QBism and the Greeks: why a quantum state does not represent an element of physical reality

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Abstract. In QBism (or Quantum Bayesianism) a quantum state does not represent an element of physical reality but an agent's personal probability assignments, reflecting his subjective degrees of belief about the future content of his experience. In this paper, we contrast QBism with hidden-variable accounts of quantum mechanics and show the sense in which QBism explains quantum correlations. QBism's agent-centered worldview can be seen as a development of ideas expressed in Schrödinger's essay "Nature and the Greeks".

1. Introduction

In 1964 John Bell derived the inequalities which now bear his name and showed that they are violated by quantum mechanics [1]. He thus established that quantum mechanics does not admit a local hidden variable account. Here and throughout this paper, the term "hidden variable" includes any mathematical object that represents an element of physical reality and determines the outcomes of experiments or their probabilities.

The assumption of locality thus rules out any hidden variable interpretation of quantum mechanics. Locality also rules out directly any interpretation that regards the quantum state as representing an element of physical reality. This can be seen by adapting an argument by Einstein (see, e.g., the detailed recent discussion by Harrigan and Spekkens [2]). Consider a maximally entangled pair of particles far removed from each other. According to quantum theory, by making measurements on one of the particles, an experimenter can choose whether the state for the other particle belongs to one or the other of two nonoverlapping sets of states (this is sometimes called "steering"). An interpretation that regards the quantum state as representing an element of physical reality would thus be nonlocal, because this element of physical reality could be manipulated at a distance.

Recently, Pusey, Barrett, and Rudolph (PBR) [3], Colbeck and Renner [4], and other authors, showed that, under certain conditions, in any hidden variable theory the quantum state must be a function of the hidden variables. Using the argument of

the preceding paragraph, it follows as a corollary that hidden variable theories must be nonlocal, which is established thus without using Bell inequalities (see also [5] for a recent discussion on the relation between PBR and Bell-type arguments). In these papers, hidden variables are introduced in the form of an "ontological model" [2]. Within that framework, a distinction is made between "psi-ontic" and "psi-epistemic" theories [2], which lets one forget easily that this is a distinction between two kinds of hidden variable theories, and that the existence of hidden variables is not implied by quantum mechanics.

QBism [6, 7, 8, 9] is an explicitly local interpretation of quantum mechanics in which there is no room for hidden variables. According to QBism, quantum mechanics is a tool any agent can use to evaluate his probabilistic expectations for his personal experience. A quantum state does not represent an element of physical reality external to the agent, but reflects the agent's personal degrees of belief about the future content of his experience.

In Section 2, we show how the agent-centered worldview of QBism arises naturally as a development of the fundamental issues identified by Schrödinger in his essay "Nature and the Greeks" [10]. Section 3 gives a short overview of QBism. Section 4 discusses the function of the Born rule in QBism and contrasts QBism with hidden variable theories. In Section 5 we show the sense in which QBism explains quantum correlations, and Section 6 concludes the paper.

2. QBism and the Greeks

In the essay "Nature and the Greeks" [10], Schrödinger writes: "Gomperz says [...] that our whole modern way of thinking is based on Greek thinking; it is therefore something special, something that has grown historically over many centuries, *not* the general, the only possible way of thinking about Nature. He sets much store on our becoming aware of this, of recognising the peculiarities as such, possibly freeing us from their well-nigh irresistible spell."

Schrödinger singles out two fundamental features of modern science that are influenced by Greek thinking in this way. One is "the assumption that the world can be understood." The other is "the simplifying provisional device of excluding the person of the 'understander' (the subject of cognizance) from the rational world-picture that is to be constructed."

About the first of these, Schrödinger remarks that "one would in this context have to discuss the questions: what does comprehensibility really mean, and it what sense, if any, does science give explanations?" The question of explanation is often brought up in discussions of the foundations of quantum mechanics. Derivations of the Bell inequalities have been phrased in terms of possible explanations of the correlated data produced in a Bell experiment [11]. We will address the issue of explanation from a QBist perspective in Section 5.

Here we focus on the second special feature of modern science identified by

Schrödinger, namely that "the scientist subconsciously, almost inadvertently, simplifies his problem of understanding Nature by disregarding or cutting out of the picture to be constructed himself, his own personality, the subject of cognizance." According to Schrödinger, this "leaves gaps, enormous lacunae, leads to paradoxes and antinomies whenever, unaware of this initial renunciation, one tries to find oneself in the picture, or to put oneself, one's own thinking and sensing mind, back into the picture."

