
100 are no lateral boundaries to impose a permeable boundary condition. As  
101 discussed above, flow in the ambient mantle couples with the internal struc-  
102 ture of the ULVZ through the top boundary. We impose a time dependent  
103 boundary condition for the matrix velocity at the top. Transient compaction  
104 is forced within the ULVZ layer by the transient mantle-ULVZ interface ve-  
105 locity. Despite the simplifications associated with the one dimensional model,  
106 this model quantifies the manner in which dynamic coupling between mantle  
107 flow and compaction within the ULVZ, modifies the spatial and temporal  
108 signature of S and P wave speeds.

### 109 *2.1. Two-phase flow in the ULVZ*

110 Consider a partially molten column within the ULVZ of height  $L$  above  
111 the CMB. Mass and momentum within this column are conserved by two  
112 coupled Partial Differential Equations (PDEs) (Bercovici et al., 2001; Hier-  
113 Majumder et al., 2006; McKenzie, 1984; Ricard et al., 2001; Richter and  
114 McKenzie, 1984). In one dimension, two PDEs – governing the conservation

115 of mass and momentum involving the melt volume fraction  $\phi(z, t)$ , and the  
 116 matrix velocity,  $w(z, t)$  are given by

$$117 \quad \frac{\partial \phi}{\partial t} = \frac{\partial}{\partial z} ((1 - \phi)w) \quad (1)$$

$$118 \quad 0 = (1 - \phi)\chi^* \left( \frac{\partial \phi}{\partial z} \right) + \frac{\partial}{\partial z} \left( \mu^* \left( \frac{K_0}{\phi} + \frac{4}{3} \right) (1 - \phi) \frac{\partial w}{\partial z} \right) \\ 119 \quad - (1 - \phi)\Delta\rho g - \frac{c(w - V(t))}{\phi^2}, \quad 0 \leq z \leq L \quad (2)$$

120 where  $\chi^*$  arises from the variation in surface tension with melt volume frac-  
 121 tion (Hier-Majumder et al., 2006),  $\mu^*$  is the melt fraction dependent vis-  
 122 cosity of the matrix (Scott and Kohlstedt, 2006),  $K_0$  is a constant  $\mathcal{O}(1)$   
 123 (Bercovici et al., 2001),  $c$  is the coefficient of frictional resistance,  $\Delta\rho$  is the  
 124 density contrast between the ULVZ matrix and the melt,  $g$  is gravity, and  
 125  $V(t) = \phi v + (1 - \phi)w$ , is the volume weighted average of matrix ( $w$ ) and  
 126 melt ( $v$ ) velocities. While mass conservation of the matrix and melt phases  
 127 requires  $V$  to be constant throughout the domain of the problem, it can vary  
 128 with time. We choose this velocity to be the transient matrix velocity at  
 129 the mantle-ULVZ interface. A consequence of this choice is that the top  
 130 boundary of the domain is rendered impermeable, as discussed in detail in  
 131 Appendix A.

132 We nondimensionalize  $z$  by  $L$ , the velocities by  $\rho g/c$ , and the surface  
 133 tension  $\chi^*$  by a constant  $\sigma/d$ , where  $\sigma$  is the grain boundary energy and  $d$   
 134 is the grain size. Following Bercovici et al. (2001), we also set  $K_0 = 4/3$ ,  
 135 leading to the nondimensional governing equations,

$$136 \quad \frac{\partial \phi}{\partial t} = \frac{\partial}{\partial z} ((1 - \phi)w) \quad (3)$$

$$137 \quad 0 = \frac{(1 - \phi)\chi^*}{\mathcal{B}} \frac{\partial \phi}{\partial z} + \frac{4}{3} \left( \frac{\delta}{L} \right)^2 \frac{\partial}{\partial z} \left( \mu^* \frac{1 - \phi^2}{\phi} \frac{\partial w}{\partial z} \right)$$

$$138 \quad -R(1 - \phi) - \frac{1}{\phi^2} (w - V(t)) \quad 0 \leq z \leq 1 \quad (4)$$

139 where  $\delta = \sqrt{\mu/c}$ , is the compaction length,  $R = \Delta\rho/\rho$  is the fractional  
 140 density contrast between the ULVZ and the melt, and the nondimensional  
 141 Bond number  $\mathcal{B} = (\rho g L d)/\sigma$  is the ratio between forces arising from buoyancy  
 142 and surface tension. Assuming that the melt resides in tubules along grain  
 143 edges, the frictional resistance,  $c$ , depends on the grain size,  $d$ , melt viscosity  
 144  $\mu_m$ , and the background melt fraction  $\phi_0$  by the relation (Hier-Majumder,  
 145 2011)

$$146 \quad c = \mu_m \frac{72\pi}{d^2 \phi_0^2}. \quad (5)$$

147 The quantity  $\mu^*$  in equation 4 arises from melt weakening of the matrix.  
 148 Currently, no direct measurement of melt weakening is available under CMB-  
 149 like conditions, as the stress levels at CMB remain poorly constrained and  
 150 deformation apparatus for rheological measurements under such conditions  
 151 are currently unavailable. As a result, following Scott and Kohlstedt (2006),  
 152 we use  $\mu^* = 7 \exp(-\alpha\phi)/3$ , where  $\alpha = 25$ , even if the measurements were  
 153 carried out at a confining pressure of 300 MPa. The melt fraction dependent  
 154 surface tension force,  $\chi^*$ , is taken from Hier-Majumder et al. (2006). In the  
 155 absence of pulsation of the boundary,  $V(t) = 0$ , and the governing equations  
 156 3 and 4 reduce to equations 15 and 16 of Hier-Majumder et al. (2006).

157 The governing PDEs were solved numerically by a finite volume dis-  
 158 cretization using 500 nodes in an object oriented Fortran 2003 suite of codes.  
 159 The velocity boundary conditions for the momentum equation were  $w(0, t) =$   
 160  $0$  and  $w(1, t) = V(t)$ . Following the definition of  $V(t)$ , as demonstrated in  
 161 Appendix A, the latter boundary condition renders the top of the ULVZ im-  
 162 permeable, an appropriate approximation for the chemically anomalous layer

163 with a sharp boundary. The boundary condition for the melt at the top and  
 164 the bottom were fixed at  $\phi(0, t) = \phi(1, t) = \phi_0$ , where  $\phi_0$  is the constant  
 165 background melt fraction. Combining the velocity and melt volume fraction  
 166 boundary conditions at the bottom, we notice that melt velocity in and out  
 167 of the bottom boundary is given by  $V/\phi_0$ . Since their signs are the same,  
 168 during the downward motion of the top boundary, melt is expelled through  
 169 the bottom, and during upward motion, melt percolates back in through the  
 170 bottom boundary. The initial condition for the melt volume fraction was  
 171  $\phi(z, 0) = \phi_0 + \bar{\phi}(z)$ , where the white noise perturbation function  $\bar{\phi}(z)$  varied  
 172 between 0 and  $10^{-5}$ . At each time step, the algebraic equations resulting  
 173 from discretization of the PDEs were solved using Linear Algebra PACKage  
 174 (LAPACK) routines available through intel Math Kernel Library. Once the  
 175 solution for matrix velocities were obtained, the melt fraction was updated  
 176 by integrating the mass conservation equation 3 in time using the Courant  
 177 criterion. The numerical solutions compare well with analytical solutions  
 178 available for simple cases. One such analytical solution, following the models  
 179 of forced compaction by Ricard et al. (2001) is compared against the nu-  
 180 merical solutions for matrix and segregation velocities in Appendix B. In  
 181 Appendix B, we also report the methods and results from a series of nu-  
 182 merical experiments testing the resolution of the model with respect to grid  
 183 size.