An example of this fundamental difficulty is provided by the quantum measurement problem: How is it that an agent experiences a single outcome when he performs a measurement on a system in a superposition state? In most accounts of the measurement problem, the quantum state is regarded as agent-independent and objective, and hence as belonging to what Schrödinger calls the rational world-picture from which the subject of cognizance is excluded. The measurement problem can be seen as a symptom of the "paradoxes and antinomies" that one finds when one tries to connect this world-picture to the experience of an agent. The many decades of ultimately unsuccessful attempts to resolve the measurement problem attest to its fundamental nature.

The following fragment, quoted twice in Schrödinger's essay, shows that the core of the problem was clearly understood by Democritus: "(Intellect:) Sweet is by convention, and bitter by convention, hot by convention, cold by convention, colour by convention; in truth there are but atoms and the void. (The Senses:) Wretched mind, from us you are taking the evidence by which you would overthrow us? Your victory is your own fall." Schrödinger comments "You simply cannot put it more briefly and clearly."

There is no measurement problem in QBism because the agent and the agent's experience are part of the story from the beginning. QBism is thus breaking free from the "irresistible spell" of Greek thinking and abandons "the simplifying provisional device of excluding the person of the 'understander' (the subject of cognizance) from the rational world-picture that is to be constructed." The next section will give details of this move. Both the locality assumption discussed in the introduction and a contemporary understanding of probability provide strong motivations for this move, independently of Schrödinger's views on the impact of Greek thinking on the presuppositions of contemporary science.

3. QBism

The fundamental primitive of QBism is the concept of experience. According to QBism, quantum mechanics is a theory that any agent can use to evaluate his expectations for the content of his personal experience.

QBism adopts the personalist Bayesian probability theory pioneered by Ramsey [12] and de Finetti [13] and put in modern form by Savage [14] and Bernardo and Smith [15] among others. This means that QBism interprets all probabilities, in particular those that occur in quantum mechanics, as an agent's personal, subjective degrees of belief. This includes the case of certainty—even probabilities 0 or 1 are degrees of belief [16]. Probabilities acquire an operational meaning through their use in decision making,

or gambling: an agent's probabilities are defined by his willingness to place or accept bets on the basis of those probabilities. In this framework, the usual probability rules can be derived from the requirement that an agent's probability assignments should not lead to a sure loss in a single instance of a bet, a requirement known as Dutch-book coherence. The probability rules are therefore of a normative character.

Dutch-book coherence for one agent does not put any constraints on another agent's probability assignments. The set of probabilities used by an agent have validity for that agent only. The general theory—degrees of belief constrained by Dutch book coherence—can be used by any agent. But one cannot mix the probability assignments made by different users of the theory.

In QBism, a measurement is an action an agent takes to elicit an experience. The measurement outcome is the experience so elicited. The measurement outcome is thus personal to the agent who takes the measurement action. In this sense, quantum mechanics, like probability theory, is a single user theory. A measurement does not reveal a pre-existing value. Rather, the measurement outcome is created in the measurement action.

According to QBism, quantum mechanics can be applied to any physical system. QBism treats all physical systems in the same way, including atoms, beam splitters, Stern-Gerlach magnets, preparation devices, measurement apparatuses, all the way to living beings and other agents. In this, QBism differs crucially from various versions of the Copenhagen interpretation. A common thread among those instead is that measuring and preparation devices, in their operation as such, must be treated as belonging to a separate classical domain outside the scope of quantum mechanics [17, Sect. 3].

An agent's beliefs and experiences are necessarily local to that agent. This implies that the question of nonlocality simply does not arise in QBism. QBist quantum mechanics is local because, for any user of quantum mechanics, quantum states encode the user's personal degrees of belief for the contents of his own experience [9].

Quantum states are represented by density operators ρ in a Hilbert space assumed to be finite dimensional. A measurement (an action taken by the agent) is described by a POVM $\{F_j\}$, where j labels the potential outcomes experienced by the agent. The agent's personalist probability $q(j)$ of experiencing outcome j is given by the Born rule,

$$q(j) = \mathrm{tr}(\rho F_j) \ . \tag{1}$$

Similar to the probabilities on the left-hand side of the Born rule, QBism regards the operators ρ and F_j on the right-hand side as judgements made by the agent, representing his personalist degrees of belief.