184 The characteristic length scale  $L$  is 20 km. Five different values of the  
 185 matrix viscosity ranging between  $10^{20}$  and  $10^{24}$  Pas were used in the simu-  
 186 lation. The nondimensional constant  $R$  was set to 0 and  $-0.03$  for the two  
 187 different cases. The volume averaged boundary velocity was prescribed as

188  $V(t) = 2\pi\omega V_0 \sin(2\pi\omega t)$ . A set of numerical experiments for four different  
189 ordinary frequencies of pulsation  $1 \times 10^{-2}$ ,  $6.6 \times 10^{-3}$ ,  $3.3 \times 10^{-3}$ , and  $1 \times 10^{-3}$ ,  
190 were carried out. The dimensional time periods corresponding to these fre-  
191 quencies range between 0.1 and 1 Ma. The dimensionless amplitude of the  
192 oscillation was fixed at  $V_0 = -5 \times 10^{-3}$ . Values of all dimensional constants  
193 and nondimensional numbers are provided in Table 1.

## 194 *2.2. Calculation of seismic velocities*

195 Two groups of parameters determine the seismic signature of partially  
196 molten rocks. The first group involves the elastic moduli and density of  
197 the matrix and the melt. The second group of parameters arise from the  
198 volume fraction and grain-scale distribution of melt. The second group of  
199 parameters are represented by contiguity, the fractional area of intergranular  
200 contact (Hier-Majumder, 2008; Park and Yoon, 1985; Takei, 1998, 2002).  
201 Contiguity in a partially molten aggregate depends strongly on melt volume  
202 fraction (von Bargen and Waff, 1986; Wimert and Hier-Majumder, 2012) and  
203 modestly on the wetting angle (Hier-Majumder and Abbott, 2010). Wetting  
204 angles under ULVZ-like conditions are currently unconstrained. This work,  
205 therefore, ignores the influence of wetting angle and focuses on the first order  
206 influence of melt volume fraction on the seismic signature.

207 For the matrix, we use bulk and shear moduli and Poisson's ratio from the  
208 PREM model under CMB condition. For the melt phase, we determined the  
209 bulk modulus of a peridotite melt using the Vinet equation of state based on  
210 data from Guillot and Sator (2007). While the presence of Fe-rich solids likely  
211 reduce the effective bulk and shear moduli of the ULVZ (Mao et al., 2006;  
212 Wicks et al., 2010), the extent of reduction depends on the volume fraction

213 of these solids (Wimert and Hier-Majumder, 2012), which is unknown. To  
 214 reduce the uncertainty, we prescribed PREM-like elastic properties to the  
 215 ULVZ matrix. The calculated seismic velocity reductions, therefore, provide  
 216 only upper limits. If the presence of Fe-rich solids are accounted for, less  
 217 melt volume fraction will be necessary to generate the seismic signature.  
 218 See Wimert and Hier-Majumder (2012) for discussions on this trade-off and  
 219 the relatively small influence of variations in the melt bulk modulus on the  
 220 seismic signature.

221 Contiguity at each point within the ULVZ was calculated from the melt  
 222 volume fraction using the parameterization from Wimert and Hier-Majumder  
 223 (2012). In their microstructural model, the melt resides in tubules. As the  
 224 melt fraction increases, the area of cross section of melt tubules increase and  
 225 intergranular contacts are wetted, reducing the contiguity. The relation be-  
 226 tween contiguity,  $\psi$ , and melt fraction,  $\phi$ , is given by the polynomial function

$$227 \quad \psi = -8065\phi^5 + 6149\phi^4 - 1778\phi^3 + 249\phi^2 - 19.77\phi + 1, \quad (6)$$

228 where  $0 \leq \phi \leq 0.25$ .

229 Relative  $S$  and  $P$  wave velocities,  $V_S/V_0^S$  and  $V_P/V_0^P$ , were calculated by  
 230 using the ‘equilibrium geometry’ model of Takei (2002). In this model, the  
 231 quantities are expressed as functions of effective elastic moduli and density,

$$232 \quad \frac{V_S}{V_0^S} = \sqrt{\frac{(N/G)}{(\bar{\rho}/\rho)}}, \quad (7)$$

233 and

$$234 \quad \frac{V_P}{V_0^P} = \sqrt{\frac{K_e/K + 4\beta/3 (N/G)}{(1 + 4\beta/3) (\bar{\rho}/\rho)}}, \quad (8)$$

235 where  $K$ ,  $G$ , and  $\rho$  are the bulk modulus, shear modulus, and density of the  
 236 solid, and  $\beta = G/K$ . The quantity  $\bar{\rho}$  is the volume averaged density of the

237 aggregate. The quantity  $N$  is the shear modulus of the intergranular skeletal  
 238 framework and  $K_e$  is the effective bulk modulus of the grain-melt aggregate.

239 The effective elastic moduli of the partially molten aggregate can be ex-  
 240 pressed in terms of contiguity  $\psi$  and the elastic moduli of the solid and the  
 241 melt as

$$242 \quad N = G(1 - \phi)g(\psi) \quad (9)$$

$$243 \quad K_e = K \left[ (1 - \phi)h(\psi) + \frac{(1 - (1 - \phi)h(\psi))^2}{(1 - \phi)(1 - h(\psi)) + \phi K/K_m} \right], \quad (10)$$

244 where  $K_m$  is the bulk modulus of the melt, and the functions  $g(\psi)$  and  $h(\psi)$   
 245 are given by,

$$246 \quad g(\psi) = 1 - (1 - \psi)^n, \quad (11)$$

$$247 \quad h(\psi) = 1 - (1 - \psi)^m, \quad (12)$$

248 where the exponents  $n$  and  $m$  depend on the contiguity,  $\psi$ , and Poisson's  
 249 ratio,  $\nu$  (Takei, 2002, App. A).

250 At each time step of the numerical solution, the melt distribution within  
 251 the ULVZ is determined by solving the coupled mass and momentum conser-  
 252 vation equations 3 and 4. Then, the parameterization in equation 6 was used  
 253 to evaluate the contiguity at each point within the ULVZ. Knowing the con-  
 254 tiguity,  $\psi$ , the effective elastic moduli in equations 9 and 10 were evaluated,  
 255 which were subsequently used to evaluate  $V_S/V_0^S$  and  $V_P/V_0^P$  from equations  
 256 7 and 8, respectively.

### 257 **3. Results**

258 The transient internal structure of the ULVZ depends strongly on tran-  
 259 sient forcing from mantle flow, density contrast between the melt and ULVZ

260 matrix, and the viscosity of the ULVZ matrix. The seismic signature varies  
261 spatially and temporally within the ULVZ differently for different melt den-  
262 sities. Distribution of neutrally buoyant melts are more strongly influenced  
263 by pulsed compaction. These results are discussed in detail below.