4. The function of the Born rule

The Born rule as written in Eq. (1) appears to connect probabilities on the left-hand side of the equation with other kinds of mathematical objects—operators—on the right-hand side. It turns out to be possible, however, to rewrite the rule entirely in terms
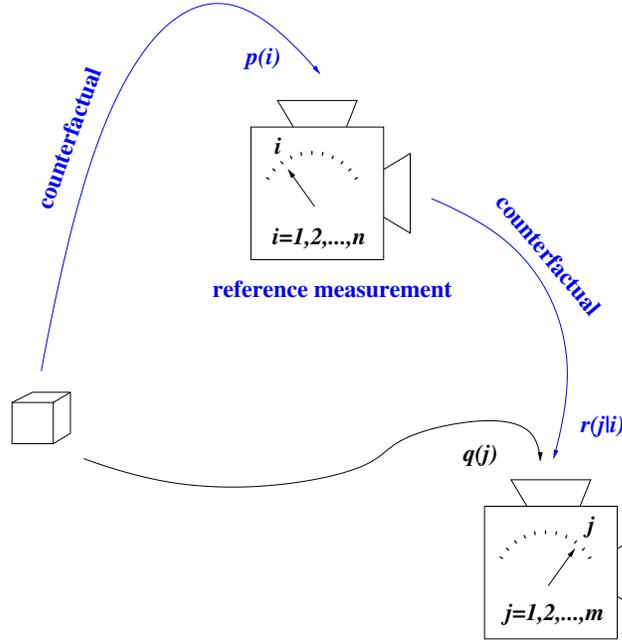
Figure 1. Analysing a measurement in an agent-centered way: the index j labels the outcomes of some actual measurement the agent intends to perform, and i labels the outcomes of a reference measurement which the agent might perform but which remains counterfactual. In both classical mechanics and quantum mechanics there exist such reference measurements for which the agent's probabilities $q(j)$ for outcome j can be expressed in terms of his probabilities $p(i)$ for outcome i and his conditional probabilities $r(j|i)$ for outcome j given outcome i.

of probabilities [7, 8]. For this, consider the scenario of Figure 1, where a reference measurement is introduced in order to characterize both the system state ρ and the POVM $\{F_j\}$.

We assume that the agent's reference measurement is an arbitrary informationally complete POVM, $\{E_i\}$, such that each E_i is of rank 1, i.e., is proportional to a one-dimensional projector Π_i. Such measurements exist for any finite Hilbert-space dimension. Furthermore, we assume that, if the agent carries out the measurement $\{E_i\}$ for an initial state ρ, upon getting outcome E_i he would update to the post-measurement state $\rho_i = \Pi_i \rho \Pi_i / \mathrm{tr}(\rho \Pi_i)$. Because the reference measurement is informationally complete, any state ρ corresponds to a unique vector of probabilities $p(i) = \mathrm{tr}(\rho E_i)$, and any POVM $\{F_j\}$ corresponds to a unique matrix of conditional probabilities $r(j|i) = \mathrm{tr}(F_j \Pi_i)$.

The operators ρ and F_j on the right-hand side of the Born rule are thus mathematically equivalent to sets of probabilities $p(i)$ and conditional probabilities $r(j|i)$, respectively. In this sense, POVMs as well as quantum states *are* probabilities. In QBism, POVMs as well as quantum states represent an agent's personal degrees of belief. The Born rule then becomes

$$q(j) = f\Big(\{p(i)\}, \{r(j|i)\}\Big) , \tag{2}$$

where the precise form of the function f depends on the details of the reference measurement. The Born rule allows the agent to calculate his outcome probabilities $q(j)$ in terms of his probabilities $p(i)$ and $r(j|i)$ defined with respect to a counterfactual reference measurement.

In QBism, the Born rule functions as a coherence requirement. Rather than setting the probabilities $q(j)$, the Born rules relates them to those defining the state ρ and the POVM $\{F_j\}$. Just like the standard rules of probability theory, the Born rule is normative: the agent ought to assign probabilities that satisfy the constraints imposed by the Born rule. Unlike the standard rules of probability theory however, which can be derived from Dutch-book coherence alone, the Born rule is empirical. It is a statement about the physical world.

We will now show that the scenario of Figure 1 captures not only the essential difference between classical physics and quantum theory, but also the essential difference between QBism and hidden variables theories. Classical physics rests on the assumption that, for every system, there exists a reference measurement such that, for every actual measurement, the following holds. As before, let $p(i)$ denote an agent's probabilities for outcome i in the reference measurement (the $p(i)$ characterize the agent's system state), let $r(j|i)$ denote his probabilities for outcome j in the actual measurement given that the reference measurement was carried out and resulted in outcome i (in a deterministic theory the $r(j|i)$ would be restricted to values 0 or 1), and let $q(j)$ denote the probabilities of outcome j in the actual measurement assuming that the reference measurement remains counterfactual. Then

$$q(j) = \sum_i p(i)r(j|i) \ . \tag{3}$$

Since in the definition of $q(j)$, the reference measurement remains counterfactual, Eq. (3) is not implied by probability theory [8]. It is a physical postulate. This formulation of the classical postulate is agent-centered. It connects an agent's degrees of belief about the outcomes of the reference measurement with his degrees of belief about the outcomes of the actual measurement.