### 264 3.1. Numerical solution

#### 265 3.1.1. Internal structure of the ULVZ

266 Pulsed compaction redistributes the neutrally buoyant melt within the  
267 ULVZ, leading to a periodic oscillation in the spatially varying seismic signa-  
268 ture. The series of plots in Figure 2 outline the melt distribution  $\phi$ , matrix  
269 velocity,  $w$ , and the relative  $S$  and  $P$  wave velocities,  $V_S/V_0^S$  and  $V_P/V_0^P$ , re-  
270 spectively. The plot in Figure 2(a) depicts a narrow, melt-rich, decompaction  
271 layer that forms near the top and a broad compacted, melt-poor region that  
272 forms near the bottom during the downward motion of the boundary. The  
273 matrix velocities in Figure 2(b) are negative throughout the column dur-  
274 ing the downward motion and change sign during the upward motion of the  
275 mantle-ULVZ interface. As the melt-rich layer forms near the top, to con-  
276 serve mass, the matrix collects near the bottom, illustrated by the downward,  
277 negative matrix velocity. Comparison between the melt fraction and veloc-  
278 ity profiles for the case B in Figures 2(a) and (b) indicates a delay between  
279 the imposition of the maximum negative matrix velocity and formation of  
280 the decompaction layer at the top. The legends on the curve in this panel  
281 indicate the time steps in a given cycle, annotated in Figure 4. Viscosity of  
282 the ULVZ matrix is  $10^{20}$  Pas for these simulations.

283 The seismic velocities within the ULVZ column reflect the spatial and  
284 temporal variations in melt volume fraction. As a result of melt redistribu-

285 tion, calculated values of  $V_S/V_0^S$  and  $V_P/V_0^P$ , in Figure 2(c) and (d) display a  
286 sharp drop near the top and a gradual increase towards the bottom following  
287 periods of downward motion of the boundary.

288 In contrast to the neutrally-buoyant melt, the dense melt percolates down  
289 the matrix, generating a melt-rich layer near the bottom of the column and  
290 a compaction layer near the top, as illustrated in Figure 3(a). Similar to  
291 Figure 2, the matrix velocity distribution within the ULVZ is forced by the  
292 prescribed velocity at the ULVZ-matrix interface, as demonstrated in panel  
293 (b). The legends on the curves in panel (b) correspond to the same times  
294 as in Figure 2(b). Notice that the magnitude of the decompaction layer in  
295 panel (a) is much smaller than the magnitude of the decompaction layer in  
296 panel (a) of Figure 2. The seismic signature within the ULVZ, depicted in  
297 Figures 3(c) and (d) display a decrease in  $S$  and  $P$  wave velocities from under  
298 the decompaction layer to the bottom of the ULVZ. The matrix viscosity for  
299 these simulations is also  $10^{20}$  Pas.

300 Both of the above cases illustrate variations in the melt distribution along  
301 the entire depth of the ULVZ with time. Evolution of the internal structure  
302 of ULVZ, discussed above, was confined within one cycle of topographic os-  
303 cillation. In the following section, we take a look at the variation of melt  
304 volume fraction and the resulting seismic signature near the top and the  
305 bottom of the ULVZ over the length of several cycles of pulsed compaction.

### 306 *3.1.2. Melt redistribution with time*

307 The internal structure of the ULVZ and the resultant seismic signature  
308 respond to the pulsation of ULVZ topography depending on the density con-  
309 trast between the melt and the matrix. This section presents results on

310 temporal variation for a matrix viscosity of  $10^{20}$  Pas.

311 The series of plots in Figure 4 illustrates the coupling between pulsed  
312 compaction and melt redistribution within the layer. Dimensional velocity  
313 of the top of the ULVZ layer,  $V$ , is plotted as a function of time in Figure  
314 4(a). A negative value of  $V$  implies periods of compaction of the ULVZ,  
315 as the mantle flow exerts a compression on the ULVZ through the mantle-  
316 ULVZ interface. Over several hundred ka, evolution of the average melt  
317 volume fraction within the ULVZ depends strongly on the density contrast  
318 between the melt and the matrix. The locally averaged melt volume fraction  
319 from the top and bottom 400 m are plotted as functions of time in Figures  
320 4(b) and (c), respectively. The top decompaction layer develops following  
321 the downward displacement of the top boundary. The average melt fraction  
322 in this layer reaches a maximum as the imposed velocity becomes zero and  
323 returns to the unperturbed state during the upward motion of the boundary.  
324 The magnitude of this oscillation is independent of the frequency of the  
325 forced pulsation of the topography. Even as the amplitude of the mantle-  
326 ULVZ interface velocity is different for different frequencies, the amplitude  
327 of the average melt volume fraction curves are insensitive to these variations.  
328 The rate of growth and decay of the decompaction and compaction layers,  
329 however, depend on the magnitude and frequency of the oscillations in the  
330 ULVZ-mantle interface velocity.

331 Redistribution of dense melts follow a distinct trend. The set of curves  
332 marked with  $\Delta\rho = -3\%$ , in Figures 4(b) and (c) illustrate this trend. The  
333 dense melt drains from the top and collects at the bottom, changing the cor-  
334 responding local averages. While these averages change over several hundred

335 ka, high frequency oscillations in the melt volume fractions are still apparent  
336 from some of the curves. Over short period of times, melt redistribution  
337 arising from such high frequency oscillations also display the formation of a  
338 melt-rich layer near the top and a compacted layer near the bottom. Over  
339 geologic timescales, however, the effect of buoyancy dominates over the effect  
340 of topographic oscillation.

341 The seismic signature arising from a neutrally buoyant melt oscillates  
342 about a mean value, as depicted in Figure 5(a)–(d). During periods of com-  
343 paction, the top 400 m of the ULVZ records up to 25% reduction in  $S$  wave  
344 speed, while the bottom 400 m records only 5% reduction at the same time.  
345 At times when the topography of the ULVZ returns to its initial state, both  
346 the top and the bottom of the ULVZ record an average reduction of 15% in  
347 the  $S$  wave velocities. A similar oscillatory behavior is observed for  $P$  wave  
348 velocities, where the magnitude of variation is much smaller, but follows the  
349 oscillation of the compaction. For all four frequencies tested in this work, the  
350 amplitude of the oscillatory seismic signal is independent of the frequency of  
351 topographic pulsation.

352 The seismic signature arising from a melt denser than the ULVZ is distinct  
353 from a neutrally buoyant melt. The series of plots in Figure 6(a)–(d) depict  
354 the variations in of  $V_S/V_0^S$  and  $V_P/V_0^P$  in the top and bottom 400 m of the  
355 ULVZ. As melt drains out from the top and pools near the bottom, the  
356 average  $S$  wave speed increases near the top and decreases near the bottom.  
357 High frequency pulsations lead to some damped oscillation in the seismic  
358 signals. Over 350 ka, however, gravitational drainage dominates the seismic  
359 signature. Over this time, the decrease in  $S$  wave speed near the top is less

360 (5–10%) compared to the decrease in the  $S$  wave speed near the bottom (15–  
361 20%), as depicted in Figures 6 (a) and (b). The decrease in  $S$  wave speed  
362 near the bottom depends on the rate of melt drainage from the top of the  
363 column to the bottom. A higher initial melt fraction will reduce the frictional  
364 resistance to melt percolation and accelerate melt drainage (Hier-Majumder,  
365 2011; Hier-Majumder and Courtier, 2011), while a stronger surface tension  
366 will reduce the drainage efficiency (Hier-Majumder et al., 2006). Similar to  
367 the  $S$  wave speed reduction, the  $P$  wave speed reduction near the top is also  
368 smaller than the bottom.