The agent (or subject) might be thought to be removable from the picture by taking the variables i to represent external states of reality that determine the probabilities $r(j|i)$. In this case $p(i)$ denotes the probability that the state of reality is i. The central assumption of classical physics now takes the form that, in principle, there is a measurement that simply reads off the value of i. The classical law Eq. (3) is then a consequence of probability theory. It is the same equation as before, but it now refers to an agent-independent reality. In a nutshell, this is the 2000 year old Greek maneuver identified by Schrödinger that excluded the subject from the world picture.

Of course the world is not classical. There is in general no reference measurement such that the classical law Eq. (3) holds. QBism takes this fact—the nonexistence of such a reference measurement—as an expression of the idea that the subject cannot be removed from the world picture.

By contrast, ontological models [2], or hidden variable models, try to preserve the

concept of an agent-independent reality. Similar to the agent-independent formulation of classical physics, they analyze measurements in terms of an external state of reality i, a probability distribution $p(i)$ over external states of reality, and a conditional probability distribution $r(j|i)$ which gives the probability for outcome j for each external state of reality i. Furthermore, the classical law Eq. (3) is assumed to hold. Since quantum mechanics rules out an interpretation of i as the outcome of a reference measurement, in ontological models Eq. (3) does not follow from probability theory but is an independent postulate. In these models, the Born rule is either a further independent postulate or follows from further assumptions about the variables i.

QBism keeps the idea of a reference measurement and thus keeps the subject in the center. Since the reference measurement is assumed to remain counterfactual, probability theory alone has nothing to say about the relation between the probabilities $q(j)$, $p(i)$ and $r(j|i)$. The Born rule can thus be seen as an addition to probability theory, a normative requirement of *quantum Bayesian coherence* [8], which applies whenever the agent contemplates a particular kind of reference measurement. The functional relationship Eq. (2) depends on the details of the reference measurement. In the special case that the reference measurement is a symmetric informationally complete POVM (SIC) [18, 19, 20], Eq. (2) takes the simple form [7, 8]

$$q(j) = \sum_i \left((d+1)p(i) - \frac{1}{d} \right) r(j|i) \,. \tag{4}$$

The authors have conjectured that this form of the Born rule may be used as an axiom in a derivation of quantum theory [8, 21, 22].

Indeed recently there have been several information-theoretic axiomatic derivations of quantum theory [23, 24, 25]. These may provide important clues and techniques for how to proceed to a full derivation of quantum theory from Eq. (4), which so far has not been complete. The key question that remains is in identifying what minimal further principles must be added to Eq. (4) for the project to be successful. What would be unique about this approach, if it proves successful, is the way it would pull the scenario depicted in Figure 1 to the front and center of the mathematical structure of quantum theory. In Ref. [26], one of us (CAF) expands on why this notion is considered key for a thorough-going QBist expression of quantum theory. In a nutshell, it is that Eq. (4) gives quantitative expression to the idea that the agent cannot be removed from the world picture.

5. Explanation

According to QBism, the quantum formalism is an addition to probability theory (see the previous section). One should therefore expect that explanations offered by quantum theory have a similar character to explanations offered by probability theory.

Here is a simple example from probability theory. Assume an agent's prior probabilities for a coin tossing experiment are such that for him the coin tosses are independent and Heads and Tails are equally likely in each toss. The agent now

considers tossing the coin 100 times, denoting by h the number of Heads. Using simple properties of the binomial distribution, the agent expects h to lie between 30 and 70 with probability close to 1. When he performs the experiment, he happens to find the value $h = 57$.

Probability theory explains the agent's expectations. The theory allows the agent to understand why, given his prior, he should be almost certain that he will find a value of h between 30 and 70. On the other hand, probability theory does not provide any explanation for why the agent found the particular value $h = 57$. This hardly limits the wide-ranging explanatory power of probability theory as witnessed by any standard probability text.