### 369 *3.1.3. The role of matrix viscosity*

370 Melt redistribution near the top and the bottom of the ULVZ is strongly  
371 modulated by the matrix viscosity. The plot in Figure 7(a) compares the  
372 melt redistribution near the top 400 m for two different matrix viscosities.  
373 The amplitude and frequency of oscillation of the ULVZ-mantle interface  
374 velocity is the same for both curves. Despite the same amount of forcing from  
375 the mantle, the peak magnitude of the decompaction layer is substantially  
376 smaller for higher matrix viscosity. The plot in Figure 7(b) compares the peak  
377 magnitude of the decompaction layer, melt volume fraction over the top 400  
378 m, for five different matrix viscosities. The magnitude of the decompaction  
379 layer drastically decreases for matrix viscosities exceeding  $10^{21}$  Pas, when  
380 the top melt fraction is nearly indistinguishable from the background melt  
381 volume fraction of 0.05.

382 High matrix viscosity increases the compaction length of the layer. As the  
383 top axis in Figure 7(b) indicates, the compaction length,  $\delta$  is more than an  
384 order of magnitude higher than the ULVZ height for a matrix viscosity of  $10^{22}$

385 Pas. For such large compaction lengths, melt segregation due to compaction  
386 is rendered inefficient over ULVZ-like length scales. Based on the scaling  
387 between ULVZ topography and viscosity, Hier-Majumder and Revenaugh  
388 (2010) suggest that the typical viscosity of the ULVZ should vary between  
389  $10^{19}$  Pas and  $10^{20}$  Pas. For such values of the ULVZ matrix viscosity, the  
390 effect of compaction should be pronounced, as suggested by the plot in Figure  
391 7(b). The qualitative behavior of melt segregation in this case is similar to  
392 the mesoscale experiments carried out by Holtzman et al. (2003).

### 393 *3.2. Analytical Solution*

394 Analysis of the governing nonlinear PDEs provide us with a wealth of in-  
395 formation regarding the behavior of the solutions. In the absence of density  
396 contrast between the melt and the matrix, growth and decay of the decom-  
397 paction layers are driven by the imposed velocity  $V$ . In this section, we  
398 present a nonlinear analysis outlining the way such a growth rate depends  
399 on the imposed velocity,  $V$  and the melt volume fraction.

400 We seek a solution to the governing mass and momentum conservation  
401 equations 3 and 4, respectively. This system of PDEs can be combined to  
402 yield a nonlinear, dispersive, and dissipative wave equation in melt volume  
403 fraction (Barcilon and Lovera, 1989; Hier-Majumder et al., 2006; Rabinowicz  
404 et al., 2002; Spiegelman, 1993). Following Hier-Majumder et al. (2006), we  
405 seek a solution for the melt volume fraction  $\phi$  in terms of a similarity variable  
406  $f = z - w_0 t$ , where  $w_0$  is a reference velocity, such that,

$$\phi = \phi(f). \tag{13}$$

407 In the following analysis, we neglect the effect of surface tension and buoy-

408 ancy. We also set  $\mu^* = 1$  in equation 4. We integrate the mass conservation  
 409 equation once to obtain,

$$w = -\frac{w_0\phi + K_1}{1 - \phi}, \quad (14)$$

410 where  $K_1$  is a constant of integration. Substituting  $w$  into the nondimensional  
 411 momentum conservation equation 4, we convert it into an ODE in  $\phi(f)$ , given  
 412 by,

$$0 = -\frac{4}{3} \left(\frac{\delta}{L}\right)^2 (w_0 + K_1) \left(\frac{1 + \phi}{\phi(1 - \phi)}\phi'\right)' + \frac{1}{\phi^2} \left(\frac{w_0\phi + K_1}{1 - \phi} + V\right), \quad (15)$$

413 where the primes indicate differentiation with respect to  $f$ . Following the  
 414 analysis outlined by Rabinowicz et al. (2002), we assume that far from the  
 415 peak of the solution, the melt volume fraction assumes a constant background  
 416 value  $\phi = \phi_0$ . This condition requires that both the gradient and the curva-  
 417 ture of the solution vanishes such that  $\phi' = \phi'' = 0$  at  $\phi = \phi_0$ . Inserting this  
 418 boundary condition into the ODE 15 leads to

$$K_1 = -(w_0\phi_0 + (1 - \phi_0)V). \quad (16)$$

419 This constant of integration is the volume averaged velocity of the melt and  
 420 the matrix. Inserting  $K_1$  into 15, multiplying by an integrating factor, and  
 421 integrating once we get

$$\frac{4}{3} \left(\frac{\delta}{L}\right)^2 \frac{(1 + \phi)^2 (1 - \phi_0)}{\phi^2 (1 - \phi)^2} (\phi')^2 = g(\phi) - \frac{K_2}{w_0 - V} \quad (17)$$

422 where  $K_2$  is the second constant of integration and the function  $g(\phi)$  is given  
 423 as,

$$g(\phi) = \frac{\phi_0}{\phi^2} - \frac{2(1 - 3\phi_0)}{\phi} + 2(3 - 5\phi_0) \ln\left(\frac{\phi}{1 - \phi}\right) + \frac{4(1 - \phi_0)}{1 - \phi}. \quad (18)$$

424 Once again, imposing  $\phi' = 0$  at  $\phi = \phi_0$ , and solving for  $K_2$ , we can rewrite  
 425 equation 17 as,

$$\phi' = \pm \left( \frac{L}{\delta} \right) \frac{\phi(1-\phi)}{1+\phi} \sqrt{\frac{3}{4} \left( \frac{g(\phi) - g(\phi_0)}{1-\phi_0} \right)}. \quad (19)$$

426 In the limit of small melt fraction,  $\phi \ll 1$ , we can ignore the first term in the  
 427 compaction rate,  $\partial((1-\phi)w)/\partial z$ , and rewrite the mass conservation equation  
 428 4 as,

$$\frac{\partial\phi}{\partial t} \approx \frac{\partial w}{\partial z} = \mp (w_0 - V) \left( \frac{L}{\delta} \right) \frac{\phi}{1-\phi^2} \sqrt{\frac{3}{4} (g(\phi) - g(\phi_0)) (1-\phi_0)}, \quad (20)$$

429 which is linear in  $V$  and inversely related to the compaction length. The  
 430 normalized magnitude of compaction rate,  $|(\partial w/\partial z)/(w_0 - V)|$  from equa-  
 431 tion 20 depends on both the compaction length and the background melt  
 432 fraction. While this normalized compaction rate at any point within the  
 433 ULVZ increases with the melt fraction at that point, the rate of increase is  
 434 modified by both the compaction length and the background, initial melt  
 435 fraction. As the series of curves in Figure 8(a) indicate, the compaction rate  
 436 is higher for smaller compaction lengths, as indicated by the inverse rela-  
 437 tionship of  $|(\partial w/\partial z)/(w_0 - V)|$  with compaction length in equation 20. For  
 438 a given compaction length, as the series of curves in Figure 8(b) indicates,  
 439 the magnitude of the growth rate is higher for a smaller background melt  
 440 fraction. In other words, decompaction layers will develop faster in response  
 441 to a forcing in a ULVZ with a smaller background melt fraction.

#### 442 4. Discussions

443 This article models internal melt redistribution within the ULVZ for both  
 444 the dense and neutrally buoyant melts. Based on seismic observations of

445 ULVZ density (Rost et al., 2006), a neutrally buoyant melt in an Fe-rich  
446 matrix is likely a better approximation to the ULVZ. The excess density of  
447 the ULVZ cannot be explained only by melting while satisfying the seismic  
448 observations and geodynamic models. For example, if the ULVZ matrix has  
449 a density similar to PREM, then for an average melt volume fraction of  
450 0.05, the melt has to be 3 times denser than a PREM-like solid to explain  
451 the observed 10% higher density of the ULVZ. Preserving an interconnected  
452 melt of such high density within the ULVZ over geologic times is physically  
453 untenable.

454 Mantle convection, through pulsed compaction, redistributes neutrally  
455 buoyant melt within a partially molten ULVZ. A few important implications  
456 of this phenomenon involve: 1. larger speed reduction near the top of the  
457 ULVZ; 2. vertical variation of seismic speed reduction that does not require  
458 a variation in the melt microstructure; and 3. spatial variation of the magni-  
459 tude of speed drop associated with ULVZs. Each of these issues are discussed  
460 below.

- 461 1. Melt distribution within the ULVZ is rarely uniform. Especially, if  
462 the dense ULVZ matrix contains an equally dense partial melt, during  
463 periods of downward motion of the ULVZ-mantle interface, wave speed  
464 reductions will be much larger near the top of the ULVZ. *If the overall  
465 seismic signature for a ULVZ patch is dominated by the signature at  
466 the top, the inferred melt volume can be larger than the average melt  
467 volume fraction in the ULVZ.*
- 468 2. Vertical variation of seismic structure within the ULVZ can be a con-  
469 sequence of pulsed compaction or stirring. To explain such observed

470 variations, Rost et al. (2006) suggested that the melt geometry changes  
471 from tubules near the top to spherical inclusions near the bottom of the  
472 ULVZ. The mechanism driving such microstructural changes, however,  
473 is not clear. This article employed the contiguity-melt volume fraction  
474 parametrization of Wimert and Hier-Majumder (2012), to calculate the  
475 seismic speed reductions. In their microgeodynamic model, melt resides  
476 within grain edge tubules through the entire range of melt volume frac-  
477 tions of interest. *It is, therefore, not necessary to invoke variation of*  
478 *melt microstructure to explain the vertical variation in seismic signa-*  
479 *ture.*

- 480 3. Signature of ULVZ patches atop the CMB vary spatially (McNamara  
481 et al., 2010; Rost et al., 2010). Previous dynamic models indicate that  
482 the topography of the ULVZ depends on the nature of the ambient  
483 mantle flow (Bower et al., 2011; Hier-Majumder and Revenaugh, 2010;  
484 McNamara et al., 2010). Additionally, *the result from this work indi-*  
485 *cates that the magnitude of speed reduction within a ULVZ patch can*  
486 *also be controlled by ambient mantle flow through pulsed compaction.*  
487 To fully understand the nature of the ULVZ, it is therefore, crucial to  
488 understand the nature of the flow in the surrounding mantle.

489 A few issues need to be investigated in greater detail. First, this work  
490 needs to be extended into higher dimensions to investigate the role of lateral  
491 pressure gradients and various patterns of ambient mantle flow. Secondly,  
492 this isothermal calculation starts with an initial homogeneous melt distri-  
493 bution. The bottom of the ULVZ is warmer, and likely subject to a larger  
494 amount of melt compared to the top. The implications for melt redistribu-

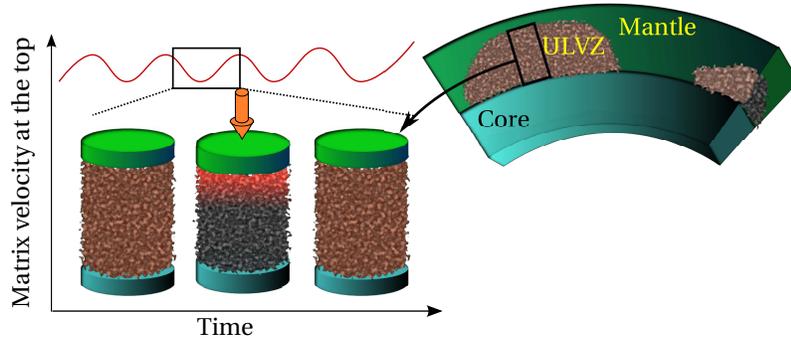


Figure 1: A schematic diagram outlining the geometry of the problem. A periodic forcing of the partially molten column redistributes the melt within the column.

495 tion and the seismic signature under such conditions need to be considered.  
 496 In addition, measurements of solidus temperatures for a variety of melt com-  
 497 positions and tighter estimates on the CMB temperature are also required.

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 504 of the manuscript.

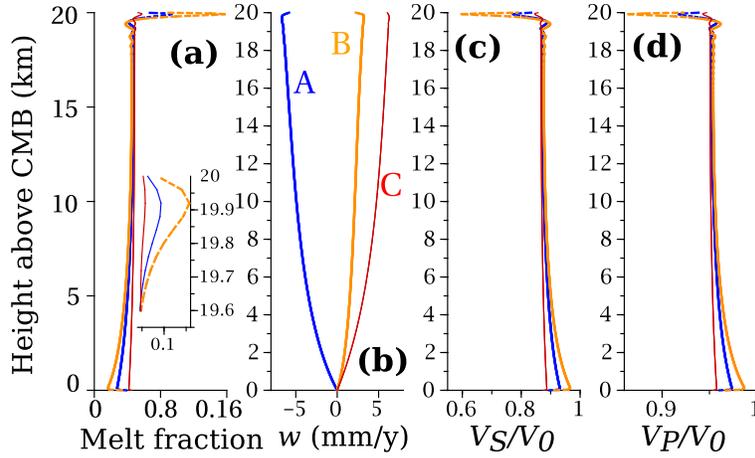


Figure 2: Internal structure and seismic signature of the ULVZ containing a neutrally buoyant melt. The vertical axis in all panels indicate the height of the ULVZ in km. (a) Melt volume fraction, (b) matrix velocity in mm/y, (c) relative  $S$  wave speed and (d) relative  $P$  wave speed for three different time steps. The legends in (b) correspond to three different stages during a compaction cycle, annotated in Figure 4(a), and apply for all panels. The inset in panel (a) displays the evolution of melt volume fraction for in the top 400 m of the ULVZ, corresponding to the three stages of the compaction cycle. The simulation corresponds to a nondimensional frequency of pulsation  $\omega = 0.01$ .

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Symbol	Quantity	Value	Unit
$\phi_0$	Background melt fraction	0.05	
$R$	Fractional density contrast	-0.03, 0.00	
$\mathcal{B}$	Bond number	$1.20 \times 10^6$	
$\rho$	Matrix density	5600.00	$\text{kgm}^{-3}$
$g$	Gravity	10.70	$\text{ms}^{-2}$
$\sigma$	Surface tension	1.00	$\text{Jm}^{-2}$
$d$	Grain size	$1.00 \times 10^{-3}$	m
$c$	Frictional resistance	$9.04 \times 10^{10}$	$\text{Pasm}^{-2}$
$\mu$	Matrix viscosity	$10^{20}, 10^{21}, 10^{22}$ $10^{23}, 10^{24}$	Pas
$L$	Length scale	$20.00 \times 10^3$	m
$v_0$	Characteristic velocity	$6.62 \times 10^{-7}$	$\text{ms}^{-1}$
$\delta$	Compaction length	33.25, 105.13, 332.5 1051.3, 3324.5	km
$K$	Matrix bulk modulus	655.60	GPa
$G$	Matrix shear modulus	293.80	GPa
$K_m$	Melt bulk modulus	583.44	GPa
$\nu$	Matrix Poisson's ratio	0.31	
$w_0$	Reference velocity		$\text{ms}^{-1}$

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Table 1: Constants used in the calculation.