Our second example is quantum mechanical. Consider an experimenter who prepares a spin-1/2 particle in an x eigenstate and performs a measurement using a Stern-Gerlach device oriented along the z axis. This setup encodes the experimenter's prior. Given this prior, quantum mechanics explains why, in order to be coherent, the experimenter should assign probability 1/2 to each of the two possible outcomes. Say the experimenter experiences the outcome "up". Quantum mechanics does not explain why he experiences "up" and not "down". Far from being a limitation of the theory, this is an expression of the QBist idea that neither the outcome of the measurement nor its probability are determined by some hidden variables: measurement outcomes or their probabilities are not a function solely of the physical reality external to the agent. The explanations provided by quantum mechanics are exactly those one would hope for in a world in which measurements are acts of creation, i.e., in a world that is unfinished and open.

The above example extends naturally to the case of repeated measurements. Assuming the experimenter has an appropriate prior, quantum mechanics explains why he should expect, with probability close to 1, that in many repetitions of the spin measurement the proportion of spin-up outcomes he experiences will be close to 1/2. Quantum mechanics does not provide any explanation for the particular proportion the agent finds—this is just as it was with the coin toss example. Yet, even more than in the case of probability theory, this does not prevent quantum mechanics from having unprecedented explanatory power.

Correlations are just a special case of more general probability assignments. To explain a correlation is therefore no different than to explain a probability assignment. Here is an example for how correlations arise in quantum mechanics. Suppose that an agent considers performing a measurement on a spin-3/2 particle. For given labels (a, b), the measurement is assumed to be of the form $\{A_x^a \otimes B_y^b\}$, where the A_x^a and B_y^b correspond to projection operators onto two complementary (two-dimensional) sub-algebras of the full four-dimensional Hilbert space. If we denote the agent's prior state for the particle by $|\psi\rangle$, the agent's probabilities $p(x, y|a, b)$ for experiencing the outcome (x, y) if he chooses to enact the measurement labeled by (a, b) are given by

$$p(x, y|a, b) = \langle\psi|A_x^a \otimes B_y^b|\psi\rangle \ . \tag{5}$$

In this way the quantum formalism explains why, given the agent's prior beliefs, he ought to assign the correlations $p(x, y|a, b)$.

If the agent performs measurements of this type on a large number n of particles for which his prior is the product state $|\psi\rangle^{\otimes n}$, he can record the frequencies with which the different outcomes occur for each setting (a, b) in a data table, $d(x, y|a, b)$. As before, quantum mechanics explains why the agent should expect the measured frequencies to lie in a certain range, but does not provide an explanation for the particular numbers the agent obtains in a given realization of the data table.

The above considerations remain unchanged in the case that the correlations $p(x, y|a, b)$ implied by the prior state and measurement operators violate a Bell inequality. Of course, Bell inequalities are not usually introduced for sub-algebras of a spin-3/2 particle, but for measurements on two space-like separated subsystems. In QBism, however, there is no important conceptual difference between these two situations.

The above considerations also remain unchanged in the case of perfect correlations, $p(x, y|a, b) \in \{0, 1\}$. Even these are an agent's personal probabilities for his future experiences. QBism treats all quantum systems and all measurements on an equal footing. That unperformed measurements have no outcomes is true for all measurements, independently of whether or not the agent assigns probability 1 to one of the outcomes. A statement such as $p(y = 0) = 1$ expresses the agent's personal belief that the measurement outcome will be $y = 0$, a belief that is given a quantitative expression through the bets he would accept on this outcome—here he would bet an arbitrary amount against the promise of an arbitrarily small gain. It has been argued by Timpson [27] that it might be irrational for an agent to make a probability assignment such as $p(y = 0) = 1$ unless the agent also believed in the existence of a "truth maker" that guarantees that the outcome will indeed be $y = 0$. Timpson's argument would lead to the introduction of an additional constraint on the assignment on probabilities, beyond the constraints imposed by the probability calculus and, via the Born rule, quantum mechanics. Such an extra constraint is not implied by quantum theory and ultimately amounts to the introduction of hidden variables. It is therefore ruled out by the QBist view of the world [28, pp. 1809–1810 and links therein].

6. Summary

According to QBism, quantum mechanics is a theory any agent can use to more safely gamble on his potential future experiences. Quantum mechanics permits any agent to quantify, on the basis of his past experiences, his probabilistic expectations for his future experiences. QBism takes measurement outcomes as well as quantum states to be personal to the agent using the theory. In QBism, there are no agent-independent elements of physical reality that determine either measurement outcomes or probabilities of measurement outcomes. Rather, every quantum measurement is an action on the world by an agent that results in the creation of something entirely new. QBism holds

this to be true not only for laboratory measurements on microscopic systems, but for any action an agent takes on the world to elicit a new experience. It is in this sense that agents have a fundamental creative role in the world.

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