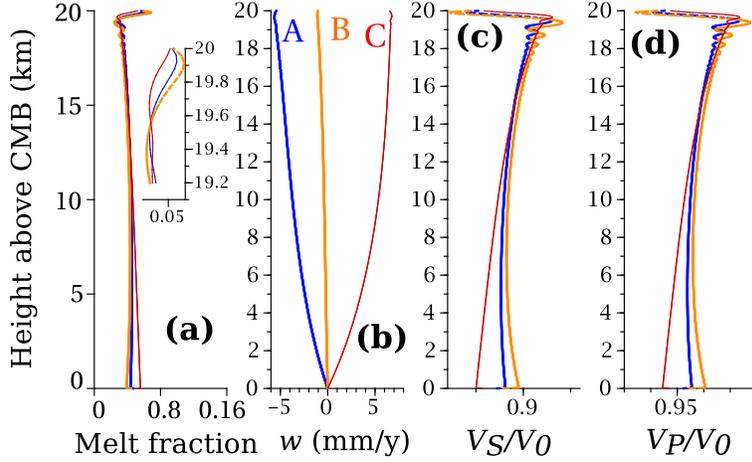


Figure 3: Internal structure and seismic signature within the ULVZ containing a melt 3% denser than the matrix. The quantities in all subfigures are similar to Figure 2. This set of simulations also correspond to a nondimensional frequency of oscillation,  $\omega = 0.01$ .

## 505 Appendix A. Derivation of the governing equations

506 In a partially molten, viscous aggregate, melt distribution is coupled  
 507 through matrix and melt velocities by a set of coupled governing equations.  
 508 If the velocities of the melt and the matrix phase are given as  $\mathbf{v}$  and  $\mathbf{w}$ , then,  
 509 in the absence of melt generation and dissolution precipitation, conservation  
 510 of the melt and matrix mass is given by

$$511 \quad 0 = \frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{v}), \quad (\text{A.1})$$

512 and

$$513 \quad \frac{\partial \phi}{\partial t} = \nabla \cdot ((1 - \phi) \mathbf{w}). \quad (\text{A.2})$$

514 Since the viscosity of the melt is many orders of magnitude smaller than that  
 515 of the matrix, we ignore the viscous stresses in the melt phase, leading to the

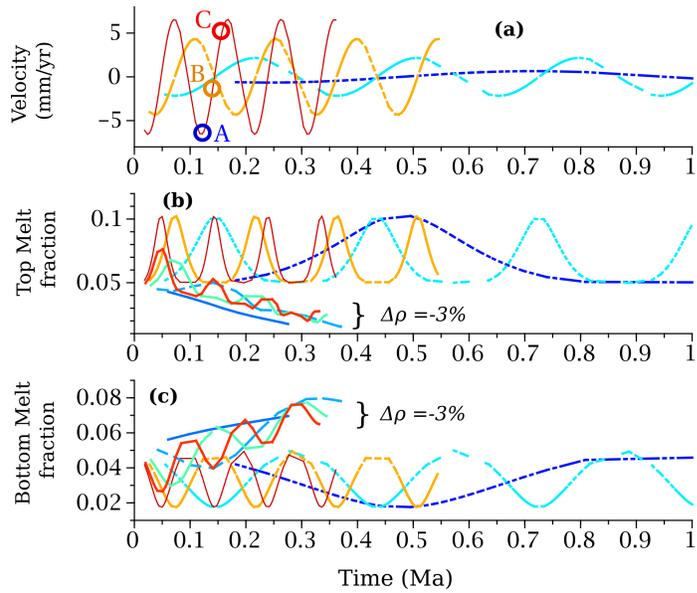


Figure 4: Transient mantle-ULVZ interfacial velocity and melt fractions within the ULVZ. The different curves correspond to four different frequencies. (a) Velocity of the ULVZ-mantle interface, as a function of time for the four different frequencies. Annotations in panel (a) correspond to the three time steps for which the vertical profiles are displayed in Figures 2 and 3. (b) Melt volume fraction averaged over the top 400 m of the ULVZ as a function of time. The plots are depicted only for the first 1 Ma. (c) Melt volume fraction averaged over the bottom 400 m of the ULVZ as a function of time in Ma.

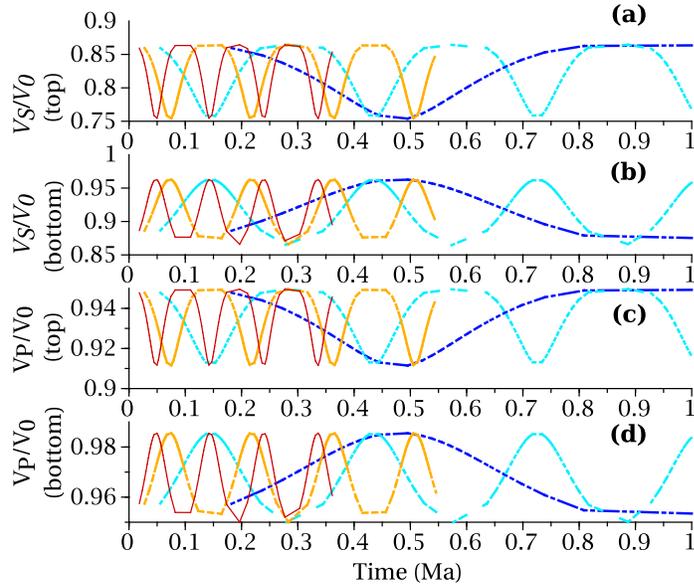


Figure 5: Transient seismic signature within the ULVZ. Relative reductions in  $S$  wave velocities near the (a) top and (b) bottom 400 m are plotted as a function of time for 4 different frequencies of pulsed compaction. The plots in Figure (c) and (d) depict the relative drops in  $P$  wave speed for the same regions within the ULVZ. In these plots, density of the melt is equal to the density of the matrix.

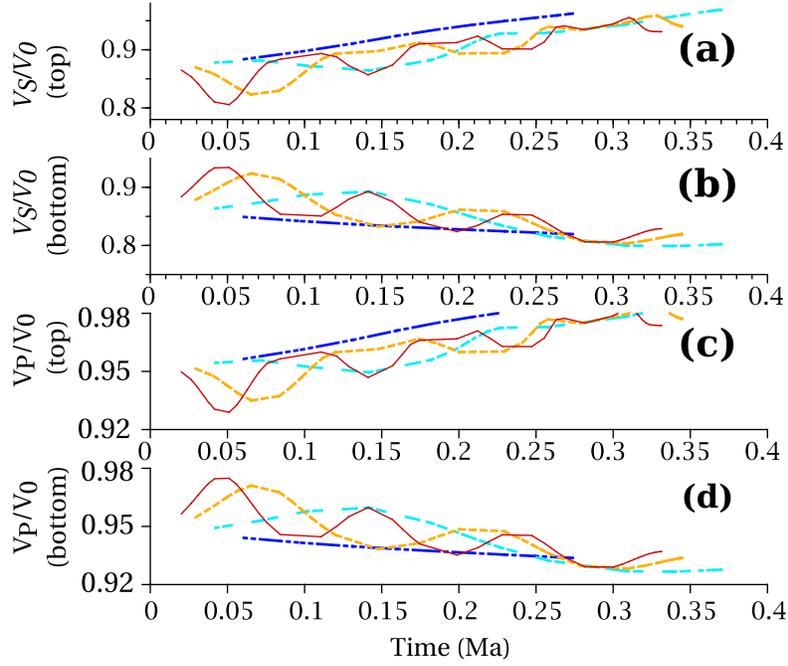


Figure 6: Transient seismic signature within the ULVZ. The physical parameters are similar to Figure 5, except for the melt density. In these plots, the melt is 3% denser than the ULVZ matrix. Relative reductions in  $S$  wave velocities near the (a) top and (b) bottom 400 m are plotted as a function of time for 4 different frequencies of pulsed compaction. The plots in Figure (c) and (d) depict the relative drops in  $P$  wave speed for the same regions within the ULVZ.

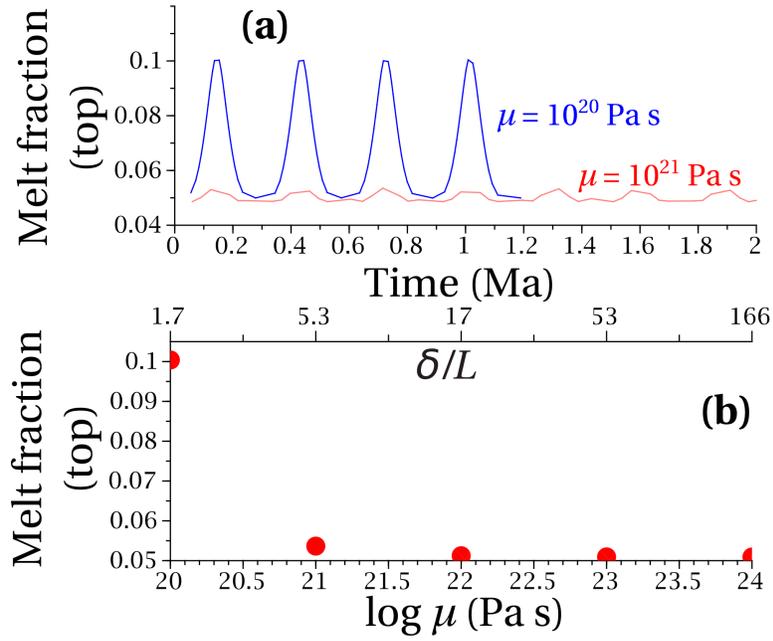


Figure 7: Role of matrix viscosity on the magnitude of the decompaction layer. (a) Plot of the averaged melt volume fraction over the top 400 m as a function of time for two different matrix viscosities, annotated next to the curves. (b) Highest magnitude of the decompaction layer as a function of logarithm of matrix viscosity. The axis on the top depicts the ratio between compaction length and the thickness of the ULVZ.

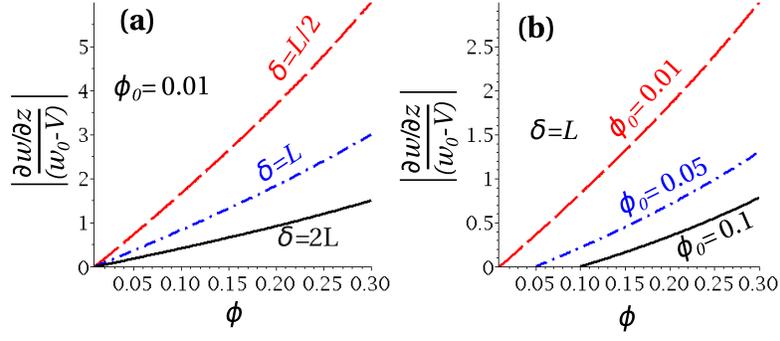


Figure 8: Plot of the magnitude of strain rate,  $|(\partial w/\partial z)/(w_0 - V)|$ , as a function of melt volume fraction,  $\phi$ , for different values of (a) compaction length and (b) background melt fraction,  $\phi_0$ . The annotations on the curves represent the value of the parameter. All curves in (a) correspond to  $\phi_0 = 0.01$ , and all curves in (b) correspond to  $\delta = L$ .

516 coupled conservation equations,

$$517 \quad 0 = -\phi(\nabla P_m + \rho_m \mathbf{g}) + c(\mathbf{w} - \mathbf{v}), \quad (\text{A.3})$$

518 and

$$519 \quad 0 = -(1 - \phi)(\nabla P + \rho \mathbf{g}) - c(\mathbf{w} - \mathbf{v}) + \nabla \cdot ((1 - \phi)\mathbf{T}) \\ 520 \quad + (\chi + P - P_m)\nabla\phi, \quad (\text{A.4})$$

521 where  $P_m$  is the melt pressure,  $P$  is the matrix pressure,  $\rho_m$  is the melt den-  
 522 sity,  $\rho$  is the matrix density,  $c$  is the frictional resistance to melt percolation,  
 523  $\chi$  is the surface tension force per unit area, and the matrix stress  $\mathbf{T}$  is given  
 524 by the constitutive relation,

$$525 \quad \mathbf{T} = \mu \left( \nabla \mathbf{w} + (\nabla \mathbf{w})^T - \frac{2}{3} (\nabla \cdot \mathbf{w}) \mathbf{I} \right), \quad (\text{A.5})$$

526 where  $\mu$  is the viscosity of the matrix and  $\mathbf{I}$  is the unit tensor. In addition to  
 527 the above relations, we need an extra closure relation between the melt and

528 the matrix pressure, given by,

$$529 \quad \chi + P - P_m = -\frac{K_0\mu}{\phi(1-\phi)} \left( \frac{\partial\phi}{\partial t} + \mathbf{w} \cdot \nabla\phi \right), \quad (\text{A.6})$$

530 where  $K_0$  is a constant  $\mathcal{O}(1)$ .

531 To obtain the one-dimensional governing equations, we first add the mass  
532 conservation equations A.1 and A.2 to obtain,

$$533 \quad \frac{\partial}{\partial z} (\phi v + (1-\phi)w) = 0, \quad (\text{A.7})$$

534 which implies the volume averaged velocity  $\phi v + (1-\phi)w$  is constant through-  
535 out the domain of calculation. We prescribe,

$$536 \quad \phi v + (1-\phi)w = V, \quad (\text{A.8})$$

537 where  $V$  is the volume averaged velocity of the aggregate, which we also set  
538 as the velocity of the ULVZ-mantle interface.

539 Next, we eliminate the pressure and melt velocity from the momentum  
540 equations multiplying equation A.3 by  $(1-\phi)$  and equation A.4 by  $\phi$ , adding,  
541 and substituting the stress, pressures, and melt velocity from equations A.5,  
542 A.6, and A.8, to obtain the one-dimensional action-reaction equation,

$$543 \quad 0 = (1-\phi)\chi^* \frac{\partial\phi}{\partial z} + \frac{\partial}{\partial z} \left( \mu \left( \frac{K_0}{\phi} + \frac{4}{3} \right) (1-\phi) \frac{\partial w}{\partial z} \right) \\ 544 \quad - (1-\phi)\Delta\rho g - \frac{c(w-V)}{\phi^2}, \quad (\text{A.9})$$

545 where  $\chi^* = (d\chi)/(d\phi)$ .

546 Thus we have two partial differential equations, A.2 and A.9 on two un-  
547 knowns  $\phi$  and  $w$ . First, we impose the impermeability condition at the top  
548 boundary  $z = h$  such that

$$549 \quad v|_{z=h} = w|_{z=h}, \quad (\text{A.10})$$

550 implying  $w = V$  at  $z = h$ .

551 Besides the impermeable boundary condition at the top, given by equa-  
 552 tion A.10, we also impose zero velocity of the matrix at the bottom boundary.

553 We prescribe the initial melt distribution, given by,

$$554 \quad \phi(z, 0) = \phi_0 + \bar{\phi}(z), \quad (\text{A.11})$$

555 where the white noise perturbation  $\bar{\phi}(z)$ , varies between 0 and  $10^{-5}$ . The  
 556 small white noise is necessary to ensure small, but nonzero gradients in melt  
 557 volume fraction, which allows time marching of the numerical solutions.

## 558 **Appendix B. Analytical solution for initial matrix velocity**

559 In the limit of a negligibly small increment in time from the beginning,  
 560 the mass and momentum conservation equations admit a simple analytical  
 561 solution, which can be compared with the numerical solution. The analytical  
 562 solutions presented here follow the forced compaction model of Ricard et al.  
 563 (2001).

564 Immediately after the beginning of the simulation, we assume that the  
 565 melt distribution is very similar to the original melt distribution. The as-  
 566 sumption applies in the limit  $t \rightarrow 0$ ,  $\partial\phi/\partial z \rightarrow 0$ . In the absence of surface  
 567 tension, the nondimensional momentum conservation equation then reduces  
 568 to the Ordinary Differential Equation (ODE) in matrix velocity  $w$ , given by,

$$0 = \frac{4}{3} \left( \frac{\delta}{L} \right)^2 \mu^* \frac{1 - \phi^2}{\phi} \left( \frac{d^2 \bar{w}}{dz^2} \right) - R(1 - \phi) - \frac{1}{\phi^2} (\bar{w} - V(t)) \quad (\text{B.1})$$

569 We set  $\mu^* = 1$ , substitute  $z = z_0 y$  and  $\bar{v} = R\phi^2(1 - \phi) - V + \bar{w}$  where,

$$z_0 = \left( \frac{\delta}{L} \right) \sqrt{\frac{4\phi}{3} (1 - \phi^2)}. \quad (\text{B.2})$$

570 This substitution reduces the ODE B.1 to

$$\frac{d^2\bar{v}}{dy^2} - \bar{v} = 0, \quad (\text{B.3})$$

571 A general solution, similar to Ricard et al. (2001) to equation B.3, is given  
572 by,

$$\bar{v} = A \cosh y + B \sinh y. \quad (\text{B.4})$$

573 We set the boundary conditions  $\bar{w}(0, t) = 0$  and  $\bar{w}(1, t) = V(t)$ , and substi-  
574 tute into equation B.3 to obtain the constants,

$$A = R\phi^2(1 - \phi) - V \quad (\text{B.5})$$

$$B = \frac{R\phi^2(1 - \phi) \left[ 1 - \cosh\left(\frac{1}{z_0}\right) \right] + V \cosh\left(\frac{1}{z_0}\right)}{\sinh\left(\frac{1}{z_0}\right)}. \quad (\text{B.6})$$

575 The analytical solution for the matrix velocity,  $\bar{w}$  and the segregation velocity,  
576  $\Delta V = (\bar{w} - V)/\phi$ , is displayed in Figure B.1 for a constant  $\phi = 0.05$ . Overlain  
577 on the plot is also the numerical solution for  $\phi = 0.05$  at time 0.

578 A number of numerical experiments were carried out to test the influence  
579 of grid resolution on the results. First, we define the residual vector

$$\boldsymbol{\epsilon} = \bar{\boldsymbol{w}} - \boldsymbol{w}, \quad (\text{B.7})$$

581 where  $\bar{\boldsymbol{w}}$  is the analytical solution and  $\boldsymbol{w}$  is the numerical solution. As a  
582 measure of convergence of the solution, we define the  $L_\infty$  norm or the largest  
583 absolute value of the residual vector within the top 1 km of the ULVZ as,

$$\|\boldsymbol{\epsilon}\|_\infty = \max |\epsilon_i|, \quad 1 < i < n_{top} \quad (\text{B.8})$$

585 where the range of the index  $i$  spans over the top 1 km of the ULVZ. We  
586 calculate the norm  $\|\boldsymbol{\epsilon}\|_\infty$  for a number of grid sizes ranging between 50 and

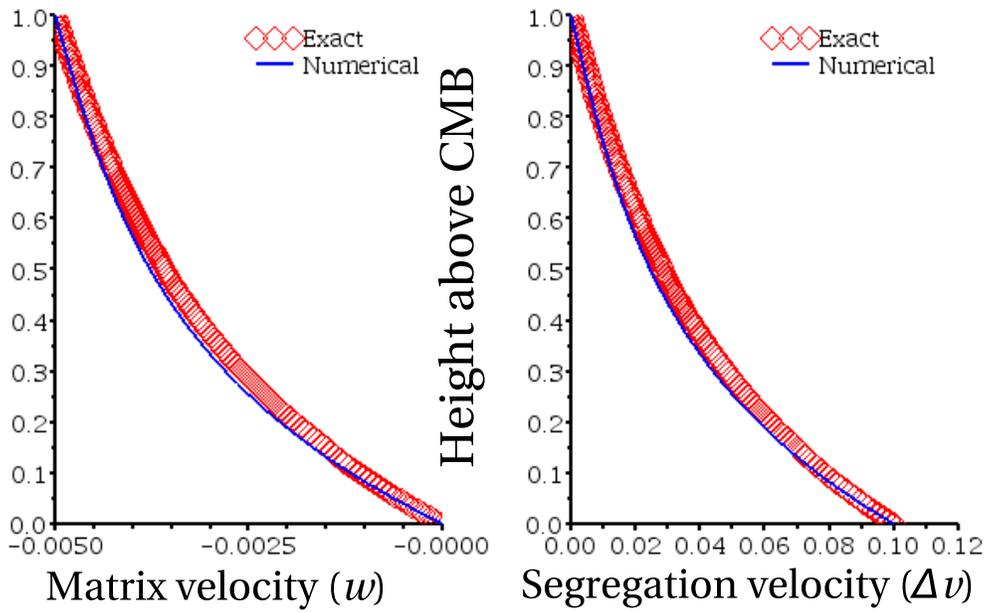


Figure B.1: Analytical solutions for nondimensional matrix and segregation velocities in open diamonds are compared with the numerical solution at time 0 . In this calculation  $V = -0.005$

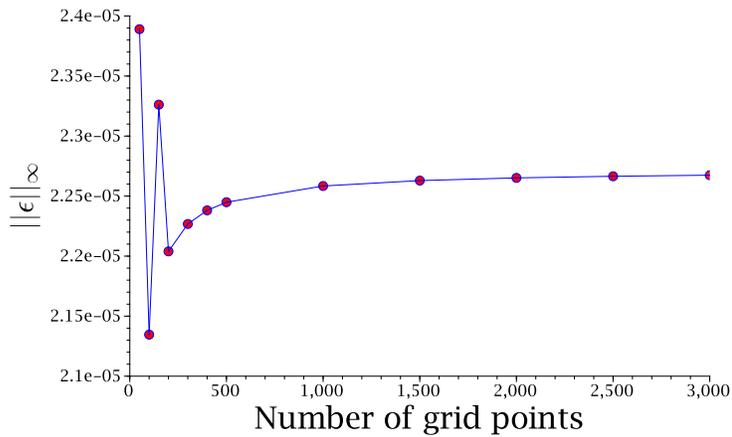


Figure B.2: A plot of the  $L_\infty$  norm of the error vector  $\epsilon$  over the top 1 km of the ULVZ, as a function of the grid size. .

587 3000. The result is plotted in Figure B.2. The error oscillates about a value  
588 of  $\sim 2.25 \times 10^{-5}$  for grids sizes smaller than 500. The oscillations in the  
589 value of the error for such low resolution grids is typically  $\mathcal{O}(10^{-6})$ , which  
590 corresponds to approximately 0.4% of the absolute maximum of  $\mathbf{w}$  within  
591 the top 1 km of the ULVZ.

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